General coarsened measurement references for revelation of a classical world*

Dong Xie(谢东)[1,†], Chunling Xu(徐春玲)[1], and Anmin Wang(王安民)[2]

[1] *Faculty of Science, Guilin University of Aerospace Technology, Guilin* 541004, *China*
[2] *Department of Modern Physics, University of Science and Technology of China, Hefei* 230026, *China*

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It has been found that for a fixed degree of fuzziness in the coarsened references of measurements, the quantum-to-classical transition can be observed independent of the macroscopicity of the quantum state. We explore a general situation that the degree of fuzziness can change with the rotation angle between two states (different rotation angles represent different references). The fuzziness of reference comes from two kinds of fuzziness: the Hamiltonian (rotation frequency) and the timing (rotation time). For the fuzziness of the Hamiltonian alone, the degree of fuzziness for the reference will change with the rotation angle between two states, and the quantum effects can still be observed with any degree of fuzziness of Hamiltonian. For the fuzziness of timing, the degree of the coarsening reference is unchanged with the rotation angle. During the rotation of the measurement axis, the decoherence environment can also help the classical-to-quantum transition due to changing the direction of the measurement axis.

Keywords: coarsened references, quantum-to-classical transition, decoherence

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1. Introduction

Since quantum phenomena are observed on the microscopic scale, the quantum-to-classical transition has attracted a lot of attention. There are two crucial elements in the framework of quantum mechanics: the state of a physical system represented by a wave function and the measurement represented by non-negative operators. Hence, explanations of the quantum-to-classical transition concentrate on the two elements.

On one hand, the decoherence of a system from a quantum state to a classical one is the main reason that the macroscopic world is classical[1,2] not quantum, and the decoherence happens due to the unavoidable interactions with environments.

On the other hand, an explanation of the quantum-to-classical transition comes from the measurement. Kofler and Brukner[3] firstly attributed the cause to coarsening of measurements. Then, a lot of works about the influence of coarsening of measurements on the quantum-to-classical transition[4–13] appeared. They mainly focused on coarsening the measuring resolution in the final detection. There are two steps for a complete measurement process: the first step is to set a measurement reference and control it, and the second step is the final detection with the corresponding projection operator. Recently, Jeong *et al.*[14] shed light upon the appearance of a classical world from another angle: coarsening the measurement reference. In their scheme, the control of the measurement reference was described by an appropriate unitary operator with a reference variable applied to the projection operator, and they considered the fixed degree of fuzziness (the degree of coarsening) in the references of measurements, which did not change with the rotation angle of the measurement axis corresponding to the different unitary operations.

In this article, we consider a more general situation: the degree of fuzziness can change in different coarsened references based on the fact that the fuzziness of the coarsened reference comes from the fuzziness of the Hamiltonian (rotation frequencies of the reference) and timing (when to do the final detection). It is found that for only the fuzziness of the Hamiltonian, the degree of fuzziness in the coarsening reference measurement increases with the rotation angle, while for only the fuzziness of timing, the degree of fuzziness is unchanged with the rotation angle. The special status of time in practical operation makes the result of rotation frequencies and timing become different. So reference [14] only considered the coarsened rotation time. Further, we show that the Bell function $|B|$ decreases with the fuzziness when $|B| \geq 2$; while it can increase with the fuzziness when $|B| < 2$. We study other ways to detect the quantum effect ($B > 2$), such as changing the angle, frequency, and steps of rotation. Moreover, we obtain the conclusion that the decoherence environments can help to detect the quantum effect in the coarsening reference during the rotation of reference. Our results will further shed light on the fuzziness in coarsening reference measurements.

The rest of this article is arranged as follows. In Sec-

*Project supported by the National Natural Science Foundation of China (Grant No. 11375168).
†Corresponding author. E-mail: xiedong@mail.ustc.edu.cn
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tion 2, we classify the fuzziness of the coarsening reference as three kinds: only Hamiltonian (subsection 2.1), only timing (subsection 2.2), and both of Hamiltonian and timing (subsection 2.3). In Section 3, the decoherence environment is studied in the coarsening reference measurements. Finally, we give a conclusion in Section 4.

2. Three kinds of fuzziness

Consider the generic example: an infinite dimensional system with the orthonormal basis set $|o_n\rangle_{n=-\infty}^{n=\infty}$. The dichotomic measurement O with eigenvalues ±1 can be given by

$$O = O_+ - O_-, \tag{1}$$

where

$$O_+ = \sum_{n=1}^{\infty} |o_n\rangle\langle o_n|, \quad O_- = \sum_{n=-\infty}^{0} |o_n\rangle\langle o_n|. \tag{2}$$

Let us consider a type of entanglement as follows:

$$|E_n\rangle = \frac{1}{\sqrt{2}}(|o_n\rangle|o_{-n}\rangle + |o_{-n}\rangle|o_n\rangle). \tag{3}$$

A unitary transform $U(\theta)$ is described by

$$\begin{aligned} U(\theta)|o_n\rangle &= \cos\theta|o_n\rangle + \sin\theta|o_{-n}\rangle, \\ U(\theta)|o_{-n}\rangle &= \sin\theta|o_n\rangle - \cos\theta|o_{-n}\rangle, \end{aligned} \tag{4}$$

where $\theta = wt$ represents the rotation angle, w denotes the rotation frequency, and t is the corresponding time of rotation. The coarsened version of the unitary operation applied to the projection operator O can be given by

$$O_\Delta(\theta_0) = \int \mathrm{d}\theta P_\Delta(\theta-\theta_0)[U^\dagger(\theta)OU(\theta)], \tag{5}$$

where

$$P_\Delta(\theta-\theta_0) = \frac{1}{\sqrt{2\pi}\Delta}\exp\left[-\frac{(\theta-\theta_0)^2}{2\Delta^2}\right]$$

is the normalized Gaussian kernel with standard deviation Δ, which quantifies the degree of fuzziness in the coarsened measurement reference.

The fuzziness of rotation angle $\theta_0 = w_0 t_0$ comes from the fuzziness of rotation frequency w_0 and rotation time t_0. So it is necessary to explore the fuzziness of the rotation angle from the following three aspects.

2.1. Coarsening the rotation frequency w_0

Due to the fuzziness of the Hamiltonian which performs the unitary operation, the rotation frequency is coarsened. In general, the corresponding fuzzy version of the rotation frequency w_0 can be written as a Gaussian distribution $P_{\Delta_{w_0}}(w-w_0) = \frac{1}{\sqrt{2\pi}\Delta_{w_0}}\exp[-\frac{(w-w_0)^2}{2\Delta_{w_0}^2}]$. Non-Gaussian types perhaps deserve further explorations in some special situations, which is beyond the scope of this article.

Then, the coarsened version of the unitary operation can be described as

$$\begin{aligned} O_{\Delta_{\theta_0}}(w_0 t) &= \int \mathrm{d}w P_{\Delta_{w_0}}(w-w_0)[U^\dagger(wt)OU(wt)] \\ &= \int \mathrm{d}\theta P_{\Delta_{\theta_0}}(\theta-\theta_0)[U^\dagger(\theta)OU(\theta)], \end{aligned} \tag{6}$$

where the rotation angle $\theta_0 = w_0 t$, and $\Delta_{\theta_0} = \Delta_{w_0}\theta_0/w_0$. It means that the degree of fuzziness in coarsening the measurement reference increases with the rotation angle θ_0.

Let us rotate the reference by many steps. For example, one first rotates the reference by angle $\theta_0/2$, then by another angle $\theta_0/2$. The reference is rotated by angle θ_0 in total. But due to the two times adjustment of the unitary operation, the degree of fuzziness $\Delta_{\theta_0}|_{0\to\theta_0/2\to\theta_0}$ will be different from a single rotation $\Delta_{\theta_0}|_{0\to\theta_0}$. For the projection operator O,

$$\begin{aligned} & O_{\Delta_{\theta_0}}(w_0 t)|_{0\to\theta_0/2\to\theta_0} \\ &= \int \mathrm{d}w' \int \mathrm{d}w P_{\Delta_{w_0}}(w'-w_0), \\ & P_{\Delta_{w_0}}(w-w_0)\left[U^\dagger\left(\frac{w+w'}{2}t\right)OU\left(\frac{w+w'}{2}t\right)\right] \\ &= \int \mathrm{d}\theta P_{\Delta_{\theta_0}|_{0\to\theta_0/2\to\theta_0}}(\theta-\theta_0)[U^\dagger(\theta)OU(\theta)], \end{aligned} \tag{7}$$

where the degree of fuzziness $\Delta_{\theta_0}|_{0\to\theta_0/2\to\theta_0} = \Delta_{\theta_0}|_{0\to\theta_0}/\sqrt{2}$. If one performs N steps of rotations, the degree of fuzziness $\Delta_{\theta_0}|_{0\to\theta_0/N\to2\theta_0/N\to\ldots\theta_0} = \Delta_{\theta_0}|_{0\to\theta_0}/\sqrt{N}$. It signifies that many steps of rotations can reduce the fuzziness of coarsening the measurement reference.

The correlation function is defined as the expectation value of the measurement operator

$$E_{\Delta_a,\Delta_b}(\theta_a,\theta_b) = \langle O_{\Delta_a}(\theta_a)\otimes O_{\Delta_b}(\theta_b)\rangle_{ab}, \tag{8}$$

where the average is taken over entangled state $|E_n\rangle_{ab}$ in Eq. (3), and Δ_a and Δ_b are the corresponding standard deviations for rotation angles θ_a and θ_b. Then, the Bell function[15,16] can be obtained as

$$\begin{aligned} B &= E_{\Delta_{a1},\Delta_{b1}}(\theta_a,\theta_b) + E_{\Delta'_{a2},\Delta_{b2}}(\theta'_a,\theta_b) \\ &\quad + E_{\Delta_{a3},\Delta'_{b3}}(\theta_a,\theta'_b) - E_{\Delta'_{a4},\Delta'_{b4}}(\theta'_a,\theta'_b), \end{aligned} \tag{9}$$

where the subscript $i = 1,2,3,4$ distinguishes the degree of fuzziness Δ for the same rotation angle θ in different joint measurements, which is possible in practical experiments. Choose the parameters $\Delta_{xi} = 0$ for $x = a,b$ and $i = 1,2,3$ (the corresponding rotation frequency w_0 close to infinity); $\Delta_{a4} = 0$ or $\Delta_{b4} = 0$ (the corresponding frequency w_0 close to 0); $\theta_a = \theta_b = 0$; and $\theta'_a = \theta'_b = 2\pi$. Based on a simple calculation, we find that the maximum of the Bell function $|B| \approx 3 > 2\sqrt{2}$ for both quantum state and classical one. It means that the Bell function following the definition in Eq. (9)

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cannot show the difference between quantum and classic. So, in the practical experiment, one must guarantee that the degree of fuzziness Δ for the same rotation angle θ is the same in different joint measurements. So the Bell function should be defined as

$$\begin{aligned} B &= E_{\Delta_a,\Delta_b}(\theta_a,\theta_b)+E_{\Delta'_a,\Delta_b}(\theta'_a,\theta_b) \\ &\quad +E_{\Delta_a,\Delta'_b}(\theta_a,\theta'_b)-E_{\Delta'_a,\Delta'_b}(\theta'_a,\theta'_b). \end{aligned} \tag{10}$$

With the above definition, the Bell function will not be larger than $2\sqrt{2}$. We obtain the same result as that in Ref. [14], i.e., the Bell function $|B|$ decreases with the degree of fuzziness Δ_{w_0} for $|B| \geq 2$. It is proved as follows:

$$\begin{aligned} \frac{\mathrm{d}B}{\mathrm{d}\Delta_{w_0}} &= -2\Delta_{w_0}/w_0^2[E_{\Delta_a,\Delta_b}(\theta_a,\theta_b)(\theta_a{}^2+\theta_b{}^2) \\ &\quad +E_{\Delta'_a,\Delta_b}(\theta'_a,\theta_b)(\theta'^2_a+\theta_b{}^2) \\ &\quad +E_{\Delta_a,\Delta'_b}(\theta_a,\theta'_b)(\theta_a{}^2+\theta'^2_b) \\ &\quad -E_{\Delta'_a,\Delta'_b}(\theta'_a,\theta'_b)(\theta'^2_a+\theta'^2_b)]. \end{aligned} \tag{11}$$

Noting that the correlation function $|E| \leq 1$ and the Bell function $|B| \geq 2$, we can obtain the differential coefficient $\mathrm{d}|B|/\mathrm{d}\Delta_{w_0} \leq 0$, which signifies the Bell function $|B|$ decreases with Δ_{w_0}.

However, when $|B| < 2$, the Bell function can also increase with Δ_{w_0}, as shown in Fig. 1. The reason is that the degree of coarsening the measurement reference changes with the rotation angle.

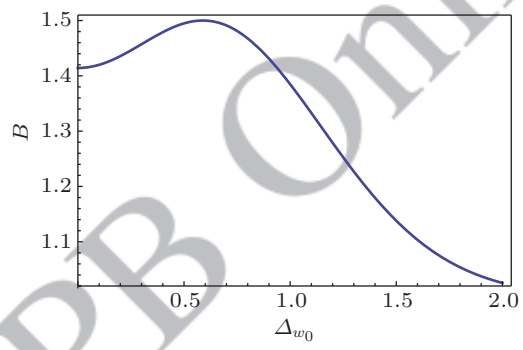


Fig. 1. (color online) The graph shows that the Bell function $|B|$ can increase with Δ_{w_0}. Here the parameters are $\theta_a = \theta_b = 0$, $\theta'_a = \pi/8$, $\theta'_b = -\pi/8$, and $|w_0| = \sqrt{2}\pi/8$.

We obtain that for any degree of fuzziness Δ_{w_0}, the quantum effects ($|B| > 2$) can be observed by reducing the rotation angle. It can be simply verified as follows. For the parameters $\theta_a = \theta_b = 0$, $\theta'_a = \theta'_b = \lambda \ll 1$, and $|w_0| = 1$, we can derive that $B = 2 + 4\lambda^2 + \mathscr{O}(\lambda^3) > 2$. Essentially, it is because that the degree of coarsening the reference increases with the rotation angle: $\Delta_{\theta_0} = \Delta_{w_0}\theta_0/w_0$.

2.2. Coarsening the rotation time t_0

When the rotation frequency w_0 can be controlled precisely, the fuzziness of reference can come from the coarsened timing. Similar to the above section, the Gaussian distribution of rotation time t is given as

$$P_{\Delta_{t_0}}(t-t_0) = \frac{1}{\sqrt{2\pi}\Delta_{t_0}} \exp\left[-\frac{(t-t_0)^2}{2\Delta_{t_0}^2}\right].$$

Then, the coarsened version of the unitary operation can be described as

$$\begin{aligned} O_{\Delta_{\theta_0}}(w_0t_0) &= \int \mathrm{d}t P_{\Delta_{t_0}}(t-t_0)[U^\dagger(w_0t)OU(w_0t)] \\ &= \int \mathrm{d}\theta P_{\Delta_{\theta_0}}(\theta-\theta_0)[U^\dagger(\theta)OU(\theta)], \end{aligned} \tag{12}$$

where the rotation angle $\theta_0 = w_0t_0$, and $\Delta_{\theta_0} = \Delta_{t_0}w_0$. It means that for the fixed rotation frequency, the degree of fuzziness in coarsening the measurement reference is unchanged with the rotation angle θ_0. So it is just the situation considered in Ref. [14]. Here, the Bell function B follows the definition in Eq. (10). It is easy to verify that the Bell function $|B|$ decreases with the degree of fuzziness Δ_{t_0}.

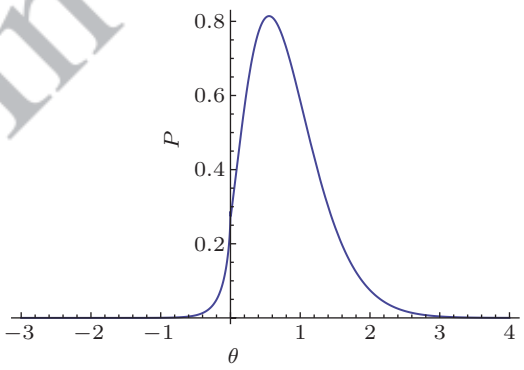


Fig. 2. (color online) Distribution $P_{\Delta_{\theta_0}}(\theta-\theta_0)$ changes with the rotation angle θ, where the parameters are $w_0 = 1$, $t_0 = \pi/4$, $\Delta_{w_0} = 0.4$, and $\Delta_{t_0} = 0.4$. The peak value is close to the expected one $\pi/4$.

Contrary to the situation in the above section, many steps of rotations will increase the fuzziness degree of the coarsened reference: $\Delta_{\theta_0}|_{0\to\theta_0/N\to 2\theta_0/N\to\cdots\theta_0} = \sqrt{N}\Delta_{\theta_0}|_{0\to\theta_0}$, where N is the steps of rotations. It is because that the many steps of rotations increase the times of timing. The effects of coarsening the frequency and the time are different. We consider the process of rotation: first step, we set up the rotation frequency (namely, the rotation Hamiltonian); second step, rotate the reference with the rotation time. This makes the rotation frequency and the rotation time become unexchangeable. In other words, the time plays a more important role. We divide the process of rotation into many steps, according to the time, not the frequency.

2.3. Coarsening both rotation frequency and time

The more general situation is that both the rotation frequency and time are coarsened. Now the coarsened version of the unitary operation can be described as

$$O_{\Delta_{\theta_0}}(w_0t_0)$$

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$$= \int \mathrm{d}w \int \mathrm{d}t P_{\Delta_{w_0}}(w-w_0) P_{\Delta_{t_0}}(t-t_0)[U^{\dagger}(wt)OU(wt)]$$
$$= \int \mathrm{d}\theta \int \mathrm{d}w \frac{1}{2\pi\Delta_{t_0}\Delta_{w_0}|w|} \exp\left[-\frac{(w-w_0)^2}{2\Delta_{w_0}^2} - \frac{(\theta/w-t_0)^2}{2\Delta_{t_0}^2}\right][U^{\dagger}(wt)OU(wt)]$$
$$= \int \mathrm{d}\theta P_{\Delta_{\theta_0}}(\theta-\theta_0)[U^{\dagger}(\theta)OU(\theta)]. \quad (13)$$

So the distribution of the rotation angle is obtained as

$$P_{\Delta_{\theta_0}}(\theta-\theta_0) = \int \mathrm{d}w \frac{1}{2\pi\Delta_{t_0}\Delta_{w_0}|w|} \exp\left[-\frac{(w-w_0)^2}{2\Delta_{w_0}^2} - \frac{(\theta/w-t_0)^2}{2\Delta_{t_0}^2}\right]. \quad (14)$$

From Figs. 2–4, we can see that function $P_{\Delta_{\theta_0}}(\theta-\theta_0)$ is not a Gaussian distribution. With the increase of the degree of fuzziness Δ_{w_0} and Δ_{t_0}, the gap between the central value and $\theta_0 = w_0 t_0$ is widening as expected.

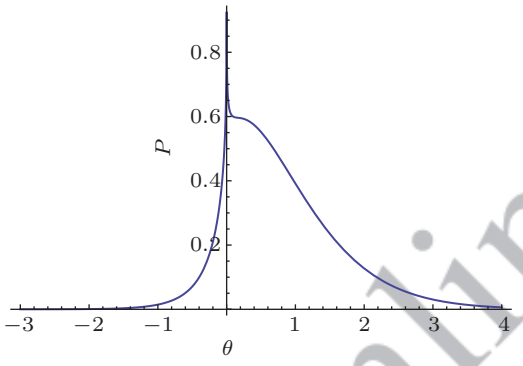


Fig. 3. (color online) Distribution $P_{\Delta_{\theta_0}}(\theta-\theta_0)$ changes with the rotation angle θ, where we choose larger parameters as Δ_{w_0} and Δ_{t_0}: $w_0 = 1$, $t_0 = \pi/4$, $\Delta_{w_0} = 0.6$, and $\Delta_{t_0} = 0.6$.

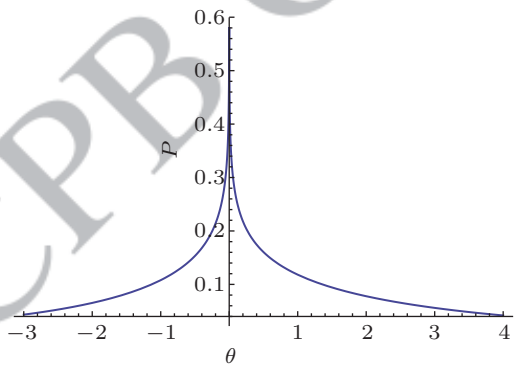


Fig. 4. (color online) Distribution $P_{\Delta_{\theta_0}}(\theta-\theta_0)$ changes with the rotation angle θ. Here the parameters are given as $w_0 = 1$, $t_0 = \pi/4$, $\Delta_{w_0} = 2$, and $\Delta_{t_0} = 2$. The function $P_{\Delta_{\theta_0}}(\theta-\theta_0)$ centers around 0, which deviates more greatly from the expected value $\theta_0 = w_0 t_0 = \pi/4$.

3. Decoherence environment

Any system is inevitably intact with the surrounding environment. We consider that during the rotation, the decoherence environment has an effect on the system.

Let us consider that a classical environment (for example, random telegraph noise) contacts with the spin system. The interaction Hamiltonian can be written as $H_{\mathrm{int}} = \beta\sigma_z$ ($\hbar \equiv 1$),[17,18] where $\beta = \pm\gamma$ with equal probability.

We first discuss the fuzziness of reference from the coarsened rotation frequency. Apply the interaction Hamiltonian H_{int} and the rotation Hamiltonian $w\sigma_x$ to the projection operator $O = \sigma_z$,

$$O_{\Delta_{\theta_0}}(w_0 t) = \int \mathrm{d}w P_{\Delta_{w_0}}(w-w_0)[\exp[\mathrm{i}(w\sigma_x + \beta\sigma_z)t]\sigma_z \times [\exp[-\mathrm{i}(w\sigma_x + \beta(t)\sigma_z)t]. \quad (15)$$

As shown in Fig. 5, without the decoherence environment ($\gamma = 0$), the Bell function $B = 1.69 < 2$; due to the influence of the environment, the Bell function B can arrive at 2.05. From Fig. 6, without the decoherence environment ($\gamma = 0$), the Bell function $B = 2.068 > 2$; due to the influence of the environment, the Bell function B can arrive at $2.1 > 2.068$. It shows that the decoherence environment can help to detect the quantum effects (increasing the value of B), whether the initial Bell function $B < 2$ or not. Intuitively, the cause is that the decoherence environment changes the direction of the measurement axis, not along the X axis. Then, it decreases the fuzziness degree of the coarsening reference to increase the Bell function B.

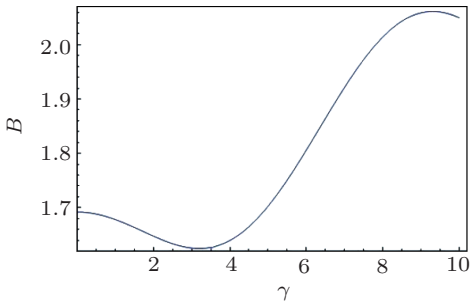


Fig. 5. (color online) The correlation between the Bell function B and the coupling strength γ, where we only consider the system suffering from the decoherence environment. Here the parameters are given as $\theta_a = -\pi/8$, $\theta_b = 0$, $\theta_a' = \pi/8$, $\theta_b' = \pi/2$, $\Delta_{w_0} = 1$, and $|w_0| = 1$.

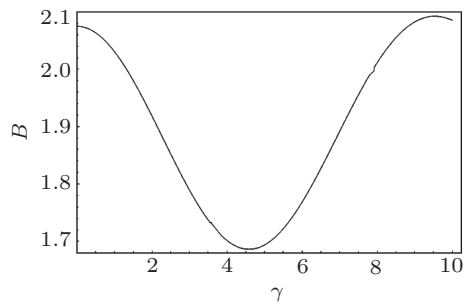


Fig. 6. (color online) The correlation between the Bell function B and the coupling strength γ, the standard deviations are reduced to $\Delta_{w_0} = 0.5$ and $|w_0| = 0.5$. We find that the decoherence environment can help to increase B when $B > 2$. Without the decoherence environment, $\gamma = 0$.

Then, we consider the situation of the fuzziness of reference from the coarsened timing. In Fig. 7, we find an abnormal

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phenomenon: the Bell function B can increase with the standard deviation Δ_{t_0}. So it is demonstrated that there are some decoherence environments that can increase the violation.

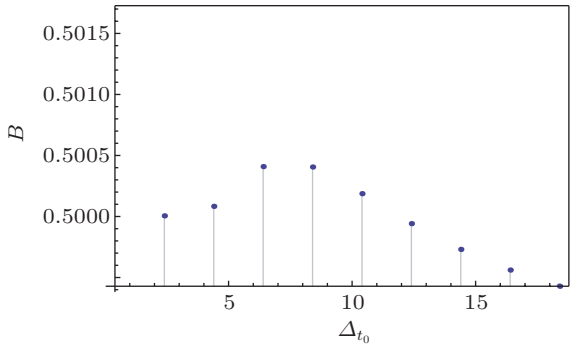


Fig. 7. (color online) The relationship between the Bell function B and Δ_{t_0}, where the parameters are given as $\theta_a = 0$, $\theta_b = 7\pi/8$, $\theta'_a = \pi/4$, $\theta'_b = \pi/8$, $\gamma = 1$, and $|w_0| = 1$.

This reflects that in an open system, the fuzziness degree of coarsening reference is modified. Therefore, it is necessary and interesting to further study how to quantify the fuzziness degree in an open system (we leave it an open question).

4. Conclusion

We explore the origination of the coarsened reference, and attribute it to the fuzziness of rotation frequency and time. In essence, two ways are equivalent. However, due to the special status of time in the practical rotation process, the final results are different. The degree of fuzziness in coarsening the reference increases with the rotation angle for the coarsened rotation frequency; for the coarsened rotation time, it is unchanged. For the coarsened rotation frequency, the degree of fuzziness of reference decreases with the central frequency and steps of rotations. On the contrary, it increases for the coarsened rotation time. For both coarsened rotation frequency and time, the distribution of rotation angle will not be Gaussian and center around the expected value, which could influence the Bell function. Finally, we study a simple classical decoherence environment during the rotation. Counterintuitively, the decoherence can help to increase the Bell function for detecting the quantum effect. It is interesting to research the complex environment and the Bell inequalities for many particles in the coarsened reference.[19–21] We believe that this article will deepen the understanding of the coarsened reference in the quantum-to-classical transition and how to control the measurement reference.

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