Asymmetric and symmetric meta-correlations in financial markets*

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In financial markets, the relation between fluctuations of stock prices and trading behaviors is complex. It is intriguing to quantify this kind of meta-correlation between market fluctuations and the synchronous behaviors. We refine the theoretical index leverage model proposed by Reigneron *et al.*, to exactly quantify the meta-correlation under various levels of price fluctuations [Reigneron P A, Allez R and Bouchaud J P 2011 *Physica A* **390** 3026]. The characteristics of meta-correlations in times of market losses, are found to be significantly different in Chinese and American financial markets. In addition, unlike the asymmetric results at the daily scale, the correlation behaviors are found to be symmetric at the high-frequency scale.

Keywords: financial market, collective behavior, complex system, asymmetry and symmetry

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1. Introduction

Interactions between individuals in complex systems can cause convergent collective behaviors under severe circumstances.[1–3] In time of emergencies, animals such as ants, birds and fishes or even bacteria would spontaneously form groups to resist the threats.[4–9] In financial markets, it is known that bubbles largely occur as long as herders have strong and common expectations of price increases. Herding behaviors can expedite the burst of bubbles, which causes market panic leading to more severe fluctuations in return.[10,11] It is natural for participants to change their trading behaviors according to various fluctuations in stock prices. However, if confronted with huge market stress like financial crises, their behaviors often converge to the same pattern. The synchronous trading behaviors can aggravate the fluctuations.[12,13] Moreover, due to the synchrony of assets prices, the diversification of portfolios would not be ensured any more.

Many recent studies on the structure and dynamics of the financial market have tried to make clear market behaviors and individual trading behaviors.[14–27] Preis *et al.* uncovered that the average correlation among these stocks could be linearly explained by normalized Dow Jones Industrial Average (DJIA) index returns at various time scales from 10 days to 60 days[18] In addition, Kenett *et al.* pointed out that the average correlation was negatively correlated with the index return through the analysis of the same DJIA index.[19] They also delivered empirical support for the trading behaviors influenced by the market index on shorter time scales, using high-frequency data.[20] However, the relationship between the market index return and the average correlation between different stocks, referred to as meta-correlation, is still worthy of more research.

In the study of index leverage effect from Reigneron *et al.*, a simplified theoretical method was proposed by some assumptions to show the index leverage.[21] If just considering the instantaneous form, the formula described the meta-correlation as a quadratic form. However, they did not consider the influences caused by instantaneous volatilities. Moreover, they also did not give convinced evidences from other financial markets. Motivated by the defectiveness, in this paper, we refine the theoretical formula by the mean standard deviation of stocks over different time intervals, to obtain a clear expression on the meta-correlation. On the other hand, Fiedor's work attached importance to the sectors behaviors rather than individual stocks behaviors.[24] Uechi *et al.* proposed an indicator called sector dominance ratio, which was also based on sectors behaviors.[25] Therefore, in the later empirical analyses, we use the sectors data instead of stocks to find supports for the meta-correlation. Most of the existing literature shows the emergence of meta-correlation in the market of Dow Jones Industrial Average (DJIA), by analyzing the 30 component stocks average correlation and the DJIA index return.[18,19] In this article, our work is based on the data from the two markets, China Securities Index 300 (CSI 300) and Standard & Poor's 500 (S & P 500).

Recently, some studies have shown that the price dynamics presents different characteristics at long-time scales and short-time scales.[28,29] Botta *et al.* found that the tails of log-return distributions of DIJA exhibit power law decays at short-time scales but exponential decays at long-time scales.[28] In addition, Jiang *et al.* found that the large-fluctuation dynamics was time-reversal symmetric at the minutes scale, while asymmetric at the daily scale.[29] Recently, agent-based modeling and cross-correlation statistics give us the answer for the asymmetric characteristics of meta-correlation at the macroscopic scale.[30,31] Considering the different characteristics of

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trading behaviors in the perspective of time scale, our work's results show good consistency with theirs. Asymmetric correlation behaviors are observed in our empirical analyses at low-frequency (daily) scales, whereas the correlation behaviors become symmetric at high-frequency (minutely) scales. To figure out the presence of the symmetric and asymmetric behaviors at different time scales, we conduct the cross-correlation analyses to show how the casual relationship behaves at those different time scales.

2. Theoretical quantification method

As is well known, stock prices fluctuate frequently and incessantly as the upward or downward trend changes. However, especially when huge fluctuations take place, different stock sectors or industries always move collectively and share almost the same behaviors.

Motivated from the work by Reigneron *et al.*,[21] we try to refine their hypothetical formula to find the underlying relationship among the index return, the stock correlations and volatility. Supposing that there are a number of N stocks (sectors) in the market, the zero-mean daily return of each stock at time t is denoted as $r_i(t)$, where $i = 1,\ldots,N$. Then the market index return $R(t)$ can be reduced as a weighted average return, i.e.,

$$R(t) = \frac{1}{N}\sum_{i=1}^{N} k_i \cdot r_i(t). \tag{1}$$

Stock returns herein are adjusted by the weight coefficient k_i, which can be related to different factors, such as aggregate market values, volumes of transaction or price volatilities, even the squared components of the eigenvector with the largest eigenvalue of the stocks cross-correlation matrix.[32,33]. In this paper, we will take the equal weight coefficients for simplification. To obtain the instantaneous relationship, we try to square the index return Eq. (1) as follows:

$$\begin{aligned} R^2(t) &= \left[\frac{1}{N}\sum_{i=1}^{N} r_i(t)\right]^2 \\ &= \frac{1}{N^2}\left[\sum_{i=1}^{N} r_i^2(t) + \sum_{i\neq j}^{N} r_i(t) r_j(t)\right]. \end{aligned} \tag{2}$$

Since $r_i(t)$ is zero-mean, it can be replaced by $\hat{r}_i(t)\cdot\sigma_i$, where $\hat{r}_i(t)$ and σ_i are the normalized return and standard deviation over the whole time period respectively. So, we can substitute them into Eq. (2),

$$R^2(t) = \frac{1}{N^2}\left[\sum_{i=1}^{N} \hat{r}_i(t)\hat{r}_i(t)\sigma_i^2 + \sum_{i\neq j}^{N} \hat{r}_i(t)\hat{r}_j(t)\sigma_i\sigma_j\right]. \tag{3}$$

Although it is difficult to get the instantaneous return $\hat{r}_i(t)$, we can use the contiguous return series with length of Δt at time t to approximate the stock i's return. That is to say, supposing that the mean of $\hat{r}_i$ equals to zero over Δt near time t, it can also be replaced by $\hat{r}_{i,\Delta t}(t)\cdot\sigma_{i,\Delta t}$, where the subscript $_{\Delta t}$ here means that the results are calculated by the Δt data at around time t. Then, the squared index return can be written as

$$\begin{aligned} R^2(t) &\approx \frac{1}{N^2}\left[\sum_{i=1}^{N} \hat{r}_i\hat{r}_i\sigma_{i,\Delta t}^2\sigma_i^2 + \sum_{i\neq j}^{N} \hat{r}_i\hat{r}_j\sigma_{i,\Delta t}\sigma_{j,\Delta t}\sigma_i\sigma_j\right] \\ &\approx \frac{1}{N^2}\left[\sum_{i=1}^{N} \langle\hat{r}_i\hat{r}_i\rangle_{\Delta t}\,\sigma_{i,\Delta t}^2\sigma_i^2 + \sum_{i\neq j}^{N} \langle\hat{r}_i\hat{r}_j\rangle_{\Delta t}\,\sigma_{i,\Delta t}\sigma_{j,\Delta t}\sigma_i\sigma_j\right] \\ &= \frac{1}{N^2}\left[\sum_{i=1}^{N} \rho_{ii,\Delta t}\sigma_{i,\Delta t}^2\sigma_i^2 + \sum_{i\neq j}^{N} \rho_{ij,\Delta t}\sigma_{i,\Delta t}\sigma_{j,\Delta t}\sigma_i\sigma_j\right] \\ &= \frac{1}{N^2}\boldsymbol{\sigma}^*\boldsymbol{\sigma}_{\Delta t}^*(t)\boldsymbol{\rho}_{\Delta t}(t)\boldsymbol{\sigma}_{\Delta t}(t)\boldsymbol{\sigma}, \end{aligned} \tag{4}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_{\Delta t}$ are the column vectors of N stocks volatilities over the whole time period and over the Δt period at time t separately, besides, $\boldsymbol{\sigma}^*$ and $\boldsymbol{\sigma}_{\Delta t}^*$ are their corresponding transposed matrices. Furthermore, if we assume that the differences of volatilities between the N stock sector are stably close to zero, then we can achieve the following simplified equation from Eq. (4),

$$R^2(t) \approx c\cdot\bar{\rho}_{\Delta t}(t)\sigma_{\Delta t}^2(t) + \mathbf{O}\left(\frac{1}{N}\right). \tag{5}$$

where $\bar{\rho}_{\Delta t}(t)$ is obtained from averaging the non-diagonal real-time correlation matrix over the Δt as

$$\frac{1}{N(N-1)}\sum_{i\neq j}^{N}\rho_{ij,\Delta t},$$

and $\sigma_{\Delta t}(t)$ stands for the volatilities of the N stock at time t over the Δt, and c is a constant term. Thereupon we can see that the squared index return can be approximately replaced with the product of the average correlation and volatility. Assuming a constant volatility over the time, a quadratic relation is found in the dependence on the index return for the average correlation. As long as the index return increases, the average correlation between pairs of stocks would increase, as well as the correlation between sectors or industries.

3. Empirical analysis

In order to observe the relevant behaviors of the overall market and various sectors, we calculate the market index returns and the time-dependent average correlations between industries in different time intervals for empirical analysis. Two primary market indices separately in Chinese and American stock markets, i.e., CSI 300 index and S & P 500 index, are taken into consideration. Both of the primary market indices include rich varieties of constituent stocks and adequate diversification on risk. However, there are problems which should be noted that some stocks would be adjusted in or out of the index constituent list regularly or irregularly, resulting that the stock price series are probably inconsecutive. "DJIA Divisor" is taken into account to ensure the decrease of influences from adjustment when handling the 30 component stocks' historical price series in DIJA.[18] The compilation methods of the CSI 300 index and the S & P 500 index are the Paasche index,

instead of the arithmetical mean as DJIA. However, as market sectors, there are 10 common industry indices related to both the two major indices respectively (see Table 1). Therefore, we intend to analyze the 10 common industry indices, which are consecutive regardless of component stocks adjustments. Furthermore, since the compilation methods of sector indices are generally consistent with those of the major indices, we can assume that the weighted summation of the 10 common industry indices is able to reflect the whole market. Historical daily closing prices from 1/1/2002 to 5/31/2015, total L =3246 consecutive trading days, are analyzed. Both of the corresponding index behaviors are shown in Figs. 1(a) and 1(b). All of these market and sector indices data are obtained from *Wind Info*,[34] a popular data-service provider in China.

Table 1. The 10 different sector indices and their corresponding trading codes in CSI 300 and S & P 500, obtained from *Wind Info*.

Sector index	Code in CSI 300	Code in S & P 500
Energy	000908.SH	SP50010.SPI
Materials	000909.SH	SP50015.SPI
Industrials	000910.SH	SP50020.SPI
Consumer Discretionary	000911.SH	SP50025.SPI
Consumer Staples	000912.SH	SP50030.SPI
Health Care	000913.SH	SP50035.SPI
Financials	000914.SH	SP50040.SPI
Information Technology	000915.SH	SP50045.SPI
Telecommunication Services	000916.SH	SP50050.SPI
Utilities	000917.SH	SP50055.SPI

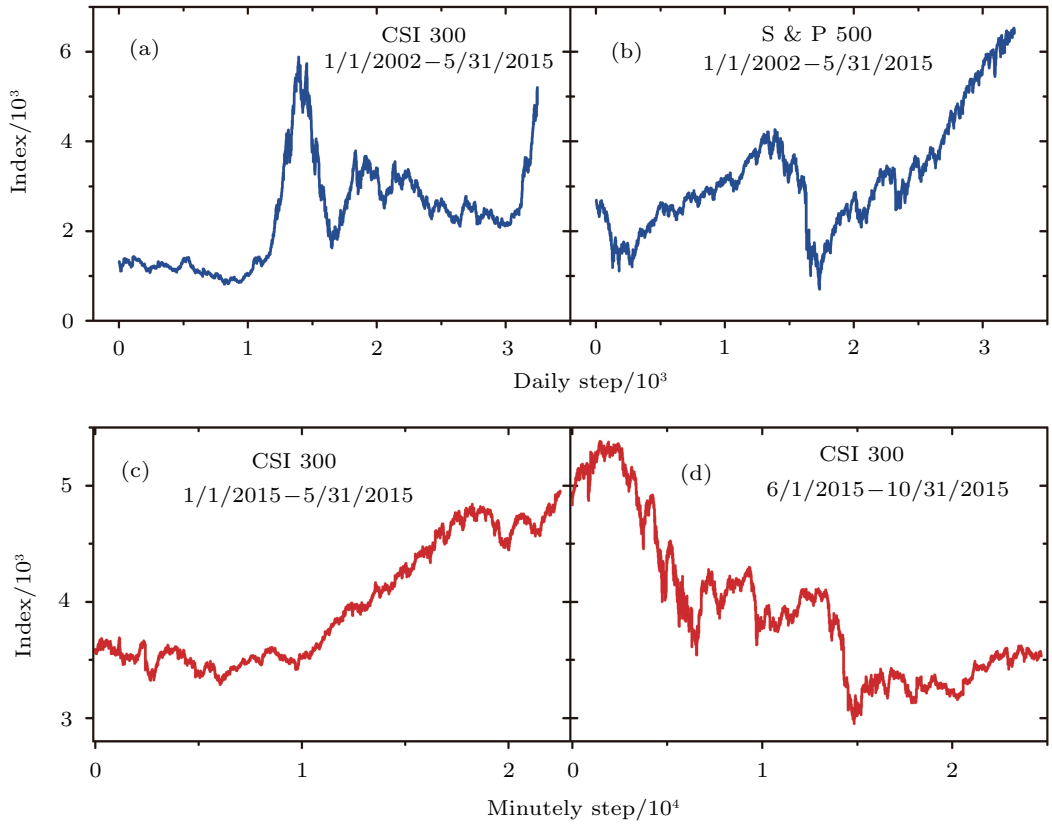


Fig. 1. (color online) Panels (a) and (b) show the daily behaviors of the market index from 1/1/2002 to 5/31/2015 in CSI 300 and S & P 500 respectively. The minute index behaviors in a upward trend in CSI 300 from 1/1/2015 to 5/31/2015 are shown in panel (c), whereas index behaviors in a downward trend from 6/1/2015 to 10/31/2015 are shown in panel (d).

3.1. Time-dependent average correlation and normalized index return

The time-dependent average correlation is measured by the average value of the Pearson product-moment matrix of the 10 sector indices in different Δt intervals. First of all, in each Δt time interval log-returns of the 10-sector indices are calculated,

$$r_i(t) = \ln[p_i(t'+1)] - \ln[p_i(t')], \tag{6}$$

where $p_i(t')$ denotes the closing price of sector index i at time t', and $t' \in [t, t+\Delta t-1]$. Since different indices vary with different levels of volatilities, it is necessary to normalize all the return time series during that Δt interval. The normalized return series are defined as

$$\tilde{r}_i(t) = \frac{r_i(t) - \langle r_i(t)\rangle_{\Delta t}}{\sigma_i(t)}, \tag{7}$$

where $\tilde{r}_i$ is the normalized return during Δt and

$$\sigma_i(t) = \sqrt{\langle r_i^2\rangle_{\Delta t} - \langle r_i\rangle_{\Delta t}^2}$$

and $\langle\cdots\rangle_{\Delta t}$ denotes the time average over Δt. During each Δt interval, we can also calculate the correlation matrix by the normalized return series of 10 different indices. $c_{i,j}$ denotes the correlation coefficient between sector index i and sector

index j, which is defined as,

$$c_{i,j}(t) = \frac{\sum_{\Delta t} \tilde{r}_i \tilde{r}_j - \sum_{\Delta t} \tilde{r}_i \sum_{\Delta t} \tilde{r}_j}{\sigma_i \sigma_j}. \quad (8)$$

More conveniently, the correlation matrix can be calculated below,

$$\boldsymbol{C}(t, \Delta t) = \frac{1}{\Delta t} \tilde{\boldsymbol{r}} \tilde{\boldsymbol{r}}^{\mathrm{T}}, \quad (9)$$

where $\tilde{\boldsymbol{r}}$ denotes the $10 \times \Delta t$ matrix of the normalized returns of 10-sector indices in the Δt interval while $\tilde{\boldsymbol{r}}^{\mathrm{T}}$ is the transposed matrix. The diagonal elements in the correlation matrix $\boldsymbol{C}$ always equal 1. Finally, we calculate the time-dependent mean correlation by averaging every non-diagonal element, i.e.,

$$\bar{C}_{\Delta t}(t) = \frac{2}{N(N-1)} \sum_{i<j}^{N} c_{ij}(t). \quad (10)$$

In addition, we also calculate the overlapping index return in the Δt interval at time t, which is expressed as,

$$r_{\mathrm{index}}(t) = \ln[p_{\mathrm{index}}(t + \Delta t)] - \ln[p_{\mathrm{index}}(t)], \quad (11)$$

where $p_{\mathrm{index}}(t)$ is the market index price at time t. Specially, we note the normalized *market index* return series as $R_{\Delta t}(t)$, which is obtained as,

$$R_{\Delta t}(t) = \frac{r_{\mathrm{index}}(t) - \langle r_{\mathrm{index}}(t) \rangle_L}{\sigma_{\mathrm{index}}(t)}, \quad (12)$$

where the standard deviation, $\sigma_{\mathrm{index}}(t)$, is obtained from $\sqrt{\langle r^2_{\mathrm{index}} \rangle_L - \langle r_{\mathrm{index}} \rangle^2_L}$ and $\langle \cdots \rangle_L$ here is the time average over the whole time period of L days.

3.2. Results

Now, we obtain several important time series: one is $R_{\Delta t}(t)$, the market index return series during Δt interval; the second one is, $\bar{C}_{\Delta t}(t)$, the time-dependent average correlation series between different sectors; the third one is the average standard deviation $\sigma(t)$. Panels (a), (b), (c) and panels (e), (f), and (g) in Fig. 2 depict the three types of time series for both CSI 300 and S & P 500 with $\Delta t = 5$ days. Moreover, to detect the stationarity and stochastic aspects of these series, we apply the method of Detrended Fluctuation Analysis (DFA)[35–38] (see Figs. 2(d) and 2(h)). As a result, all of the series are possessing high values of the power-law exponents, which are larger than 0.5 and less than 1, indicating they are strongly long-range correlated in time scale. Augmented Dickey–Fuller (ADF) test[39] is also applied, resulting that we should reject the null hypothesis of non-stationarity for all of the series (see Table. 2).

Table 2. The power-law exponent α from Detrended Fluctuation Analysis and the statistics and p-value from Augmented Dickey-Fuller test on $R(t)$, $\bar{C}(t)$ and $\sigma(t)$ for both CSI 300 and S & P 500, where $\Delta t = 5$ days.

Time series	α	DF	p-value
R (CSI 300)	0.72	−18.38	<0.01
$\bar{C}$ (CSI 300)	0.80	−5.85	<0.01
σ (CSI 300)	1.02	−2.80	<0.01
R (S & P 500)	0.62	−21.66	<0.01
$\bar{C}$ (S & P 500)	0.79	−6.62	<0.01
σ (S & P 500)	1.16	−3.60	<0.01

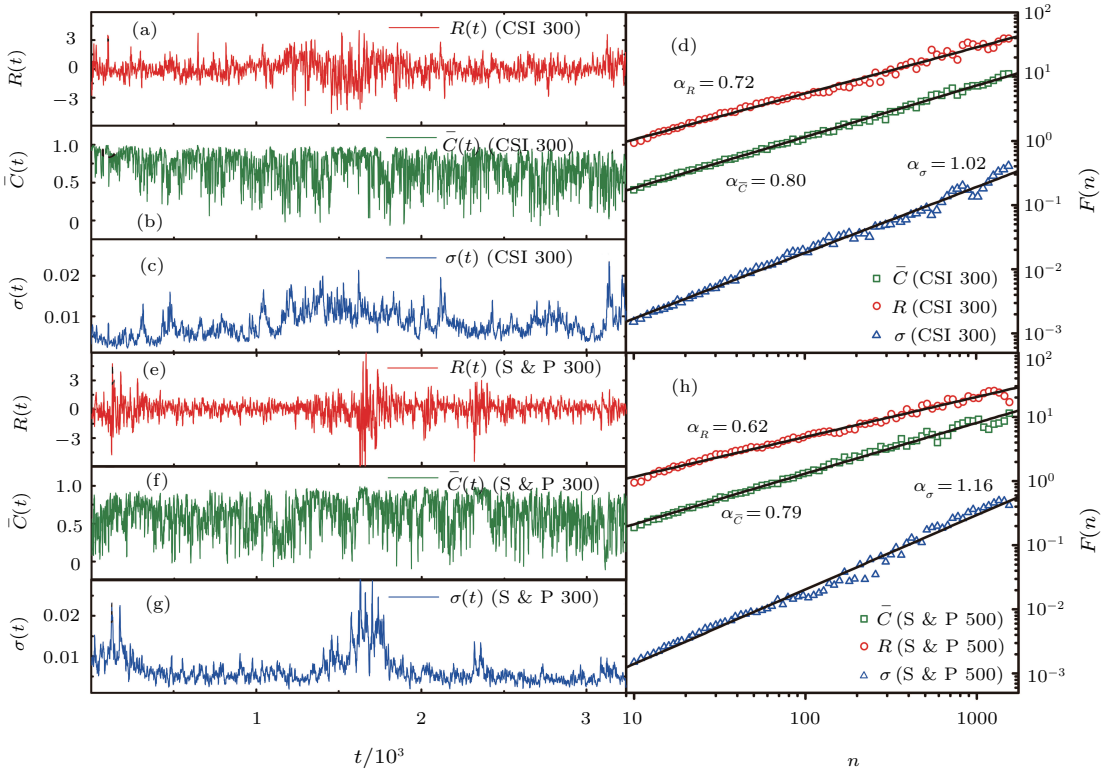


Fig. 2. (color online) Three time series for CSI 300, normalized index return R, the average correlation between industries $\bar{C}$ and the corresponding average standard deviation σ are graphed in panels (a), (b), and (c) respectively. Panels (e), (f), and (g) are graphs of the three time series for S & P 500. Panels (d) and (h) show the Detrended Fluctuation Analysis results in both two markets.

3.2.1. Asymmetric meta-correlation at daily scale

To quantify the meta-correlation between the normalized index return and industry indices average correlation in time intervals of Δt days, we graph the scatters of the two series of $R_{\Delta t}(t)$ and $\bar{C}_{\Delta t}(t)$ for different Δt days. As for the fact that the intervals actually have no interference on the relationship,[18] we get the average $\bar{C}$ at every window of R. Figures 3(a) and 3(b) show the empirical meta-correlation in CSI 300 and S & P 500 separately. Equation (5) has already shown us that the robustness of up-opening parabolic meta-correlation depends on the standard deviations during Δt interval at time t. So our empirical results of the time-dependent average correlations in Δt have been adjusted by the mean standard deviation σ over that interval. Huge fluctuations of market index return are always accompanied by an increase of correlation between various industries. The relationship between normalized index return R and $\bar{C}$ adjusted by σ is independent of the interval Δt, showing that this characteristic always exists in the markets no matter for long terms or short terms in the trading day scale.

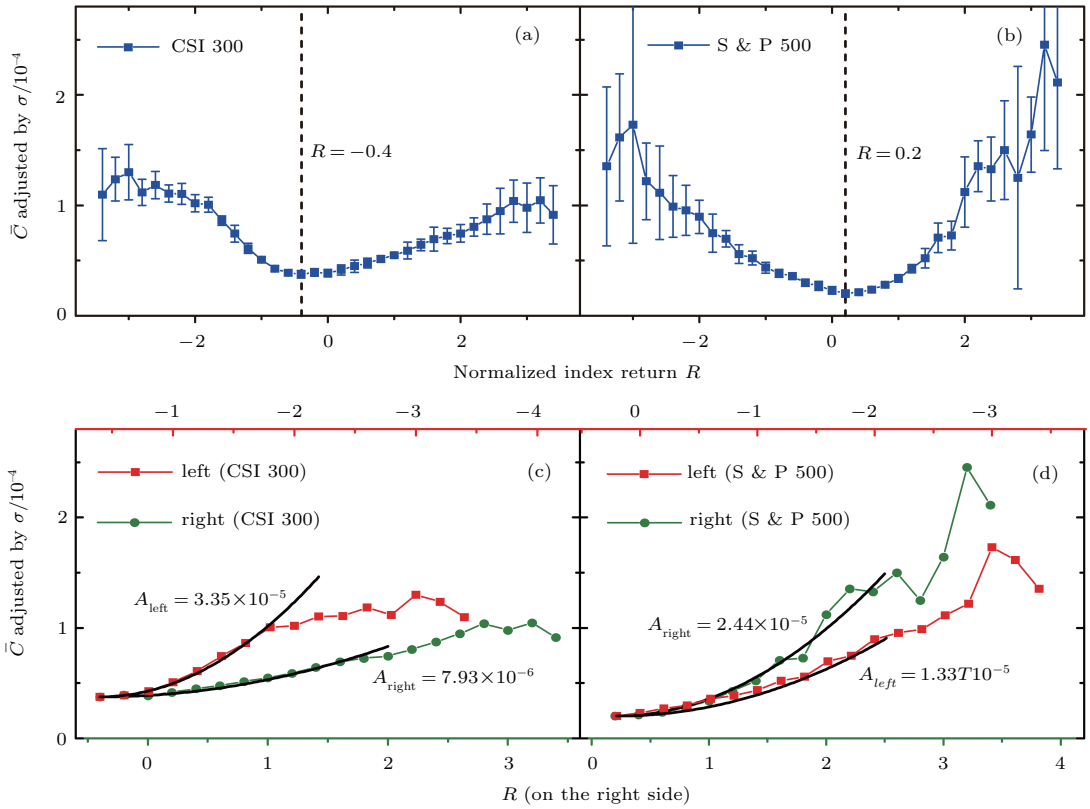


Fig. 3. (color online) Panels (a) and (b) show the mean meta-correlation result separately with different Δt ($\Delta t = 2, 4, \ldots, 20$) for CSI 300 and for S & P 500. The standard deviation is calculated as an error bar. Panels (c) and (d) depict the merge graph of the two sides of the lowest point in corresponding meta-correlation graph. The top red axis and the red data points are shown for the left side, while the bottom axis and green data points are for the right side. The black lines are the best fitting parabolas.

We find that the empirical meta-correlations are not strictly symmetric when the market index goes upward or downward. It results that the minimal correlation between sector indices does not take place at the market neutral state. Actually, this asymmetry is found in both two countries' main markets, and the lowest point is at $R = -0.4$ for CSI 300 and $R = 0.2$ for S & P 500. That means the diversification of industries is most obvious when the normalized market index return is deviated negatively from zero in China. Whereas, in America, the state of most sufficient diversification of industries is found at a little positive market index return. Furthermore, we also see that the growing trends of the meta-correlation are totally different on both sides at the lowest point.

For clarity, we flip the left side of the lowest point to the right side as shown in Figs. 3(c) and 3(d). As mentioned above in the theoretical analysis, the underlying relationship would be an up-opening parabola. So we try to fit the empirical data by using $y = A \cdot (x - x_0)^2 + y_0$, where (x_0, y_0) are the coordinates of the lowest point. The data for fitting do not include the data of higher index returns. Because these data are too noisy with huge deviation. As a result, the significant coefficient A is 3.35×10^{-5} on the left side and 7.93×10^{-6} on the right side for CSI 300. While for S & P 500, A is 1.33×10^{-5} on the left side and 2.44×10^{-5} on the right side. The fitting parameter A, goodness of fit (R^2), and fitting error are shown in Table. 3. The changing trend on the left side is faster than that on the right side for CSI 300. That means the correlation of industries grows more easily when the normalized index return is negative. In other words, it is more possible that the sectors in the Chinese market behave the same when the index goes down than when the index increases. This correlation asymmetry is caused by the biased individuals' psychological

behaviors on both rising or falling situations. However, the biased behaviors are just opposite in the American stock market.

Table 3. The up-opening parabolic fitting results for the empirical curves in Fig. 3 for CSI 300 and S & P 500 respectively.

Time series	Fitting parameter A	R^2	Fitting error
CSI 300 (left)	3.35×10^{-5}	0.99	6.80×10^{-7}
CSI 300 (right)	7.93×10^{-6}	0.97	2.46×10^{-7}
S & P 500 (left)	1.33×10^{-5}	0.96	4.52×10^{-7}
S & P 500 (right)	2.44×10^{-5}	0.97	8.04×10^{-7}

3.2.2. Symmetric meta-correlation at the minutely scale

Now, high-frequency price dynamics sparks a lot of interest due to the better development of technologies.[20,28] Do there also exist the biased characteristics of the meta-correlations at high-frequency time scale or not? As is well known, Chinese stock markets were in the rising trend before June 2015. After that time, the markets blew out in a rapid downward trend. To make comparisons between the meta-correlations in both bull market and bear market, two pieces of minutely trading data are made use of. One of the two pieces is from 1/1/2015 to 5/31/2015 (22506 minutes) and the other from 6/1/2015 to 10/31/2015 (24684 minutes). Figures 4(a) and 4(b) show the relationship between the normalized index return and the time-dependent average correlation for the two pieces of data respectively. No matter whether the minutely data is from a rising trend or a falling trend, the meta-correlations behave symmetrically with the lowest point located at the position where $R=0$. Similarly, if the left side of the graph is flipped over to the right side, as shown in Figs. 4(c) and 4(d), most data of the two sides can coincide together very well. This indicates that the characteristic of meta-correlation at high-frequency time scales is symmetric, and independent of the high-frequency data's inherent trends as well.

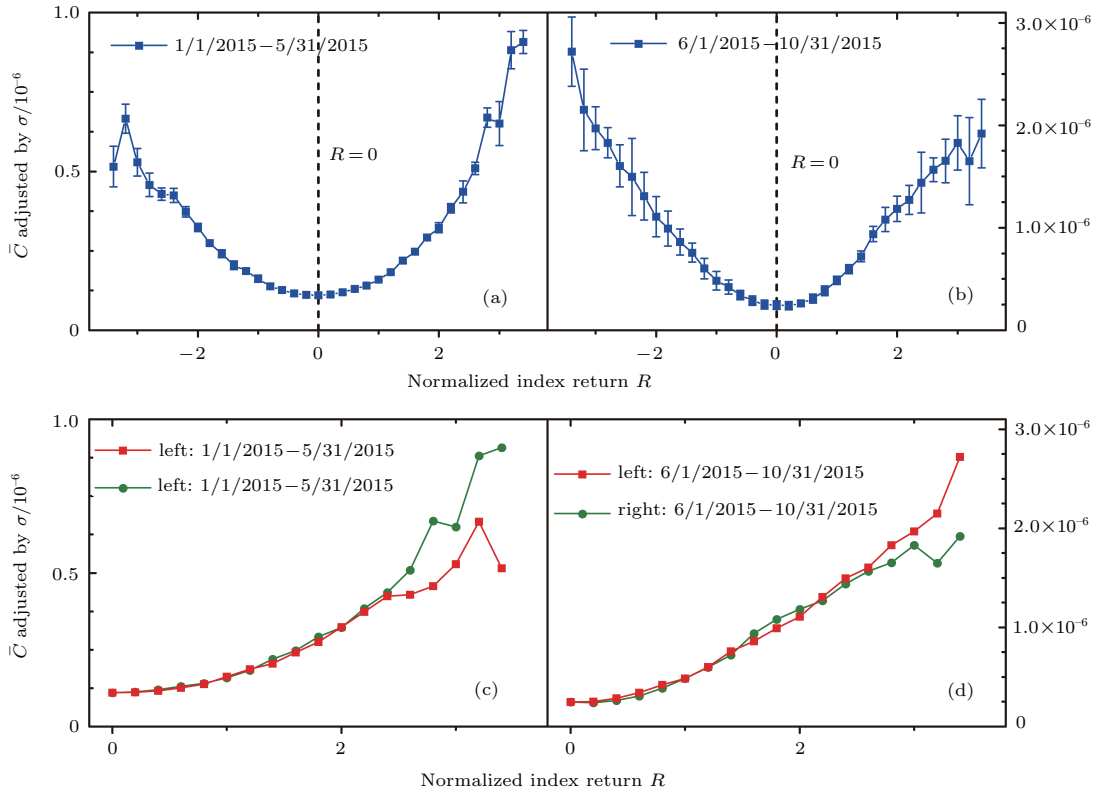


Fig. 4. (color online) Same as Fig. 3, but the data for calculation are minutely from 1/1/2015 to 5/31/2015 in panels (a), (c) and from 6/1/2015 to 10/31/2015 in panels (b), (d).

3.2.3. Causality analyses for results at different time scales

However, it is still questionable for the presence of the biased characteristics of meta-correlations. In the work of Jiang *et al.*, they pointed out that macroscopic exogenous events led to the asymmetry at long-time scales, through the analyses of remnant and anti-remnant volatilities.[29] Here, we try to uncover the reason by the analyses of cross-correlation between the normalized index return and the time-dependent average sectors correlation. The cross-correlation, deemed to be an index leverage effect,[19,21] is defined as,

$$C_{R\bar{C}}(m,\Delta t)=\frac{\left\langle (R_{\Delta t}(t)-\langle R_{\Delta t}\rangle)(\bar{C}_{\Delta t}(t+m)-\langle \bar{C}_{\Delta t}\rangle)\right\rangle}{\sigma_{R_{\Delta t}}\sigma_{\bar{C}_{\Delta t}}}, \quad (13)$$

where $\langle\cdots\rangle$ is defined as the expectation value and m stands for time lag. Since

$$C_{R\bar{C}}(m,\Delta t)=\left\langle \hat{R}_{\Delta t}(t)\hat{\bar{C}}_{\Delta t}(t+m)\right\rangle=\left\langle \hat{R}_{\Delta t}(t+m)\hat{\bar{C}}_{\Delta t}(t)\right\rangle,$$

thus, we can fix the $R_{\Delta t}$ series and shift the $\bar{C}_{\Delta t}$ by time lag m. The cross-correlation results are shown in Fig. 4, where the

range of the time lag m is $[-100, 100]$.

Figures 5(a) and 5(b) depict various cross-correlations between $R_{\Delta t}$ and $\bar{C}_{\Delta t}$ for different Δt, in CSI 300 and S & P 500 respectively. For the results based on daily data, the cross-correlations are obviously different for $m > 0$ and $m < 0$, showing a kind of asymmetry in time. Especially, the cross-correlations are significantly negative when $m < 0$ for CSI 300, whereas they are significantly positive when $m > 0$ for S & P 500. The distinct negative index correlation leverage in CSI 300 indicates that the past sectors correlations are negatively related with the future index return. The situations are opposite in S & P 500. Thus, the biased characteristics of meta-correlations of the two markets have been well explained. In order to be convincing, we also inspect the index correlation leverage at the minutely scale, as shown in Figs.5(c) and 5(d). The two pieces of minutely data in an up trend and a down trend respectively, are used to calculate cross-correlations in the same way. Like the results in Figs. 5(a) and 5(b), the cross-correlations are all strongly negative when m approximately equals to 0. However, this kind of property has already been found to be significant for different Δt.[19] Actually, the strong negative correlation between the normalized index return $R(t)$ and the average correlation $C(t)$ when time lag is negatively close to 0, indicates the remarkable index leverage effect. That means as long as the time lag $|m|(m < 0)$ becomes small enough, the average correlation between industries $C(t)$ is significantly anti-correlated with future market return $R(t+|m|)$. In other words, the occurrence of this property is decided by the directional information flow, which can lead to the universal time-reversal asymmetry in financial markets. In addition, we find that the cross-correlations present no clear differences for $m > 0$ and $m < 0$. We therefore consider that the symmetric meta-correlations at the high-frequency level can also be well interpreted by the causality analyses.

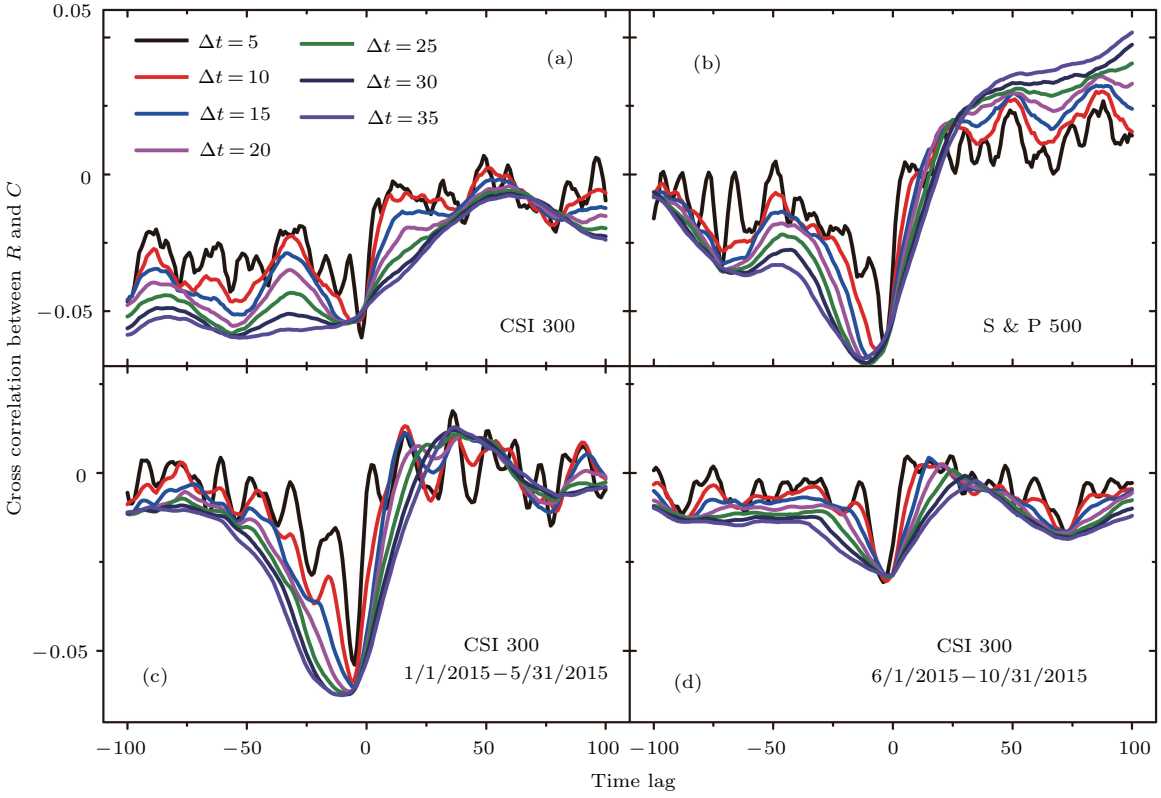


Fig. 5. (color online) Panels (a) and (b) show the cross-correlations between the normalized index return R and the time-dependent average correlation $\bar{C}$ for the daily data in CSI 300 and S & P 500 respectively. Panels (c) and (d) depict the cross-correlations for the two pieces of CSI 300 minutely data in an up trend and in a down trend respectively. Different colors in the four figures stand for different Δt.

4. Conclusion

In summary, human behaviors are ever-changing and unpredictable in financial markets. When they are encountered with drastic fluctuations (bubbles or crises), their behaviors usually become more convergent, such like herding behaviors. This work gives a method to quantify the complex correlations.

For the aim of inspecting and better quantifying the collective behaviors under different levels of fluctuations, we have refined the theoretical formula proposed by Reigneron *et al.*[21] The new theoretical work implies a universal meta-correlation which should be revised by the term of instantaneous average volatilities. The meta-correlation here has been described as the further correlation between the normalized market index return $R_{\Delta t}(t)$ and the time-dependent average sectors correlation $\bar{C}_{\Delta t}(t)$ in the interval of Δt trading days. The improved formula suggests a quadratic expression between $R_{\Delta t}(t)$ and the product of $\bar{C}_{\Delta t}(t)$ and $\sigma_{\Delta t}(t)$. Differ-

ent from earlier researches, our approach points out that the average correlation between sectors should be revised by the standard deviation.

According to the relationship, symmetric meta-correlations are expected no matter how the index fluctuates. However, empirical analyses of daily data from the Chinese and American markets (i.e., CSI 300 and S & P 500) show asymmetric meta-correlations. The behaviors of market sectors in CSI 300, prefer to be more convergent in negative circumstances rather than in positive circumstances, whereas the situations are opposite for those in S&P 500. The asymmetric characteristics of meta-correlations of the two markets can be explained by the causality analyses of cross-correlations. In addition, we also observe the meta-correlations by tracking the high-frequency minutely data. As a result, for both upward and downward trends, the meta-correlations become unbiased and balanced. It means that the meta-correlations are symmetric at high-frequency time scales but asymmetric at low-frequency time scales. In the same way, the analyses of cross-correlation can help us better understand the presence of the symmetric property at high-frequency time scales.

Overall, the significance of this work is that it shows what the complex meta-correlations behave like in detail and gives reasonable explanations for the emergence of the asymmetric characteristics at different time scales. Making sense of the meta-correlations can help build more robust portfolios under various fluctuations.

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