Theory of specific heat of vortex liquid of high T_c superconductors*

Chen Bai(白晨), Cheng Chi(迟诚), and Jiangfan Wang(王江帆)†

Institute of Theoretical Physics, School of Physics, Peking University, Beijing 100871, *China*

(Received 18 May 2016; revised manuscript received 7 June 2016; published online 25 August 2016)

Superconducting thermal fluctuation (STF) plays an important role in both thermodynamic and transport properties in the vortex liquid phase of high T_c superconductors. It was widely observed in the vicinity of the critical transition temperature. In the framework of Ginzburg–Landau–Lawrence–Doniach theory in magnetic field, a self-consistent analysis of STF including all Landau levels is given. Besides that, we calculate the contribution of STF to specific heat in vortex liquid phase for high T_c cuprate superconductors, and the fitting results are in good agreement with experimental data.

Keywords: superconducting thermal fluctuation, vortex liquid, self-consistent approximation, specific heat

PACS: 74.40.–n, 74.25.Uv, 31.15.xr, 65.40.Ba **DOI:** 10.1088/1674-1056/25/10/107404

1. Introduction

The superconducting thermal fluctuation effects on the vortex matter have attracted great attention since the discovery of the high T_c superconductor.[1,2] The strength of thermal fluctuations on the mesoscopic scale can be characterized by the Ginzburg number[2–5]

$$Gi = \frac{1}{2}\left(8\,e^2\kappa^2\xi T_c^{mf}\gamma/c^2\hbar^2\right)^2.$$

In conventional low T_c type-II superconductors, it is rather difficult to observe STF effects. The mixed state between the upper and lower critical field is described by Abrikosov lattice in the framework of the mean field (MF) theory given by Abrikosov in 1957.[6,7] In high T_c superconductors (HTSC), the STF effects play a crucial role in inducing precursor phenomena of superconductivity. For example, the paraconductivity phenomenon, i.e., the decrease of the resistance of superconductor in the normal phase, was first theoretically calculated by Aslamazov and Larkin[8,9] and then experimentally observed by Glover.[10] The appearance of STF leads to the preformed pairs in the range of the critical transition temperature and the MF critical transition temperature where there are Cooper pairs without phase coherence. There are strong STF in HTSCs due to (i) higher critical transition temperature, (ii) larger value of Kappa, (iii) high anisotropy, and (iv) quasi-two-dimensionality. Strong STF lead to a rather different phase diagram compared to the MF phase diagram. In this sense, it is important to take into account STF contribution to study the thermodynamic and transport behavior of vortex matter.

Strong STF have significant effects on H–T phase diagram of type-II superconductors, especially changing the Abrikosov lattice solid state near the MF phase transition line $H_{c2}(T)$ to the vortex liquid state.[11] Without STF, vortices form a stable Abrikosov lattice phase between the lower and upper critical phase transition lines, $H_{c1}(T)$ and $H_{c2}(T)$. The vortex lattice will be melted into vortex liquid due to strong STF near the upper critical phase transition line $H_{c2}(T)$. The vortex melting transition in HTSC is the first-order phase transition, which was confirmed by the jumps of magnetization measurements[12–16] and the spikes and jumps in specific heat experiments.[14,17–21] The crossover from the normal phase to the vortex liquid phase is a real phase transition and occurs near the location of the MF transition line $H_{c2}(T)$. Due to strong STF of HTSC, a substantial portion of the phase diagram below $H_{c2}(T)$ could be occupied by the vortex liquid phase. The materials with large Gi typically have relatively large critical regions.

We calculate specific heat of the vortex liquid phase in the framework of Ginzburg–Landau (GL) theory. In order to study the vortex liquid phase, we need to go beyond the simple Gaussian theory. We employ self-consistent fluctuation theory (sometimes referred to as the Hartree–Fock approximation)[22] to study the Ginzburg–Landau–Lawrence–Doniach model where all Landau levels are considered instead of the lowest Landau level (LLL) approximation.[23]

For low T_c weak STF superconductors, specific heat will have a sharp jump crossing the upper critical transition line $H_{c2}(T)$, when the temperature decreases from the high temperature to the low temperature with fixed magnetic field.[20] But for HTSC, the specific heat jump is smeared out and the specific heat peak is blurred.[17,24–27] Motivated by experimental data of HTSC, we want to provide a theoretical understanding of the STF effect on the specific heat in the vicinity of critical transition temperature. We will analyze the specific heat data of high T_c cuprates including $YBa_2Cu_3O_{7-\delta}$, Bi-based materials, and Tl-based materials.

The paper is organized as follows. In Section 2, the Ginzburg–Landau–Lawrence–Doniach model is introduced.

*Project supported by the National Natural Science Foundation of China (Grant No. 11274018).

†Corresponding author. E-mail: haihunshan@yeah.net

© 2016 Chinese Physical Society and IOP Publishing Ltd

We present the self-consistent approximation in Section 3. Theoretical calculation of specific heat will be carried out in Section 4, and we give a conclusion in Section 5.

2. Ginzburg–Landau–Lawrence–Doniach model

We review GL theory in a magnetic field for layered superconductors. The GL free energy functional in Lawrence–Doniach model near T_c is

$$F_{\rm GL} = s'\sum_n \int {\rm d}\boldsymbol{r}' \left(\frac{\hbar^2}{2m^*} |\boldsymbol{D}\Psi_n(\boldsymbol{r})|^2 + \frac{\hbar^2}{2m_c^* d'^2} |\Psi_n(\boldsymbol{r}) - \Psi_{n+1}(\boldsymbol{r})|^2 + \alpha\left(T - T_c^{\rm mf}\right)|\Psi_n(\boldsymbol{r})|^2 + \frac{\beta}{2}|\Psi_n(\boldsymbol{r})|^4 \right), \quad (1)$$

where $\Psi_n(\boldsymbol{r})$ is a complex order parameter field denoting wave function of the Cooper pairs in the n-th layer. The covariant derivative $\boldsymbol{D}$ is

$$\boldsymbol{\nabla} + {\rm i}\frac{2\pi}{\Phi_0}\boldsymbol{A},$$

$\boldsymbol{A} = (By, 0)$ is the vector potential in Landau gauge, describing a magnetic field perpendicular to the superconducting plane. $\Phi_0 = hc/e^*$ is the magnetic flux quantum, and $e^* = 2e$ is the charge of a Cooper pair. s' is the layer thickness and d' is the interplanar separation. $T_c^{\rm mf}$ is the mean-field critical transition temperature.

This paper will use the MF units as follows: the unit of length is the coherence length $\xi = \hbar/\sqrt{2m^*\alpha T_c^{\rm mf}}$, the unit of magnetic field is the upper critical field $H_{c2} = \Phi_0/2\pi\xi^2$ with $\Phi_0 = hc/2e$, and the unit of order parameter is $\Psi_0 = \sqrt{2\alpha T_c^{\rm mf}/\beta}$. The dimensionless quantities can be defined with the units as follows: $t_{\rm mf} = T/T_c^{\rm mf}$, $b = B/H_{c2}$, $\psi^2 = \Psi^2/|\Psi_0|^2$.

The Boltzmann factor in these units is

$$\frac{F_{\rm GL}}{T} = \frac{s}{2\omega_{\rm mf} t_{\rm mf}} \sum_n \int {\rm d}\boldsymbol{r} \left[|\boldsymbol{D}\psi_n|^2 + d^{-2}|\psi_n - \psi_{n+1}|^2 + (t_{\rm mf} - 1)|\psi_n|^2 + |\psi_n|^4 \right], \quad (2)$$

where the dimensionless coefficient $\omega_{\rm mf} = \sqrt{2Gi_{\rm mf}}\pi$ and the dimensionless Ginzburg number

$$Gi_{\rm mf} = \frac{1}{2}\left(8e^2\kappa^2\xi T_c^{\rm mf}\gamma/c^2\hbar^2\right)^2$$

determines the strength of STF. The anisotropy ratio γ can be defined as $\sqrt{m_c^*/m^*}$. The dimensionless length $s = s'/\xi_c = s'\gamma/\xi$, $d = d'/\xi_c = d'\gamma/\xi$.

3. the self-consistent approximation

Now we will introduce the self-consistent approximation theory to study vortex liquid. We employ a freedom to choose "the best" quadratic part to divide the Hamiltonian into a non-perturbation and a perturbation based on the idea of "principle of minimal sensitivity".[28,29]

Within the self-consistent approximation,[22] we add and subtract a quadratic term $(2\varepsilon - b)|\psi_n|^2$ with the employment of a variational parameter ε in the Boltzmann factor. The Boltzmann factor is divided into the large quadratic part $K(\varepsilon)$ and a small perturbation $W(\varepsilon)$. The physical meaning of ε can be explained as the excitation energy gap of the vortex liquid in zero field.

$$\frac{F_{\rm GL}}{T} = K(\varepsilon, \psi) + W(\varepsilon, \psi), \quad (3)$$

where

$$K(\varepsilon, \psi) = \frac{s}{2\omega_{\rm mf} t_{\rm mf}} \sum_n \int {\rm d}\boldsymbol{r} \left[|\boldsymbol{D}\psi_n|^2 + d^{-2}|\psi_n - \psi_{n+1}|^2 + (2\varepsilon - b)|\psi_n|^2 \right], \quad (4)$$

$$W(\varepsilon, \psi) = \frac{s}{2\omega_{\rm mf} t_{\rm mf}} \sum_n \int {\rm d}\boldsymbol{r} \left[(b + t_{\rm mf} - 2\varepsilon - 1)|\psi_n|^2 + |\psi_n|^4 \right]. \quad (5)$$

Considering STF contribution on the mesoscopic scale, we calculate the free energy F from $F = -T\ln Z$, where the partition function

$$Z = \int \mathscr{D}\psi(x)\mathscr{D}\psi^*(x) \exp\left(-\frac{F_{\rm GL}}{T}\right).$$

$\mathscr{D}\psi(x)$ denotes a functional integral. The approximation of the free energy by expanding it to the first order in $W(\varepsilon, \psi)$ is derived below,

$$\begin{aligned} F &= -T\ln Z = -T\ln \int \mathscr{D}\psi\mathscr{D}\psi^* {\rm e}^{-K} {\rm e}^{-W} \\ &= -T\ln \int \mathscr{D}\psi\mathscr{D}\psi^* {\rm e}^{-K}(1 - W) \\ &= -T\ln Z_0 + T\langle W\rangle_0, \end{aligned} \quad (6)$$

where the zero-order partition function Z_0 and the first-order perturbation term $\langle W\rangle_0$ are

$$Z_0 = \int \mathscr{D}\psi\mathscr{D}\psi^* {\rm e}^{-K}, \quad (7)$$

$$\langle W\rangle_0 = \frac{\int \mathscr{D}\psi\mathscr{D}\psi^* {\rm e}^{-K} W}{Z_0}. \quad (8)$$

Here, $\langle W\rangle_0$ can be expressed as

$$\langle W\rangle_0 = \frac{Vs/d}{\omega t}\left[\left(\frac{b + t_{\rm mf} - 2\varepsilon - 1}{2}\right)\left\langle |\psi_n|^2 \right\rangle_0 + \left(\left\langle |\psi_n|^2 \right\rangle_0\right)^2 \right], \quad (9)$$

by using $\left\langle |\psi_n|^4 \right\rangle_0 = 2\left\langle |\psi_n|^2 \right\rangle_0^2$, where the dimensionless $V = Nd \cdot S$, and S is the dimensionless area of the layers.

The average of the superfluid density can be written as

$$\left\langle |\psi_n|^2 \right\rangle_0 = Z_0^{-1} \int \mathscr{D}\psi\mathscr{D}\psi^* |\psi_n|^2 {\rm e}^{-K}$$

$$= -\left(\frac{Vs/d}{\omega_{\rm mf}t_{mf}}\right)^{-1}\frac{\partial \ln Z_0}{\partial \varepsilon}. \tag{10}$$

The total dimensionless variational free energy is

$$f = \left[f_0 + \left(\frac{b+t_{\rm mf}-1}{2} - \varepsilon\right)\frac{\partial f_0}{\partial \varepsilon} + \left(\frac{\partial f_0}{\partial \varepsilon}\right)^2\right], \tag{11}$$

and

$$f_0 = -\left(\frac{Vs/d}{\omega t}\right)^{-1}\ln Z_0, \tag{12}$$

where

$$\left\langle |\psi_n|^2 \right\rangle_0 = \frac{\partial f_0}{\partial \varepsilon}.$$

The particular form of $\left\langle |\psi_n|^2 \right\rangle_0$ will be given in the next section.

Minimizing the free energy $F(\varepsilon)$ with respect to ε, i.e.,

$$\frac{\partial F(\varepsilon)}{\partial \varepsilon} = 0. \tag{13}$$

The gap equation is obtained as follows:

$$\begin{aligned}\varepsilon &= \frac{b+t_{\rm mf}-1}{2} + 2\frac{\partial f_0}{\partial \varepsilon} \\ &= \frac{b+t_{\rm mf}-1}{2} + 2\left\langle |\psi_n|^2 \right\rangle_0.\end{aligned} \tag{14}$$

4. Theoretical results of specific heat

4.1. Calculation of specific heat

In order to calculate Z_0, we expand the order parameter field via the Landau level,

$$\psi_n(\boldsymbol{r}) = \frac{d^{1/2}}{(2\pi)^{3/2}}\int_0^{2\pi/d} {\rm d}k_z \sum_{l=0}^{N_{\rm f}} \int {\rm d}\boldsymbol{q}\varphi_{l,\boldsymbol{q}}(\boldsymbol{r})\,\psi_{l,\boldsymbol{q},k_z}\,{\rm e}^{-{\rm i}ndk_z}, \tag{15}$$

where k_z is the wave vector in c axis, $N_{\rm f}$ is the upper limit of Landau level, and $\varphi_{l,\boldsymbol{q}}$ is the Landau's quasimomentum basis with the Landau level index l and quasimomentum q, integrated over the Abrikosov lattice Brillouin zone with the area $2\pi b$.[22,30] This basis is the analogue of solid-state Bloch waves in the case of nonzero magnetic field. In this basis,

$$Z_0 = \int \mathscr{D}\psi\mathscr{D}\psi^* \exp\left(-\frac{s}{\omega_{\rm mf}t_{\rm mf}d}\sum_{l=0}^{N_{\rm f}}\int {\rm d}\boldsymbol{q}\,{\rm d}k_z\left[nb + \frac{1-\cos(k_z d)}{d^2} + \varepsilon\right]\psi_{l,\boldsymbol{q},k_z}\psi^*_{l,\boldsymbol{q},k_z}\right). \tag{16}$$

$\ln Z_0$ results in

$$\ln Z_0 = -\frac{Vb}{(2\pi)^2}\int_0^{2\pi/d}{\rm d}k_z\left[\frac{\Lambda}{b}\ln b + \ln\Gamma\left(\frac{j(k_z)+\varepsilon+\Lambda}{b}\right) - \ln\Gamma\left(\frac{j(k_z)+\varepsilon}{b}\right)\right], \tag{17}$$

where

$$j(k_z) = \frac{1}{d^2}(1-\cos(k_z d)).$$

The relationship between maximum Landau level and energy cutoff satisfies[22]

$$(N_{\rm f}+1)\,b = \Lambda. \tag{18}$$

Then we can write the gap equation in terms of Λ

$$\varepsilon = \frac{b+t_{\rm mf}-1}{2} + 2\omega_{\rm mf}t_{mf}U_\varepsilon(\Lambda), \tag{19}$$

where

$$\begin{aligned}U_\varepsilon(\Lambda) &= U(b,\varepsilon,\Lambda) \\ &= \frac{1}{(2\pi)^2}\int_0^{2\pi/d}{\rm d}k_z\left[\psi\left(\frac{j(k_z)+\varepsilon+\Lambda}{b}\right) - \psi\left(\frac{j(k_z)+\varepsilon}{b}\right)\right].\end{aligned} \tag{20}$$

Hence

$$\left\langle |\psi_n|^2 \right\rangle_0 = \frac{\partial f_0}{\partial \varepsilon} = \omega_{\rm mf}t_{\rm mf}U_\varepsilon(\Lambda). \tag{21}$$

The critical transition temperature corresponding to the zero energy gap $\varepsilon = 0$ at zero magnetic field can be given as

$$\frac{T_{\rm c}/T_{\rm mf}-1}{2} + 2\omega U_0(\Lambda) = 0, \tag{22}$$

where $\omega = \omega_{\rm mf}(T_{\rm c}/T_{\rm c}^{\rm mf})$. The solution of Eq. (22) is $T_{\rm c} = T_{\rm c}^{\rm mf}r_\Lambda$, and

$$\begin{aligned}r_\Lambda &= T_{\rm c}/T_{\rm c}^{\rm mf} = 1 - 4\omega U(0,0,\Lambda) \\ &= 1 - 4\omega\frac{1}{2\pi d}\left[2\ln\left(\sqrt{\Lambda d^2} + \sqrt{\Lambda d^2+2}\right) - \ln 2\right].\end{aligned} \tag{23}$$

The gap equation can be written with the renormalized temperature $t = T/T_{\rm c}$ as

$$\varepsilon = \frac{b+t-1}{2} + 2\omega t\left(U_\varepsilon(\Lambda) - U_0(\Lambda)\right). \tag{24}$$

When the reduced magnetic field b and the energy gap ε are far less than energy cutoff Λ, most vortex liquid region satisfies this condition. The gap equation can be approximated by taking Λ to infinite limit

$$\varepsilon = \frac{b+t-1}{2} - \frac{\omega t d}{2\pi^2 s}\int_0^{2\pi/d}{\rm d}k_z\left[\psi\left(\frac{j(k_z)+\varepsilon}{b}\right) + \frac{2\pi}{d}\ln\left(2bd^2\right)\right]. \tag{25}$$

The free energy can be expressed as a function of f,

$$F(b,t,\varepsilon) = T\frac{Vf}{\omega t}. \tag{26}$$

Near $T_{\rm c}$, the correlation length is large due to the small liquid energy gap, therefore we take $s = d$ as the maximum value.[22] The specific heat per volume C_V can be obtained by the second derivative of F with respect to T, i.e.,

$$C_V = -\frac{T}{V'}\frac{\partial^2 F}{\partial T^2} = -\frac{T_{\rm c}H_{\rm c2}^2}{2\pi\kappa^2}t\frac{\partial^2 f}{\partial t^2} = \frac{T_{\rm c}H_{\rm c2}^2}{2\pi\kappa^2}C_v, \tag{27}$$

where $V' = \xi^3 V/\gamma$, and the dimensionless specific heat per volume is

$$C_V = -t\frac{\partial^2 f}{\partial t^2} = t \cdot r_\Lambda^2 \left[-\frac{1}{8} + \frac{2\left[\omega U_\varepsilon / r_\Lambda + 1/4\right]^2}{1 - 2\omega t(\partial U_\varepsilon/\partial\varepsilon)}\right]. \quad (28)$$

C_V as a function of t, b, Λ is

$$C_V(t,b,\Lambda) = \frac{TH_{c2}^2}{2\pi\kappa^2} r_\Lambda^2 \left[-\frac{1}{8} + \frac{2\left[\omega U_\varepsilon / r_\Lambda + 1/4\right]^2}{1 - 2\omega t(\partial U_\varepsilon/\partial\varepsilon)}\right]. \quad (29)$$

The renormalized upper critical field $H_{c2}^{(r)}$ is

$$H_{c2}^{(r)} = H_{c2} r_\Lambda = \Phi_0 2m^* \alpha T_c / 2\pi\hbar^2. \quad (30)$$

The final result of $C_V(t,b,\Lambda)$ is

$$C_V(t,b,\Lambda) = \frac{TH_{c2}^{(r)2}}{2\pi\kappa^2} \left[-\frac{1}{8} + \frac{2\left[\omega U_\varepsilon / r_\Lambda + 1/4\right]^2}{1 - 2\omega t(\partial U_\varepsilon/\partial\varepsilon)}\right]. \quad (31)$$

4.2. Fitting with experimental data

In order to fit experimental data with theoretical calculation of Eq. (31), the vortex liquid contribution to specific heat, we should substract the specific heat in vortex liquid from the background. The experimental data of specific heat is $C(T,B) = C_{\rm STF}(T,B) + C_{\rm N}(T,B)$, where $C_{\rm STF}(T,B)$ is the STF contribution which can be expressed by Eq. (31) and $C_{\rm N}(T,B)$ is the normal contribution. The normal part of specific heat is difficult to calculate because there is no reliable microscopic calculation. If the normal part of specific heat is weakly dependent on the small magnetic field, we can use $C_{\rm N}(T,B_1) = C_{\rm N}(T,B_2)$ in this case. The normal contribution can be canceled through the difference form $C(T,B_1) - C(T,B_2)$ of experimental data.[31] The specific heat curves (see the solid curves) including all Landau levels are shown in Fig. 1 as well as the experiment data (see the scattered curves).

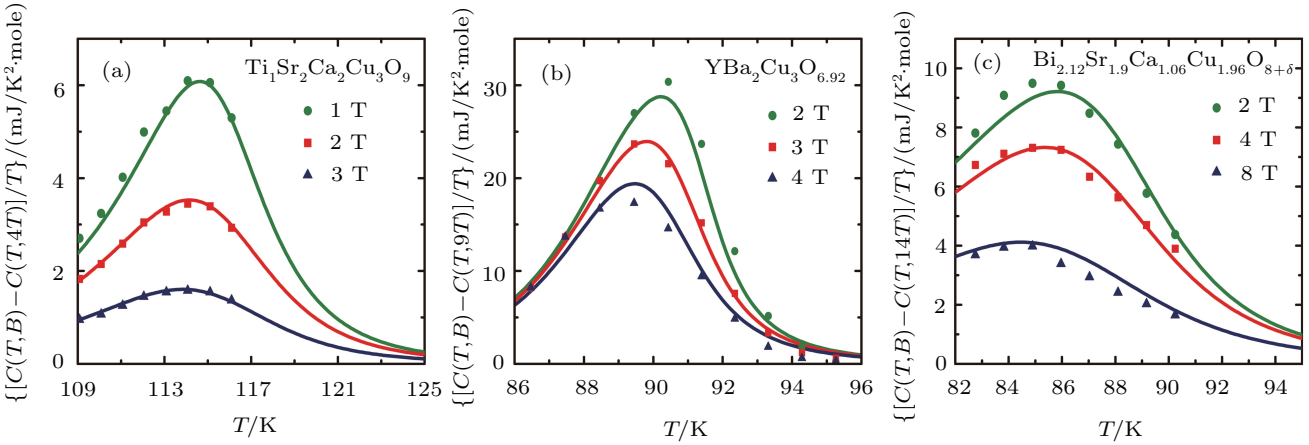


Fig. 1. (color online) Experimental data (scattered lines) and corresponding fitting results (solid lines). Three major types of high T_c cuprate superconductors are represented: (a) Tl-1223, (b) optimally doped Y-123, (c) Bi-2212. Each set of curves uses just three fitting parameters, H_{c2}, γ, and κ. The experimental data can be found in Refs. [18], [32], and [33].

The applied magnetic field can be neither too weak nor too strong. We compare the experimental data from 1 T to 8 T with the theoretical results. If the magnetic field is too small, the state will close to critical fluctuation region where non-perturbation theory is needed, for example the renormalization group theory. The energy cutoff Λ obtained in this paper is consistent with the fitting results.[22]

Three major families of HTSC materials, including $Tl_1Sr_2Ca_2Cu_3O_9$ (Tl-1223),[32] $YBa_2Cu_3O_{6.92}$ (Y-123),[18] and $Bi_{2.12}Sr_{1.9}Ca_{1.06}Cu_{1.96}O_{8+\delta}$ (Bi-2212)[33] are employed, respectively. The fitting parameters are shown in Table 1. The fitting parameters of different materials are in reasonable range compared with the corresponding measurements of experiments.[34] Gi is a theoretical value characterizing the strength of STF and is significantly larger in HTSC compared to conventional superconductors ($\approx 10^{-10}$). For relatively weak anisotropic high T_c cuprates such as YBCO, the magnitude of Gi is about 10^{-3} in contrast to 10^{-2} for strong anisotropic BSCCO.[30] In order to make our calculation fit better with experimental data, we find the reasonable fitting parameters and calculate Gi as the function of H_{c2}, γ, and κ. Tl-1223 and Bi-2212 with larger γ compared with YBCO will be better described by the two-dimensional layered model. The specific heat jump is smeared out and the specific heat peak is rounded.

Table 1. Fitting parameters for Tl-1223, Y-123, and Bi-2212.

Material	T_c/K	d/Å	H_{c2}/T	$T_c^{\rm mf}$/K	Λ	ω	γ	κ	Gi
Tl-1223	116	15.3	180	124.4	0.3	0.35	13.5	45	0.0062
Y-123	93	11.7	257.2	97	0.22	0.14	6.3	51.3	0.001
Bi-2212	85	30.6	200	93.7	0.25	1.06	22	74	0.057

5. Conclusion

We have calculated specific heat of vortex liquid by using the self-consistent approximation theory based on Ginzburg–Landau–Lawrence–Doniach model. The theoretical calculation agrees well with experimental data in the vortex liquid region. The Ginzburg–Landau–Lawrence–Doniach model can be used to well describe specific heat, magnetization,[22] conductivity,[35] etc.

Acknowledgment

We thank Professor Dingping Li for useful discussions.

References

[1] Tinkham M 1996 *Introduction to Superconductivity* (New York: McGraw-Hill)
[2] Larkin A and Varlamov A 2005 *Theory of Fluctuations in Superconductors* (New York: Oxford University Press)
[3] Levanyuk A P 1959 *Zh. Eksp. Teor. Fiz.* **36** 810
[4] Ginzburg V L 1960 *Fiz. Tverd Tela* **2** 203
[5] Ginzburg V L 1961 *Sov. Phys. Solid State* **2** 1824
[6] Abrikosov A A 1957 *Zh Eksp Teor Fiz* **32** 1442
[7] Abrikosov A A 1957 *Sov. Phys. JETP* **5** 1174
[8] Aslamazov L G and Larkin A I 1968 *Sov. Solid State Phys.* **10** 875
[9] Aslamazov L G and Larkin A I 1968 *Phys. Lett. A* **26** 238
[10] Glover R E 1967 *Phys. Lett. A* **25** 542
[11] Fisher D S, Fisher M A and Huse D 1991 *Phys. Rev. B* **43** 130
[12] Zeldov E, Majer D, Konczykowski M, Geshkenbein V B, Vinokur V M and Shtrikman H 1995 *Nature* **375** 373
[13] Willemin M, Schilling A, Keller H, Rossel C, Hofer J, Welp U, Kwok W K, Olsson R J and Crabtree G W 1998 *Phys. Rev. Lett.* **81** 4236
[14] Nishizaki T, Shibata K, Sasaki T and Kobayashi N 2000 *Physica C* **341–348** 957
[15] Beidenkopf H, Avraham N, Myasoedov Y, Shtrikman H, Zeldov E, Rosenstein B, Brandt E H and Tamegai T 2005 *Phys. Rev. Lett.* **95** 257004
[16] Beidenkopf H, Verdene T, Myasoedov Y, Shtrikman H, Zeldov E, Rosenstein B, Li D and Tamegai T 2007 *Phys. Rev. Lett.* **98** 167004
[17] Schilling A, Fisher R A, Phillips N E, Welp U, Dasgupta D, Kwok W K and Crabtree G W 1996 *Nature* **382** 791
[18] Schilling A, Fisher R A, Phillips N E, Welp U, Kwok W K and Crabtree G W 1997 *Phys. Rev. Lett.* **78** 4833
[19] Bouquet F, Marcenat C, Steep E, Calemczuk R, Kwok W K, Welp U, Crabtree G W, Fisher R A, Phillips, N E and Schilling A 2001 *Nature* **411** 448
[20] Lortz R, Lin F, Musolino N, Wang Y, Junod A, Rosenstein B and Toyota N 2006 *Phys. Rev. B* **74** 104502
[21] Lortz R, Musolino N, Wang Y, Junod A and Toyota N 2007 *Phys. Rev. B* **75** 094503
[22] Jiang X, Li D and Rosenstein B 2014 *Phys. Rev. B* **89** 064507
[23] Li D and Rosenstein B 1999 *Phys. Rev. B* **60** 9704
[24] Deligiannis K, Charalambous M, Chaussy J, Liang R, Bonn D and Hardy W N 2000 *Physica C* **341–348** 1329
[25] Roulin M, Junod A, Erb A and Walker E 1996 *J. Low Temp. Phys.* **105** 1099
[26] Roulin M, Junod A, and Walker E 1996 *Science* **273** 1210
[27] Ebner C and Stroud D 1989 *Phys. Rev. B* **39** 789
[28] Stevenson P W 1981 *Phys. Rev. D* **23** 2916
[29] Okopinska A 1987 *Phys. Rev. D* **35** 1835
[30] Li D and Rosenstein B 2010 *Rev. Mod. Phys.* **82** 109
[31] Junod A, Erb A and Renner C 1999 *Physica C* **317–318** 333
[32] Triscone G, Junod A and Gladyshevskii R E 1996 *Physica C* **264** 233
[33] Junod A, Wang K Q, Tsukamoto T, Triscone G, Revaz B, Walker E and Muller J 1994 *Physica C* **229** 209
[34] Poole C P Jr, Farach H A, Creswick R J and Prozorov R 2007 *Superconductivity*, 2nd edn. (Elsevier)
[35] Tinh B D, Li D and Rosenstein B 2010 *Phys. Rev. B* **81** 224521