Effect of Bohm quantum potential in the propagation of ion–acoustic waves in degenerate plasmas

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A theoretical investigation has been carried out on the propagation of the ion–acoustic (IA) waves in a relativistic degenerate plasma containing relativistic degenerate electron and positron fluids in the presence of inertial non-relativistic light ion fluid. The Korteweg-de Vries (K-dV), modified K-dV (mK-dV), and mixed mK-dV (mmK-dV) equations are derived by adopting the reductive perturbation method. In order to analyze the basic features (phase speed, amplitude, width, etc.) of the IA solitary waves (SWs), the SWs solutions of the K-dV, mK-dV, and mmK-dV are numerically analyzed. It is found that the degenerate pressure, inclusion of the new phenomena like the Fermi temperatures and quantum mechanical effects (arising due to the quantum diffraction) of both electrons and positrons, number densities, etc., of the plasma species remarkably change the basic characteristics of the IA SWs which are found to be formed either with positive or negative potential. The implication of our results in explaining different nonlinear phenomena in astrophysical compact objects, e.g., white dwarfs, neutron stars, etc., and laboratory plasmas like intense laser–solid matter interaction experiments, etc., are mentioned.

Keywords: Bohm quantum potential, degenerate relativistic plasma, electron and positron quantum effect

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1. Introduction

Nowadays, the quantum effects associated with the plasma particles have become very interesting due to their ubiquitous presence in different astrophysical as well as different space dense plasma environments.[1,2] Since dense electron–positron–ion (EPI) plasmas may exist in astrophysical environments, e.g., white dwarfs, neutron stars, active galactic nuclei, etc.[1–4] as well as in laboratory plasma viz intense laser–solid matter interaction experiments,[5–7] it is of practical interest to investigate the properties of the IA waves in such kinds of plasmas. In the case of dense EPI plasmas, the quantum effect plays a significant role in governing the dynamics of charged particles.[8,9] On the other hand, it is found that quantum plasmas occur abundantly in dense astrophysical environments such as white dwarf and magnetars.[10]

The equation of state for degenerate plasma occurring in the dense astrophysical compact objects (e.g., white dwarfs, neutron stars, etc.) was explained by Chandrasekhar.[11,12] The degenerate pressure exerted by plasma fluids can be given as[4,13]

$$P_s = K_s n_s^{\alpha} = K_s n_s^{\gamma}, \tag{1}$$

where s is the plasma species ($s = i$ for ions; $s = e$ for electrons; and $s = p$ for positrons),

$$\alpha = \frac{5}{3}, \quad K_s = \frac{3}{5}\left(\frac{\pi}{3}\right)^{1/3}\frac{\pi\hbar^2}{m_s} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2}$$

for the non-relativistic limit ($\Lambda_c = \pi\hbar/m_s c = 1.2 \times 10^{-10}$ cm, and $\hbar = h/2\pi$);

$$\gamma = \frac{4}{3}, \quad K_s = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{1/3}\hbar c \simeq \frac{3}{4}\hbar c, \tag{3}$$

for the ultra-relativistic limit.

Recently, a number of authors have studied the nonlinear propagation of the IA waves in EPI plasmas either considering non-degenerate plasma[14–19] or degenerate plasmas.[13,20–25] In a degenerate dense plasma, Mamun and Shukla[13] studied the SWs, and implied their results in white dwarfs. Rahman *et al.*[22] investigated an EPI plasma, and showed that the amplitude, width and phase velocity of the IA waves are significantly modified due to the positron concentration as well as the relativistic plasma parameters. In degenerate plasma, the thermal de Broglie wavelength of the plasma species becomes comparable to the dimension of the system due to its high density. As a consequence, the quantum mechanical (such as tunneling associated with the Bohm potential) along with statistical (Fermi–Dirac pressure) effect come into play in such kinds of dense plasma. So, the quantum effects of the plasma particles viz electrons, positrons or ions must be taken into account to clearly describe the dynamics of these plasma systems. As far as we know, none of the earlier works on degenerate plasma considered the quantum mechanical effects (due to Bohm potential) of the plasma particles.

A number of works[26–32] have been done considering the quantum effects of plasma particles in different plasma

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medium. The behavior of high-densities and low-temperatures quantum plasmas was first studied by Pines.[33] A few years ago, Haas *et al.*[29] found in their investigation that the Bohm potentials associated with the plasma particles observably modify the basic features of the nonlinear IA waves. In a quantum electron-ion (EI) plasma, Bhowmik *et al.*[30] investigated the effects of the quantum diffraction parameter (H), and the equilibrium density ratio of the plasma species in modifying the electron–acoustic waves. While analyzing the IA waves in an EPI plasma, Ali *et al.*[34] found that the nonlinear properties of the IA waves are significantly affected by the inclusion of the quantum corrections of the corresponding plasma species. As stated earlier, in dense as well as degenerate plasma shows a crucial dependency on quantum effects of the plasma species, none of the earlier works have considered this case.

In our present communication, we have considered a degenerate EPI plasma (composed of inertial non-relativistic light ions, and degenerate electrons as well as positrons) considering the quantum mechanical (such as tunneling associated with the Bohm potential) effects for both electrons and positrons. The K-dV, mK-dV, and mmK-dV equations are derived, and their solutions are analyzed to study the basic properties of the IA waves propagating in such kinds of EPI plasma which abundantly occurs in different astrophysical situations (viz white dwarfs, neutron stars, active galactic nuclei, etc.[1–4]) and laboratory plasmas like intense laser-solid matter interaction experiments.[5–7]

The article is organized in the following way. The normalized governing equations are presented in Section 2. The K-dV, mK-dV, and mmK-dV equations for the SWs are derived on Section 3 (Subsections 3.1–3.3), respectively, and the SWs solution of the mmK-dV equation are derived in Subsection 3.4. The brief results and some discussions are given in Section 4. Finally, this paper ends with a summary in Section 5.

2. Theoretical model

The propagation of the IA waves in a degenerate dense EPI plasma system containing degenerate electron and positron fluids (both non-relativistic and ultra-relativistic), and an inertial non-relativistic light ion fluid has been considered. At equilibrium, the quasi-neutrality condition can be expressed as $n_{p0}+n_{i0}=n_{e0}$, where n_{p0}, n_{i0}, and n_{e0} are the equilibrium number densities of positrons, ions, and electrons, respectively. The dynamics of the electrostatic IA waves propagating in such a plasma system is governed by the following set of normalized equations

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0, \tag{4}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial \phi}{\partial x}-\frac{K_1}{n_i}\frac{\partial n_i^{\alpha}}{\partial x}, \tag{5}$$

$$n_e\frac{\partial \phi}{\partial x}-K_2\frac{\partial {n_e}^{\gamma}}{\partial x}+\beta\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial x^2}\sqrt{n_e}\right)=0, \tag{6}$$

$$n_p\frac{\partial \phi}{\partial x}+K_2\eta\frac{\partial n_p^{\gamma}}{\partial x}-\lambda\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_p}}\frac{\partial^2}{\partial x^2}\sqrt{n_p}\right)=0, \tag{7}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\mu n_e-n_i-\sigma n_p, \tag{8}$$

where n_s is the number densities of plasma species (s) normalized by their equilibrium value n_{s0}; u_i is the ion fluid speed normalized by the IA wave speed $c_i=(k_B T_{Fe}/m_i)^{1/2}$ (with m_i being the ion rest mass, k_B being the Boltzmann constant, and T_{Fe} being the Fermi temperature of the electron gas); ϕ is the electrostatic wave potential normalized by $k_B T_{Fe}/e$ (with e being the magnitude of an electron charge); $\eta=T_{Fp}/T_{Fe}$ (with T_{Fp} being the Fermi temperature of positron); $\mu=n_{e0}/n_{i0}$, $\sigma=n_{p0}/n_{i0}$. The non-dimensional quantum parameter for electron (positron) which is proportional to quantum diffraction, is $\beta=n_{i0}{H_e}^2/2n_{e0}$ ($\lambda=n_{i0}{H_p}^2/2n_{p0}$), where $H_e=\hbar\omega_{pe}/k_B T_{Fe}$ ($H_p=\hbar\omega_{pp}/k_B T_{Fp}$). We note that the last terms in Eqs. (6) and (7) arise due to the quantum Bohm potential or tunneling effect.[32] The time variable (t) is normalized by ${\omega_{pm}}^{-1}=(m_i/4\pi e^2 n_{i0})^{1/2}$, the space variable ($x$) is normalized by $\lambda_{Di}=(k_B T_{Fe}/4\pi e^2 n_{i0})^{1/2}$. We have also defined as $K_1=n_{i0}^{\alpha-1}K_i/m_i c_i^2$ and $K_2=n_{j0}^{\gamma-1}K_j/m_i c_i^2$, where $j=$e (p) for electrons (positrons).

3. Nonlinear equations for the SWs

3.1. Derivation of the K-dV equation

In order to examine the characteristics of the IA SWs propagating in our considered plasma system, we derive the K-dV equation employing the reductive perturbation method.[35] So, we first introduce the stretched coordinates[35] as

$$\xi=\varepsilon^{1/2}(x-V_p t), \tag{9}$$

$$\tau=\varepsilon^{3/2}t, \tag{10}$$

where V_p [$=\omega/k$ (ω is angular frequency and k is the wave number)] is the phase speed of the IA wave, and ε ($0<\varepsilon<1$) is a smallness parameter measuring the weakness of the dispersion. We then expand the variables n_s, u_i and ϕ, in a power series of ε as

$$n_s=1+\varepsilon n_s^{(1)}+\varepsilon^2 n_s^{(2)}+\varepsilon^3 n_s^{(3)}+\cdots, \tag{11}$$

$$u_i=\varepsilon u_i^{(1)}+\varepsilon^2 u_i^{(2)}+\varepsilon^3 u_i^{(3)}+\cdots, \tag{12}$$

$$\phi=\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+\cdots. \tag{13}$$

Now, after substituting Eqs. (9)–(13) in Eqs. (4)–(8), the lowest order coefficients of ε can be written as

$$u_i^{(1)}=\frac{\phi^{(1)}+{k_1}' n_i^{(1)}}{V_p}, \tag{14}$$

$$n_{\rm i}^{(1)} = \frac{\phi^{(1)}}{V_{\rm p}^2 - {k_1}'}, \tag{15}$$

$$n_{\rm e}^{(1)} = \frac{\phi^{(1)}}{{k_2}'}, \tag{16}$$

$$n_{\rm p}^{(1)} = \frac{-\phi^{(1)}}{\eta {k_2}'}, \tag{17}$$

$$V_{\rm p} = \sqrt{\left[\frac{\eta {k_2}'}{(\eta\mu+\sigma)} + {k_1}'\right]}, \tag{18}$$

where ${k_1}' = \alpha K_1$, and ${k_2}' = \gamma K_2$. The equation (18) represents the dispersion relation for the IA-type electrostatic waves propagating in the degenerate EPI plasma under consideration. Again, substituting Eqs. (9)–(13) into Eqs. (4)–(8), and equating the coefficient of $\varepsilon^{5/2}$ from Eqs. (4)–(7), and the coefficient of ε^2 from Eq. (8), we obtain a set of equations that can be simplified as

$$\begin{aligned}\frac{\partial n_{\rm i}^{(2)}}{\partial \xi} &= \frac{V_{\rm p}}{V_{\rm p}^2 - {k_1}'}\frac{\partial n_{\rm i}^{(1)}}{\partial \tau} + \frac{V_{\rm p}}{V_{\rm p}^2 - {k_1}'}\frac{\partial}{\partial \xi}\left[n_{\rm i}^{(1)} u_{\rm i}^{(1)}\right] \\ &+ \frac{1}{V_{\rm p}^2 - {k_1}'}\frac{\partial u_{\rm i}^{(1)}}{\partial \tau} + \frac{1}{V_{\rm p}^2 - {k_1}'} u_{\rm i}^{(1)}\frac{\partial u_{\rm i}^{(1)}}{\partial \xi} \\ &+ \frac{{k_1}'(\alpha-2)}{V_{\rm p}^2 - {k_1}'} n_{\rm i}^{(1)}\frac{\partial n_{\rm i}^{(1)}}{\partial \xi} + \frac{1}{V_{\rm p}^2 - {k_1}'}\frac{\partial \phi^{(2)}}{\partial \xi}, \end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial n_{\rm e}^{(2)}}{\partial \xi} &= \frac{1}{{k_2}'} n_{\rm e}^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + \frac{1}{{k_2}'}\frac{\partial \phi^{(2)}}{\partial \xi} \\ &- (\gamma-1) n_{\rm e}^{(1)}\frac{\partial n_{\rm e}^{(1)}}{\partial \xi} + \frac{\beta}{2{k_2}'}\frac{\partial^3 n_{\rm e}^{(1)}}{\partial \xi^3}, \end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\partial n_{\rm p}^{(2)}}{\partial \xi} &= -\frac{1}{\eta {k_2}'} n_{\rm p}^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} - \frac{1}{\eta {k_2}'}\frac{\partial \phi^{(2)}}{\partial \xi} \\ &- (\gamma-1) n_{\rm p}^{(1)}\frac{\partial n_{\rm p}^{(1)}}{\partial \xi} + \frac{\lambda}{2\eta {k_2}'}\frac{\partial^3 n_{\rm p}^{(1)}}{\partial \xi^3}, \end{aligned} \tag{21}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \mu n_{\rm e}^{(2)} - n_{\rm i}^{(2)} - \sigma n_{\rm p}^{(2)}. \tag{22}$$

Now, simplifying Eqs. (19)–(22) by using Eqs. (14)–(18), and combining each other, we finally obtain our desired equation in the form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0. \tag{23}$$

Equation (23) is the well-known K-dV equation describing the dynamics of the IA SWs propagating in the degenerate EPI plasma, where the nonlinear coefficient (A) and the dispersion coefficient (B) are given as

$$\begin{aligned} A &= \left[\frac{(V_{\rm p}^2 - {k_1}')^2}{2V_{\rm p}}\right] \\ &\times \left[\frac{(\eta^2\mu - \sigma)(\gamma-2)}{(\eta {k_2}')^2} + \frac{3V_{\rm p}^2 + {k_1}'(\alpha-2)}{(V_{\rm p}^2 - {k_1}')^3}\right], \end{aligned} \tag{24}$$

$$B = \left[\frac{(V_{\rm p}^2 - {k_1}')^2}{2V_{\rm p}}\right] \times \left[1 - \frac{\mu\beta}{2{k_2}'^2} - \frac{\sigma\lambda}{2(\eta {k_2}')^2}\right]. \tag{25}$$

Now, to investigate the properties of the IA SWs, we are interested in the SWs solution of the K-dV equation. To do so, we introduce another stretched coordinate, $\zeta = \xi - U_0\tau$. After the coordinate transformation, the steady state ($\partial/\partial\tau = 0$) solution of the SWs can be written as (by taking $\phi^{(1)} = \Phi$)

$$\Phi = \Phi_{\rm m}\,{\rm sech}^2\left(\frac{\zeta}{\delta_{\rm l}}\right), \tag{26}$$

where the amplitude $\Phi_{\rm m} = 3U_0/A$, and the width $\delta_{\rm l} = \sqrt{4B/U_0}$. It is seen that the quantum parameters both β and λ are present in $\delta_{\rm l}$, whereas $\Phi_{\rm m}$ is totally independent of these parameters.

3.2. Derivation of the mK-dV equation

The K-dV equation [Eq. (23)] is the result of the second-order calculation in the smallness parameter ε, where the quadratic nature has been revealed by the nonlinear term $A\phi^{(1)}\partial\phi^{(1)}/\partial\xi$. For plasmas with more than two species as like our system, however, there can arise cases where (A) vanishes at a particular value of a certain parameter μ, and equation (23) fails to describe the nonlinear evolution of the perturbation. So, a higher order calculation (i.e., derivation of the mK-dV equation) is needed to describe the dynamics of the system at the critical value of a certain parameter. So, at the critical value $\mu = \mu_{\rm c}$, we derive the mK-dV equation by applying the following stretched coordinates[36]

$$\xi = \varepsilon(x - V_{\rm p}t), \tag{27}$$

$$\tau = \varepsilon^3 t. \tag{28}$$

By using Eqs. (27) and (28) in Eqs. (4)–(8), we found the same values of $n_{\rm i}^{(1)}$, $n_{\rm p}^{(1)}$, $n_{\rm e}^{(1)}$, $u_{\rm i}^{(1)}$, and $V_{\rm p}$ as those of the K-dV equation. To the next higher order of ε, we obtain a set of equations, which, after using the values of $n_{\rm i}^{(1)}$, $n_{\rm p}^{(1)}$, $n_{\rm e}^{(1)}$, $u_{\rm i}(1)$, and $V_{\rm p}$ can be simplified as

$$n_{\rm i}^{(2)} = \frac{3V_{\rm p}^2 + {k_1}'(\alpha-2)}{2(V_{\rm p}^2 - {k_1}')^3}[\phi^{(1)}]^2 + \frac{1}{V_{\rm p}^2 - {k_1}'}\phi^{(2)}, \tag{29}$$

$$n_{\rm e}^{(2)} = \frac{(2-\gamma)}{2{k_2}'^2}[\phi^{(1)}]^2 + \frac{1}{{k_2}'}\phi^{(2)}, \tag{30}$$

$$n_{\rm p}^{(2)} = \frac{(2-\gamma)}{2(\eta {k_2}')^2}[\phi^{(1)}]^2 - \frac{1}{\eta {k_2}'}\phi^{(2)}, \tag{31}$$

$$\rho^{(2)} = -\frac{1}{2}A_1[\phi^{(1)}]^2 = 0, \tag{32}$$

where

$$A_1 = \left[\frac{(\eta^2\mu - \sigma)(\gamma-2)}{(\eta {k_2}')^2} + \frac{3V_{\rm p}^2 + {k_1}'(\alpha-2)}{(V_{\rm p}^2 - {k_1}')^3}\right]. \tag{33}$$

To the next higher order of ε, we obtain a set of equations as follows:

$$\frac{\partial n_{\rm i}^{(3)}}{\partial\xi}=\frac{2V_{\rm p}}{(V_{\rm p}^2-k_1{}')^2}\frac{\partial\phi^{(1)}}{\partial\tau}+\left[\frac{V_{\rm p}^2}{(V_{\rm p}^2-k_1{}')^4}+\frac{3V_{\rm p}^2k_1{}'}{(V_{\rm p}^2-k_1{}')^5}+\frac{k_1{}'^2(\alpha-2)}{(V_{\rm p}^2-k_1{}')^5}+\frac{k_1{}'(\alpha-2)}{(V_{\rm p}^2-k_1{}')^4}\right.$$
$$\left.+\frac{3V_{\rm p}^4}{(V_{\rm p}^2-k_1{}')^5}+\frac{k_1{}'V_{\rm p}^2(\alpha-2)}{2(V_{\rm p}^2-k_1{}')^5}+\frac{k_1{}'^2(\alpha-2)^2}{2(V_{\rm p}^2-k_1{}')^5}+\frac{k_1{}'^2(\alpha-2)(\alpha-3)}{6(V_{\rm p}^2-k_1{}')^4}\right]\frac{\partial[\phi^{(1)}]^3}{\partial\xi}$$
$$+\left[\frac{2}{(V_{\rm p}^2-k_1{}')^2}+\frac{2k_1{}'}{(V_{\rm p}^2-k_1{}')^3}+\frac{v_{\rm p}^2}{(V_{\rm p}^2-k_1{}')^3}+\frac{k_1{}'(\alpha-2)}{(V_{\rm p}^2-k_1{}')^3}\right]\frac{\partial}{\partial\xi}[\phi^{(1)}\phi^{(2)}]+\frac{1}{V_{\rm p}^2-k_1{}'}\frac{\partial\phi^{(3)}}{\partial\xi},\tag{34}$$

$$\frac{\partial n_{\rm e}^{(3)}}{\partial\xi}=\frac{\beta}{2k_2{}'^2}\frac{\partial^3\phi^{(1)}}{\partial\xi^3}+\left[\frac{(\gamma-2)(2\gamma-3)}{6k_2{}'^3}\right]\frac{\partial[\phi^{(1)}]^3}{\partial\xi}+\frac{(2-\gamma)}{k_2{}'^2}\frac{\partial}{\partial\xi}[\phi^{(1)}\phi^{(2)}]+\frac{1}{k_2{}'}\frac{\partial\phi^{(3)}}{\partial\xi},\tag{35}$$

$$\frac{\partial n_{\rm p}^{(3)}}{\partial\xi}=-\frac{\lambda}{2(\eta k_2{}')^2}\frac{\partial^3\phi^{(1)}}{\partial\xi^3}-\left[\frac{(\gamma-2)(2\gamma-3)}{6(\eta k_2{}')^3}\right]\frac{\partial[\phi^{(1)}]^3}{\partial\xi}+\frac{(2-\gamma)}{(\eta k_2{}')^2}\frac{\partial}{\partial\xi}[\phi^{(1)}\phi^{(2)}]-\frac{1}{\eta k_2{}'}\frac{\partial\phi^{(3)}}{\partial\xi},\tag{36}$$

$$\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}=\mu n_{\rm e}^{(3)}-n_{\rm i}^{(3)}-\sigma n_{\rm p}^{(3)}.\tag{37}$$

Now, combining Eqs. (34)–(37) after simplifying by using Eqs. (29)–(32), we obtain the well-known mK-dV equation as follows:

$$\frac{\partial\phi^{(1)}}{\partial\tau}+C[\phi^{(1)}]^2\frac{\partial\phi^{(1)}}{\partial\xi}+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}=0,\tag{38}$$

where

$$C=\left[\frac{(V_{\rm p}^2-k_1{}')^2}{2V_{\rm p}}\right]\times\left[\frac{(\eta^3\mu+\sigma)(\gamma-2)(3-2\gamma)}{2(\eta k_2{}')^3}\right.$$
$$\left.+\frac{Q}{2(V_{\rm p}^2-k_1{}')^4}+\frac{Z}{2(V_{\rm p}^2-k_1{}')^5}\right],\tag{39}$$

$$B=\left[\frac{(V_{\rm p}^2-k_1{}')^2}{2V_{\rm p}}\right]\times\left[1-\frac{\mu\beta}{2k_2{}'^2}-\frac{\sigma\lambda}{2(\eta k_2{}')^2}\right],\tag{40}$$

where

$$Q=6V_{\rm p}^2+6k_1{}'(\alpha-2)+k_1{}'(\alpha-2)(\alpha-3),$$

and

$$Z=18V_{\rm p}^2k_1{}'+6k_1{}'^2(\alpha-2)+18V_{\rm p}^4+3k_1{}'V_{\rm p}^2(\alpha-2)$$
$$+3k_1{}'^2(\alpha-2)^2.$$

Now, taking the same stretching as that employed in the K-dV solution, the stationary SWs solution of Eq. (38) can be directly given as (by taking $\phi^{(1)}=\Psi$)

$$\Psi=\Psi_{\rm m}\,\mathrm{sech}\left(\frac{\zeta}{\delta_2}\right),\tag{41}$$

where the amplitude $\Psi_{\rm m}=\sqrt{6U_0/C}$, and the width $\delta_2=\sqrt{B/U_0}$. It is seen from δ_2 that it depends on both β and λ; on the other hand, $\Psi_{\rm m}$ shows no dependency on these parameters.

3.3. Derivation of the mmK-dV equation

For μ in the vicinity of $\mu_{\rm c}$, however, (A) is small and can be of the order ε. Then, the term $A\phi^{(1)}\partial\phi^{(1)}/\partial\xi$ as a whole becomes of the same order as the terms in Eq. (38). Including this term in the mK-dV equation, we need to derive the mmK-dV. So, for μ around its critical value $\mu_{\rm c}$, $A=A_0$ can be expressed as

$$A_0=s\left(\frac{\partial A}{\partial\mu}\right)_{\mu=\mu_{\rm c}}|\mu-\mu_{\rm c}|=sc_1\varepsilon,\tag{42}$$

where $|\mu-\mu_{\rm c}|$ is a small and dimensionless parameter, also known as the expansion parameter ε, i.e., $|\mu-\mu_{\rm c}|\simeq\varepsilon$, and $s=1$ for $\mu>\mu_{\rm c}$ and $s=-1$ for $\mu<\mu_{\rm c}$. c_1 is a constant depending on plasma parameter γ and $k_2{}'$ which is given by

$$c_1=\frac{\gamma-2}{k_2{}'^2},\tag{43}$$

and $\rho^{(2)}$ can be expressed as

$$\varepsilon^2\rho^{(2)}\simeq-\varepsilon^3\frac{1}{2}c_1s[\phi^{(1)}]^2.\tag{44}$$

Now taking the coefficient of ε^3 from Eq. (8), we get

$$\frac{\partial\rho^{(3)}}{\partial\xi}=\sigma\frac{\partial n_{\rm p}{}^{(3)}}{\partial\xi}+\frac{\partial n_{\rm i}^{(3)}}{\partial\xi}-\mu\frac{\partial n_{\rm e}^{(3)}}{\partial\xi}.\tag{45}$$

Now we can find the value of $\rho^{(3)}$ from Eq. (45), where the values of $n_{\rm p}^{(3)}$, $n_{\rm n}^{(3)}$, and $n_{\rm e}^{(3)}$ can be found from Eqs. (34)–(36). Therefore, combining these equations into Eq. (45), we can finally write the following equation as

$$\frac{\partial\phi^{(1)}}{\partial\tau}+sc_1D\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+C[\phi^{(1)}]^2\frac{\partial\phi^{(1)}}{\partial\xi}$$
$$+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}=0,\tag{46}$$

where $D=(V_{\rm p}^2-k_1{}')^2/2V_{\rm p}$. It is called the mmK-dV equation[36] which describes the propagation of the IA waves in the EPI plasma system in the vicinity of the critical value of the plasma parameter μ.

3.4. SWs solution of the mmK-dV equation

The stationary SWs solution of the mmK-dV equation [Eq. (46)] can be derived by the same transformation done in case of the solution of the K-dV and mK-dV equation. To do so, we introduce a transformation $\zeta = \xi - U_0\tau$ which allows us to write Eq. (46), under the steady state condition ($\partial/\partial\tau$=0), as (by taking $\phi^{(1)} = \Phi$)

$$\frac{1}{2}\left(\frac{\partial\Phi}{\partial\zeta}\right)^2 + V(\Phi) = 0, \tag{47}$$

where the pseudo-potential $V(\Phi)$ is

$$V(\Phi) = -\frac{U_0}{2B}\Phi^2 + \frac{sc_1D}{6B}\Phi^3 + \frac{C}{12B}\Phi^4, \tag{48}$$

we can write from Eq. (48) that

$$V(\Phi)\big|_{\Phi=0} = \frac{\partial V(\Phi)}{\partial\Phi}\Big|_{\Phi=0} = 0, \tag{49}$$

$$\frac{\partial^2 V(\Phi)}{\partial\Phi^2}\Big|_{\Phi=0} < 0. \tag{50}$$

The conditions of Eqs. (49) and (50) imply that the SWs solution of Eq. (47) exists if

$$V(\Phi)\,|_{\Phi=\Phi_m} = 0. \tag{51}$$

The latter can be solved as

$$U_0 = \frac{sc_1D}{3}\Phi_{m1,2} + \frac{C}{6}\Phi^2_{m1,2}, \tag{52}$$

$$\Phi_{m1,2} = \Phi_m\left[1 \mp \sqrt{1 + \frac{U_0}{V_0}}\right], \tag{53}$$

where $\Phi_m = -sc_1D/C$ and $V_0 = (sc_1D)^2/6C$. Now, using Eqs. (48) and (53) in Eq. (47), we have

$$\left(\frac{\partial\Phi}{\partial\zeta}\right)^2 + \kappa\Phi^2(\Phi - \Phi_{m1})(\Phi - \Phi_{m2}) = 0, \tag{54}$$

where $\kappa = C/6B$. The SWs solution of Eq. (47) or Eq. (54) is, therefore, directly given by

$$\Phi = \left[\frac{1}{\Phi_{m2}} - \left(\frac{1}{\Phi_{m2}} - \frac{1}{\Phi_{m1}}\right)cosh^2\left(\frac{\zeta}{\Delta}\right)\right]^{-1}, \tag{55}$$

where $\Phi_{m1,2}$ are given in Eq. (53), and the SWs width Δ is

$$\Delta = \frac{2}{\sqrt{-\kappa\Phi_{m1}\Phi_{m2}}}. \tag{56}$$

4. Results and discussions

In this section, we numerically investigate the dependency of the nonlinear wave potential $\phi^{(1)}$ on the densities of the degenerate electrons (n_{e0}), positrons (n_{p0}), and ions (n_{i0}); the Fermi temperatures of electrons (T_{Fe}) and positrons (T_{Fp}); and the quantum diffraction parameters of both electrons (β) and positrons (λ). For our purpose, we have numerically derived the SWs solutions from our derived K-dV, mK-dV, and mmK-dV equations, and analyzed their stationary SWs solutions based on some typical plasma parameters relevant to different astrophysical and laboratory plasma situations. We consider some typical plasma species density which is consistent with the relativistic degenerate astrophysical plasmas, e.g., $n_{i0} = 1.1\times10^{29}$ cm^{-3}, $n_{e0} = 9.1\times10^{29}$ cm^{-3}, and $n_{p0} = 1.5\times n_{e0}$ cm^{-3},[13,24,37] and the other quantum parameters e.g., $\lambda = 0.1$–0.9, $\beta = 0.1$–0.9, $\eta = 0.2$–0.6, and $H = 0.2$–0.9[27,32,38] related to the different EPI plasma environments existing in some published works. The values of the parameters may change depending on different plasma situations. The results, we have found from this investigation can be summarized as follows.

(i) The phase speed, (V_p) of the IA waves decreases with the increase of μ (Fig. 1). This happens due to the increase of the inertia of the IA waves (where the inertia comes from the mass of the ions, and the restoring force comes from the degenerate pressure of both electrons and positrons) with the increase of the value of μ.

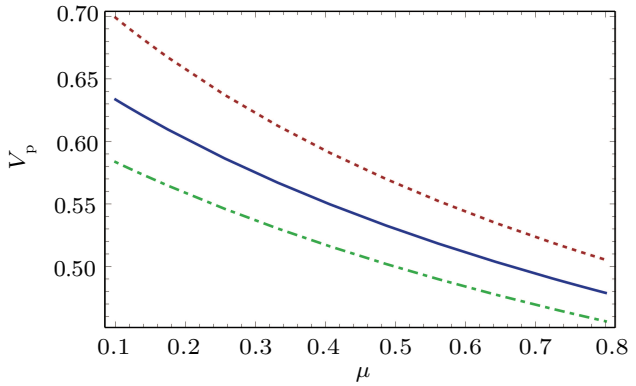


Fig. 1. (color online) The variation of V_p with μ for $\gamma = 4/3$, $\eta = 0.6$, $n_{i0} = 1.1\times10^{29}$ cm^{-3}, $n_{e0} = 9.1\times10^{29}$ cm^{-3}, and $n_{p0} = 1.5\times n_{e0}$ cm^{-3}. The dotted curve for $\sigma = 0.4$, the solid curve for $\sigma = 0.5$, and the dot–dashed curve for $\sigma = 0.6$.

(ii) Figure 2 represents the variation of amplitude (Φ_m) with the variations of μ and σ (both of which are responsible for giving the restoring force to our investigated IA waves). As we increase the value of positrons number density (σ), their interaction with electrons increases due to its opposite charge properties, and its contribution to the restoring force becomes high. On the other hand, in case of the electron number density (μ), the amplitude increases with the increase of μ from 0.2–0.35. In the case of electrons, due to its small number density, it can contribute largely to the restoring force. As the density increases from 0.35–0.8 the electrostatic interaction between electrons and positrons increases, and thus their contribution to the restoring force increases. As the restoring force increases, the amplitude decreases significantly.

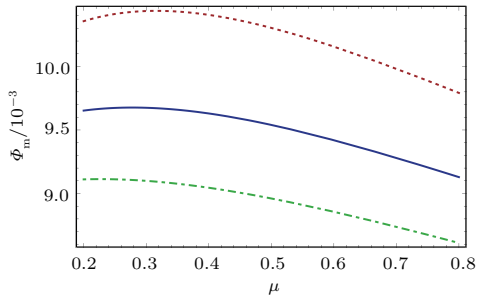


Fig. 2. (color online) The variation of $\Phi_{\rm m}$ with μ for $U_0 = 0.01$, $\gamma = 4/3$, $\alpha = 5/3$, $\eta = 0.6$, $n_{\rm i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{\rm e0} = 9.1 \times 10^{29}$ cm^{-3}, $n_{\rm p0} = 1.5 \times n_{\rm e0}$ cm^{-3}, and $\beta = 0.1$. The dotted curve for $\sigma = 0.4$, the solid curve for $\sigma = 0.5$, and the dot–dashed curve for $\sigma = 0.6$.

(iii) The effect of electron quantum parameter (β) on the width of the K-dV SWs are displayed in Fig. 3. As β arises due to the Bohm potential which is solely responsible for the tunneling effect of the corresponding plasma component, with the increase of β the tunneling effect also increases. So, the electrons interact more actively with the ions causing the reduction in the width.

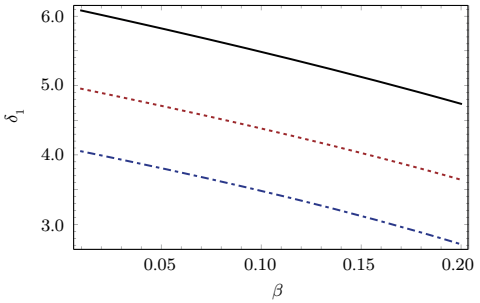


Fig. 3. (color online) The variation of δ_1 with β for $U_0 = 0.01$, $\mu = 0.4$, $\lambda = 0.1$, $\gamma = 4/3$, $\eta = 0.6$, $n_{\rm i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{\rm e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{\rm p0} = 1.5 \times n_{\rm e0}$ cm^{-3}. The solid curve for $\sigma = 0.3$, the dotted curve for $\sigma = 0.4$, and the dot–dashed curve for $\sigma = 0.5$.

(iv) The inertia of the K-dV SWs comes from the ion number density where the electrons and positrons number density give the restoring force. So, as σ increases, the restoring force of the K-dV SWs increases significantly causing the reduction in the K-dV SWs profiles depicted in Fig. 4. Our findings nicely match the findings of Popel *et al.*[14]

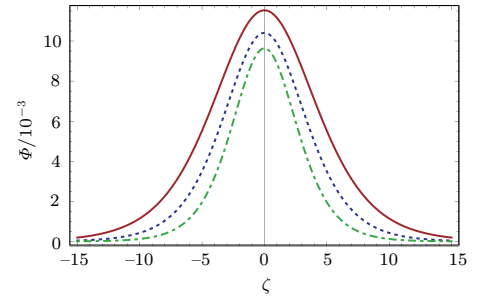


Fig. 4. (color online) The effects of σ on the SWs profiles (Φ versus ζ) for $U_0 = 0.01$, $\mu = 0.4$, $\lambda = 0.1$, $\gamma = 4/3$, $\alpha = 5/3$, $\beta = 0.1$, $\eta = 0.6$, $n_{\rm i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{\rm e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{\rm p0} = 1.5 \times n_{\rm e0}$ cm^{-3}. The solid curve for $\sigma = 0.3$, the dotted curve for $\sigma = 0.4$, and the dot–dashed curve for $\sigma = 0.5$.

(v) The effects of the non-relativistic $(\gamma = 5/3)$ and ultra relativistic $(\gamma = 4/3)$ cases of the degenerate pressure on the δ_1 and $\Phi_{\rm m}$ of the K-dV SWs are shown in Figs. 5 and 6, respectively. Since the number density of plasma species in the non-relativistic case is always less than the ultra relativistic case, plasma particles can move more freely in the non-relativistic plasma than in the ultra relativistic plasma. For this reason $\Phi_{\rm m}$ as well as δ_1 of the IA SWs are noticeably higher for $\gamma = 5/3$ than for $\gamma = 4/3$.[23]

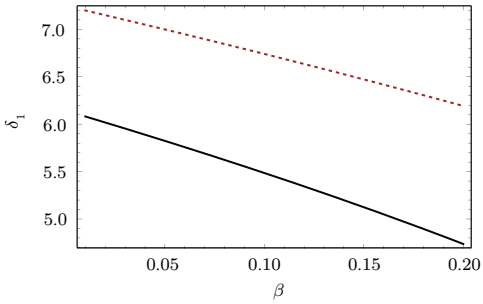


Fig. 5. (color online) The variation of δ_1 with β for $U_0 = 0.01$, $\mu = 0.4$, $\lambda = 0.1$, $\sigma = 0.3$, $\eta = 0.6$, $n_{\rm i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{\rm e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{\rm p0} = 1.5 \times n_{\rm e0}$ cm^{-3}. The upper dotted (red) curve represents the non-relativistic case ($\gamma = 5/3$), and the lower solid (black) curve represents the ultra-relativistic case ($\gamma = 4/3$).

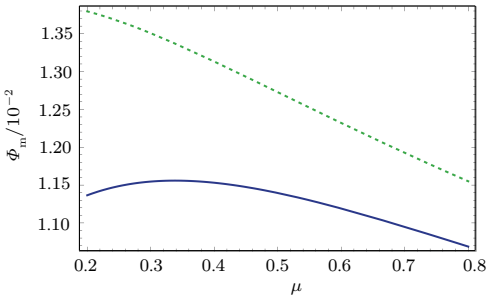


Fig. 6. (color online) The variation of Φ_m with μ for $U_0 = 0.01$, $\alpha = 5/3$, $\beta = 0.1$, $\sigma = 0.3$, $\eta = 0.6$, $n_{\rm i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{\rm e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{\rm p0} = 1.5 \times n_{\rm e0}$ cm^{-3}. The dotted (upper green) curve for $\gamma = 5/3$, represents the non-relativistic case and the solid (lower blue) curve for $\gamma = 4/3$, represents the ultra-relativistic case.

(vi) The influences of μ and σ on the mK-dV SWs profile are depicted in Figs. 7 and 8. It is found that the IA SWs profiles decrease significantly due to the increases in the values of both μ and σ (the physics behind this is described earlier).

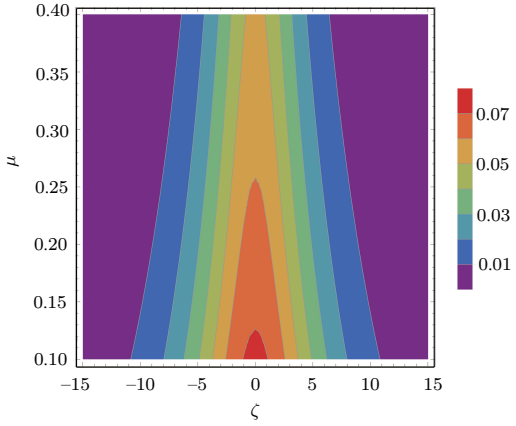


Fig. 7. (color online) The effects of μ on the electrostatic potential Ψ of the mK-dV SWs for $U_0 = 0.01$, $\alpha = 5/3$, $\beta = 0.1$, $\gamma = 4/3$, $\sigma = 0.3$, $\lambda = 0.1$, $\eta = 0.6$, $n_{\rm i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{\rm e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{\rm p0} = 1.5 \times n_{\rm e0}$ cm^{-3}.

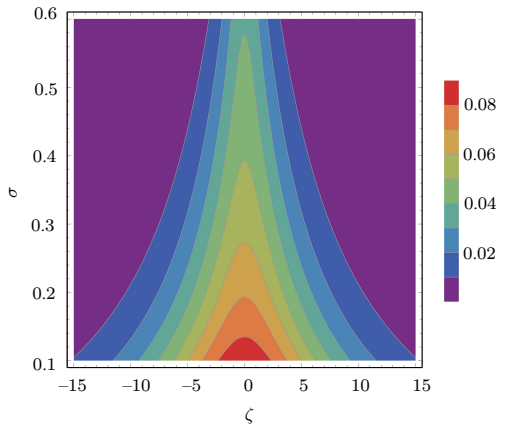


Fig. 8. (color online) The effects of σ on the electrostatic potential Ψ of the mK-dV SWs for $U_0 = 0.01$, $\alpha = 5/3$, $\beta = 0.1$, $\gamma = 4/3$, $\mu = 0.3$, $\lambda = 0.1$, $\eta = 0.6$, $n_{i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{p0} = 1.5 \times n_{e0}$ cm^{-3}.

(vii) It is important to note here that the mmK-dV equation, which contains higher-order nonlinearity, can have different types of solutions depending on different boundary conditions. In our present work, we are interested only in the SWs solution. We have found that depending on the degenerate plasma parameters, the SWs with positive (compressive) for $s = -1$ or negative (rarefactive) for $s = 1$ potential exist (Figs. 9 and 10).

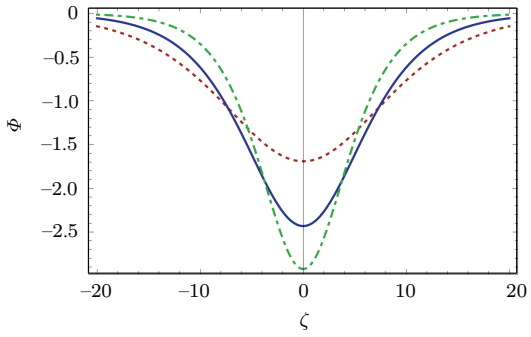


Fig. 9. (color online) The effects of σ on the electrostatic potential Φ of the mmK-dV SWs for $U_0 = 0.01$, $\alpha = 5/3$, $s = 1$, $\beta = 0.1$, $\gamma = 4/3$, $\mu = 0.3$, $\lambda = 0.1$, $\eta = 0.6$, $n_{i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{p0} = 1.5 \times n_{e0}$ cm^{-3}. The dotted curve shows for $\sigma = 0.1$, the solid curve shows for $\sigma = 0.2$ and the dotdashed curve shows for $\sigma = 0.3$.

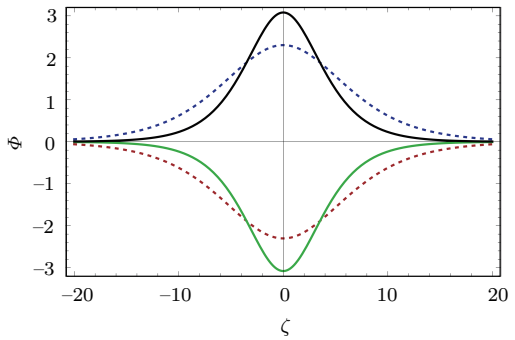


Fig. 10. (color online) The positive and negative profile (Φ versus ζ) of the mmK-dV SWs for $U_0 = 0.01$, $\alpha = 5/3$, $\beta = 0.1$, $\gamma = 4/3$, $\lambda = 0.1$, $\sigma = 0.3$, $\eta = 0.6$, $n_{i0} = 1.1 \times 10^{29}$ cm^{-3}, $n_{e0} = 9.1 \times 10^{29}$ cm^{-3}, and $n_{p0} = 1.5 \times n_{e0}$ cm^{-3}. The upper curves for s= -1 (μ=0.1 for blue dotted curve, and μ=0.4 for black solid curve), and the lower curves for s=1 (μ=0.1 for red dotted curve, and μ=0.4 for green solid curve).

(viii) The Φ_m of the mmK-dV rarefactive ($s = 1$) SWs, decreases with the increase of the value of σ. It is also obvious from Fig. 9 that as the value of σ increases, the SWs become more spiky.

(ix) The mmK-dV SWs with both positive (compressive) and negative (rarefactive) potential are depicted in Fig. 10. The Φ_m of the mmK-dV SWs increases with the increase of μ for s= -1 curves but shows opposite phenomena for s= 1 curves.

We have considered the non-relativistic case for the degenerate inertial light ion fluids in our model. As light ions may have a little/significant contribution to the degenerate pressure due to their light mass, in our present work we did not neglect this possibility.[20,39] It should be remarked here that we have neglected the quantum effect for ions since ions are heavier than electrons and positrons.[32] We have also neglected the electron-positron annihilation process by considering the conditions that $\omega_{pj}^{-1} \ll T_{ann}$, where ω_{pj} is the plasma frequency of the j-th species, and T_{ann} is the annihilation time. Many authors have neglected the annihilation process in the relativistic dense plasmas.[40–42]

5. Summary

We have numerically analyzed, and discussed the effects of relativity (both ultra-relativistic and non-relativistic cases), quantum parameters, Fermi temperatures, and number densities of the plasma species on the basic features (viz phase speed, polarity, amplitude, width, etc.) of the IA SWs propagating in a degenerate EPI plasma. It is found that: the degenerate pressure of electrons, positrons, and ions; the inclusion of the new phenomena such as Fermi temperatures and quantum effects of both electrons and positrons; the number densities of electrons, positrons, and ions, etc. remarkably change the basic characteristics of the IA SWs. Finally, it should be noted here that our lower order to higher order analysis for the SWs formulation, which includes not only at the critical values $\mu = \mu_c$ of certain plasma parameters but also in the vicinity of the critical value, will be very much helpful in analyzing the IA SWs propagating in the different astrophysical compact objects viz. white dwarfs, neutron stars, etc.[1,2,4] and also in the laboratory plasmas, e.g., intense laser-solid matter interaction experiments.[5–7]

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