K–P–Burgers equation in negative ion-rich relativistic dusty plasma including the effect of kinematic viscosity

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(Received 23 January 2016; revised manuscript received 8 June 2016; published online 25 August 2016)

The stationary solution is obtained for the K–P–Burgers equation that describes the nonlinear propagations of dust ion acoustic waves in a multi-component, collisionless, un-magnetized relativistic dusty plasma consisting of electrons, positive and negative ions in the presence of charged massive dust grains. Here, the Kadomtsev–Petviashvili (K–P) equation, three-dimensional (3D) Burgers equation, and K–P–Burgers equations are derived by using the reductive perturbation method including the effects of viscosity of plasma fluid, thermal energy, ion density, and ion temperature on the structure of a dust ion acoustic shock wave (DIASW). The K–P equation predictes the existences of stationary small amplitude solitary wave, whereas the K–P–Burgers equation in the weakly relativistic regime describes the evolution of shock-like structures in such a multi-ion dusty plasma.

Keywords: K–P–Burgers equation, negative ion, relativistic plasma, reductive perturbation method

PACS: 52.27.Lw, 52.35.Fp, 52.35.Mw, 52.35.Tc **DOI:** 10.1088/1674-1056/25/10/105202

1. Introduction

Amongst all nonlinear phenomena in plasma physics, the study of ion acoustic soliton has emerged as one of the most important promising areas in the arena of fundamental and laboratory applications throughout the last few decades. The properties of ion-acoustic solitary waves have been studied by using the reductive perturbation technique in different types of multicomponent plasma conditions.[1–4] On the other hand, with the growing demand in the study of dust plasma interaction, the linear and nonlinear propagation of electrostatic excitations in dusty plasmas have received considerable attention in the last few decades. The existence of the low frequency dust-ion-acoustic (DIA) wave, whose phase speed is much smaller than the electron thermal speed and larger than the ion thermal speed, was first studied theoretically by Shukla and Silin,[1] and later its existence was confirmed experimentally by Barkan *et al.*[5] Since then numerous investigations have been reported on different aspects of nonlinear plasma wave propagation in experimental and theoretical plasma physics.[6–12] As dust is a common species in a wide range of space and astrophysical plasmas, such as the cometary tails, interstellar clouds, Earth mesosphere and ionosphere, Saturn's rings, the gossamer ring of Jupiter, and in laboratory experiments,[5,13–15] the study of dust-plasma interaction has become a hub of interest for recent research in plasma physics.[16,17]

The propagation of a solitary wave is important as it describes the characteristic nature of the interaction of the wave with the plasma constituents. However, once the ion velocity approaches to the velocity of light, the amplitude, width, and energy of the wave will each experience a drastic modification as the relativistic effect becomes dominant. Relativistic plasmas occur in a variety of situations, such as in the earth magnetosphere,[18] laser–plasma interaction,[19] and in different environments of space-plasmas.[20] Very high speed and energetic ion beams with streaming energy in a range from 0.1 MeV to 100 MeV are frequently observed in the solar atmosphere and interplanetary space. It is seen that the relativistic effect plays an important role in the propagations of such energetic ion waves in interplanetary space.[21] For example, Lu *et al.* observed an exact relativistic plasma wave which is completely different from a soliton in an electron–positron plasma which the Sagdeev potential approach was applied to Ref. [22]. The one-dimensional (1D) propagation of ion acoustic solitary wave has been studied in different plasma systems by using the Korteweg–de Vries (KdV) equation.[23–28] These plasmas have been of wide interest for the experimental as well as the theoretical research. However, in space and even in laboratory devices the observations are not always confined in 1D geometry. In the auroral region, especially at higher polar altitudes a purely 1D model cannot account for all observed features.[29] In many observations it was reported that transverse perturbations always exist in higher-dimensional systems, and the wave structure and many properties of the solitary waves are modified by these transverse perturbations.[30–32] In two-dimensional (2D) systems, Kadomtsev and Petviashvili made the first at-

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tempt to model solitary waves.[33] They obtained a 2D differential equation which is known as the famous Kadomtsev–Petviashvili (K–P) equation. The K–P equation has been studied extensively[34–36] by the reductive perturbation method for different plasma environments.

Numerous investigations on the nonlinear solitary waves (solitons) in relativistic plasmas have been done for the last several years.[25–28,37–39] In order to study the electromagnetic waves, the nonlinear interactions of circularly polarized waves[40] and the influence of high-power laser radiation[41] have been investigated theoretically by using the relativistic speed of plasma particle. In the field of laser–plasma interaction[42] and space plasma phenomena,[19] relativistic Langmuir and electromagnetic waves have also been studied. Singh and Honzawa[36] obtained the K–P equation for an ion acoustic soliton in a weakly relativistic plasma, with considering the finite temperature of ions. It is shown that both the relativistic effect and the ion temperature greatly influence the phase velocity, amplitude, and the width of the solitary wave. It was observed that the behaviors of the 2D ion acoustic solitons are considerably influenced by the relativistic effect in a weakly relativistic plasma with cold ions.[37] In the weakly relativistic 2D thermal plasma, the existences of ion-acoustic solitary waves have been discussed by Han *et al.*[43] and they concluded that the amplitude, width of the newly formed nonlinear waves are significantly controlled by the relativistic factor. The effects of density gradient and electron inertia in relativistic plasma containing electrons and ions have been investigated by Malik,[44] and he concluded that a soliton is possible only for the values of ion drift velocity under a certain limit, and this limit of the ion drift velocity is determined by the ion temperature. The K–P equation in ion acoustic wave in weakly relativistic plasma with nonthermal electron, positron, and warm ion has been studied by Pakzad and it was found that the amplitude (width) of soliton decreases (increases) as the velocity of the ion approaches to light speed.[45] Therein it was also reported that the amplitude and width of solitons in three components (ion, positronm, and electron) are less than those of two components (electron and ion).

However, the study of the K–P–Burger equation in an unmagnetized negative ion-rich relativistic plasma with considering the effect of kinematic viscosity in dusty plasma has not been considered in detail to date. Here, in this paper we investigate the features of ion acoustic solitary waves in an unmagnetized negative ion-rich dusty plasma in a weakly relativistic regime by analyzing the solutions of the K–P equation and K–P–Burgers equation, derived with the help of a reductive perturbation method.

2. Basic equations and formulation of the problem

We consider the propagation of a dust ion acoustic wave in a multi-component, collisionless, unmagnetized relativistic dusty plasma consisting of electrons, positive and negative ions, and highly charged dust grains. In the present study, the dynamics of charged dusts are not taken into account because dusts are too heavy to respond to the time scale of the ion-acoustic wave. We also assume that the negative ions are heavier than the positive ions. The immobile dust particles carry some charges so as to maintain the overall charge neutrality condition given by

$$n_{e0} + n_{n0} = n_{p0} - n_d z_{d0}, \tag{1}$$

where n_{j0} is the unperturbed number density of charged species j ($j = p, n$ respectively, stand for positive ions, negative ions) and n_{e0} represents electron density, n_{d0} refers to the density of static dusts, and $z_{d0}\,(>0)$ is the dust charge state. The dynamics of dust–ion acoustic wave in such a dusty plasma system can be expressed by the following basic equations: the continuity and momentum equation for both positive and negative ions, which can be described in three-dimensional (3D) space as follows:

$$\frac{\partial n_j}{\partial t} + \nabla.(n_j v_j) = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + v_j \nabla\right)(\gamma_j v_j) = \mp \frac{Z_j q_j}{m_j} \nabla\varphi - \frac{\nabla p_j}{m_j n_j} + \delta_j \nabla^2 v_j. \tag{3}$$

The physical quantities $n_j v_j$ and m_j respectively, denote the number density, velocity, and mass of j-species particles with $Z_{p(n)} = e(-e)$ and T_j is the particle thermodynamic temperature. The relativistic factor is given as $\gamma_j = (1 - v_j^2/c^2)^{-1/2}$, where c is the speed of light and $v_j^2 = v_{xj}^2 + v_{yj}^2 + v_{zj}^2$. The relativistic factor for a weakly relativistic regime can be approximated as $\gamma_j = 1 + (v_j^2/2c^2)$. We have used the adiabatic state in Eq. (3), namely $P_j/P_{j0} = (n_j/n_{j0})^\gamma = N_j^\gamma$, $P_{j0} = n_{j0} k_B T_j$. The adiabatic index $\gamma = 5/3$ [$(2+D)/D$, D is the number of degrees of freedom] is due to the 3D geometry of the system. The electron density is given by

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{k_B T_e}\right), \tag{4}$$

where φ is the electrostatic potential and k_B is the Boltzmann constant and finally the Poisson equation is given by

$$\nabla^2 \varphi = 4\pi e \left(n_e - Z_p n_p + Z_d n_{d0} + Z_n n_n\right), \tag{5}$$

where the physical quantities are normalized: $\phi = e\varphi/k_B T_e$, $N_j = n_j/n_{j0}$, $V_j = v_j/c_i$, and $c_i = \sqrt{k_B T_e/m_n}$ is the ion-acoustic speed. The space and time variables are normalized by the Debye length λ_D and the inverse of the negative-ion plasma frequency ω_{pn}, respectively. $\rho_j = \delta_j/\omega_{pn}\lambda_D^2$ is the kinematic viscosity where $\omega_{pn} = \left(4\pi n_n e^2/m_n\right)^{1/2}$ and $\lambda_D = \left(k_B T/4\pi n_n e^2\right)^{1/2}$ thus, $\omega_{pn}\lambda_d = c_i$.

Therefore, from Eqs. (2), (3), (4), and (5) we obtain the following normalized set of equations:

$$\frac{\partial\left(N_p\right)}{\partial\tau} + \nabla.\left(N_p \times V_P\right) = 0, \tag{6}$$

$$\frac{\partial\left(N_{\mathrm{n}}\right)}{\partial\tau}+\nabla.\left(N_{\mathrm{n}}\times V_{\mathrm{n}}\right)=0, \tag{7}$$

$$N_{\mathrm{p}}^{1/3}\frac{\partial\left(\gamma_{\mathrm{p}}V_{\mathrm{P}}\right)}{\partial\tau}+N_{\mathrm{p}}^{1/3}\left\{V_{\mathrm{P}}\times\nabla\left(\gamma_{\mathrm{p}}V_{\mathrm{P}}\right)\right\} =-\beta N_{\mathrm{p}}^{1/3}\nabla\phi-\frac{5\sigma_{\mathrm{p}}\beta\,\nabla N_{\mathrm{P}}}{3}+\rho_{\mathrm{p}}N_{\mathrm{p}}^{1/3}\nabla^{2}\left(V_{\mathrm{P}}\right), \tag{8}$$

$$N_{\mathrm{n}}^{1/3}\frac{\partial\left(\gamma_{\mathrm{n}}V_{\mathrm{n}}\right)}{\partial\tau}+N_{\mathrm{n}}^{1/3}\left\{V_{\mathrm{n}}\times\nabla\left(\gamma_{\mathrm{n}}V_{\mathrm{n}}\right)\right\} =N_{\mathrm{n}}^{1/3}\nabla\phi-\frac{5\sigma_{\mathrm{n}}\nabla N_{\mathrm{n}}}{3}+\rho_{\mathrm{n}}N_{\mathrm{n}}^{1/3}\nabla^{2}\left(V_{\mathrm{n}}\right), \tag{9}$$

$$\nabla^{2}\phi=\left(\mu_{\mathrm{e}}\exp\left(\phi\right)+N_{\mathrm{n}}-\mu_{\mathrm{p}}N_{\mathrm{p}}+\mu_{\mathrm{d}}\right), \tag{10}$$

where $\mu_{\mathrm{e}}=n_{\mathrm{e0}}/n_{\mathrm{n_{n0}}}$, $\mu_{\mathrm{p}}=n_{\mathrm{p0}}/n_{\mathrm{n0}}$, $\mu_{\mathrm{d}}=Z_{\mathrm{d}}n_{\mathrm{d0}}/n_{\mathrm{n0}}$, $\sigma_{\mathrm{p}}=T_{\mathrm{p}}/T_{\mathrm{e}}$, $\sigma_{\mathrm{n}}=T_{\mathrm{n}}/T_{\mathrm{e}}$, $\beta=m_{\mathrm{n}}/m_{\mathrm{p}}$, $\rho_{\mathrm{p}}=\delta_{\mathrm{p}}/\omega_{\mathrm{pn}}\lambda_{\mathrm{D}}^{2}$, and $\rho_{\mathrm{n}}=\delta_{\mathrm{n}}/\omega_{\mathrm{pn}}\lambda_{\mathrm{D}}^{2}$.

3. Derivation of K–P–Burgers equation

In order to derive the K–P–Burgers equation from Eqs. (4) and (6)–(10) by using the reductive perturbation method, we take the stretched coordinates:

$$\xi=\varepsilon^{1/2}\left(x-\lambda t\right),\ \eta=\varepsilon y,\ \varsigma=\varepsilon z,\ \tau=\varepsilon^{3/2}t,\ \rho_{j}=\varepsilon^{1/2}\rho_{j0};$$

where ε is the smallness parameter measuring the weakness of the dispersion and λ is the phase speed (normalized by C_i). The physical variables in equations namely N_j, V_j, and ϕ are expanded into a power series in terms of the expansion parameter ε as

$$\left.\begin{aligned}N_{j}&=1+\varepsilon N_{j}^{(1)}+\varepsilon^{2}N_{j}^{(2)}+\varepsilon^{3}N_{j}^{(3)}+\cdots,\\V_{jx}&=V_{0}+\varepsilon V_{jx}^{(1)}+\varepsilon^{2}V_{jx}^{(2)}+\varepsilon^{3}V_{jx}^{(3)}+\cdots,\\V_{jy,z}&=\varepsilon^{\frac{3}{2}}V_{jy,z}^{(1)}+\varepsilon^{\frac{5}{2}}V_{jy,z}^{(2)}+\varepsilon^{\frac{7}{2}}V_{jy,z}^{(3)}+\cdots,\\\phi&=\varepsilon\,\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+\varepsilon^{3}\phi^{(3)}+\cdots.\end{aligned}\right\} \tag{11}$$

Putting the stretched co-ordinates and expansion series into Eqs. (4) and (6)–(10) and after equating the lowest order of ε, we obtain the following relations for the first order perturbations:

$$N_{\mathrm{p}}^{(1)}=a_{1}\phi^{(1)},\ a_{1}=\beta\Big/\left[\left(\lambda^{2}-\lambda V_{0}\right)\gamma_{1}-\frac{5}{3}\beta\sigma_{\mathrm{p}}\right], \tag{12}$$

$$V_{\mathrm{p}}^{(1)}=\lambda a_{1}\phi^{(1)}, \tag{13}$$

$$N_{\mathrm{n}}^{(1)}=-a_{2}\phi^{(1)},\ a_{2}=1\Big/\left[\left(\lambda^{2}-\lambda V_{0}\right)\gamma_{1}-5\sigma_{\mathrm{n}}/3\right], \tag{14}$$

$$V_{\mathrm{nx}}^{(1)}=-\lambda a_{2}\phi^{(1)}, \tag{15}$$

where $\gamma_1=(1+(3/2)\gamma^2)$ with $\gamma=V_0/c$.

Together with the expression for the phase velocity in the moving frame of reference as

$$\lambda=\frac{1}{2\gamma_{1}}\left[\gamma_{1}V_{0}\pm\sqrt{\left(\gamma_{1}V_{0}\right)^{2}-4\gamma_{1}\left(\frac{u\pm\sqrt{u^{2}-4\mu_{\mathrm{e}}v}}{2\mu_{\mathrm{e}}}\right)}\right], \tag{16}$$

where

$$u=\left\{\frac{5}{3}\mu_{\mathrm{e}}\left(\sigma_{\mathrm{p}}\beta+\sigma_{\mathrm{n}}\right)+\left(1+\mu_{\mathrm{p}}\beta\right)\right\};$$

$$v=\frac{5}{3}\beta\left\{\left(\sigma_{\mathrm{p}}+\sigma_{\mathrm{n}}\mu_{\mathrm{p}}\right)+\frac{5}{3}\sigma_{\mathrm{p}}\sigma_{\mathrm{n}}\mu_{\mathrm{e}}\right\},$$

proceeding to the next higher order of ε, we obtain the following set of equations for the second-order perturbed quantities:

$$\frac{\partial N_{\mathrm{p}}^{(1)}}{\partial\tau}-\lambda\frac{\partial N_{\mathrm{p}}^{(2)}}{\partial\xi}+\frac{\partial V_{\mathrm{px}}^{(2)}}{\partial\xi}+\frac{\partial\left(N_{\mathrm{p}}^{(1)}V_{\mathrm{px}}^{(1)}\right)}{\partial\xi} +\frac{\partial V_{\mathrm{py}}^{(1)}}{\partial\eta}+\frac{\partial V_{\mathrm{pz}}^{(1)}}{\partial\zeta}=0, \tag{17}$$

$$\frac{\partial N_{\mathrm{n}}^{(1)}}{\partial\tau}-\lambda\frac{\partial N_{\mathrm{n}}^{(2)}}{\partial\xi}+\frac{\partial V_{\mathrm{nx}}^{(2)}}{\partial\xi}+\frac{\partial\left(N_{\mathrm{n}}^{(1)}V_{\mathrm{nx}}^{(1)}\right)}{\partial\xi} +\frac{\partial V_{\mathrm{ny}}^{(1)}}{\partial\eta}+\frac{\partial V_{\mathrm{nz}}^{(1)}}{\partial\zeta}=0, \tag{18}$$

$$\gamma_{1}\frac{\partial V_{\mathrm{px}}^{(1)}}{\partial\tau}+\left(V_{0}-\lambda\right)\gamma_{2}\frac{\partial V_{\mathrm{px}}^{(1)^{2}}}{\partial\xi}+\left(V_{0}-\lambda\right)\gamma_{1}\frac{\partial V_{\mathrm{px}}^{(2)}}{\partial\xi} +\gamma_{1}V_{\mathrm{px}}^{(1)}\frac{\partial V_{\mathrm{px}}^{(1)}}{\partial\xi}+\frac{\left(V_{0}-\lambda\right)}{3}\gamma_{1}N_{\mathrm{p}}^{(1)}\frac{\partial V_{\mathrm{px}}^{(1)}}{\partial\xi}+\beta\frac{\partial\phi^{(2)}}{\partial\xi} +\frac{\beta}{3}N_{\mathrm{p}}^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+\frac{5}{3}\beta\sigma_{\mathrm{p}}\frac{\partial N_{\mathrm{p}}^{(2)}}{\partial\xi}+\rho_{\mathrm{p0}}\frac{\partial^{2}V_{\mathrm{px}}^{(1)}}{\partial\xi^{2}}=0, \tag{19}$$

$$\gamma_{1}\frac{\partial V_{\mathrm{nx}}^{(1)}}{\partial\tau}-\frac{\lambda}{3}\gamma_{1}N_{\mathrm{n}}^{(1)}\frac{\partial V_{\mathrm{nx}}^{(1)}}{\partial\xi}-\lambda\gamma_{1}\frac{\partial V_{\mathrm{nx}}^{(2)}}{\partial\xi}+\left(V_{0}-\lambda\right)\gamma_{2}\frac{\partial V_{\mathrm{nx}}^{(1)^{2}}}{\partial\xi} +V_{0}\gamma_{1}\frac{\partial V_{\mathrm{nx}}^{(2)}}{\partial\xi}+\gamma_{1}V_{\mathrm{nx}}^{(1)}\frac{\partial V_{\mathrm{nx}}^{(1)}}{\partial\xi}+\frac{V_{0}}{3}\gamma_{1}N_{\mathrm{n}}^{(1)}\frac{\partial V_{\mathrm{nx}}^{(1)}}{\partial\xi}-\frac{\partial\phi^{(2)}}{\partial\xi} -\frac{1}{3}N_{\mathrm{n}}^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+\frac{5}{3}\sigma_{\mathrm{n}}\frac{\partial N_{\mathrm{n}}^{(2)}}{\partial\xi}+\rho_{\mathrm{n0}}\frac{\partial^{2}V_{\mathrm{nx}}^{(1)}}{\partial\xi^{2}}=0, \tag{20}$$

with $\gamma_2=3\gamma/(2c)$, $\rho_{\mathrm{p0}}=\delta_{\mathrm{p0}}/(\omega_{\mathrm{pn}}\lambda_{\mathrm{D}}^{2})$, and $\rho_{\mathrm{n0}}=\delta_{\mathrm{n0}}/(\omega_{\mathrm{pn}}\lambda_{\mathrm{D}}^{2})$.

$$\frac{\partial}{\partial\xi}\left(\frac{\partial V_{\mathrm{py}}}{\partial\eta}\right)+\frac{\partial}{\partial\xi}\left(\frac{\partial V_{\mathrm{pz}}}{\partial\zeta}\right) =-\frac{\beta}{\left(V_{0}-\lambda\right)}\left(1+\frac{5}{3}\sigma_{\mathrm{p}}a_{1}\right)\left\{\frac{\partial^{2}\phi}{\partial\zeta^{2}}+\frac{\partial^{2}\phi}{\partial\eta^{2}}\right\}, \tag{21}$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial V_{\mathrm{ny}}}{\partial\eta}\right)+\frac{\partial}{\partial\xi}\left(\frac{\partial V_{\mathrm{nz}}}{\partial\zeta}\right) =\frac{1}{\left(V_{0}-\lambda\right)}\left(1+\frac{5}{3}\sigma_{\mathrm{n}}a_{2}\right)\left\{\frac{\partial^{2}\phi}{\partial\eta^{2}}+\frac{\partial^{2}\phi}{\partial\zeta^{2}}\right\}, \tag{22}$$

$$\frac{\partial^{2}\phi^{(1)}}{\partial\xi^{2}}=\mu_{\mathrm{e}}\phi^{(2)}+\frac{1}{2}\mu_{\mathrm{e}}\left(\phi^{(1)}\right)^{2}+N_{\mathrm{n}}^{(2)}-\mu_{\mathrm{p}}N_{\mathrm{p}}^{(2)}. \tag{23}$$

Eliminating the second-order quantities from Eqs. (17)–(23) with the help of first order Eqs. (12)–(16), we obtain the following evolution equation of the K–P–Burgers equation:

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi^{(1)}}{\partial\tau}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+B\frac{\partial^{3}\phi^{(1)}}{\partial\xi^{3}}-C\frac{\partial^{2}\phi^{(1)}}{\partial\xi^{2}}\right) +D\left\{\frac{\partial^{2}\phi}{\partial\zeta^{2}}+\frac{\partial^{2}\phi}{\partial\eta^{2}}\right\}=0, \tag{24}$$

where $A=q/p$, $B=r/p$, $C=s/p$, $D=t/p$,

$$p = \frac{(\lambda a_2\gamma_1-(V_0-\lambda)\gamma_1 a_2)\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)+\mu_{\rm p}(\lambda\gamma_1 a_1-(V_0-\lambda)\gamma_1 a_1)(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})}{(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)}, \tag{25}$$

$$q = \frac{\mu_{\rm p}\left\{6(V_0-\lambda)\gamma_2\lambda^2a_1^2+3\gamma_1\lambda^2a_1^2-5(V_0-\lambda)\gamma_1\lambda a_1^2+\beta a_1\right\}}{\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)} - \frac{\left(6(V_0-\lambda)\gamma_2\lambda^2a_2^2+3\gamma_1\lambda^2a_2^2+a_2-5\lambda(V_0-\lambda)\gamma_1a_2^2\right)}{(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})}-\mu_{\rm e}, \tag{26}$$

$$s = \frac{3\lambda a_2\rho_{\rm n0}\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)+3\mu_{\rm p}\lambda a_1\rho_{\rm p0}(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})}{(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)}, \tag{27}$$

$$t = \frac{\gamma_1(3+5\sigma_{\rm n}a_2)\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)+\mu_{\rm p}\gamma_1\beta(3+5\sigma_{\rm p}a_1)(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})}{(3\lambda(\lambda-V_0)\gamma_1-5\sigma_{\rm n})\left(3\lambda(\lambda-V_0)\gamma_1-5\beta\sigma_{\rm p}\right)}, \tag{28}$$

and $r=1$.

Neglecting the dissipation term form Eq. (25), we obtain the K–P equation as

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi^{(1)}}{\partial\tau}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}\right) +D\left\{\frac{\partial^2\phi}{\partial\zeta^2}+\frac{\partial^2\phi}{\partial\eta^2}\right\}=0. \tag{29}$$

The stationary solution of the K–P equation by using the well-known tanh-method is obtained as

$$\phi^{(1)}=\phi_{m1}\,\mathrm{sech}^2\left(\frac{\chi}{\omega_1}\right), \tag{30}$$

where quantities $\phi_{m1}=3\left\{Ul+D\left(m^2+n^2\right)\right\}/Al^2$, $\omega_1=2\left[Bl^3/\left\{Ul+D\left(m^2+n^2\right)\right\}\right]^{1/2}$, and $\chi=(l\xi+m\eta+n\zeta-U\tau)$ are the amplitude, width, and independent variable of the solitary waves, and l, m, n representing the direction cosines of the angle included between the propagation direction and x axis, y axis, and z axis respectively.

Similarly, if we neglect the dispersion term from Eq. (24), we obtain the 3D Burgers equation as

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi^{(1)}}{\partial\tau}+A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi}-C\frac{\partial^2\phi^{(1)}}{\partial\xi^2}\right) +D\left\{\frac{\partial^2\phi}{\partial\zeta^2}+\frac{\partial^2\phi}{\partial\eta^2}\right\}=0, \tag{31}$$

and the stationary solution of the 3D Burgers equation by using a similar method as

$$\phi^{(2)}=\phi_{m2}\left\{1-\tanh\left(\frac{\chi}{\omega_2}\right)\right\}, \tag{32}$$

where $\phi_{m2}=\left\{D\left(m^2+n^2\right)+Ul\right\}/Al^2$ and $\omega_2=2Cl^3/\left\{D\left(m^2+n^2\right)+Ul\right\}$ are the height and thickness of the shock wave respectively with l, m, n, representing the direction cosines of the angles included between the propagation direction and the x axis, K–P y axis, and z axis, respectively.

To find the solution of the K–P–Burgers equation (24), we use the transformation relation $\chi=(l\xi+m\eta+n\zeta-U\tau)$, where l, m, n are the direction cosines along the x, y, z axes respectively. Finally the stationary solution of this equation by using the same method as that stated earlier with independent variable χ can be obtained as

$$\phi^{(1)} = \frac{D\left(1-l^2\right)+Ul}{Al^2}-\frac{12Cl}{5A}\tanh(\chi) +\frac{12Bl^2}{A}\left[1-\tanh^2(\chi)\right], \tag{33}$$

where $C=10Bl$ and $\left(Dl^2+Ul-D\right)=24Bl^4$. Equation (33) is a solution of the K–P–Burgers equation (24), which provides the evolution of small-amplitude DIA shock wave in a dusty plasma system.

4. Result and discussion

The main aim of the theoretical model presented here, is to examine the properties of the dust ion acoustic solitary waves in an un-magnetized negative ion-rich dusty plasma in the weakly relativistic regime. We have numerically analyzed the relativistic effect on the propagation of a dust ion acoustic solitary wave in a multicomponent dusty plasma with both positive and negative ions in a viscous dusty plasma system. Here, the dust particles in plasma are considered to be uniform in size and negatively charged with equal charged state, while the other components of background plasma are singly charged positive and negative ions. In the subsequent sections, the dependences of different dusty plasma parameters on the course of evolution solitary waves in the relativistic regime are studied with considering both the K–P and the 3D Burger equation.

4.1. K–P equation

The stationary solution obtained from the K–P equation given in expression (30), clearly resembles a typical evolution equation of a solitary wave. Figures 1 and 2 represent the dependences of the wave amplitude on the temperatures of

plasma constituents and the relativistic parameter $\gamma\,(=v/c)$.

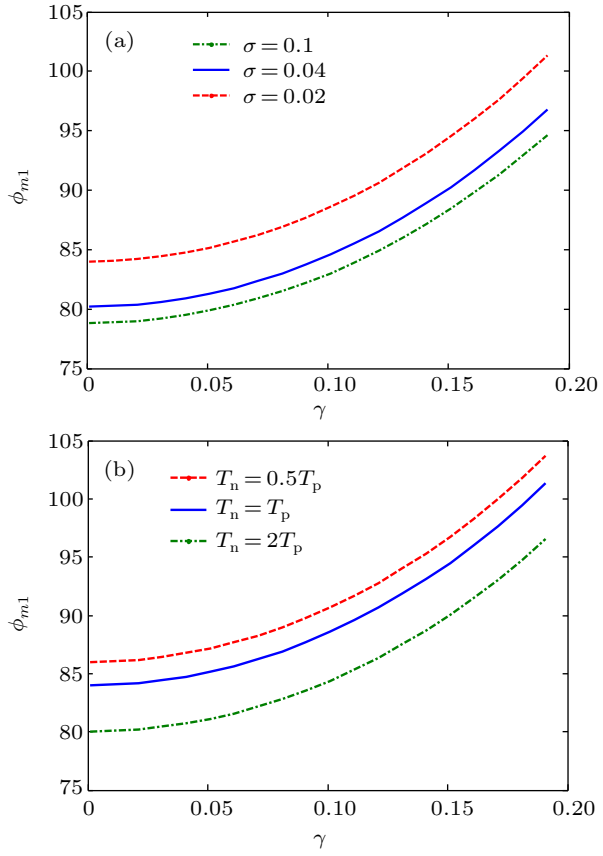


Fig. 1. (color online) (a) Effect of relativistic parameter along with the influence of positive ion temperature on the solitary wave amplitude, (b) variation of wave amplitude with the relativistic effect for different ratios of negative ion temperature (T_n) to positive ions temperature (T_p). Here, $m_p = 40\times10^{-27}$ kg (argon positive ion), $m_n = 16\times10^{-27}$ kg (oxygen negative ion), $m_d = 10^{-18}$ kg, dust temperature, $T_d = 0.06$ eV, $n_{e0} = 3.8\times10^{14}$, $n_{p0} = 5.5\times10^{14}$, $n_{n0} = 0.3\times10^{14}$, $n_{d0} = 2\times10^{10}$, $v_0 = 1$, $z_{d0} = 1.5\times10^4 e$, and $r_d = 2.0\times10^{-6}$ m. The dash line is for $T_n = 0.5T_p$, the solid line is for $T_n = T_p$, and the dash–dotted line is for $T_n = 2T_p$.

The effect of the positive ion temperature on the wave amplitude is shown in Fig. 1(a), which implies that with increasing the positive ion temperature, the wave amplitude ϕ_{m1} is reduced, showing that the depth of the localized potential structure also decreases. But, for all the cases wave amplitudes increase whith increasing the relativistic parameter γ and it is also seen that the relative depth reduction is less effective as the relativistic parameter of the plasma is enhanced. It is also observed that the wave amplitude depends on the temperature ratio of both the ions, and as shown in Fig. 1(b), the wave amplitudes decrease with increasing the ratio (σ) of the negative ion temperature (T_n) to positive ions temperature (T_p). Here the other plasma parameters are considered to be $m_p = 40\times10^{-27}$ kg (argon positive ion), $m_n = 16\times10^{-27}$ kg (oxygen negative ion), $m_d = 10^{-18}$ kg, dust temperature, $T_d = 0.06$ eV, $n_{e0} = 3.8\times10^{14}$, $n_{p0} = 5.5\times10^{14}$, $n_{n0} = 0.3\times10^{14}$, $n_{d0} = 2\times10^{10}$, $v_0 = 1$, $z_{d0} = 1.5\times10^4$e, and $r_d = 2.0\times10^{-6}$ m. The dash line is for $T_n = 0.5T_p$, the solid line is for $T_n = T_p$, and the dash–dotted line is for $T_n = 2T_p$.

The influence of the mass ratio $\beta\,(=m_n/m_p)$ of the ions on the amplitude of the solitary wave under the relativistic condition is shown in Fig. 2, the dash line is for $\beta = 5$, the solid line is for $\beta = 3$, and the dash–dotted line is for $\beta = 1$; which implies that the wave amplitude increases with mass ratio. The spatial evolution of solitary wave governed by Eq. (30) is plotted in Fig. 3 against the relativistic parameter, and it is seen that with increasing the relativistic parameter γ from 0.01 to 0.2, the amplitude of the solitary wave increases appreciably. So it can be concluded that the relativistic streaming factor energizes the localized potential profile of the solitary wave significantly.

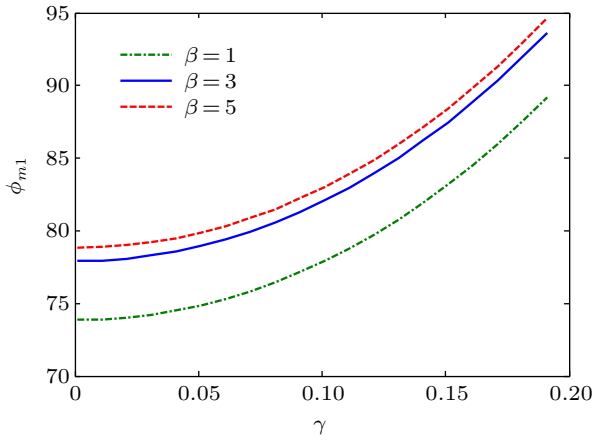


Fig. 2. (color online) Influences of the mass ratio $\beta\,(=m_n/m_p)$ of the ions on the amplitude of the solitary waves under the relativistic condition, the dash line is for $\beta = 5$, the solid line is for $\beta = 3$, and the dash–dotted line is for $\beta = 1$.

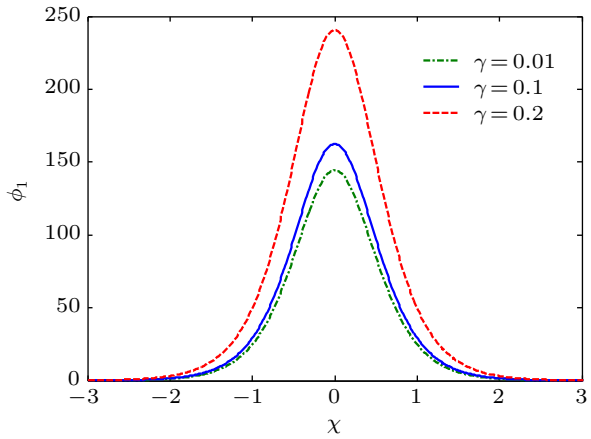


Fig. 3. (color online) Spatial variations of the solitary wave amplitude for different relativistic plasma conditions, the dash line is for $\gamma = 0.2$, the solid line is for $\gamma = 0.1$, and the dash–dotted line is for $\gamma = 0.01$.

4.2. 3D Burgers equation

We have also solved the 3D Burger equation in an unmagnetized negative ion-rich relativistic plasma in the presence of kinematic viscosity of dusty plasma in a weakly relativistic regime by using the reductive perturbation method. The stationary solution of the 3D Burger equation is given in Eq. (32). Analyzing the evolution equation of a viscous dusty plasma

system governed by Eq. (32), we see a shock-like potential profile as shown in Figs. 4(a) and 4(b).

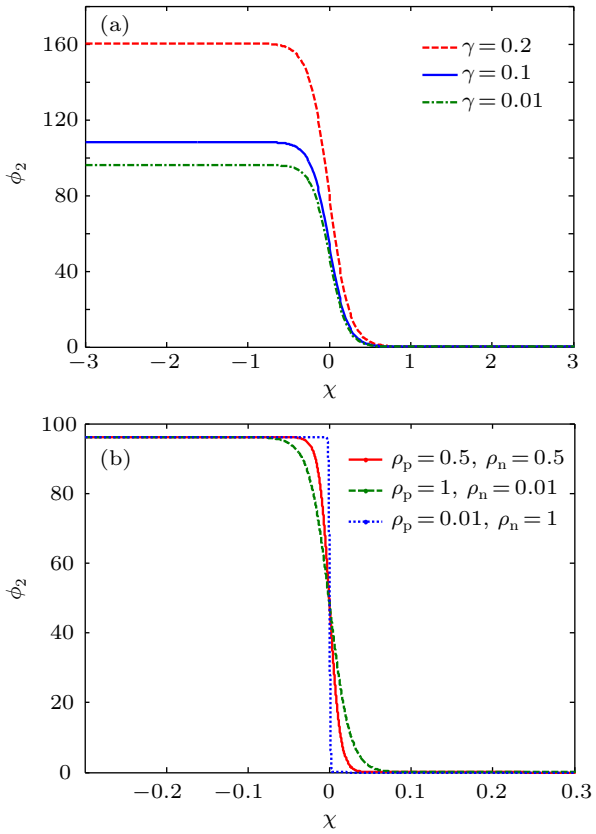


Fig. 4. (color online) (a) Spatial variations of the shock wave, obtained from the solution of the 3D Burgers equation for different relativistic parameters, the dash line is for $\gamma = 0.2$, the solid line is for $\gamma = 0.1$, and the dash–dotted line is for $\gamma = 0.01$; (b) spatial variations of the shock variation, obtained from the solution of 3D Burgers equation for different kinematic viscosity effects at $\gamma = 0.01$, the dotted line is for $\rho_p = \rho_n = 0.5$, the solid line is for $\rho_p = 1$, $\rho_n = 0.01$, and the dash line is for $\rho_p = 0.01$, $\rho_n = 1$.

Figure 4(a) shows the spatial variations of the shock potential profile obtained from the solution of the 3D Burger equation for different relativistic parameters, where the dash line is for $\gamma = 0.2$, the solid line is for $\gamma = 0.1$ and the dash–dotted line is for $\gamma = 0.01$, and it is seen that the relativistic effect predominantly enhances the amplitude of the shock wave. On the other hand, the effect of kinematic viscosity of both the ions also plays a key role in the structure of the shock wave as shown in Fig. 4(b). Here the dotted line is for $\rho_p = \rho_n = 0.5$, the solid line is for $\rho_p = 1$, $\rho_n = 0.01$, and the dash line is for $\rho_p = 0.01$, $\rho_n = 1$ and we have considered $l = 0.4$, $m^2 + n^2 = 1 - l^2$, and $\gamma = 0.01$ with other parameters to be the same as those stated above in the case of Fig. 1. Now as seen from Fig. 4(b), the kinematic viscosity plays an imperative role in controlling the shock potential profile. In the presence of kinematic viscosity a diminishing shock potential is seen compared with the case in the absence of kinematic viscosity shown in Fig. 4(a). Thus it can be inferred that the effect of dissipation becomes dominant in determining the nature as well as the magnitude of the shock potential profile in relativistic viscous dusty plasma.

4.3. K–P–Burgers equation

For the present plasma model under discussion, we have also examined the behavior of the K–P–Burgers equation. The stationary solution is the same as that stated in Eq. (33). It is seen that the stationary solution in Eq. (33) contains three terms. Here the coefficients of tanh and $\tanh^2$ terms determine the structure of the wave. So analyzing Eq. (33) numerically with other parameters as stated above, we have found a shock-like pattern of the potential profile (Fig. 5(a)). Thus as shown in Fig. 5(a), the spatial evolution of the shock wave profile for the K–P–Burgers equation with relativistic effect shows an increase in amplitude as γ is increased from 0.01 to 0.2.

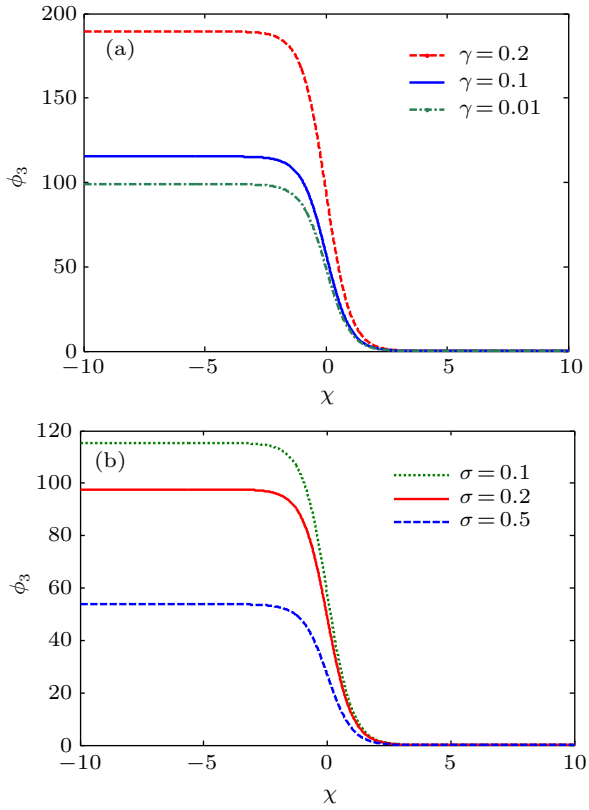


Fig. 5. (color online) (a) Spatial shock wave profiles for the K–P–Burgers equation with different relativistic effects, the dash line is for $\gamma = 0.2$, the solid line is for $\gamma = 0.1$, and the dash–dotted line is for $\gamma = 0.01$; (b) spatial variations of the shock wave with ion temperature ratio, the dotted line is for $\sigma = 0.1$, the solid line is for $\sigma = 0.2$, and the dash line is for $\sigma = 0.5$ at $\gamma = 0.1$.

On the other hand the spatial variations of the shock wave potential with ion temperature ratio (σ) are plotted in Fig. 5(b) for different values of σ ranging from 0.1 to 0.5 with keeping $\gamma = 0.1$. It is evident from Fig. 5(b) that σ has a substantial role in the amplitude of the shock wave. It is also clear that the thermal velocity of each of the ion species has a crucial effect on the height of the shock potential profile. It can be concluded that with increasing (decreasing) the thermal velocity of negative (positive) ion, a reduction in the shock wave potential is seen.

5. Conclusions

In this work, we attempt to solve the problem of nonlinear dust ion acoustic waves in a multi-ion dusty plasma with considering the relativistic effect. Here the K–P and K–P–Burgers equations are derived by using the reductive perturbation method including the effects of viscosity of plasma fluid, thermal energy, ion density, and ion temperature on the structure of dust ion acoustic shock wave (DIASW). The existence of stationary soliton-like small amplitude solitary wave is predicted by the K–P equation whereas the K–P–Burgers equation predicts the evolution of the shock-like structure. From the stationary solution of the KP equation, the dependences of the solitary wave amplitude on the temperature of the plasma constituent and the relativistic parameter $\gamma\,(=v/c)$ are studied. Also in the presence of kinematic viscosity of dusty plasma and negative ions in a weakly relativistic regime, the 3D Burgers equation is derived. Analyzing the stationary solution of the 3D Burgers equation numerically, a shock-like structure is seen. It is also seen that the relativistic effect enhances the amplitude of the shock wave and the kinematic viscosities of both the ions also play a key role in the structure of the shock wave. Finally, in the same plasma system we also examine the behavior of the K–P–Burgers equation. It is seen that the stationary solution of the K–P–Burgers equation contains three terms but the coefficients of the tanh and $\tanh^2$ term determine the structure of the wave and here also a shock-like structure is observed. Also it is seen that the relativistic factor and ion temperature ratio control the structure and the amplitude of the shock wave. We hope this work will encourage researchers to carry out further work on the 3D relativistic multi-component dusty plasma related problems.

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