Absorption of laser radiation in ultracold plasma

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In this paper, we analyze the spectral behavior (optical thickness, shape and linewidth) of laser radiation absorption under the correlation heating of ions in an ultracold plasma. The Voigt formula is used to find the absorption coefficient. The spectral line width is shown to grow with time while the optical thickness reduces. Our modeling results are in good agreement with the experimental findings reported in the literature.

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1. Introduction

Ultracold plasma (UCP) was first obtained in Ref. [1] where the method of near-threshold two-stage laser ionization proposed in Ref. [2] was used to turn laser-cooled ^{88}Sr atoms into the state of UCP. In such a plasma, the ion temperature T_i varies from 10^{-2} K to 1 K, the electron temperature T_e ranges from 1 K to 1000 K and the concentration of charged particles is $n \leq 10^{10}$ cm^{-3}.[3] The availability of such a medium opens up unique prospect for studying macroscopic and microscopic processes in ultracold plasma. In Refs. [4] and [5] a method of cooling ions was proposed aiming at obtaining non-ideal ion-ion interaction in electron-ion plasma. Later it was shown that the nonlinear dependence of laser friction force on the velocity of ions is an important consideration for laser cooling[6] and the spectrum of cooling laser radiation was described.[7]

Photoionization of atoms induces ion–ion interaction resulting in correlation heating of ions.[8] Absorption spectroscopy is one of the instruments to study gases.[9] In this paper we use this instrument to study spectral characteristics of absorption (optical thickness and line shape and width) in UCP under correlation heating and compare our results with the experimental findings.[3,10,11]

2. Absorption model

Formation of UCP starts with two-stage photoionization of an ensemble of cooled atoms having spherical geometry; therefore the initial space distribution of ions is a replica of atomic distribution and is described by the following expression:[3]

$$n(r) = n_0 \exp\left\{-\frac{r^2}{2\sigma^2}\right\}, \tag{1}$$

where n_0 is the ion concentration in the center of the sphere, the size of the latter being dependent on σ.

Radiation absorption $I(\nu, r)$ obeys the Beer–Bouger–Lambert law

$$\mathrm{d}I(\nu, r) = -k(\nu, r) I(\nu, r)\,\mathrm{d}r, \tag{2}$$

where $k(\nu, r)$ is the absorption coefficient at ν frequency at point r.

The shape of absorption profile in such a plasma is determined by Lorentz broadening resulting from spontaneous decay and Doppler broadening due to thermal motion of ions. Under the joint influence of these broadenings the absorption coefficient has the Voigt profile[12] and, considering Eq. (1), will be given by

$$k(\nu, r) = k_0 \exp\left\{-\frac{r^2}{2\sigma^2}\right\} H(a, \omega), \tag{3}$$

where the Voigt function $H(a, \omega)$ is as follows:

$$H(a, \omega) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-y^2}\,\mathrm{d}y}{a^2 + (\omega - y)^2}, \tag{4}$$

$$\omega = \frac{(\nu - \nu_0)}{\nu_0} \frac{c}{V_0}, \tag{5}$$

$$a = \frac{\lambda_0(\gamma + \gamma_\mathrm{L})}{4\pi V_0}, \tag{6}$$

$$k_0 = \frac{\lambda_0^3}{8\pi} n_0 \frac{g_2}{g_1} \frac{\gamma}{\sqrt{\pi} V_0}. \tag{7}$$

Here c is the speed of light, ν_0 and λ_0 are the wave frequency and length for the absorption line center, g_2 and g_1 are the statistical weights of the excited and ground states, γ is the spontaneous decay probability, $V_0 = \sqrt{2k_\mathrm{B}T_\mathrm{i}/m}$ is the thermal velocity of ions, k_B is the Boltzmann constant, and m is the ion mass. In expression (6), the laser spectral width γ_L has been taken into account for comparison between the theoretical and experimental results.

Considering that k_0 and a depend on V_0, we rewrite expression (7) as

$$k_0 = k_0' a, \tag{8}$$

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where

$$k_0' = \frac{\lambda_0^2}{2} n_0 \frac{g_2}{g_1} \frac{1}{\sqrt{\pi}} \frac{\gamma}{(\gamma+\gamma_L)}. \quad (9)$$

Then expression (3) acquires the form

$$k(\nu, r) = \frac{k_0'}{\pi} e^{-r^2/(2\sigma^2)} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{1+\left(4\pi\frac{(\nu-\nu_0)}{(\gamma+\gamma_L)} - \frac{y}{a}\right)^2} dy. \quad (10)$$

The Voigt profile turns into the Lorentz profile for large c, and the absorption coefficient becomes

$$k_L(\nu, r) = \frac{k_0'}{\sqrt{\pi}} e^{-r^2/(2\sigma^2)} \cdot \frac{1}{1+\left[\frac{4\pi(\nu-\nu_0)}{(\gamma+\gamma_L)}\right]^2}, \quad (11)$$

while for small a it is determined by the Doppler profile

$$k_D(\nu, r) = k_0 e^{-r^2/(2\sigma^2)} \cdot \exp\left[-\left(\frac{(\nu-\nu_0)}{\nu_0}\frac{c}{V_0}\right)^2\right]. \quad (12)$$

3. Results

To analyze the situation, we take the origin of the coordinate system to be the center of the sphere. Then for the Voigt profile, using expression (10), we write an expression for the optical thickness along the diameter when $\nu = \nu_0$, with assuming T_i to be constant over space, as follows:

$$\begin{aligned}\tau_0 &= \frac{k_0'}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{1+(y/a)^2} \exp\left[-z^2/(2\sigma^2)\right] dz dy \\ &= \frac{k_0'}{\pi} \sqrt{2\pi}\sigma \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{1+(y/a)^2} dy. \end{aligned} \quad (13)$$

The optical thickness for Lorentz broadening is found from expression (11) followed by integration and the resulting expression is

$$\tau_0^L = \frac{\lambda_0^2}{2\pi} n_0 \frac{g_2}{g_1} \frac{\gamma}{(\gamma+\gamma_L)} \sqrt{2\pi}\sigma, \quad (14)$$

and the one for Doppler broadening, according to expression (12), is

$$\tau_0^D = k_0' \cdot a \cdot \sqrt{2\pi}\sigma. \quad (15)$$

Our calculations are based on the experimental data for strontium ions:[3,10,11] $\lambda_0 = 421.7$ nm ($^2S_{1/2}$–$^2P_{1/2}$ transition), $\gamma/2\pi = 21$ MHz, $\gamma_L/2\pi = 5$ MHz, $n_0 = 2\cdot 10^{10}$ cm^{-3}, $\sigma = 0.6$ mm, initial $T_i = 10^{-2}$ K, $T_e = 56$ K. Note that for the given parameters $a = 0.39/\sqrt{T_i}$.

The variations of optical thickness with a derived from expressions (13)–(15) for various types of broadenings, are shown in Fig. 1. The results obtained indicate that the optical thickness for Voigt broadening is the same as that for Doppler broadening if $a < 0.06$ and as that for Lorentz broadening if $a > 3$. The optical thickness for $0.06 < a < 3$ is affected by both Lorentz and Doppler broadening.

According to the numerical data (Fig. 1), the optical thickness τ_0 varies from 0.65 for $a = 3.9$, $T_i = 10^{-2}$ K to 0.26 for $a = 0.39$, $T_i = 1$ K. Due to correlation heating the ion temperature grows linearly with time increasing from $T_i = 10^{-2}$ K until it reaches its maximum $T_i \simeq 1$ K at $t \simeq 0.3$ μs.[8] Hence $T_i \simeq 0.3$ K, $a \simeq 0.7$ at $t \simeq 0.1$ μs. Then for $a \simeq 0.7$ from Fig. 1 we have $\tau_0 = 0.36$. The experimental optical thickness under these conditions is $\tau_0 = 0.28$.[3,10] However, considering the ±40% accuracy of the measured ion concentration n_0,[3] the error for the optical thickness will be $\Delta\tau_0 \approx \pm 0.1$, which means that the experimental and calculated data are in good agreement.

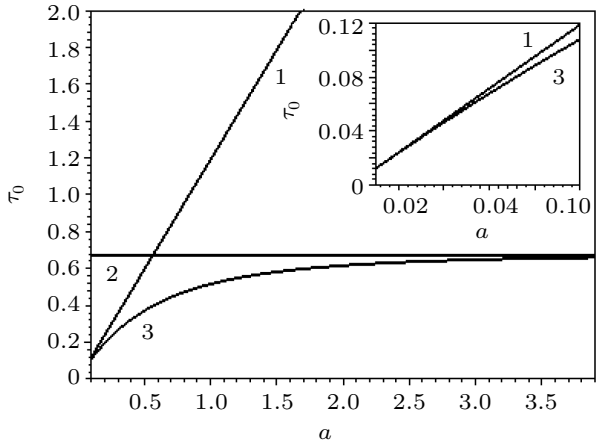


Fig. 1. Plots of optical thickness versus a for various types of broadenings: 1: Doppler broadening (15), 2: Lorentz broadening (14), 3: Voigt broadening (13).

To find an integral optical thickness τ^i, we will use a sphere of radius $R \gg \sqrt{2}\sigma$. Consider the propagation of radiation along the chord parallel to the vector of the direction of propagation e. The chord location is determined by the angle φ from the sphere center between the direction e and the endpoint of the chord on the surface of the sphere (for $\varphi = 0$, radiation propagates along the sphere diameter). So

$$r^2 = R^2 \sin^2\varphi + z^2, \quad (16)$$

where z is the distance along the chord, which varies from $-R\cos\varphi$ to $+R\cos\varphi$. Then

$$\begin{aligned}\tau^i(\nu) = 2k_0' \int_0^{\pi/2} \exp\left[-\frac{R^2\sin^2\varphi}{2\sigma^2}\right] \int_{-R\cos\varphi}^{R\cos\varphi} \exp\left[-\frac{z^2}{2\sigma^2}\right] dz \cdot \\ \times \int_{-\infty}^{\infty} \frac{\exp[-y^2]dy}{1+\left(4\pi\frac{(\nu-\nu_0)}{(\gamma+\gamma_L)} - \frac{y}{a}\right)^2} R^2 \sin\varphi\cos\varphi \cdot d\varphi, \end{aligned} \quad (17)$$

and $\tau^i(\nu_0) = \tau_0^i$ for the line center will be equal to

$$\tau_0^i \approx 2\pi\sigma^2\tau_0. \quad (18)$$

Figure 2 shows the integrated optical thickness τ_0^i as a function of a. The experimental value τ_0^i for $t = 75$ nsec equals 0.8 mm^2.[11] For this timescale, $T_i \simeq 0.25$ K, $a \simeq 0.8$; and

hence from expression (18) and Fig. 2 we have $\tau_0^{\rm i} \simeq 0.9$ mm^2, which agrees with the experimental value.

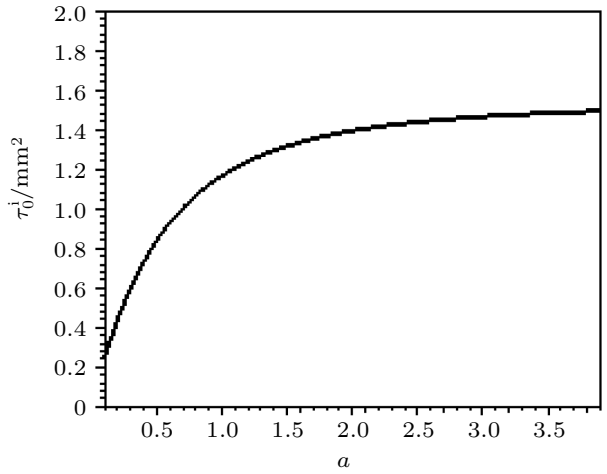


Fig. 2. Optical thickness integrated over sphere for the Voigt profile.

The absorption line profiles for various types of broadenings are given by expressions (10)–(12) and can be seen in Figs. 3–5. Here $\Delta\nu = \nu - \nu_0$. Plotted on the Y-coordinate are absorption coefficients for $\Delta\nu = 0$ divided by $k_0'(\gamma+\gamma_{\rm L})/(\gamma\sqrt{2\pi}\sigma)$.

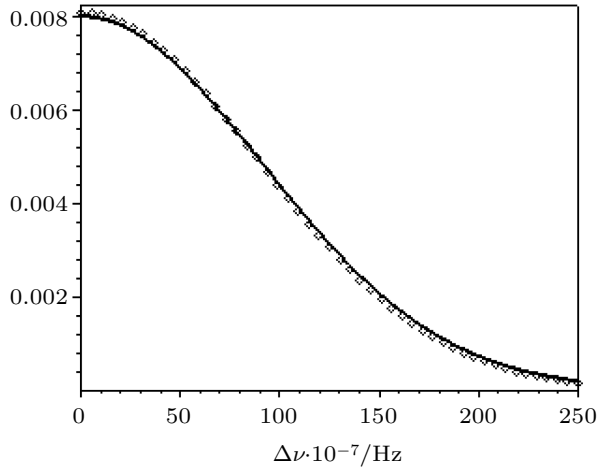


Fig. 3. Absorption line profiles ($a = 0.01$), solid line Voigt broadening, ♢: Doppler broadening.

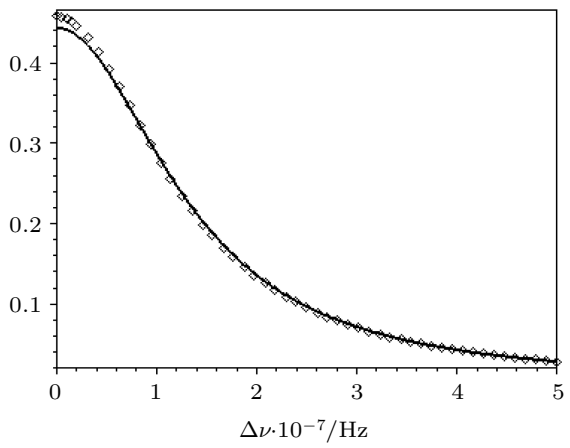


Fig. 4. Absorption profiles ($a = 3.9$), ♢: Lorentz, solid line: Voigt.

The Voigt and Doppler profiles coincide with each other for small a (Fig. 3) while the Lorentz and Voigt absorption curves coincide with each other for large a (Fig. 4). The absorption line profile for any a in between these values is given by the Voigt formula (Fig. 5).

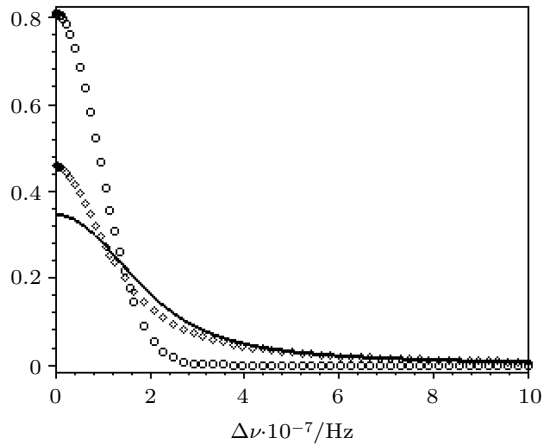


Fig. 5. Absorption profiles ($a = 1$), ♢: Lorentz, o: Doppler, solid line: Voigt.

The temperature $T_{\rm i}$ and a being independent of the coordinates, the absorption line profile for radiation propagating along the sphere diameter will coincide with the one integrated over sphere. It should be noted that the frequency dependence of the absorption coefficient is an important factor to be considered when dealing with laser cooling of ions.[7]

The spectral width of the absorption line $\Delta\nu$ equals double the value $\Delta\nu = \nu_1 - \nu_0$, where ν_1 is the frequency at which the absorption is half the absorption in the center of the line at $\nu = \nu_0$.[12] So the absorption line spectral width is

$$\Delta\nu_{\rm L} = \frac{(\gamma+\gamma_{\rm L})}{2\pi} \tag{19}$$

for the Lorentz profile (see Eq. (11)) and

$$\Delta\nu_{\rm D} = \frac{\sqrt{\ln 2}(\gamma+\gamma_{\rm L})}{2\pi a} \tag{20}$$

for the Doppler profile (see Eq. (12)).

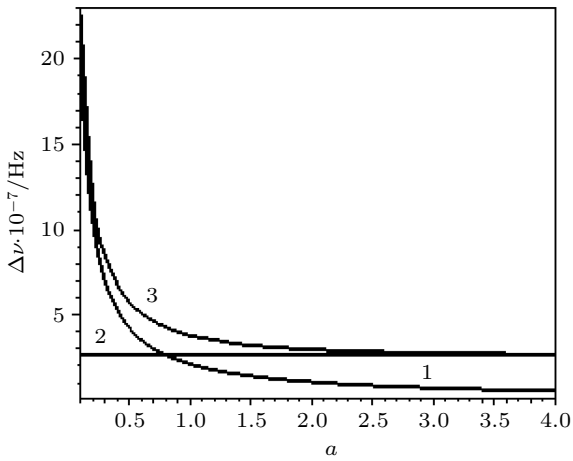


Fig. 6. Absorption line widths versus a for different broadenings, 1: Doppler broadening, 2: Lorentz broadening, 3: Voigt broadening.

The variations of line width with a for various types of broadening are shown in Fig. 6. It is clearly seen that the Lorentz and Voigt line widths coincide with each other for $a > 3$ and the Doppler and Voigt line widths coincide with each other when $a < 0.06$. For $t = 75$ ns, the experimental value is $\Delta\nu \approx 62$ MHz[3,11] and the calculated one is 52 MHz.

4. Conclusions

In this paper, we deal with the absorption of laser radiation in UCP under correlation heating of ions. The spectral characteristics (optical thickness and the line shape and width) are calculated for the absorption coefficient given by the Voigt formula. The spectral data for Doppler and Voigt broadenings are shown to coincide with each other if $a < 0.06$, and the spectral data for Lorentz and Voigt broadenings also coincide with each other when $a > 3$. The increase in the ion temperature due to correlation heating results in a reduced optical thickness and increased absorption line width. The numerical estimates obtained are in good agreement with the experimental results reported in Refs. [3], [10], and [11].

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