Controlled remote preparation of an arbitrary four-qubit cluster-type state*

Wei-Lin Chen(陈维林)[1], Song-Ya Ma(马松雅)[1,†], and Zhi-Guo Qu(瞿治国)[2]

[1] School of Mathematics and Statistics, Henan University, Kaifeng 475004, China

[2] Jiangsu Engineering Center of Network Monitoring, Nanjing University of Information Science and Technology, Nanjing 210044, China

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Two schemes are proposed to realize the controlled remote preparation of an arbitrary four-qubit cluster-type state via a partially entangled channel. We construct ingenious measurement bases at the sender's and the controller's locations, which play a decisive role in the proposed schemes. The success probabilities can reach 50% and 100%, respectively. Compared with the previous proposals, the success probabilities are independent of the coefficients of the entangled channel.

Keywords: controlled remote state preparation, partially entangled channel, measurement basis, quantum network communication

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1. Introduction

Information security is regularly regarded as one of the most essential issues for network technology, especially when cloud computing technology is widely spreading.[1–4] In order to resolve this issue, quantum networks are rapidly developing as the next generation networks, in which the secure transmission of any quantum state is a fundamental requirement. The application of quantum entanglement in quantum communication provides some novel ways for the transmission of a quantum state. One celebrated protocol is quantum teleportation (QT) which transfers an unknown qubit from a sender to a distant receiver by dual usage of local operation and classical communication.[5] In the case that the sender knows about the original state completely, another remarkable method called remote state preparation (RSP) was proposed,[6] which uses the same entanglement resource as in QT but with simpler local operation and less classical communication cost for some special ensembles of states. In virtue of its unique advantage, various theoretical aspects have been investigated for the generalization of RSP[7–30] and some experimental implementations have been reported.[31–36]

In addition to the traditional RSP with two parties, multi-party RSP plays a significant role in the general quantum network communication and quantum distributed computation. One branch of multi-party RSP is called joint remote state preparation (JRSP).[15–24] Different from conventional RSP protocols in that all the secret information is provided to one sender which may lead to leakage of information, JRSP deals with the situation that the complete knowledge of the quantum state is independently shared among a number of senders who must collaborate to complete the task. Another branch of multi-party RSP is called controlled remote state preparation (CRSP).[25–30] In CRSP, one or several controllers are introduced in addition to the sender and the receiver. The controller in CRSP has no information about the prepared state which is different from the added sender in JRSP. The controlling party determines whether the quantum state is constructed for the receiver or not. In order to achieve the optimal success probability and fidelity, most of the two-party and multi-party RSP proposals assume the quantum channels are maximally entangled. However, it is most of the time not possible to generate or maintain a maximally entangled state at one's disposal. Therefore, it is important to investigate the RSP via non-maximally entangled quantum channel. Inevitably, both the success probability and the fidelity decrease with the degree of entanglement of the quantum channel.

Cluster state is one of the most important entangled resources in quantum information and can be efficiently applied to information processing tasks, such as quantum teleportation,[37] quantum dense coding,[38] quantum secret sharing,[39] quantum computation,[40] and quantum secure direct communication.[41] Especially, the cluster state has the properties of the GHZ-class and the W-class entangled states, and it is harder to be destroyed by local operations.[42] Thus, many authors proceed to focus on RSP or JRSP for cluster-type state by exploring various novel methods.[12,13,18–20] For example, Ma and Zhan have presented a scheme to remotely prepare a four-qubit cluster-type state.[12] Their scheme can be successfully realized with the probability 25% via four EPR pairs as the quantum channel and by a set of four-particle orthogonal basis projective measurements. Moreover, they also investigated that the quantum channel is composed of non-maximally entangled states. It is shown that the receiver can reestablish the original state with a certain probability. To

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†Corresponding author. E-mail: masongya0829@126.com

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achieve high success probability, the remote preparation of the cluster-type state is re-investigated.[13] The authors also considered the case that the quantum channel is partially entangled. Like many RSP schemes via non-maximally entangled channels, auxiliary qubits and two-qubit unitary transformations are required to prepare the target state. The RSP schemes succeed with certain probabilities which depend on the degree of entanglement of the quantum channels.

Recently, Wang *et al.* put forward two CRSP schemes for arbitrary single-qubit and two-qubit states via partially entangled channels.[29] Neither auxiliary qubits nor two-qubit unitary transformations are required in their schemes. The schemes succeed with the probabilities 50% and 25% for arbitrary single- and two-qubit states, respectively. The success probabilities are independent of the coefficients of the quantum channel. Chen *et al.* further proposed a deterministic scheme to realize the CRSP of an arbitrary two-qubit state by using the method of information splitting.[30] The key technology in their schemes is that single-qubit measurement basis is constructed for the controller.[29,30] Following some of the ideas of Refs. [29] and [30] we examine the implementations of controlled remote preparation for a family of four-qubit cluster-type states with the aid of a partially entangled quantum channel. Two CRSP schemes are proposed. In Section 2, we construct a set of ingenious measurement bases which play a decisive role in our scheme. The sender and the controller collaborate with each other to perform projective measurements on their own particles, and the receiver performs appropriate local unitary operations to recover the prepared state. Without introducing any auxiliary particles and two-qubit unitary transformations, the success probability can reach 50% which is independent of the coefficients of the entangled channel. It is worth mentioning that the measurement basis we construct for the controller is simpler than that in Refs. [29] and [30]. In Section 3, the other deterministic CRSP scheme of an arbitrary four-qubit cluster-type state is considered. Discussions and conclusions are given in the last section.

2. The first scheme with the success probability 50%

Suppose the sender Alice wants to help the receiver Bob remotely prepare an arbitrary four-qubit cluster-type state

$$f(x_0,x_1,x_2,x_3)=x_0|0000\rangle+x_1|0011\rangle+x_2|1100\rangle-x_3|1111\rangle \tag{1}$$

under the control of the supervisor Charlie, where the complex numbers x_j, $j\in\{0,\ldots,3\}$ satisfy the normalization condition $\sum_{j=0}^{3}|x_j|^2=1$. Alice knows the parameters x_j exactly, while Bob and Charlie do not have any information about the prepared state.

Assume all the participants share two partially entangled states:

$$\begin{aligned}|\Psi\rangle_{A_1B_1B_2C_1}&=\frac{1}{\sqrt{2}}(|0000\rangle+a_1|1111\rangle+b_1|1110\rangle)_{A_1B_1B_2C_1},\\ |\Psi\rangle_{A_2A_3B_3B_4C_2}&=\frac{1}{\sqrt{2}}(|00000\rangle+a_2|11111\rangle+b_2|11110\rangle)_{A_2A_3B_3B_4C_2},\end{aligned} \tag{2}$$

where $a_j^2+b_j^2=1$, $j=0,1$. The particles (A_1,A_2,A_3) belong to the sender Alice, the particles (B_1,B_2,B_3,B_4) belong to the receiver Bob, and the controller Charlie owns the particles (C_1,C_2). Hence, the initial state of the whole system can be written as

$$\begin{aligned}|\Psi\rangle_{A_1B_1B_2C_1}\otimes|\Psi\rangle_{A_2A_3B_3B_4C_2}=\ &\frac{1}{2}[|000000000\rangle+|0110011\rangle(a_2|01\rangle+b_2|00\rangle)+|1001100\rangle(a_1|10\rangle+b_1|00\rangle)\\ &+|1111111\rangle(a_1a_2|11\rangle+a_1b_2|10\rangle+a_2b_1|01\rangle+b_1b_2|00\rangle)]_{A_1A_2A_3B_1B_2B_3B_4C_1C_2}.\end{aligned} \tag{3}$$

For implementing CRSP, the procedure can be divided into the following steps.

Step 1 The sender Alice performs a three-particle projective measurement on her particles (A_1,A_2,A_3) under the basis $\{|\zeta_0\rangle,\ldots,|\zeta_7\rangle\}$ which is determined as

$$\begin{pmatrix}|\zeta_0\rangle\\|\zeta_1\rangle\\|\zeta_2\rangle\\|\zeta_3\rangle\\|\zeta_4\rangle\\|\zeta_5\rangle\\|\zeta_6\rangle\\|\zeta_7\rangle\end{pmatrix}=\begin{pmatrix}\boldsymbol{F}&\boldsymbol{F}^*\\ \boldsymbol{F}^*&-(\lambda^*/\lambda)\boldsymbol{F}\end{pmatrix}^*\begin{pmatrix}|000\rangle\\|011\rangle\\|100\rangle\\|111\rangle\\|001\rangle\\|010\rangle\\|101\rangle\\|110\rangle\end{pmatrix}, \tag{4}$$

where

$$\boldsymbol{F}=\begin{pmatrix}x_0&-x_1&-x_2&-x_3\\ x_1&x_0&-x_3&x_2\\ x_2&x_3&x_0&-x_1\\ x_3&-x_2&x_1&x_0\end{pmatrix} \tag{5}$$

and

$$\lambda=\frac{1}{2}\sum_{j=0}^{3}x_j^2$$

where λ^*/λ is defined by 1 if $\lambda=0$. After the measurement, Alice broadcasts the classical message m if the measurement result is $|\zeta_m\rangle$, $m\in\{0,\ldots,7\}$. The initial state in Eq. (3) can be rewritten in terms of the measurement basis as

$$|\Psi\rangle_{A_1B_1B_2C_1}\otimes|\Psi\rangle_{A_2A_3B_3B_4C_2}=\frac{1}{2}[|\zeta_0\rangle d(x_0,-x_1,-x_2,-x_3)+|\zeta_1\rangle d(x_1,x_0,-x_3,x_2)$$
$$+|\zeta_2\rangle d(x_2,x_3,x_0,-x_1)-|\zeta_3\rangle d(-x_3,x_2,-x_1,-x_0)$$
$$+|\zeta_4\rangle d(x_0^*,-x_1^*,-x_2^*,-x_3^*)+|\zeta_5\rangle d(x_1^*,x_0^*,-x_3^*,x_2^*)$$
$$+|\zeta_6\rangle d(x_2^*,x_3^*,x_0^*,-x_1^*)-|\zeta_7\rangle d(-x_3^*,x_2^*,-x_1^*,-x_0^*)]. \quad (6)$$

Here for convenience, we use the following short notation

$$d(u_0,u_1,u_2,u_3)=[u_0|000000\rangle+u_1|0011\rangle(a_2|01\rangle+b_2|00\rangle)+u_2|1100\rangle(a_1|10\rangle+b_1|00\rangle)$$
$$+u_3|1111\rangle(a_1a_2|11\rangle+a_1b_2|10\rangle+a_2b_1|01\rangle+b_1b_2|00\rangle)]_{B_1B_2B_3B_4C_1C_2}. \quad (7)$$

Step 2 If the controller Charlie consents to provide the assistance, he performs a two-qubit measurement on his particles (C_1,C_2) under the basis $|\mu_l\rangle$, $l\in\{0,\ldots,3\}$, which has the following relationship to the computation basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$:

$$\begin{pmatrix}|\mu_0\rangle\\|\mu_1\rangle\\|\mu_2\rangle\\|\mu_3\rangle\end{pmatrix}=\boldsymbol{W}_1\otimes\boldsymbol{W}_2\begin{pmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{pmatrix}, \quad (8)$$

Here

$$\boldsymbol{W}_j=t_j\begin{pmatrix}s_j&-a_j\\a_j&s_j\end{pmatrix}, \quad (9)$$

where $t_j=1/\sqrt{(1-b_j)^2+a_j^2}$, $s_j=1-b_j$, $j=1,2$. In fact,

$$\boldsymbol{W}_1\otimes\boldsymbol{W}_2=t_1t_2\begin{pmatrix}s_1s_2&-s_1a_2&-a_1s_2&a_1a_2\\s_1a_2&s_1s_2&-a_1a_2&-a_1s_2\\a_1s_2&-a_1a_2&s_1s_2&-s_1a_2\\a_1a_2&a_1s_2&s_1a_2&s_1s_2\end{pmatrix}. \quad (10)$$

After the measurement, Charlie tells Bob the classical message l if the measurement result is $|\mu_l\rangle$, $l\in\{0,\ldots,3\}$.

According to Charlie's measurement basis, the state in Eq. (7) can be rewritten as

$$d(u_0,u_1,u_2,u_3)=t_1t_2[s_1s_2|\mu_0\rangle(u_0|0000\rangle-u_1|0011\rangle-u_2|1100\rangle+u_3|1111\rangle)$$
$$+s_1a_2|\mu_1\rangle(u_0|0000\rangle+u_1|0011\rangle-u_2|1100\rangle-u_3|1111\rangle)$$
$$+a_1s_2|\mu_2\rangle(u_0|0000\rangle-u_1|0011\rangle+u_2|1100\rangle-u_3|1111\rangle)$$
$$+a_1a_2|\mu_3\rangle(u_0|0000\rangle+u_1|0011\rangle+u_2|1100\rangle+u_3|1111\rangle)]. \quad (11)$$

Step 3 The receiver Bob performs the corresponding unitary operations on his particles (B_1,B_2,B_3,B_4) to recover the desired state conditioned on the measurement results $|\zeta_m\rangle$ and $|\mu_l\rangle$.

Table 1. The relation among Alice's and Charlie's measurement outcomes (m,l) $(m,l\in\{0,\ldots,3\})$, the collapsed state (CS) of the qubits (B_1,B_2,B_3,B_4), and Bob's recovery unitary operation (BRUO) on the collapsed state under the assumption the state can be prepared successfully. Here I,X,Y,Z are the four Pauli operations.

(m,l)	CS	BRUO
$(0,0)$	$f(x_0,x_1,x_2,x_3)$	$I\otimes I\otimes I\otimes I$
$(0,1)$	$f(x_0,-x_1,x_2,-x_3)$	$I\otimes I\otimes Z\otimes I$
$(0,2)$	$f(x_0,x_1,-x_2,-x_3)$	$I\otimes Z\otimes I\otimes I$
$(0,3)$	$f(x_0,-x_1,-x_2,x_3)$	$I\otimes Z\otimes Z\otimes I$
$(1,0)$	$f(x_1,-x_0,x_3,-x_2)$	$I\otimes Z\otimes Y\otimes X$
$(1,1)$	$f(x_1,x_0,x_3,x_2)$	$I\otimes Z\otimes X\otimes X$
$(1,2)$	$f(x_1,-x_0,-x_3,x_2)$	$I\otimes I\otimes Y\otimes X$
$(1,3)$	$f(x_1,x_0,-x_3,-x_2)$	$I\otimes I\otimes X\otimes X$
$(2,0)$	$f(x_2,-x_3,-x_0,x_1)$	$X\otimes Y\otimes I\otimes I$
$(2,1)$	$f(x_2,x_3,-x_0,-x_1)$	$X\otimes Y\otimes Z\otimes I$
$(2,2)$	$f(x_2,-x_3,x_0,-x_1)$	$X\otimes X\otimes I\otimes I$
$(2,3)$	$f(x_2,x_3,x_0,x_1)$	$X\otimes X\otimes Z\otimes I$
$(3,0)$	$f(x_3,x_2,-x_1,-x_0)$	$X\otimes X\otimes Y\otimes X$
$(3,1)$	$f(x_3,-x_2,-x_1,x_0)$	$X\otimes X\otimes X\otimes X$
$(3,2)$	$f(x_3,x_2,x_1,x_0)$	$X\otimes Y\otimes Y\otimes X$
$(3,3)$	$f(x_3,-x_2,x_1,-x_0)$	$X\otimes Y\otimes X\otimes X$

Due to the lack of the values $x_j,j=0,\ldots,3$, equations (6) and (11) reveal that the desired state can be reconstructed by Bob only if Alice's measurement outcome lies in $\{|\zeta_0\rangle,\ldots,|\zeta_3\rangle\}$. Here we do not depict them one by one any more. Bob's recovery unitary operations conditioned on the measurement results $|\zeta_m\rangle$ and $|\mu_l\rangle$ $(m,l=0,\ldots,3)$ are summarized in Table 1.

As each measurement outcome of Alice occurs with an equal probability of $1/8$, the total success probability is $4\times1/8=50\%$ which is independent of the coefficients of the partially entangled channels.

3. The second scheme with the unit success probability

In this section, we propose the other deterministic scheme for CRSP of an arbitrary four-qubit cluster-type state via the partially entangled channel.

To achieve the CRSP with unit success probability. The coefficients $\{x_j\}$ of prepared state are divided into phase information $\{r_j\}$ and amplitude information $\{\theta_j\}$, respectively.

In detail, the state in Eq. (1) can be rewritten as

$$r_0 e^{i\theta_0}|0000\rangle + r_1 e^{i\theta_1}|0011\rangle + r_2 e^{i\theta_2}|1100\rangle + r_3 e^{i\theta_3}|1111\rangle, \tag{12}$$

where $x_j = r_j e^{i\theta_j}$, real coefficients $r_j \geq 0$ satisfy the normalization condition $\sum_{j=0}^{3} r_j^2 = 1$, and $\theta_j \in [0, 2\pi)$. Since the sender Alice knows x_j, she is aware of r_j and θ_j completely, $j = 0, \ldots, 3$.

Suppose the quantum channels are two five-particle partially entangled states

$$\begin{aligned} |\Psi\rangle_{A_1A_3B_1B_2C_1} &= \frac{1}{\sqrt{2}}(|00000\rangle + a_1|11111\rangle + b_1|11110\rangle)_{A_1A_3B_1B_2C_1}, \\ |\Psi\rangle_{A_2A_4B_3B_4C_2} &= \frac{1}{\sqrt{2}}(|00000\rangle + a_2|11111\rangle + b_2|11110\rangle)_{A_2A_4B_3B_4C_2}, \end{aligned} \tag{13}$$

where $a_j^2 + b_j^2 = 1$, $j = 0, 1$. The particles (A_1, A_2, A_3, A_4) belong to the sender Alice, particles (B_1, B_2, B_3, B_4) belong to the receiver Bob, and the controller Charlie owns the particles (C_1, C_2). Hence, the initial state of the whole system can be written as

$$\begin{aligned} |\Psi\rangle &= |\Psi\rangle_{A_1A_4B_1B_2C_1} \otimes |\Psi\rangle_{A_2A_3B_3B_4C_2} \\ &= \frac{1}{2}[|0000000000\rangle + |01010011\rangle(a_2|01\rangle + b_2|00\rangle) + |10101100\rangle(a_1|10\rangle + b_1|00\rangle) \\ &\quad + |11111111\rangle(a_1a_2|11\rangle + a_1b_2|10\rangle + a_2b_1|01\rangle + b_1b_2|00\rangle)]_{A_1A_2A_3A_4B_1B_2B_3B_4C_1C_2}. \end{aligned} \tag{14}$$

We present the process of CRSP protocol as follows:

Step 1 The sender Alice performs a two-particle projective measurement on her particles (A_1, A_2) under the basis $\{|\xi_0\rangle, \ldots |\xi_3\rangle\}$ which is determined by $\{r_j\}$

$$\begin{pmatrix} |\xi_0\rangle \\ |\xi_1\rangle \\ |\xi_2\rangle \\ |\xi_3\rangle \end{pmatrix} = \begin{pmatrix} r_0 & -r_1 & -r_2 & -r_3 \\ r_1 & r_0 & -r_3 & r_2 \\ r_2 & r_3 & r_0 & -r_1 \\ r_3 & -r_2 & r_1 & r_0 \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}. \tag{15}$$

After the measurement, the entangled channel in Eq. (14) can be expanded in terms of the measurement basis as the following

$$\begin{aligned} |\Psi\rangle = \frac{1}{2}[&|\xi_0\rangle g(r_0, -r_1, -r_2, -r_3) + |\xi_1\rangle g(r_1, r_0, -r_3, r_2) \\ &+ |\xi_2\rangle g(r_2, r_3, r_0, -r_1) + |\xi_3\rangle g(r_3, -r_2, r_1, r_0)]. \end{aligned} \tag{16}$$

Here for convenience, we use the following short notation

$$\begin{aligned} g(u_0, u_1, u_2, u_3) = [&u_0|00000000\rangle + u_1|010011\rangle(a_2|01\rangle + b_2|00\rangle) + u_2|101100\rangle(a_1|10\rangle + b_1|00\rangle) \\ &+ u_3|111111\rangle)(a_1a_2|11\rangle + a_1b_2|10\rangle + b_1a_2|01\rangle + b_1b_2|00\rangle)]_{A_3A_4B_1B_2B_3B_4C_1C_2}. \end{aligned} \tag{17}$$

Step 2 The sender Alice does not perform the projective measurement immediately but performs a unitary operation U^n on her particles (A_3, A_4) conditioned on the first-step measurement outcome $|\xi_n\rangle, n \in \{0, \ldots, 3\}$, where

$$U^0 = I \otimes I, U^1 = I \otimes X, U^2 = X \otimes I, U^3 = X \otimes X. \tag{18}$$

Then Alice measures her particles (A_3, A_4) under the complete orthogonal basis $\{|\eta_m\rangle\}, m \in \{0, \ldots, 3\}$ which are determined by $\{\theta_j\}$

$$\begin{pmatrix} |\eta_0\rangle \\ |\eta_1\rangle \\ |\eta_2\rangle \\ |\eta_3\rangle \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{-i\theta_0} & e^{-i\theta_1} & e^{-i\theta_2} & e^{-i\theta_3} \\ -e^{-i\theta_0} & e^{-i\theta_1} & e^{-i\theta_2} & -e^{-i\theta_3} \\ -e^{-i\theta_0} & -e^{-i\theta_1} & e^{-i\theta_2} & e^{-i\theta_3} \\ -e^{-i\theta_0} & e^{-i\theta_1} & -e^{-i\theta_2} & e^{-i\theta_3} \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}. \tag{19}$$

After the measurements, Alice will broadcast the classical message n, m to the receiver Bob if her measurement results are $|\xi_n\rangle, |\eta_m\rangle, n, m \in \{0, \ldots, 3\}$.

According to Alice's measurement basis, the initial state in Eq. (14) can be rewritten as

$$\begin{aligned} |\Psi\rangle = \frac{1}{4}\{&|\xi_0\rangle[|\eta_0\rangle g_1(x_0, -x_1, -x_2, -x_3) - |\eta_1\rangle g_1(x_0, x_1, x_2, -x_3) \\ &+ |\eta_2\rangle g_1(-x_0, x_1, -x_2, -x_3) + |\eta_3\rangle g_1(-x_0, -x_1, x_2, -x_3)] + |\xi_1\rangle[|\eta_0\rangle g_1(x_1, x_0, -x_3, x_2) - |\eta_1\rangle g_1(-x_1, x_0, -x_3, -x_2) \\ &+ |\eta_2\rangle g_1(-x_1, -x_0, -x_3, x_2) + |\eta_3\rangle g_1(x_1, -x_0, -x_3, -x_2)] + |\xi_2\rangle[|\eta_0\rangle g_1(x_2, x_3, x_0, -x_1) - |\eta_1\rangle g_1(-x_2, x_3, x_0, x_1) \\ &+ |\eta_2\rangle g_1(x_2, x_3, -x_0, x_1) + |\eta_3\rangle g_1(-x_2, x_3, -x_0, -x_1)] + |\xi_3\rangle[|\eta_0\rangle g_1(x_3, -x_2, x_1, x_0) + |\eta_1\rangle g_1(-x_3, -x_2, x_1, -x_0) \\ &+ |\eta_2\rangle g_1(x_3, -x_2, -x_1, -x_0) - |\eta_3\rangle g_1(-x_3, -x_2, -x_1, x_0)]\}. \end{aligned} \tag{20}$$

Here we use the following short notation

$$g_1(u_0, u_1, u_2, u_3) = [u_0|000000\rangle + u_1|0011\rangle(a_2|01\rangle + b_2|00\rangle) + u_2|1100\rangle(a_1|10\rangle + b_1|00\rangle)$$

$$+u_3|1111\rangle(a_1a_2|11\rangle+a_1b_2|10\rangle+a_2b_1|01\rangle+b_1b_2|00\rangle)]_{B_1B_2B_3B_4C_1C_2}. \quad (21)$$

Step 3 The controller Charlie performs a two-qubit measurement on his particles (C_1, C_2) under the basis $|\mu_l\rangle, l \in \{0,\ldots,3\}$ which is defined by Eq. (8). After the measurement, Charlie informs Bob his measurement outcome $|\mu_l\rangle$ by the classical message l.

According to Charlie's measurement basis, equation (21) can be rewritten as

$$\begin{aligned} g_1(u_0,u_1,u_2,u_3) = t_1t_2[&s_1s_2|\mu_0\rangle(u_0|0000\rangle-u_1|0011\rangle-u_2|1100\rangle+u_3|1111\rangle) \\ &+s_1a_2|\mu_1\rangle(u_0|0000\rangle+u_1|0011\rangle-u_2|1100\rangle-u_3|1111\rangle)+a_1s_2|\mu_2\rangle(u_0|0000\rangle-u_1|0011\rangle \\ &+u_2|1100\rangle-u_3|1111\rangle)+a_1a_2|\mu_3\rangle(u_0|0000\rangle+u_1|0011\rangle+u_2|1100\rangle+u_3|1111\rangle)]. \end{aligned} \quad (22)$$

Step 4 The receiver Bob performs the corresponding unitary operations on his particles (B_1, B_2, B_3, B_4) to recover the desired state corresponding to the measurement results $|\xi_n\rangle|\eta_m\rangle$ and $|\mu_l\rangle$ $(n,m,l \in \{0,\ldots,3\})$. All of the recovery operations are shown in Table 2.

Table 2. The relation among Alice's and Charlie's measurement outcomes (nm,l), the collapsed state (CS) of the qubits (B_1,B_2,B_3,B_4) after the measurements and Bob's recovery unitary operation (BRUO) on the collapsed state.

(nm,l)	CS	BRUO
(00,0) (01,3) (02,2) (03,1)	$f(x_0,x_1,x_2,x_3)$	$I\otimes I\otimes I\otimes I$
(00,1) (01,2) (02,3) (03,0)	$f(x_0,-x_1,x_2,-x_3)$	$I\otimes I\otimes Z\otimes I$
(00,2) (01,1) (02,0) (03,3)	$f(x_0,x_1,-x_2,-x_3)$	$I\otimes Z\otimes I\otimes I$
(00,3) (01,0) (02,1) (03,2)	$f(x_0,-x_1,-x_2,x_3)$	$I\otimes Z\otimes Z\otimes I$
(10,0) (11,3) (12,2) (13,1)	$f(x_1,-x_0,x_3,-x_2)$	$I\otimes Z\otimes Y\otimes X$
(10,1) (11,2) (12,3) (13,0)	$f(x_1,x_0,x_3,x_2)$	$I\otimes Z\otimes X\otimes X$
(10,2) (11,1) (12,0) (13,3)	$f(x_1,-x_0,-x_3,x_2)$	$I\otimes I\otimes Y\otimes X$
(10,3) (11,0) (12,1) (13,2)	$f(x_1,x_0,-x_3,-x_2)$	$I\otimes I\otimes X\otimes X$
(20,0) (21,3) (22,2) (23,1)	$f(x_2,-x_3,-x_0,x_1)$	$X\otimes Y\otimes I\otimes I$
(20,1) (21,2) (22,3) (23,0)	$f(x_2,x_3,-x_0,-x_1)$	$X\otimes Y\otimes Z\otimes I$
(20,2) (21,1) (22,0) (23,3)	$f(x_2,-x_3,x_0,-x_1)$	$X\otimes X\otimes I\otimes I$
(20,3) (21,0) (22,1) (23,2)	$f(x_2,x_3,x_0,x_1)$	$X\otimes X\otimes Z\otimes I$
(30,0) (31,3) (32,2) (33,1)	$f(x_3,x_2,-x_1,-x_0)$	$X\otimes X\otimes Y\otimes X$
(30,1) (31,2) (32,3) (33,0)	$f(x_3,-x_2,-x_1,x_0)$	$X\otimes X\otimes X\otimes X$
(30,2) (31,1) (32,0) (33,3)	$f(x_3,x_2,x_1,x_0)$	$X\otimes Y\otimes Y\otimes X$
(30,3) (31,0) (32,1) (33,2)	$f(x_3,-x_2,x_1,-x_0)$	$X\otimes Y\otimes X\otimes X$

Therefore, the success probability is 100% for the CRSP of an arbitrary four-qubit cluster-type state although the quantum channels is non-maximally entangled.

4. Conclusion

In conclusion, two CRSP schemes via partially entangled channels for an arbitrary four-qubit cluster-type state are proposed. It needs emphasizing that to accomplish the CRSP tasks with high success probabilities, we construct a series of ingenious measurement bases at the sender's and the controller's locations. In the first scheme, through the measurements under the measurement bases, the original state can be successfully prepared with the probability 50%. In the second scheme, another set of useful measurement bases are constructed. Under the bases, the sender performs two-step two-particle projective measurements on her particles. After achieving the sender's and the controller's measurement results, the receiver can recover the prepared state deterministically. Different from the previous RSP schemes via a partially entangled channel, our schemes have the following features: (i) neither auxiliary qubits nor two-qubit unitary transformations are required; (ii) the success probabilities are independent of the degree of entanglement of the quantum channels. Compared to the first scheme in Section 2, the second scheme has unfavorable and favorable aspects. On the one hand, the sender Alice needs to use the information splitting and divide the complex coefficients x_j before constructing the measurement basis. Also, before performing the second-step two-particle projective measurement, Alice needs to perform an additional unitary operation conditioned on her first-step measurement outcome. From this point of view, the first scheme can be looked as a desired transmission protocol. On the other hand, two-qubit projective measurement is more easily performed than three-qubit projective measurement, and the success probability is twice as that in the first one. Although the partially entangled channels are employed in our schemes, the success probability can even reach 100% as well as the case of maximally entangled channels. It implies that a partially entangled channel may have massive advantages in the applications of quantum communication. In fact, our schemes can be reduced to two RSP schemes if the sender has the controller's particle and performs her operations. In this sense, the success probabilities of the proposed RSP schemes for an arbitrary four-qubit cluster-type state are higher than those of the schemes via partially entangled channel in Refs. [12] and [13].

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