A novel scheme of hybrid entanglement swapping and teleportation using cavity QED in the small and large detuning regimes and quasi-Bell state measurement method

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We outline a scheme for entanglement swapping based on cavity QED as well as quasi-Bell state measurement (quasi-BSM) methods. The atom–field interaction in the cavity QED method is performed in small and large detuning regimes. We assume two atoms are initially entangled together and, distinctly two cavities are prepared in an entangled coherent–coherent state. In this scheme, we want to transform entanglement to the atom-field system. It is observed that, the fidelities of the swapped entangled state in the quasi-BSM method can be compatible with those obtained in the small and large detuning regimes in the cavity QED method (the condition of this compatibility will be discussed). In addition, in the large detuning regime, the swapped entangled state is obtained by detecting and quasi-BSM approaches. In the continuation, by making use of the atom–field entangled state obtained in both approaches in a large detuning regime, we show that the atomic as well as field states teleportation with complete fidelity can be achieved.

Keywords: entanglement swapping, quantum teleportation, cavity QED, quasi-Bell state measurement

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1. Introduction

Entanglement swapping and teleportation are among the most important concepts in quantum information theory. Entanglement swapping which has been proposed by Zukowski *et al.*[1] is known as entangling particles that have not shared any common past.[2–4] The entangling process of these particles may be performed via the cavity QED method[5–8] or a joint measurement (such as BSM).[2,9] The entanglement swapping can be utilized in several interesting areas such as quantum secret sharing,[10] quantum signature,[11] quantum steganography,[12] secure direct communication,[13] and specially quantum teleportation.[14,15] The quantum teleportation which was first suggested by Bennett *et al.*,[16] provides a mechanism to transmit the quantum information from transmitter to receiver using a quantum channel. In this process, via transferring additionally classical information through a classical channel by the transmitter, the teleported state[17] is reconstructed in the receiver's side. To demonstrate the similarity between the reconstructed state and teleported state, the fidelity measure may be used. The fidelity as a measure of the closeness of two quantum states $|\phi\rangle$ and $|\psi\rangle$ is defined as $F = |\langle\phi|\psi\rangle|^2$.[18] There are different methods to perform the teleportation process including the BSM method[19] and by using atom–field interaction in cavity QED.[20–23] Teleportation of atomic and field states by using atom–field interaction in cavity QED has been discussed in Refs. [24] and [25]. Also, teleportation of a three-particle entangled state has been investigated in Refs. [26], [27], and [28].

In this paper, we firstly study a scheme for entanglement swapping by using atom-field interaction in cavity QED and quasi-BSM (instead of BSM). To achieve this purpose, we suppose two atoms labeled 1 and 2 are entangled and distinctly two cavities labeled 3 and 4 each contains a coherent state are also entangled with each other. In the cavity QED method we consider two different regimes, i.e., small detuning[7,25,29] and large detuning regimes.[17] The employed atom in the small (large) detuning regime is a three-level (two-level) one which interacts with a single-mode field. In this respect, after the interaction of atom 2 and cavity 3 in these two regimes the entangled state of atom 1 and cavity 4 is generated. Therefore, the entanglement of two systems has been performed, while they never have interaction with each other.

As another method for the generation of entanglement of atom 1–cavity 4, we can point out the quasi-BSM on the states of atom 2–cavity 3. Note here that due to the employment of the coherent states as the field states, we call this method the quasi-BSM method. The expression "quasi" is used to distinguish this method from ones which the number states are extremely used in the literature. In the mentioned methods, the fidelity of the atom–field entangled state (which is obtained from the entanglement swapping process) is calculated relative to a maximally entangled state. The value of

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this fidelity in the quasi-BSM method for enough intensities of the initial field, has a good compatibility with the cavity QED method in both small and large detuning regimes. In addition, we have obtained the swapped entangled state in the large detuning regime in two approaches, i.e., detecting and quasi-BSM approaches. Finally, the resulted state in both approaches which is exactly the same, has been used to construct a quantum channel to teleport unknown atom and field states with complete fidelity.

This paper is organized as follows. In the next section, as the first stage, we take a brief review on the interaction of the atom with a single-mode quantized field in small and large detuning regimes. In Section 3, we first suppose that two atoms (atoms 1 and 2) are prepared in an entangled state, and two cavities (cavities 3 and 4) which are initially prepared in the coherent state, are also entangled together. Then, by interacting atom 2 with cavity 3 in the cavity QED method or performing the quasi-BSM on them, we generate entanglement between atom 1 and cavity 4. In Section 4, we investigate the quantum teleportation of the unknown atomic and field states by making use of the created quantum channel (in the large detuning regime). Finally, we present a summary and concluding remarks in the last section.

2. The model

In this section, we review the interaction of a three-level (two-level) atom with a single-mode field in a cavity through the Jaynes–Cummings (JC) model in the small (large) detuning regime.

2.1. The JC model with small detuning

We consider the Ξ-type three-level atom with the ground state $|g\rangle$ and the excited states $|f\rangle$ and $|e\rangle$, with the only allowed transitions $|e\rangle \longleftrightarrow |f\rangle$ and $|f\rangle \longleftrightarrow |g\rangle$. Having assumed the interaction of this atomic subsystem with a single-mode quantized field in a cavity, the corresponding Hamiltonian in the interaction picture reads as

$$\begin{aligned}\hat{\mathrm{V}}(t) = {} & \hbar g_1(\hat{\sigma}_+^{ef}\hat{a}\,\mathrm{e}^{-\mathrm{i}\delta t} + \hat{\sigma}_-^{fe}\hat{a}^\dagger\,\mathrm{e}^{\mathrm{i}\delta t}) \\ & + \hbar g_2(\hat{\sigma}_+^{fg}\hat{a}\,\mathrm{e}^{\mathrm{i}\delta t} + \hat{\sigma}_-^{gf}\hat{a}^\dagger\,\mathrm{e}^{-\mathrm{i}\delta t}),\end{aligned} \tag{1}$$

where

$$\hat{\sigma}_+^{ef} = |e\rangle\langle f| \quad \text{and} \quad \hat{\sigma}_+^{fg} = |f\rangle\langle g|$$

are the atomic raising operators, and

$$\hat{\sigma}_-^{fe} = |f\rangle\langle e| \quad \text{and} \quad \hat{\sigma}_-^{gf} = |g\rangle\langle f|$$

are the atomic lowering operators. Also, g_1 and g_2 are the atom–field coupling coefficients and

$$\delta = \Omega - (\omega_e - \omega_f) = (\omega_f - \omega_g) - \Omega,$$

where Ω is the cavity–field frequency and ω_e, ω_f and ω_g are the frequencies associated with the atomic levels $|e\rangle$, $|f\rangle$, and $|g\rangle$, respectively. The state vector of the above-mentioned system at any time t can be expressed as

$$\begin{aligned}|\psi(t)\rangle = {} & \sum_{n=0}^{\infty} c_{e,n}(t)|e,n\rangle + c_{f,n+1}(t)|f,n+1\rangle \\ & + c_{g,n+2}(t)|g,n+2\rangle.\end{aligned} \tag{2}$$

Assuming

$$|\psi(0)\rangle_{\text{field}} = \sum_n c_n(0)|n\rangle$$

and

$$|\psi(0)\rangle_{\text{atom}} = c_e|e\rangle + c_f|f\rangle + c_g|g\rangle,$$

the time-dependent coefficients in Eq. (2) have been obtained as follows:[7,25,29]

$$\begin{aligned}c_{e,n}(t) = {} & \left[\frac{g_1^2(n+1)}{\Lambda_n\xi_n^2}\gamma_n(t) + 1\right] c_e c_n(0) \\ & - \mathrm{i}\left(\frac{g_1\sqrt{n+1}}{\Lambda_n}\right)\sin(\Lambda_n t)\,\mathrm{e}^{-\mathrm{i}\delta t/2} c_f c_{n+1}(0) \\ & + \left[\frac{g_1 g_2\sqrt{(n+1)(n+2)}}{\Lambda_n\xi_n^2}\gamma_n(t)\right] c_g c_{n+2}(0),\end{aligned} \tag{3}$$

$$\begin{aligned}c_{f,n+1}(t) = {} & -\mathrm{i}\left(\frac{g_1\sqrt{n+1}}{\Lambda_n}\right)\sin(\Lambda_n t)\,\mathrm{e}^{\mathrm{i}\delta t/2} c_e c_n(0) \\ & + \left[\cos(\Lambda_n t) - \mathrm{i}\left(\frac{\delta}{2\Lambda_n}\right)\sin(\Lambda_n t)\right]\mathrm{e}^{\mathrm{i}\delta t/2} c_f c_{n+1}(0) \\ & - \mathrm{i}\left(\frac{g_2\sqrt{n+2}}{\Lambda_n}\right)\sin(\Lambda_n t)\,\mathrm{e}^{\mathrm{i}\delta t/2} c_g c_{n+2}(0),\end{aligned} \tag{4}$$

$$\begin{aligned}c_{g,n+2}(t) = {} & \left[\frac{g_1 g_2\sqrt{(n+1)(n+2)}}{\Lambda_n\xi_n^2}\gamma_n(t)\right] c_e c_n(0) \\ & - \mathrm{i}\left(\frac{g_2\sqrt{n+2}}{\Lambda_n}\right)\sin(\Lambda_n t)\,\mathrm{e}^{-\mathrm{i}\delta t/2} c_f c_{n+1}(0) \\ & + \left[\frac{g_2^2(n+2)}{\Lambda_n\xi_n^2}\gamma_n(t) + 1\right] c_g c_{n+2}(0).\end{aligned} \tag{5}$$

Here,

$$\Lambda_n = \sqrt{\frac{\delta^2}{4} + \xi_n^2}$$

is the Rabi frequency with

$$\xi_n = \sqrt{g_1^2(n+1) + g_2^2(n+2)}$$

and

$$\gamma_n(t) = \left[\Lambda_n\cos(\Lambda_n t) + \mathrm{i}\frac{\delta}{2}\sin(\Lambda_n t) - \Lambda_n\,\mathrm{e}^{\mathrm{i}\delta t/2}\right]\mathrm{e}^{-\mathrm{i}\delta t/2}.$$

In the above mentioned model, the frequencies of the allowed atomic transitions are close to the frequency of field (near-resonant), so that the interaction of atom–field leads to atomic transitions. This type of interaction is known as a "small detuning regime" in which the detuning is approximately equal

to the atom–field coupling ($\delta \simeq g$).[30] On the other hand, if the atomic transition frequency is far from the field frequency (far-resonant), the interaction of atom–field is dispersive and this type of interaction is known as a "large detuning regime".

2.2. The JC model with large detuning

In this subsection, we turn our attention to a two-level atom with the ground state $|g\rangle$ and the excited state $|e\rangle$ which is out of resonance with the quantized field frequency, i.e., the interaction of atom–field is dispersive. The effective atom–field interaction Hamiltonian in this limit has been given by[17]

$$\hat{H}_{\rm eff} = \hbar\chi(\hat{\sigma}_+\hat{\sigma}_- + \hat{a}^\dagger\hat{a}\hat{\sigma}_3), \tag{6}$$

where $\chi = \lambda^2/\delta$ describes the strength of the atom–light coupling with λ as the atom–field coupling coefficient, $\hat{\sigma}_+\hat{\sigma}_- = |e\rangle\langle e|$ and $\hat{\sigma}_3 = |e\rangle\langle e| - |g\rangle\langle g|$. Let us suppose that the initial state of the atom–field system is $|\psi(0)\rangle = |e\rangle|\pm\alpha\rangle$, that is, the atom is in its excited state and the field is in a coherent state. Noticing that the laser behaves like a coherent state far from threshold,[31], the considered situation may be easily prepared. Then, according to the Hamiltonian in Eq. 6 the time-evolved state is given by

$$|\psi(t)\rangle_\pm = {\rm e}^{-{\rm i}\hat{H}_{\rm eff}t/\hbar}|\psi(0)\rangle_\pm = {\rm e}^{-{\rm i}\chi t}|e\rangle|\pm\alpha\,{\rm e}^{-{\rm i}\chi t}\rangle. \tag{7}$$

Similarly, for the initial state $|\psi(0)\rangle_\pm = |g\rangle|\pm\alpha\rangle$ we obtain

$$|\psi'(t)\rangle_\pm = {\rm e}^{-{\rm i}\hat{H}_{\rm eff}t/\hbar}|\psi(0)\rangle_\pm = |g\rangle|\pm\alpha\,{\rm e}^{{\rm i}\chi t}\rangle. \tag{8}$$

The phase factor in Eq. (7) may be ignored. Also, by an appropriate phase rotation in Eqs. (7) and (8) one has

$$\begin{aligned} |\psi(t)\rangle_\pm &= |e\rangle|\pm\alpha\rangle, \\ |\psi'(t)\rangle_\pm &= |g\rangle|\pm\alpha\,{\rm e}^{2{\rm i}\chi t}\rangle. \end{aligned} \tag{9}$$

Based on the small and large detuning regimes which were presented in the last two subsections, now we can perform the entanglement swapping process in our system.

3. Hybrid entanglement swapping

In a hybrid version of entanglement swapping, the interaction in the cavity QED and BSM are performed on the atom–field hybrid system. Here, we study the hybrid entanglement swapping by making use of two methods, i.e., i) the atom-field interaction in cavity QED in small and large detuning regimes and ii) the quasi-BSM method (instead of BSM). To achieve this purpose, let us suppose that the two atoms are initially prepared in an entangled state as

$$|\phi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2\right), \tag{10}$$

and two cavities are prepared in an entangled coherent–coherent state as follows:[32,33]

$$|\phi\rangle_{34} = N_+\left(|\alpha\rangle_3|\alpha\rangle_4 + |-\alpha\rangle_3|-\alpha\rangle_4\right), \tag{11}$$

where

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

and $N_+ = [2(1+{\rm e}^{-4|\alpha|2})]^{-1/2}$ is the normalization factor. Accordingly, the initial state of the whole system reads as

$$\begin{aligned} |\psi(0)\rangle_{1234} &= |\phi\rangle_{12}|\phi\rangle_{34} \\ &= \frac{N_+}{\sqrt{2}}\big[|e\rangle_1|g\rangle_2|\alpha\rangle_3|\alpha\rangle_4 + |e\rangle_1|g\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4 \\ &\quad + |g\rangle_1|e\rangle_2|\alpha\rangle_3|\alpha\rangle_4 + |g\rangle_1|e\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4\big]. \end{aligned} \tag{12}$$

In the continuation, the entanglement swapping, i.e., the entanglement generation between atom 1 and cavity 4 will be obtained by cavity QED and quasi-BSM methods.

3.1. The cavity QED method

In this method, atom 2 is imported into cavity 3 in which the governing interaction Hamiltonian is expressed in two different regimes, small (Hamiltonian in Eq. (1)) and large (Hamiltonian in Eq. (6)) detuning regimes.

3.1.1. Small detuning regime

After entering atom 2 into cavity 3 and passing the interaction time t, the state of the whole system via Hamiltonian (1) reads as

$$\begin{aligned} |\psi(t)\rangle_{1234} = \frac{N_+}{\sqrt{2}}\big(&|e\rangle_1|\alpha\rangle_4\sum_{n=0}^{\infty}\sum_i C_{i,n}^{(g,\alpha)}(t)|i\rangle_2|n\rangle_3 \\ &+ |e\rangle_1|-\alpha\rangle_4\sum_{n=0}^{\infty}\sum_i C_{i,n}^{(g,-\alpha)}(t)|i\rangle_2|n\rangle_3 \\ &+ |g\rangle_1|\alpha\rangle_4\sum_{n=0}^{\infty}\sum_i C_{i,n}^{(e,\alpha)}(t)|i\rangle_2|n\rangle_3 \\ &+ |g\rangle_1|-\alpha\rangle_4\sum_{n=0}^{\infty}\sum_i C_{i,n}^{(e,-\alpha)}(t)|i\rangle_2|n\rangle_3\big). \end{aligned} \tag{13}$$

Here $C_{i,n}^{(j,\pm\alpha)}$ denote the probability amplitudes given by Eqs. (3)–(5) with $i = e, f, g$, where the initial field state is as coherent state $|\pm\alpha\rangle$ and the initial atomic state as $|j\rangle$ with $j = e, g$. Now, if atom 2 is detected in its ground state $|g\rangle_2$, the state of the whole system collapses to

$$\begin{aligned} |\psi(t)\rangle_{134} = \frac{N_+}{\sqrt{2}}\big(&|e\rangle_1|\alpha\rangle_4\sum_{n=0}^{\infty}C_{g,n}^{(g,\alpha)}(t)|n\rangle_3 \\ &+ |e\rangle_1|-\alpha\rangle_4\sum_{n=0}^{\infty}C_{g,n}^{(g,-\alpha)}(t)|n\rangle_3 \\ &+ |g\rangle_1|\alpha\rangle_4\sum_{n=0}^{\infty}C_{g,n}^{(e,\alpha)}(t)|n\rangle_3 \\ &+ |g\rangle_1|-\alpha\rangle_4\sum_{n=0}^{\infty}C_{g,n}^{(e,-\alpha)}(t)|n\rangle_3\big). \end{aligned} \tag{14}$$

However, the success probability for detecting atom 2 in its ground state may be evaluated (see Fig. 1), where we have assumed $g_1 = g_2 = g = 1$ MHz.[7,34] As is seen, for small intensities of the initial field, the success probability versus time has oscillatory behavior between 0.5 and 1.0. For large enough values of intensity, the success probability remains at about 0.5 (with less oscillations around this value).

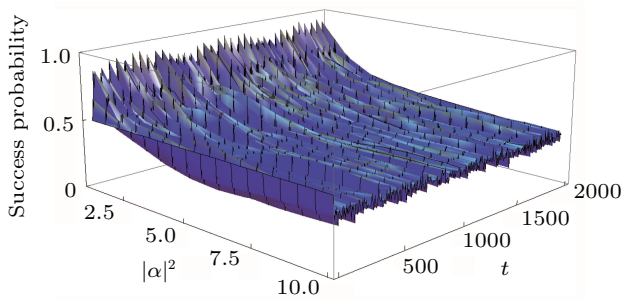


Fig. 1. (color online) Success probability for detecting atom 2 in its ground state versus the intensity of the initial field and time t with $g = 1$ MHz and $\delta = g$.

Now we make a measurement on the state in Eq. (14) to obtain the coherent state $|\beta\rangle_3$. In this way, the reduced state reads as

$$|\psi(t)\rangle_{14} = N\big[a_1(\alpha,\beta,t)|\alpha\rangle_4 + a_2(\alpha,\beta,t)|-\alpha\rangle_4\big]|e\rangle_1 + N\big[a_3(\alpha,\beta,t)|\alpha\rangle_4 + a_4(\alpha,\beta,t)|-\alpha\rangle_4\big]|g\rangle_1, \quad (15)$$

where N is an appropriate normalization factor, and

$$a_1(\alpha,\beta,t) = \frac{N_+}{\sqrt{2}}\exp\left(-\frac{|\beta|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}C_{g,n}^{(g,\alpha)}(t), \quad (16)$$

$$a_3(\alpha,\beta,t) = \frac{N_+}{\sqrt{2}}\exp\left(-\frac{|\beta|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}C_{g,n}^{(e,\alpha)}(t), \quad (17)$$

and $a_2(\alpha,\beta,t) = a_1(-\alpha,\beta,t)$ and $a_4(\alpha,\beta,t) = a_3(-\alpha,\beta,t)$. In this respect, the success probability to detect the coherent state $|\beta\rangle$ for cavity 3 is plotted in Fig. 2. As is seen, for small intensities of the initial field, the plot of success probability versus time has oscillatory behavior between 0.5 and 1.0 and vanishes for the values of $|\alpha|^2 \gtrsim 5$. The fidelity of the state (15) relative to the following (normalized) maximally entangled state

$$|\Psi\rangle_{14} = \frac{1}{2}\big[|e\rangle_1|\alpha,+\rangle_4 + |g\rangle_1|\alpha,-\rangle_4\big], \quad (18)$$

is calculated (see Fig. 3). Notice that, in Eq. (18) $|\alpha,\pm\rangle_4 = |\alpha\rangle_4 \pm |-\alpha\rangle_4$ is the non-normalized even (odd) cat state has been introduced by Yurke and Stoler[35] (its nonlinear form is recently generated by one of us in Ref. [36]). As is clear from this figure, the value of fidelity is initiated from 1 for small values of the field intensity with no oscillation and tends to 0.5 for $|\alpha|^2 \gtrsim 3$.

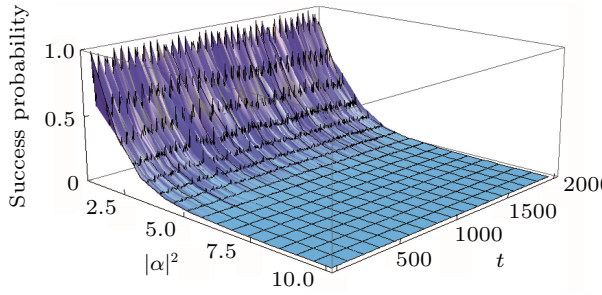


Fig. 2. (color online) Success probability for obtaining the coherent state $|\beta\rangle_3$ versus the intensity of the initial field and time t with $\beta = 0.1$. Other parameters are chosen as in Fig. 1.

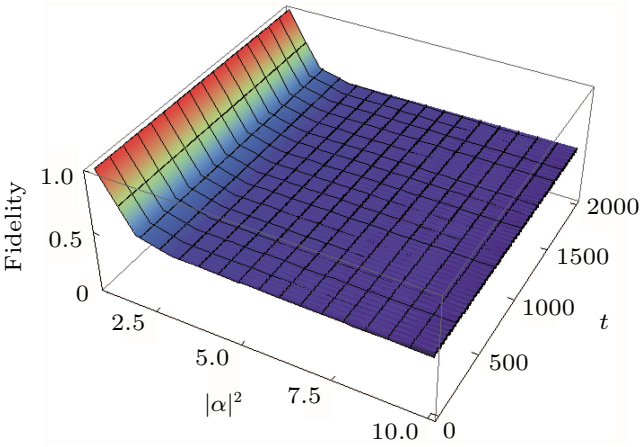


Fig. 3. (color online) Fidelity versus the intensity of the initial field and time t with $\beta = 0.1$. Other parameters are chosen as in Fig. 1.

3.1.2. Large detuning regime

In this section, we suppose that atom 2 interacts with cavity 3 in which the interaction Hamiltonian is expressed by Eq. (6) (in the large detuning). After the interaction time t' between atom 2 and cavity 3, the state of the whole system reads as

$$|\psi'(t')\rangle_{1234} = \frac{N_+}{\sqrt{2}}\big[|e\rangle_1|g\rangle_2|\alpha e^{2i\chi t'}\rangle_3|\alpha\rangle_4 + |e\rangle_1|g\rangle_2|-\alpha e^{2i\chi t'}\rangle_3|-\alpha\rangle_4 + |g\rangle_1|e\rangle_2|\alpha\rangle_3|\alpha\rangle_4 + |g\rangle_1|e\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4\big]. \quad (19)$$

Now, to generate entanglement between atom 1 and cavity 4, we use two different approaches, namely detecting and quasi-BSM methods.

(i) Detecting approach

If $2\chi t' = \pi$, then the state of the total system (Eq. (19)) reduces to

$$|\psi'(\pi/2\chi)\rangle_{1234} = \frac{N_+}{\sqrt{2}}\big[|e\rangle_1|g\rangle_2|-\alpha\rangle_3|\alpha\rangle_4 + |e\rangle_1|g\rangle_2|\alpha\rangle_3|-\alpha\rangle_4 + |g\rangle_1|e\rangle_2|\alpha\rangle_3|\alpha\rangle_4 + |g\rangle_1|e\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4\big]. \quad (20)$$

Now, if we detect cavity 3 in the coherent state $|\alpha\rangle_3$, then the state of the two atoms and cavity 4 becomes

$$|\psi'\rangle_{124} = \frac{N_+}{\sqrt{2}}\big[|e\rangle_1|g\rangle_2|-\alpha\rangle_4 + |g\rangle_1|e\rangle_2|\alpha\rangle_4\big]. \quad (21)$$

After the operation of a Hadamard gate with unitary operator

$$\hat{U}_{\rm H} = \frac{1}{\sqrt{2}}\big[|e\rangle\langle e| + |e\rangle\langle g| + |g\rangle\langle e| - |g\rangle\langle g|\big]$$

on the atom 2,[18] one has

$$|\psi'\rangle_{124} = \frac{N_+}{2}\big[\big(|e\rangle_1|-\alpha\rangle_4 + |g\rangle_1|\alpha\rangle_4\big)|e\rangle_2 + \big(-|e\rangle_1|-\alpha\rangle_4 + |g\rangle_1|\alpha\rangle_4\big)|g\rangle_2\big]. \quad (22)$$

By performing the measurement in the atomic basis, if atom 2 is detected in an exited state, then the state of atom 1 and cavity 4 arrives at

$$|\psi'\rangle_{14} = \frac{N'N_+}{2}(|e\rangle_1|-\alpha\rangle_4 + |g\rangle_1|\alpha\rangle_4), \quad (23)$$

where N' is a normalization factor. Interestingly, the fidelity of state (23) in comparison with the following maximally entangled state

$$|\Psi'\rangle_{14} = \frac{1}{\sqrt{2}}(|e\rangle_1|-\alpha\rangle_4 + |g\rangle_1|\alpha\rangle_4), \quad (24)$$

is precisely 1.

(ii) Quasi-BSM approach

If $2\chi t' = 0$, then the total state of the system, i.e. Eq. (19), reduces to

$$|\psi'(0)\rangle_{1234} = \frac{N_+}{\sqrt{2}}\big[|e\rangle_1|g\rangle_2|\alpha\rangle_3|\alpha\rangle_4 + |e\rangle_1|g\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4 + |g\rangle_1|e\rangle_2|\alpha\rangle_3|\alpha\rangle_4 + |g\rangle_1|e\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4\big]. \quad (25)$$

Now, we consider a quasi-Bell state as[37]

$$|\psi\rangle_{\text{quasi-Bell}} = \frac{1}{\sqrt{2}}(|e\rangle_2|\alpha\rangle_3 + |g\rangle_2|-\alpha\rangle_3), \quad (26)$$

and perform the quasi-BSM on the state (25). Then, the state of atom 2 and cavity 3 is projected onto the introduced quasi-Bell state and the state of atom 1 and cavity 4 is entangled in the form of Eq. (23). Interestingly, the fidelity of this state relative to the maximally entangled state in Eq. (24) is also precisely 1.

3.2. Quasi-BSM method

As is stated in Section 3 of this paper, the whole initial state of the system is expressed by Eq. (12). By making use of the quasi-Bell state of the form (26) and performing the quasi-BSM on the state (12), the state of atom 2 and cavity 3 is projected onto the introduced quasi-Bell state and the state of atom 1 and cavity 4 results in

$$|\psi\rangle_{14} = \frac{N'N_+}{2}\big[\big(\mathrm{e}^{-2|\alpha|^2}|\alpha\rangle_4 + |-\alpha\rangle_4\big)|e\rangle_1 + \big(|\alpha\rangle_4 + \mathrm{e}^{-2|\alpha|^2}|-\alpha\rangle_4\big)|g\rangle_1\big], \quad (27)$$

where N' is a normalization factor. The fidelities of the above state relative to Eqs.(18) and (24) are respectively plotted in Fig. 4 in the dotted and solid lines. As is clear, the behavior of fidelity in both plots (dotted and solid lines) is similar to each other, i.e., it begins from a specific value (1.0 and 0.5, respectively) and with increasing the intensity of the initial field, it firstly evolves a bit, decreases and then after coming back to its maximum value, remains constant with increasing the intensity ($|\alpha|^2 \gtrsim 3$). As is observed, the value of fidelity in the solid line is precisely twice of its values in the dotted line at any $|\alpha|^2$.

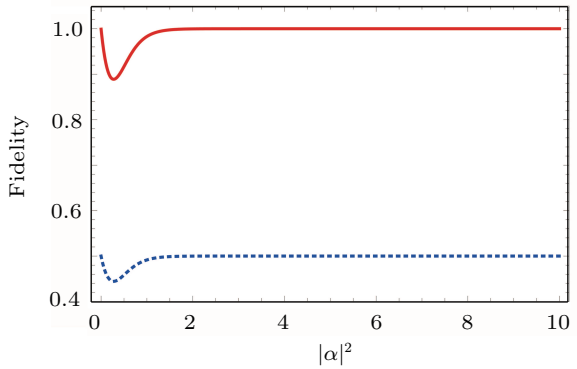


Fig. 4. (color online) The fidelity of state (27) relative to Eq. (18) (blue dotted line) and relative to Eq. (24) (red solid line), versus the intensity of the initial field.

The fidelity which is obtained in the quasi-BSM method (Fig. 4) differs from the fidelity in the cavity QED method in the following cases:

i) Comparing the fidelity of state (27) relative to state (18) (dotted line) with the fidelity in the small detuning regime, it is observed that the behavior of the dotted line for $|\alpha|^2 \gtrsim 3$ is similar to Fig. 3, where in both, the value of fidelity becomes 0.5 for the mentioned range of $|\alpha|^2$.

ii) The value of fidelity of the state (27) relative to state (24) (solid line) for $|\alpha|^2 \gtrsim 3$ is equal to the value of fidelity in the large detuning regime, i.e., 1.

So, the value of fidelities in the quasi-BSM method for $|\alpha|^2 \gtrsim 3$, has a good compatibility with the cavity QED method (in both small and large detuning regimes).

4. Atomic and field state teleportation

We end the paper with paying attention to the atomic and field state teleportation. As is stated, in teleportation protocol, Alice intends to transmit an unknown state to Bob. For this purpose, she requires a quantum channel. So, our proposal

allows her to use the entangled state (23) (in both detecting and quasi-BSM approaches) to transmit an unknown atomic or field state to Bob. In this paper, the teleportation scheme of both atomic and field states is presented by making use of the cavity QED method in the large detuning regime.

4.1. Atomic state teleportation

Let us assume that atom 2 has been initially prepared in the unknown state

$$|\varphi\rangle_2 = \eta|e\rangle_2 + \zeta|g\rangle_2, \tag{28}$$

where the coefficients η and ζ satisfy the normalization condition. We are going to teleport the atomic state (28) to atom 1 through the quantum channel which is expressed by $|\psi'\rangle_{14}$ in Eq. (23). To achieve this goal, we consider the initial state of the total system as

$$|\Phi\rangle_{124} = |\psi'\rangle_{14}|\varphi\rangle_2 = \frac{N_+}{2}\big[\eta|e\rangle_1|e\rangle_2|-\alpha\rangle_4 + \eta|g\rangle_1|e\rangle_2|\alpha\rangle_4 + \zeta|e\rangle_1|g\rangle_2|-\alpha\rangle_4 + \zeta|g\rangle_1|g\rangle_2|\alpha\rangle_4\big]. \tag{29}$$

After the interaction of atom 2 with cavity 4 (in the large detuning regime) at the interaction time $t = \pi/2\chi$, by making use of Eq. (9) one obtains

$$|\Phi\rangle_{124} = \frac{N_+}{2}\big[\eta|e\rangle_1|e\rangle_2|-\alpha\rangle_4 + \eta|g\rangle_1|e\rangle_2|\alpha\rangle_4 + \zeta|e\rangle_1|g\rangle_2|\alpha\rangle_4 + \zeta|g\rangle_1|g\rangle_2|-\alpha\rangle_4\big]. \tag{30}$$

Now, if we detect cavity 4 in the coherent state $|-\alpha\rangle_4$, then the latter state collapses to

$$|\Phi\rangle_{12} = \frac{N_+}{2}\big[\eta|e\rangle_1|e\rangle_2 + \zeta|g\rangle_1|g\rangle_2\big], \tag{31}$$

in which operating a Hadamard gate on atom 2 arrives us at

$$|\Phi\rangle_{12} = \frac{N_+}{2\sqrt{2}}\big[(\eta|e\rangle_1 + \zeta|g\rangle_1)|e\rangle_2 + (\eta|e\rangle_1 - \zeta|g\rangle_1)|g\rangle_2\big]. \tag{32}$$

At last, by detecting atom 2 in the excited state $|e\rangle_2$, the state of atom 1 is exactly in the initial state of atom 2 and therefore, the teleportation process becomes successful (with complete fidelity). If the state of atom 2 is $|g\rangle_2$, atom 1 needs to perform a phase transformation (by performing the rotation operation $\hat{\sigma}_z$ on atom 1) to reconstruct the initial state of atom 2 and to succeed the teleportation process, again. Thus, the probability of successful teleportation is 1.

4.2. Field state teleportation

Now, let us assume cavity 2 has been initially prepared in the state

$$|\varphi'\rangle_2 = \gamma|\alpha\rangle_2 + \lambda|-\alpha\rangle_2, \tag{33}$$

where the unknown coefficients γ and λ satisfy the normalization condition. We are going to teleport cavity state (33) to cavity 4 through the quantum channel which is expressed by $|\psi'\rangle_{14}$ in Eq (23). To achieve this goal, we consider the initial state of the whole system as

$$|\Phi'\rangle_{124} = |\psi'\rangle_{14}|\varphi'\rangle_2 = \frac{N_+}{2}\big[\gamma|e\rangle_1|\alpha\rangle_2|-\alpha\rangle_4 + \gamma|g\rangle_1|\alpha\rangle_2|\alpha\rangle_4 + \lambda|e\rangle_1|-\alpha\rangle_2|-\alpha\rangle_4 + \lambda|g\rangle_1|-\alpha\rangle_2|\alpha\rangle_4\big]. \tag{34}$$

After the interaction of atom 1 with cavity 2 (in the large detuning regime) at the interaction time $t = \pi/2\chi$, with the help of Eq. (9) one obtains

$$|\Phi'\rangle_{124} = \frac{N_+}{2}\big[\gamma|e\rangle_1|\alpha\rangle_2|-\alpha\rangle_4 + \gamma|g\rangle_1|-\alpha\rangle_2|\alpha\rangle_4 + \lambda|e\rangle_1|-\alpha\rangle_2|-\alpha\rangle_4 + \lambda|g\rangle_1|\alpha\rangle_2|\alpha\rangle_4\big]. \tag{35}$$

Now, if we detect cavity 2 in the coherent state $|-\alpha\rangle_2$, then the latter state collapses to

$$|\Phi'\rangle_{12} = \frac{N_+}{2}\big[\gamma|g\rangle_1|\alpha\rangle_4 + \lambda|e\rangle_1|-\alpha\rangle_4\big]. \tag{36}$$

By operating a Hadamard gate on atom 1, we have

$$|\Phi'\rangle_{12} = \frac{N_+}{2\sqrt{2}}\big[(\gamma|\alpha\rangle_4 + \lambda|-\alpha\rangle_4)|e\rangle_1 - (\gamma|\alpha\rangle_4 - \lambda|-\alpha\rangle_4)|g\rangle_1\big]. \tag{37}$$

If we detect atom 1 in the excited state $|e\rangle_1$, the state of cavity 4 is exactly in the initial state of cavity 2 and the teleportation process becomes successful (with complete fidelity). If the state of atom 1 is $|g\rangle_1$, the teleportation fails. Thus, the probability of successful teleportation reduced to 0.5.

5. Summary and conclusion

In this paper, we started with a brief review on the interaction of a single-mode quantized field with a Ξ-type three-level atom in the small detuning regime and with a two-level atom in the large detuning regime, based on the JC model. Then, we propose a scheme in which two atoms are entangled and also two cavities are prepared in an entangled coherent-coherent state. Then, the entanglement swapping based on the above-mentioned two regimes and also the quasi-BSM method was studied. The swapped entangled state in the large detuning regime is obtained in two approaches: i.e., detecting and quasi-BSM methods. In this line, by using the atom-field entangled state obtained by both approaches in the large detuning regime, we illustrated that one can teleport unknown atomic and field states with complete fidelity.

In comparison with Refs. [5]–[8], we have employed a coherent–coherent entangled state (see Eq. (11)) for two cavities instead of some entangled states including two number states that have been previously used in the literature, such as the state $|2\rangle_3|0\rangle_4 + |0\rangle_3|2\rangle_4$ in Refs. [6] and [7]. In addition, it has been observed that the value of fidelities in the quasi-BSM

method for $|\alpha|^2 \gtrsim 3$, has a good compatibility with the cavity QED method (in both small and large detuning regimes). Since the fidelity is a measure of the closeness of two quantum states,[18] so it has been seen that the "atom 1-cavity 4" entangled state in the quasi-BSM method is closer to the maximally entangled state that has been utilized in the large detuning regime. Indeed, the amount of this closeness is twice of its closeness to the maximally entangled state that has been utilized in the small detuning regime. As is expressed, we have employed the quasi-BSM method in our work, while the references [2] and [9] have used the BSM method. Via this approach, our considered cavities contain the coherent states and we defined the quasi-Bell state (as Eq. (26)) instead of the Bell state. Finally, we have employed the "atom 1–cavity 4" entangled state (as a quantum channel), which is generated in both approaches in the large detuning regime to teleport atomic and field states with the help of the interaction of atom–field again in the JC model with a large detuning regime. We end this conclusion section with emphasizing the fact that what briefly makes our work distinguishable from Refs. [24] and [25] is as follows. 1) Our quantum channel is an atom–field entangled state which is obtained via the entanglement swapping process, instead of an atom–field entangled state which was obtained via the interaction of an atom with a cavity in the JC model. 2) The fidelity of teleportation in our scheme, for both atomic and field states teleportation is complete, while the value of fidelity is less than 1 in the previous works.[24,25]

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