Cavity linewidth narrowing with dark-state polaritons*

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We present a quantum-theoretical treatment of cavity linewidth narrowing with intracavity electromagnetically induced transparency (EIT). By means of intracavity EIT, the photons in the cavity are in the form of cavity polaritons: bright-state polariton and dark-state polariton. Strong coupling of the bright-state polariton to the excited state induces an effect known as vacuum Rabi splitting, whereas the dark-state polariton decoupled from the excited state induces a narrow cavity transmission window. Our analysis would provide a quantum theory of linewidth narrowing with a quantum field pulse.

Keywords: cavity electromagnetically induced transparency, linewidth narrowing, dark-state polariton

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1. Introduction

Electromagnetically induced transparency (EIT) is a coherent interference effect where the absorption and dispersion of a resonant medium are strongly modified by a coupling field acting on the linked transition.[1–4] The EIT medium in an optical cavity known as intracavity EIT was first discussed by Lukin *et al.*[5] They have shown that the cavity response is drastically modified by intracavity EIT, resulting in frequency pulling and a substantial narrowing of spectral features. By following this seminal work, significant experimental advance has been made in narrowing the cavity linewidth[6–9] and enhancing the cavity lifetime.[10]

The previous semi-classical treatments of cavity linewidth narrowing and lifetime enhancing with intracavity EIT were based on the solution of the susceptibility of EIT system. Here we present a quantum-theoretical treatment of cavity linewidth narrowing with intracavity EIT. By means of intracavity EIT, the photons in the cavity are in the form of cavity polaritons: bright-state polariton and dark-state polariton. Strong coupling of the bright-state polariton to the excited state induces an effect known as vacuum Rabi splitting, whereas the dark-state polariton decoupled from the excited state induces a narrow cavity transmission window $\upsilon = \cos^2\theta\,\upsilon_0$, with υ_0 being the empty-cavity linewidth and θ the mixing angle of the dark-state polariton.[11] When the atom-cavity system is in the collective strong coupling regime, a weak control field is sufficient for avoiding the absorption owing to spontaneous emission of the excited state, and then the dark-state polariton induces a very narrow cavity linewidth.

2. Quantum dynamics of intracavity EIT

Now we try to solve the quantum dynamics of EIT in an optical standing-wave cavity with input–output processes (Fig. 1(a)). We consider an ensemble of N atoms with three levels, two ground states $|g\rangle$, $|s\rangle$, and an excited state $|e\rangle$, forming a Λ-configuration system (see Fig. 1(b)). The two transitions $|g\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |e\rangle$ are resonantly coupled by a cavity mode and a laser field, respectively. The interaction Hamiltonian for these coherent processes is described by $H_{\rm I} = \sum_{j=1}^{N}(g\,|e\rangle_j\langle g|\,a + \Omega\,|s\rangle_j\langle e| + {\rm H.c.})$, where a is the annihilation operator of the cavity mode, g (Ω) is the coupling strength of quantized cavity mode (laser field) to the corresponding transition. After defining the collective atomic operators $C_\mu^\dagger = \sum_{j=1}^{N}|\mu\rangle_j\langle g|/\sqrt{N}$ with $\mu = e,s$, we rewritten $H_{\rm I}$ as[12]

$$H_{\rm I}' = \sqrt{N}gC_e^\dagger a + C_{\rm s}^\dagger C_e\Omega + {\rm H.c.}, \tag{1}$$

where only the coupling constant g between the atoms and the quantized cavity mode is collectively enhanced by a factor $\sqrt{N}$, since we have assumed that all atoms are initially in their ground states, i.e., $|G\rangle = \prod_{j=1}^{N}|g_j\rangle$, and the average photon number n inputted into the cavity is so small that almost all atoms are in the ground state $|g\rangle$ at all times. One can define two standing-wave cavity polaritons:[13] a dark-state polariton $m_{\rm D} = \cos\theta a - \sin\theta C_{\rm s}$, and a bright-state polariton $m_{\rm B} = \sin\theta a + \cos\theta C_{\rm s}$, with $\cos\theta = \Omega/\sqrt{Ng^2+\Omega^2}$ and $\sin\theta = \sqrt{N}g/\sqrt{Ng^2+\Omega^2}$. In terms of these cavity po-

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laritons the Hamiltonian $H_{\rm I}'$ can be represented by

$$H_{\rm I}'' = \sqrt{Ng^2+\Omega^2}(C_e^{\dagger}m_{\rm B}+C_e m_{\rm B}^{\dagger}). \tag{2}$$

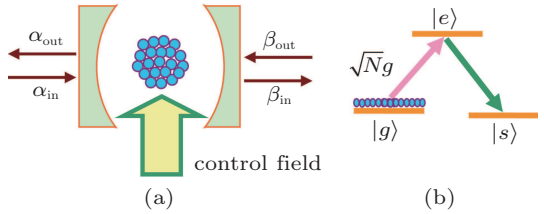


Fig. 1. (color online) (a) Schematic setup to cavity linewidth narrowing with dark-state polaritons. (b) Relevant atomic level structure and transitions.

The cavity dark-state polariton is decoupled from the collective excited state $C_e^{\dagger}|G\rangle$. We note that the dark-state polariton in intracavity Λ-configuration system with optomechanical resonator have been observed in the experiment.[13] The dark state polarition is valid when the condition, $R = [1+\eta_2+\eta_1(\kappa/\gamma_{\rm s})]^2\Omega^2/(Ng^2) \gg 1$,[13] is satisfied, where $\eta_1 = N|g|^2/(\kappa\gamma_e)$ ($\eta_2 = |\Omega|^2/(\gamma_{\rm s}\gamma_e)$) is the cooperativity of cavity mode a (ground state $|s\rangle$), with κ, γ_e, and $\gamma_{\rm s}$ being the decay rate of bare cavity mode a, excited state $|e\rangle$, and the ground state $|s\rangle$, respectively.

The external fields interact with cavity mode a through two input ports $\alpha_{\rm in}$, $\beta_{\rm in}$, and two output ports $\alpha_{\rm out}$, $\beta_{\rm out}$. The Hamiltonian for the cavity input–output processes is described by[14]

$$\begin{aligned} H_{\text{in-out}} &= \sum_{\Theta=\alpha,\beta}\int_{-\infty}^{+\infty}\omega {\rm d}\omega\Theta^{\dagger}(\omega)\Theta(\omega) \\ &+ \sum_{\Theta=\alpha,\beta}\left[{\rm i}\int_{-\infty}^{+\infty}{\rm d}\omega\sqrt{\frac{\kappa}{2\pi}}\Theta^{\dagger}(\omega)a+{\rm H.c.}\right], \end{aligned} \tag{3}$$

where ω is the frequency of the external field, and $\Theta(\omega)$ with the standard relation $[\Theta(\omega),\Theta^{\dagger}(\omega')] = \delta(\omega-\omega')$ denotes the one-dimensional free-space mode. We express the Hamiltonian $H_{\text{in-out}}$ in the polariton bases as

$$\begin{aligned} &H_{\text{in-out}}' \\ &= \sum_{\Theta=\alpha,\beta}\int_{-\infty}^{+\infty}\omega {\rm d}\omega\Theta^{\dagger}(\omega)\Theta(\omega) \\ &+ \sum_{\Theta=\alpha,\beta}\left[{\rm i}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}{\rm d}\omega\Theta^{\dagger}(\omega)\left(\sqrt{\frac{\kappa_{\rm D}}{2\pi}}m_{\rm D}+\sqrt{\frac{\kappa_{\rm B}}{2\pi}}m_{\rm B}\right)\right. \\ &\left.+{\rm H.c.}\right], \end{aligned} \tag{4}$$

with $\kappa_{\rm D} = \cos^2\theta\kappa$ and $\kappa_{\rm B} = \sin^2\theta\kappa$.

In the intracavity EIT system, only the bright-state polariton $m_{\rm B}$ resonantly couples to the excited state, leading to the splitting of the cavity transmission peak for the bright-state polariton $m_{\rm B}$ into a pair of resolvable peaks at $\omega = \omega_0 \pm \sqrt{Ng^2+\Omega^2}$,[15] with ω_0 being the cavity resonant frequency. For the cavity transmission near the cavity resonant frequency ω_0, the probability $P_{\rm B}$ for excitation of bright-state polariton $m_{\rm B}$ scales with $1/(1+\eta_1+\eta_2)$.[13] Under the condition

$$\sqrt{Ng^2+\Omega^2} \gg \kappa, \gamma_e, \gamma_{\rm s}, \tag{5}$$

the probability $P_{\rm B} = 1/[1+Ng^2/(\kappa\gamma_e)+\Omega^2/(\gamma_{\rm s}\gamma_e)] \ll 1$, thus one can neglect bright-state polariton $m_{\rm B}$ to calculate the cavity transmission spectrum near the cavity resonant frequency ω_0.

According to quantum Langevin equation, the evolution equation of the cavity dark-state polariton $m_{\rm D}$ is given by[14]

$$\dot{m}_{\rm D} = -{\rm i}\omega_0 m_{\rm D} - \kappa_{\rm D}m_{\rm D} + \sqrt{\kappa_{\rm D}}\alpha_{\rm in} + \sqrt{\kappa_{\rm D}}\beta_{\rm in}. \tag{6}$$

Using the relationships between the input and output modes at each mirror[14]

$$\alpha_{\rm out}(t) + \alpha_{\rm in}(t) = \sqrt{\kappa_{\rm D}}m_{\rm D}, \tag{7}$$

and

$$\beta_{\rm out}(t) + \beta_{\rm in}(t) = \sqrt{\kappa_{\rm D}}m_{\rm D}, \tag{8}$$

and the Fourier transformations

$$\Lambda = \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{+\infty}{\rm d}\omega\Lambda(\omega){\rm e}^{-{\rm i}\omega t},$$

with $\Lambda = m_{\rm D}$, $\alpha_{\rm in}$, $\alpha_{\rm out}$, $\beta_{\rm in}$, $\beta_{\rm out}$, we can find

$$\alpha_{\rm out}(\omega) = \frac{\kappa_{\rm D}\beta_{\rm in}(\omega)}{\kappa_{\rm D}-{\rm i}\Delta(\omega)}, \tag{9}$$

where $\Delta(\omega) = \omega-\omega_0$, and we have assumed that the photons enter into the cavity from the input port $\beta_{\rm in}$ ($\alpha_{\rm in} = 0$). Then the transmission spectrum for intracavity EIT is described by

$$T(\omega) = \frac{|\alpha_{\rm out}(\omega)|^2}{|\beta_{\rm in}(\omega)|^2} = \frac{\kappa_{\rm D}^2}{\kappa_{\rm D}^2+\Delta^2(\omega)}. \tag{10}$$

As depicted in Fig. 2, the transmission spectrum T can be controlled by the external coherent field. Then the calculation of cavity linewidth υ, i.e., the full width at half height of $T(\omega)$, gives

$$\upsilon = 2\kappa_{\rm D} = 2\kappa\cos^2\theta = \cos^2\theta\upsilon_0, \tag{11}$$

where $\upsilon_0 = 2\kappa$ is the empty-cavity linewidth.[14]

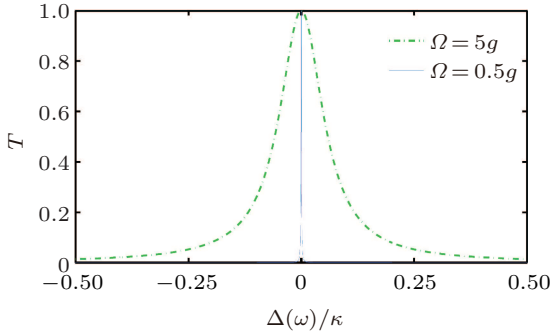


Fig. 2. (color online) Transmission spectrum T as a function of $\Delta(\omega)/\kappa$ for $\Omega = \{5g, 0.5g\}$ on the assumption that $N = 400$ and $g = \kappa = \gamma_e$.

To obtain the expression of cavity linewidth υ in Eq. (11), we have ignored the bright-state polariton $m_{\rm B}$ and the atomic spontaneous emission. Next we perform the numerical simulation of the cavity transmission spectrum with the master equation.[16] From $H'_{\text{in-out}}$ in Eq. (4), we see that both the polaritons $m_{\rm D}$ and $m_{\rm B}$ are coupled to the continuous modes $\int_{-\infty}^{+\infty} {\rm d}\omega\Theta^{\dagger}(\omega)$, and thus dissipate in the same way as the bare cavity mode a, with the decay process[16]

$$\Gamma\rho = \sum_{\Lambda={\rm D,B}} \kappa_\Lambda (2m_\Lambda \rho m_\Lambda^\dagger - m_\Lambda^\dagger m_\Lambda \rho - \rho m_\Lambda^\dagger m_\Lambda), \quad (12)$$

where ρ is the reduced density operator for the intracavity system. When a weak probe field is applied to the cavity–atom system, the dynamics evolution of the system is governed by the master equation

$$\begin{aligned}\partial_t \rho \;=\; & -{\rm i}[H_{\rm I}'' + H_{\rm pro}, \rho] + \Gamma\rho \\ & + \gamma_e (2C_e \rho C_e^\dagger - C_e^\dagger C_e \rho - \rho C_e^\dagger C_e), \end{aligned} \quad (13)$$

where $H_{\rm pro} = \sum_{\Lambda={\rm D,B}} \sqrt{2\kappa_\Lambda}(E^* {\rm e}^{{\rm i}\Delta(\omega)t} b_0 + E\,{\rm e}^{-{\rm i}\Delta(\omega)t} b_0^\dagger)$[17] describes the driving probe fields with the effective amplitude E in natural units and the frequency detuning $\Delta(\omega)$, the last term in Eq. (13) describes the atomic spontaneous emission from the excited state, and we have ignored the decay of the ground states since $\gamma_{\rm s} \ll \kappa, \gamma_e$. We calculate the cavity transmission, which is proportional to the mean intracavity photon number.[18] Figure 3 shows the transmission spectrum T versus the normalized frequency detuning $\Delta(\omega)/\kappa$, assuming the relevant parameters $(g, \kappa, \gamma_e)/2\pi \approx (16, 3.8, 2.6)$ MHz,[19] $\Omega = 2k$, $\Omega_{\rm dri} = \sqrt{2\kappa}E = 0.5k$, and $\sqrt{N}g = 10k$. From Fig. 3, we see that for cavity EIT system, there are three transmission peaks, i.e., a very narrow middle peak and two very small side peaks. The cavity dark-state polariton $m_{\rm D}$ induces the narrow peak in the middle of transmission spectrum, and the bright-state polariton $m_{\rm B}$ resonantly coupled to the excited state leads to the splitting of the cavity transmission peak for itself into a pair of small side peaks at $\omega = \omega_0 \pm \sqrt{Ng^2 + \Omega^2}$.[15]

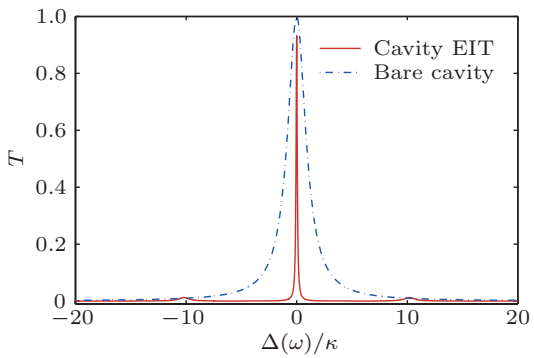


Fig. 3. (color online) Transmission spectrum as a function of $\Delta(\omega)/\kappa$, with cavity EIT and bare cavity, assuming the relevant parameters $(g, \kappa, \gamma_e)/2\pi \approx (16, 3.8, 2.6)$ MHz,[19] $\Omega = 2k$, $\Omega_{\rm dri} = \sqrt{2\kappa}E = 0.5k$, and $\sqrt{N}g = 10k$.

3. Brief discussion

Now we briefly discuss the results of our theoretical treatment of intracavity EIT. If $\Omega \ll \sqrt{N}g$, the dark-state polariton will induce a very narrow cavity linewidth $\upsilon = \upsilon_0 \cos^2\theta \approx \upsilon_0 \Omega^2/(Ng^2) \ll \upsilon_0$. However, in order that the dark state polarition is valid, we require the condition[13] $R = [1 + \eta_2 + \eta_1(\kappa/\gamma_{\rm s})]^2 \Omega^2/(Ng^2) \gg 1$. When Ω is very small, $R \approx [\eta_1(\kappa/\gamma_{\rm s})]^2 \Omega^2/(Ng^2) \gg 1$, i.e.,

$$\Omega \gg \frac{\gamma_e \gamma_{\rm s}}{\sqrt{N}g}. \quad (14)$$

Thanks to the long ground-state coherence with a cold atomic ensemble,[20] $\gamma_{\rm s} \approx 2\pi$ kHz is two order smaller than other parameters g, γ_e, and κ, thus the condition in Eq. (14) can be easily satisfied. If the atom–cavity system is in the collective strong coupling regime $\sqrt{N}g \gg \kappa, \gamma_e$, even when the control field is so weak that $\Omega < g$, both the required conditions in Eqs. (5) and (14) could be satisfied. For example, $\Omega = 0.1g < g$, $g = \gamma_e = \kappa = 10^2\gamma_{\rm s}$, and $N = 100$, then both conditions $\Omega \gg \gamma_e\gamma_{\rm s}/(\sqrt{N}g) = 10^{-3}g$ and $\sqrt{N}g = 10g \gg \kappa, \gamma_e$ are satisfied. The cavity linewidth υ directly links to cavity Q-factor.[21] Thus, the cavity linewidth narrowing with quantum field offers a potential approach to enhance the cavity Q-factor and prolong the cavity lifetime of single photons by changing Ω.

4. Conclusion

In summary, we have performed a theoretical investigation of intracavity EIT quantum mechanically. In intracavity EIT system, the cavity photons are in the form of cavity polaritons: bright-state polariton and dark-state polariton. Strong coupling of the bright-state polariton to the excited state leads to an effect known as vacuum Rabi splitting, whereas the dark-state polariton decoupled from the excited state induces a narrow transmission window. If the atom–cavity system is in the weak coupling regime, a strong control field is required for avoiding the absorption owing to spontaneous emission of the excited state. However, if atom-cavity system is in the collective strong coupling regime, a weak control field is sufficient for avoiding the absorption, and then the dark-state polariton induces a very narrow cavity linewidth. Our quantum-theoretical treatment of intracavity EIT would provide an alternative theory of linewidth narrowing with quantum field pulses.

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