Kuznetsov–Ma soliton and Akhmediev breather of higher-order nonlinear Schrödinger equation*

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In terms of Darboux transformation, we have exactly solved the higher-order nonlinear Schrödinger equation that describes the propagation of ultrashort optical pulses in optical fibers. We discuss the modulation instability (MI) process in detail and find that the higher-order term has no effect on the MI condition. Under different conditions, we obtain Kuznetsov–Ma soliton and Akhmediev breather solutions of higher-order nonlinear Schrödinger equation. The former describes the propagation of a bright pulse on a continuous wave background in the presence of higher-order effects and the soliton's peak position is shifted owing to the presence of a nonvanishing background, while the latter implies the modulation instability process that can be used in practice to produce a train of ultrashort optical soliton pulses.

Keywords: Kuznetsov–Ma soliton, Akhmediev breather

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1. Introduction

During the last four decades, optical solitons have been the object of intensive theoretical and experimental studies.[1–4] This is because that optical soliton can be treated as a natural data bit in ultrahigh-speed optical telecommunication systems.[5,6] The all-optical communication link will play the major role in optical fiber transmission systems such as all optical networks, optical communication, and optical logic devices. There are many nonlinear dynamic models in different branches of science which have soliton-like solutions. Based on the balance of the group velocity dispersion and the self-phase modulation, an optical soliton can be formed in fiber. The nonlinear Schrödinger (NLS) equation was reported theoretically by Hasegawa and Tappert[1,2] to describe self-focusing and self-phase modulation of light waves[7–14] in nonlinear optics. The propagation of optical solitons[15–18] are generated by the NLS equation.[19–21] In recent years, the femtosecond optical pulse transmission technology in optical fibers has a rapid development and potential prospect. As early as 1987, the higher-order NLS equation was derived by using the multiple-scale method,[11] which is the transmission evolution equation of femtosecond optical pulses in optical fibers. The higher-order NLS equation includes the terms caused by third-order dispersion,[22] self-steepening,[23] self-frequency shift effects,[24] and so on. However, it is more difficult to deal with both in physics and mathematics because the higher-order NLS equation is a higher-order nonlinear partial differential equation. Many authors have obtained dark and bright solitary wave solutions[25–29] and W-shaped solitary wave solutions[30] of a higher-order equation by using various methods, such as the Hirota algorithm,[31] traveling wave transformation,[32] and Darboux transformation.[33–35] Some types of exact N-soliton solutions[36–41] are reported for the higher-order nonlinear equation. In this paper, we will investigate Kuznetsov–Ma soliton and Akhmediev breather solutions of the higher-order NLS equation. In terms of Darboux transformation we obtain the analytical soliton solutions and breather solutions on a continuous wave (cw) background. The higher-order effects on the breather solution are discussed in detail.

2. Modulation instability for higher-order nonlinear equation

The higher-order NLS equation[42] takes the form

$$\begin{aligned} 0 = {} & \mathrm{i}\frac{\partial q}{\partial z} + \frac{1}{2}\frac{\partial^2 q}{\partial t^2} + |q|^2 q - \mathrm{i}\alpha\left(\frac{\partial^3 q}{\partial t^3} + 6|q|^2\frac{\partial q}{\partial t}\right) \\ & + \gamma\left[\frac{\partial^4 q}{\partial t^4} + 6\left(\frac{\partial q}{\partial t}\right)^2 \overline{q} + 4q\left|\frac{\partial q}{\partial t}\right|^2 + 8|q|^2\frac{\partial^2 q}{\partial t^2}\right. \\ & \left. + 2q^2\frac{\partial^2 \overline{q}}{\partial t^2} + 6q|q|^4\right], \end{aligned} \tag{1}$$

where z is the propagation variable and t is the time in the moving frame, with the function $q(z,t)$ being the envelop of a wave. When $\alpha = \gamma = 0$, the higher-order integrable evolution equation (1) reduces to the standard NLS equation.[43] If only $\alpha = 0$, the equation is found to be integrable and known as the 'Lakshmanan–Porsezian–Daniel' equation.[44,45] Furthermore, when only $\gamma = 0$, the equation is also integrable and known as the Hirota equation.[46,47]

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Modulation instability (MI)[48] as a parametric process exists by the interplay between dispersive and nonlinearity effects, in which a continuous or quasi-continuous wave experience a modulation of its amplitude or phase in the presence of weak perturbation.[49] Therefore, it may act as a precursor for the formation of bright solitons. Conversely, the stable dark solitons will exist if the absence of MI is under the constant intensity background. The phenomenon of MI has been discussed and studied in many physical systems. The MI process of the steady-state solutions can be discussed by introducing the following perturbed solution:

$$q = (A_c + A(z,t))\, e^{i(kz+\omega t)}, \tag{2}$$

where $A(z,t)$ is a weak perturbation. The general expression A can be assumed as $A = u + i\upsilon$ with

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{i(Kz-\Omega t)},$$

where u_0 and υ_0 are real amplitudes of the perturbation, Ω denotes the real frequency of modulation perturbations, and K is the real disturbance wave number. Substituting Eq. (2) into Eq. (1) and solving the linearized equation for A, we can obtain the dispersion relation of K and Ω, which determines the modulation instability process of the steady-state cw solution as

$$\begin{aligned} K &= \Omega\left[\omega + 2\alpha\omega^2 + (\alpha - 4\gamma\omega)\left(\Omega^2 + \omega^2 - 6A_c^2\right)\right] \\ &\quad \pm \frac{1}{2}\Omega\sqrt{\varsigma^2(\Omega^2 - 4A_c^2)}, \end{aligned} \tag{3}$$

where $\varsigma = \left(1 + 6\alpha\omega - 2\gamma\left(\Omega^2 + 6\omega^2 - 6A_c^2\right)\right)$. When the inequality $\Omega^2 < 4A_c^2$ is satisfied, K becomes complex. This means that the steady-state cw solution is unstable since A experiences an exponential growth along the fiber length. This phenomenon is referred to as modulation instability since it leads to the modulation of the steady-state amplitude. From Eq. (3) we find that the higher-order term has no effect on the MI condition.

3. Kuznetsov–Ma soliton and Akhmediev breather solution

The higher-order NLS equation can be presented by the condition of compatibility of the Lax equation as

$$\frac{\partial}{\partial t}\psi = L\psi, \frac{\partial}{\partial z}\psi = B\psi, \tag{4}$$

where $\psi = (\psi_1, \psi_2)^{\mathrm{T}}$, and the matrices L and B have the following forms:

$$\begin{aligned} \boldsymbol{L} &= \lambda\boldsymbol{J} + \boldsymbol{U}, \\ \boldsymbol{B} &= \lambda^2\boldsymbol{J} + \lambda\boldsymbol{U} + \frac{1}{2}\boldsymbol{V} - \alpha\boldsymbol{M} + \gamma\boldsymbol{V}_{\mathrm{p}}, \end{aligned} \tag{5}$$

where

$$\boldsymbol{J} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \boldsymbol{U} = \begin{pmatrix} 0 & i\overline{q} \\ iq & 0 \end{pmatrix}, \quad \boldsymbol{V} = \begin{pmatrix} -i|q|^2 & \dfrac{\partial\overline{q}}{\partial t} \\ -\dfrac{\partial q}{\partial t} & i|q|^2 \end{pmatrix},$$

$$\boldsymbol{K}_{\mathrm{a}} = \begin{pmatrix} A_{\mathrm{n}} & -\overline{B}_{\mathrm{n}} \\ B_{\mathrm{n}} & -A_{\mathrm{n}} \end{pmatrix}, \quad \boldsymbol{V}_{\mathrm{p}} = \begin{pmatrix} iA_{\mathrm{p}} & B_{\mathrm{p}} \\ -C_{\mathrm{p}} & -iA_{\mathrm{p}} \end{pmatrix},$$

$$\boldsymbol{M} = 4\lambda^3\boldsymbol{J} + 4\lambda^2\boldsymbol{U} + 2\lambda\boldsymbol{V} + \boldsymbol{K}_{\mathrm{a}},$$

with

$$\begin{aligned} A_{\mathrm{n}} &= \overline{q}\frac{\partial q}{\partial t} - q\frac{\partial\overline{q}}{\partial t}, B_{\mathrm{n}} = -i\left(2q|q|^2 + \frac{\partial^2 q}{\partial t^2}\right), \\ A_{\mathrm{p}} &= -3|q|^4 + \left|\frac{\partial q}{\partial t}\right|^2 - 2i\lambda\left(q\frac{\partial\overline{q}}{\partial t} - \overline{q}\frac{\partial q}{\partial t}\right) \\ &\quad -\overline{q}\frac{\partial^2 q}{\partial t^2} - q\frac{\partial^2\overline{q}}{\partial t^2} + 4\lambda^2|q|^2 - 8\lambda^4, \\ B_{\mathrm{p}} &= 2\overline{q}\left(3q\frac{\partial\overline{q}}{\partial t} - 4i\lambda^3\right) + 2i\lambda\frac{\partial^2\overline{q}}{\partial t^2} \\ &\quad + \frac{\partial^3\overline{q}}{\partial t^3} + 4i\lambda\overline{q}|q|^2 - 4\lambda^2\frac{\partial\overline{q}}{\partial t}, \\ C_{\mathrm{p}} &= 2q\left(3\overline{q}\frac{\partial q}{\partial t} + 4i\lambda^3\right) - 2i\lambda\frac{\partial^2 q}{\partial t^2} \\ &\quad + \frac{\partial^3 q}{\partial t^3} - 4i\lambda q|q|^2 - 4\lambda^2\frac{\partial q}{\partial t}. \end{aligned}$$

In the above equations the overbar stands for the complex conjugate. It is easy to verify that equation (1) can be recovered under the compatibility condition $\partial\boldsymbol{B}/\partial\boldsymbol{t} - \partial\boldsymbol{L}/\partial z + [\boldsymbol{B},\boldsymbol{L}] = 0$. It is well known that the bright solitary wave solutions exist for Eq. (1), which is given by the vanishing boundary condition. In addition, the bright soliton solutions can be obtained on the non-vanishing cw background. To this propose, we take the initial "seed" solution as $q = A_c e^{i(k_c z + \omega_c t)}$, which is a cw solution and satisfies the nonlinear dispersion relation

$$\begin{aligned} k_c &= -\frac{1}{2}\omega_c^2 + A_c^2 + \alpha\left(6\omega_c A_c^2 - \omega_c^3\right) \\ &\quad + \gamma\left(\omega_c^4 - 12\omega_c^2 A_c^2 + 6A_c^4\right), \end{aligned}$$

where A_c and k_c are arbitrary real constants, respectively.

Following the standard procedure of Darboux transformation,[50–52] we obtain the soliton solution of Eq. (1) on a cw background corresponding to eigenvalue $\lambda_1 = \omega_s + iA_s$. This solution takes the following form:

$$\begin{aligned} q_1 &= e^{i\delta}\left(A_c + 2A_s\frac{a_1\cosh\Theta_1 + \sin\Phi_1}{\cosh\Theta_1 + a_1\sin\Phi_1}\right. \\ &\quad \left. + i2A_s\frac{a_2\sinh\Theta_1 - a_3\cos\Phi_1}{\cosh\Theta_1 + a_1\sin\Phi_1}\right), \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Theta_1 &= \frac{1}{2}\left(\beta_1 N_1 - \overline{\beta}_1\overline{N}_1\right)z - \frac{1}{2}\left(N_1 - \overline{N}_1\right)t + t_0, \\ \Phi_1 &= \frac{1}{2}\left(\beta_1\overline{N}_1 + N_1\overline{\beta}_1\right)z + \frac{1}{2}\left(N_1 + \overline{N}_1\right)t + \varphi_0, \end{aligned}$$

$$
\begin{aligned}
\delta &= k_c z + \omega_c t,\\
a_1 &= \mathrm{i}A_c\left(Z_1^* - Z_1\right)/\left(1+|Z_1|^2A_c^2\right),\\
a_2 &= A_c\left(Z_1 + Z_1^*\right)/\left(1+|Z_1|^2A_c^2\right),\\
a_3 &= \left(1-|Z_1|^2A_c^2\right)/\left(1+|Z_1|^2A_c^2\right),\\
Z_1 &= 1/\left(\frac{1}{2}\omega_c - \frac{1}{2}N_1 + \omega_s + \mathrm{i}A_s\right),\\
N_1 &= \sqrt{(\omega_c + 2\omega_s + \mathrm{i}2A_s)^2 + 4A_c^2},
\end{aligned}
$$

with

$$
\begin{aligned}
\beta_1 &= \omega_c/2 - \omega_s + \gamma(l_0 + \mathrm{i}p_0) - \mathrm{i}A_s\\
&\quad + \alpha\left(m_0 + \mathrm{i}A_s\left(8\omega_s - 2\omega_c\right)\right),
\end{aligned}
$$

where

$$
\begin{aligned}
m_0 &= \left(\omega_c - 2\omega_s\right)^2 + 2\left(\omega_c\omega_s - A_c^2 - 2A_s^2\right),\\
l_0 &= 2\omega_c\left(3A_c^2 + 2A_s^2\right) + 4\omega_s\left(2\omega_s^2 - A_c^2 - 6A_s^2\right)\\
&\quad - \omega_c\left(\omega_c^2 + 4\omega_s^2 - 2\omega_c\omega_s\right),
\end{aligned}
$$

and $p_0 = 2A_s\left(\omega_c - 2\omega_s\right)^2 + 4A_s\left(4\omega_s^2 - A_c^2 - 2A_s^2\right)$, where t_0 and φ_0 are arbitrary real constants. With the different parameters in Eq. (6), we mainly discuss the properties of bright solitary wave solution which are affected by the cw background. When $A_s = 0$, the solution of Eq. (6) reduces to the cw solution $q = A_c\,\mathrm{e}^{\mathrm{i}\delta}$. As $A_c = 0$, i.e., the initial seed for Eq. (1) is zero, the solution in Eq. (6) becomes

$$
q_1 = \frac{2A_s}{\cosh\Theta_1}\,\mathrm{e}^{\mathrm{i}(\Phi_1 - \pi/2)}, \tag{7}
$$

where

$$
\begin{aligned}
\Phi_1 &= 2\left(\omega_s^2 - A_s^2\right)z + 8\alpha\omega_s\left(3A_s^2 - \omega_s^2\right)z\\
&\quad + 16\gamma\left(6\omega_s^2A_s^2 - \omega_s^4 - A_s^4\right)z + 2\omega_s t + \varphi_0,\\
\Theta_1 &= -4\omega_sA_sz + 8\alpha A_sz\left(3\omega_s^2 - A_s^2\right)\\
&\quad + 64A_s\omega_s\gamma\left(\omega_s^2 - A_s^2\right)z - 2A_st + t_0.
\end{aligned}
$$

Therefore, equation (6) denotes a soliton solution embedded in a cw background. With the increase of A_s, the cw background is gradually localized and forms Akhmediev breather[53,54] due to the interaction of soliton and cw background.

3.1. Kuznetsov–Ma soliton

When $A_c \neq 0$, $\omega_c = -2\omega_s$, and $A_s^2 > A_c^2$, the solution in Eq. (6) reduces to

$$
q_1 = \left(\zeta_1\frac{2\zeta_1\sin\Phi_1 + 2\mathrm{i}A_s\cos\Phi_1}{A_s\cosh\Theta_1 - A_c\sin\Phi_1} - A_c\right)\mathrm{e}^{\mathrm{i}(k_cz + \omega_ct)}, \tag{8}
$$

where $\Theta_1 = -2\zeta_1 t - 2\zeta_1\left(2\omega_s - F\alpha - G\gamma\right)z + t_0$, $\Phi_1 = -2\zeta_1\left(A_s - 12\omega_sA_s\alpha - E\gamma\right)z + \varphi_0$, with $\zeta_1 = \sqrt{A_s^2 - A_c^2}$, $F = 12\omega_s^2 - 4A_s^3 - 2A_c^2$, $G = 32\omega_s^3 - 16\omega_sA_c^2 - 32\omega_sA_s^2$, and $E = 48\omega_s^2A_s - 4A_sA_c^2 - 8A_s^3$. From Eq. (8) we see that the main characteristic properties of a bright soliton is aperiodic in t coordinate and periodic in x coordinate, which is usually called the Kuznetsov–Ma soliton. The propagation of the Kuznetsov–Ma soliton is depicted in Fig. 1 on a cw background with the higher-order effects. Also, it is easy to verify that equation (8) can reduce to the special form $q_1 = \mathrm{e}^{\mathrm{i}\delta}\left(2\mathrm{i}\zeta_1/\cosh\Theta_1 - A_c\right)$, at any position $z_0 = (2n\pi)/\left[2\zeta_1\left(12\omega_sA_s\alpha + E\gamma - A_s\right)\right]$, $n = 0, \pm1, \pm2, \ldots$. This result shows that this solution can be generated by coherently adding in quadrature a bright soliton to a cw background. Furthermore, from Eq. (8) we can find that the soliton intensity $|q_1|^2$ has a maximum value $|q_1|^2_{\max} = A_c^2 + 4A_s\zeta_1^2/\left(A_s - A_c\sin\Phi_1\right)$ at $\Theta_1 = 0$. However, it has a minimum value $|q_1|^2_{\min} = A_c^2 - 4A_c^2\zeta_1^2\left(\sin^2\Phi_1\right)/\left(4A_s^2 - 4A_c^2\sin^2\Phi_1\right)$, at $\cosh\Theta_1 = 2A_s/A_c\sin\Phi_1 - A_c\sin\Phi_1/A_s$.

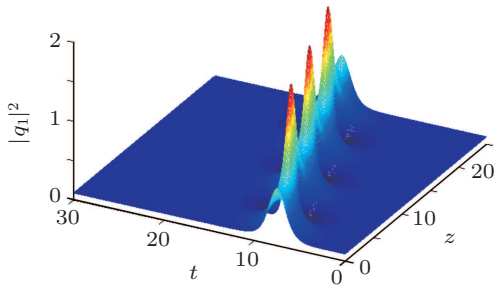


Fig. 1. (color online) Evolution of Kuznetsov–Ma soliton solution in Eq. (8). Parameters are as follows: $A_c = 0.3$, $A_s = 0.5$, $\kappa_c = \omega_c = 0.2$, $t_0 = 6$, $\varphi_0 = 0$, $\alpha = 0.05$, and $\gamma = 0.025$.

From Fig. 2(a), one can find that the maximum and minimum intensities evolve along the propagation direction. The location of maximum and minimum intensity in the time–space plane is shown in Fig. 2(b). It is clear that the narrower the soliton, the sharper the peak and the deeper the two dips at the wings of the soliton. This phenomenon indicates the characteristic breather behavior of the Kuznetsov–Ma soliton. It is easy to find that the peak position of the soliton is moved by the presence of the amplitude A_c. With careful analysis we can find the soliton's peak position is moved from its initial value $t_s = (F'\alpha + G'\gamma - 2\omega_s)z + t_0/2A_s$ to the value $t_{st} = (F\alpha + G\gamma - 2\omega_s)z + t_0/2\zeta_1$ with $F' = 12\omega_s^2 - 4A_s^2$ and $G' = 32\omega_s^3 - 32\omega_sA_s^2$. Thus, the amount of shift for the soliton's peak position is given by

$$
t_{st} - t_s = -2\left(A_c^2\alpha + 8\omega_sA_s^2\gamma\right)z + (1/\zeta_1 - 1/A_s)t_0/2. \tag{9}
$$

Figure 3 describes the location of the peak of the soliton under two different situations ($A_c \neq 0$ and $A_c = 0$). From Eq. (9) we can find that the higher-order terms give the shift $-2A_c^2\alpha$ and $-16\omega_sA_c^2\gamma$, respectively. For the NLS equation ($\alpha = \gamma = 0$), the value of shift of the soliton's peak position is $t_0/(2\zeta_1) - t_0/(2A_s)$. Compared with the NLS equation we can see that the main feature of the solution has no change except

for a move of the soliton's peak position. This phenomenon shows that the injection of a cw pulse can be used for parametric control of soliton timing.[55]

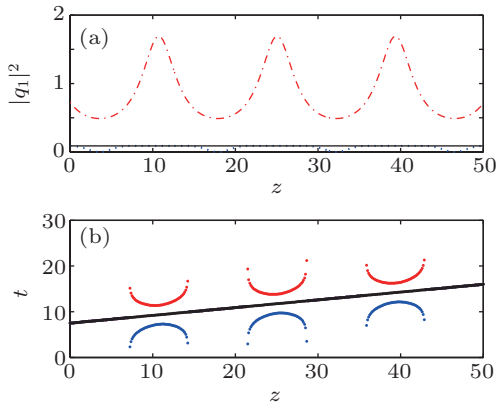


Fig. 2. (color online) (a) Evolution of the maximum intensity (red curve), the minimum intensity (blue curve), and the intensity of cw background (solid curve) of the solution in Eq. (8); (b) location of the maximum intensity (solid curve) and the minimum intensity (dotted curve) in the time-propagation-distance plane. The parameters are the same as in Fig. 1.

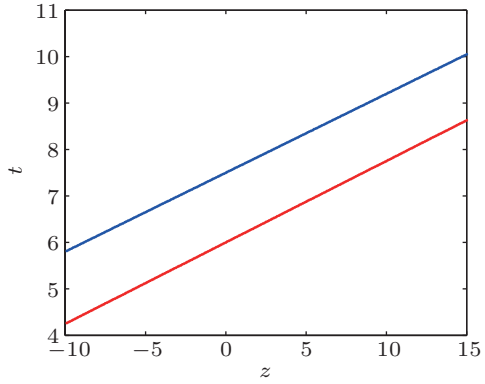


Fig. 3. (color online) Evolution of Akhmediev breather in Eq. (10) with the soliton's peak position of the one-soliton solution q_1 in Eq. (7) (red line) and the Kuznetsov–Ma soliton solution (8) (blue line). The parameters are the same as in Fig. 1.

3.2. Akhmediev breather

When $A_c \neq 0$, $\omega_c = -2\omega_s$, and $A_s^2 < A_c^2$, the solution in Eq. (6) reduces to the form

$$q_1 = D_2\, e^{i k_c z + i \omega_c t}, \tag{10}$$

where

$$D_2 = A_c - A_s \frac{2A_s \cosh \Theta_1}{A_c \cosh \Theta_1 - A_s \sin \Phi_1} - A_s \frac{i N_1 \sinh \Theta_1 - 2A_c \sin \Phi_1}{A_c \cosh \Theta_1 - A_s \sin \Phi_1},$$

with $\Theta_1 = (-A_s + (12\omega_s A_s)\,\alpha + E\gamma) N_1 z + t_0$, and $\Phi_1 = N_1 t + (2\omega_s - F\alpha - G\gamma) N_1 z + \varphi_0$, with $N_1 = 2\sqrt{A_c^2 - A_s^2}$. The solution of Eq. (10) is aperiodic in the z direction and periodic in the t direction, which is known as Akhmediev breather. Therefore, the solution of Eq. (10) can be used to describe the modulation instability process.[50,51] Figure shows the higher-order effects on the modulation instability process. We can observe that the main features of solitons have no change under different values of z except for the time shift.

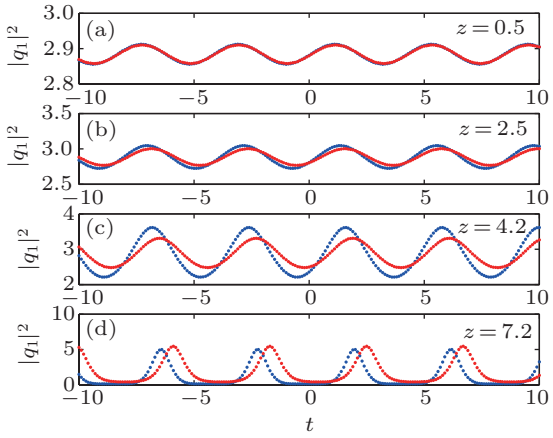


Fig. 4. (color online) Evolution of the soliton's peak position shift of the single soliton in Eq. (10) with and without the higher-order effects at (a) $z = 0.5$, (b) $z = 2.5$, (c) $z = 4.2$, and (d) $z = 7.2$, respectively. The parameters are $\alpha = 0.05$, $\gamma = 0.025$ (blue curves); and $\alpha = \gamma = 0$ (red curve), $A_s = 0.5$, $A_c = 0.9$. The other parameters are the same as in Fig. 1.

4. Conclusions

In summary, we have exactly solved the higher-order integrable NLS equation which describes the propagation of ultrashort optical pulse in optical fibers. We have discussed the modulation instability process in detail and obtained the modulation instability conditions for a higher-order nonlinear equation. Based on the Lax pair and Darboux transformation method, bright solitary soliton solutions on a continuous wave background have been presented. With the different parameters, we give the Kuznetsov–Ma soliton and Akhmediev breather solutions of the higher-order nonlinear Schrödinger equation. The propagation characteristics of those solutions on a cw background have been analyzed in detail. When the amplitude of cw light exceeds the half of the soliton amplitude, this one-soliton solution describes the modulation instability process that can be used in practice to produce a train of ultrashort optical soliton pulses. However, the amplitude of cw light is less than half of the soliton amplitude, the one-soliton solution describes the propagation of a bright pulse on the cw background in the presence of higher-order effects, and the soliton's peak position is shifted owing to the presence of a nonvanishing background field amplitude.

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