Entanglement and non-Markovianity of a multi-level atom decaying in a cavity*

Zi-Long Fan(范子龙), Yu-Kun Ren(任玉坤), and Hao-Sheng Zeng(曾浩生)†

Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Department of Physics, Hunan Normal University, Changsha 410081, *China*

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We present a paradigmatic method for exactly studying non-Markovian dynamics of a multi-level V-type atom interacting with a zero-temperature bosonic bath. Special attention is paid to the entanglement evolution and the dynamical non-Markovianity of a three-level V-type atom. We find that the entanglement negativity decays faster and non-Markovianity is smaller in the resonance regions than those in the non-resonance regions. More importantly, the quantum interference between the dynamical non-Markovianities induced by different transition channels is manifested, and the frequency domains for constructive and destructive interferences are found.

Keywords: entanglement negativity, non-Markovianity, quantum interference

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1. Introduction

Quantum non-Markovian dynamics, due to its wide existence in the quantum optical system,[1] quantum dot,[2] superconductor system,[3] quantum chemistry,[4] and biological system[5] and some possible applications in quantum metrology[6] and quantum communication,[7–10] has received much attention in recent years. Several proposals[11–17] for the measure of non-Markovianity have been presented and various dynamical properties[18–39] of non-Markovian processes have been investigated. Experimentally, the simulation of non-Markovianity[40,41] under controlled environments has been realized.

The nature of a quantum non-Markovian process is the flow of lost information from the environment back to the open system, which leads to the oscillations of some pivotal physical quantities such as quantum trace distance,[11] quantum correlation,[12,14] and quantum Fisher information.[13] Although the properties of these oscillations are not in complete agreement, each of them can serve as the signature of non-Markovian dynamics. We will study the dynamical non-Markovianity in terms of the entanglement negativity.[42,43]

The study of non-Markovian dynamics of open quantum systems is typically very involved. Most of the previous works about non-Markovian dynamics mainly concentrated on the two-level systems. The high-dimensional open quantum systems, especially the high-dimensional dissipative quantum systems, due to their complexity, have been seldom investigated. In this paper, we present a paradigmatic method for exactly studying the non-Markovian dynamics of a multi-level V-type atom decaying in a zero-temperature bosonic bath. As a special example, we will emphatically study the case of a three-level V-type atom. The evolution of entanglement, the non-Markovianity of the quantum process, and the interference effect between different transition channels will be the main contents investigated.

The paper is organized as follows. In Section 2, we introduce the microscopic model for a multi-level V-type atom interacting with a zero-temperature bosonic bath and present the schematic method for treating the problem. In Section 3 and Section 4, we study respectively the evolution of entanglement (measured by negativity) and the non-Markovianity of the dynamics for the special case of a three-level V-type atom. Finally, the conclusion is given in Section 5.

2. Microscopic model

Consider an atom A embedded in a zero-temperature bosonic bath modeled by an infinite chain of quantum harmonic oscillators (see Fig. 1(a)). The atom has V-type $(n+1)$ levels, labeled as $|0\rangle, |1\rangle, \ldots, |n\rangle$, with transition frequencies ω_i, $i = 1, 2, \ldots, n$ (Fig. 1(b)). The Hamiltonian of this system in the Schrödinger picture is given by

$$H = \sum_{i=1}^{n} \omega_i |i_{\rm A}\rangle\langle i_{\rm A}| + \sum_k \omega_k a_k^\dagger a_k + \sum_{i,k} g_{ik}(a_k^\dagger |0_{\rm A}\rangle\langle i_{\rm A}| + {\rm h.c.}), \quad (1)$$

where ω_k, a_k, and $a_k^\dagger$ are respectively the frequency, annihilation, and creation operators for the k-th harmonic oscillator of the bath. For simplicity but without loss of generality, we assume the coupling strengths g_{ik} to be real.

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†Corresponding author. E-mail: hszeng@hunnu.edu.cn

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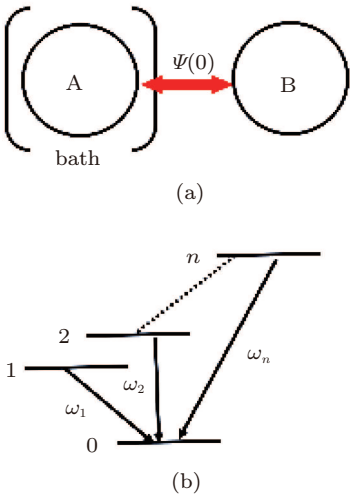


Fig. 1. (color online) (a) Schematic diagram of the model. The atom A (main system) is embedded in a bosonic bath and entangled initially with another identical ancillary atom B. The total initial state is $|\Psi(0)\rangle$. (b) Energy levels of the multi-level V-type atom.

For the following study, we now assume another identical ancillary atom B which is free from the environment but initially entangled with atom A. The initial state of the two atoms plus the environment is assumed to be

$$|\Psi(0)\rangle = \frac{1}{\sqrt{n+1}}[|0_A 0_B\rangle + \cdots + |n_A n_B\rangle] \otimes |\mathbf{0}\rangle, \tag{2}$$

where $|\mathbf{0}\rangle = |00\cdots0\rangle$ denotes the vacuum state of the bath. Due to the feature of Hamiltonian (1), the evolved state at any time t may be written as

$$|\Psi(t)\rangle = \frac{1}{\sqrt{n+1}}\left[|0_A 0_B \mathbf{0}\rangle + \sum_{i,j=1}^{n} \alpha_{ij}|i_A j_B \mathbf{0}\rangle + \sum_{j,k} \beta_{jk}|0_A j_B \mathbf{1}_k\rangle\right], \tag{3}$$

where $|\mathbf{1}_k\rangle = |00\cdots1_k\cdots0\rangle$ denotes the state that a photon is in mode k of the bath. As the state of atom B is unchangeable, we only need to consider the conservation of the total excitation number of atom A plus its bath. The first term in the above equation has total excitation number of zero; the second term has total excitation number of one, in which the state of atom A has altered from level j_A to i_A through the interaction with the bath; the last term also has total excitation number of one but the excitation in atom A has been transferred to the bath.

The state $|\Psi(t)\rangle$ must satisfy the Schrodinger equation governed by Hamiltonian (1), which leads to the following equations about coefficients $\alpha_{ij}(t)$ and $\beta_{jk}(t)$:

$$\mathrm{i}\dot{\alpha}_{ij} = \omega_i \alpha_{ij} + \sum_k g_{ik}\beta_{jk}, \tag{4}$$

$$\mathrm{i}\dot{\beta}_{jk} = \omega_k \beta_{jk} + \sum_i g_{ik}\alpha_{ij}. \tag{5}$$

Integrating Eq. (5) formally and plugging it into Eq. (4), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\alpha_{ij}(t) = -\mathrm{i}\omega_i \alpha_{ij}(t) - \int_0^t \mathrm{d}t' \sum_{i'=1}^{n} G_{ii'}(t-t')\alpha_{i'j}(t'), \tag{6}$$

where the correlation function is defined as

$$G_{ii'}(t-t') = \int_0^\infty \mathrm{d}\omega J_{ii'}(\omega)\exp[-\mathrm{i}\omega(t-t')]. \tag{7}$$

In the above derivation, we have used the continuum limit $\sum_k g_{ik}g_{i'k} \to \int \mathrm{d}\omega J_{ii'}(\omega)$, with $J_{ii'}(\omega)$ being the transition-dependent spectral density. In this paper, we assume the spectral density to be a Lorentzian distribution

$$J_{ii'}(\omega) = \frac{1}{2\pi}\frac{\Omega_i \Omega_{i'}\lambda^2}{(\omega-\omega_0)^2+\lambda^2}, \tag{8}$$

with central frequency ω_0 and width λ. The parameter Ω_i describes the transition-dependent strength. For this spectral distribution, the correlation function may be further written as

$$G_{ii'}(t-t') = \frac{\Omega_i \Omega_{i'}\lambda}{2}\exp[-(\mathrm{i}\omega_0+\lambda)(t-t')]. \tag{9}$$

Plugging this correlation function into Eq. (6) and performing the Laplace transformation, we have

$$(p+\mathrm{i}\omega_i)\alpha_{ij}(p) = \alpha_{ij}(0) - \sum_{i'=1}^{n}\frac{\Omega_i\Omega_{i'}}{2}\frac{\lambda}{p+\lambda+\mathrm{i}\omega_0}\alpha_{i'j}(p), \tag{10}$$

where $i, j = 1, \ldots, n$. This is a set of linear equations with respect to $\alpha_{ij}(p)$, which can be solved through programmatic procedures. Furthermore, we find that the inverse Laplace transformation on $\alpha_{ij}(p)$ is in principle feasible, because they can be written in the form of rational functions (see Eq. (12) in the next section). Combining with Eq. (4), we can thus obtain all the coefficients $\alpha_{ij}(t)$ and $\beta_{jk}(t)$ needed for the study of system dynamics.

3. Entanglement negativity

In this section, we take the three-level V-type atom as a paraphrastic example to demonstrate the procedure for exactly solving the dynamics. We study the entanglement evolution in terms of negativity. The time evolution of entanglement in a one-sided noisy environment is an important model, which may be used to simulate the procedure of a real entanglement distribution. Suppose Alice wants to establish an entanglement resource with distant Bob. She first produces a local entangled state of two atoms A and B, and then sends atom B to Bob through a quantum channel and keeps atom A in her hand. If we suppose only the sent atom suffered from noise, then the procedure corresponds to the time evolution of entanglement in a one-sided environment. The non-local entanglement of high-dimensional quantum systems is very relevant because of the security in quantum cryptography.[44,45]

Tracing over the bath degrees of freedom, from Eq. (3), we obtain the reduced density matrix for atoms A and B

$$\rho_{\rm AB}(t)=\frac{1}{3}\begin{pmatrix}
1 & 0 & 0 & 0 & \alpha_{11}^* & \alpha_{12}^* & 0 & \alpha_{21}^* & \alpha_{22}^* \\
0 & \sum_k|\beta_{1k}|^2 & \sum_k\beta_{1k}\beta_{2k}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sum_k\beta_{2k}\beta_{1k}^* & \sum_k|\beta_{2k}|^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_{11} & 0 & 0 & 0 & |\alpha_{11}|^2 & \alpha_{11}\alpha_{12}^* & 0 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{22}^* \\
\alpha_{12} & 0 & 0 & 0 & \alpha_{12}\alpha_{11}^* & |\alpha_{12}|^2 & 0 & \alpha_{12}\alpha_{21}^* & \alpha_{12}\alpha_{22}^* \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_{21} & 0 & 0 & 0 & \alpha_{21}\alpha_{11}^* & \alpha_{21}\alpha_{12}^* & 0 & |\alpha_{21}|^2 & \alpha_{21}\alpha_{22}^* \\
\alpha_{22} & 0 & 0 & 0 & \alpha_{22}\alpha_{11}^* & \alpha_{22}\alpha_{12}^* & 0 & \alpha_{22}\alpha_{21}^* & |\alpha_{22}|^2
\end{pmatrix}, \tag{11}$$

where the basis has been set as $\{|0_{\rm A}j_{\rm B}\rangle;|1_{\rm A}j_{\rm B}\rangle;|2_{\rm A}j_{\rm B}\rangle;|j=0,1,2\}$. All the matrix elements may be determined through Eqs. (5) and (10). Firstly, equation (10) with the initial conditions $\alpha_{11}(0)=\alpha_{22}(0)=1$ and $\alpha_{12}(0)=\alpha_{21}(0)=0$ leads to

$$\begin{aligned}
\alpha_{11}(p) &= \frac{(p-\mu_1)(p-\mu_2)}{(p-\nu_1)(p-\nu_2)(p-\nu_3)}, \\
\alpha_{12}(p) &= \frac{-\lambda\Omega_1\Omega_2}{(p-\nu_1)(p-\nu_2)(p-\nu_3)}, \\
\alpha_{21}(p) &= \alpha_{12}(p), \\
\alpha_{22}(p) &= \frac{(p-\mu_3)(p-\mu_4)}{(p-\nu_1)(p-\nu_2)(p-\nu_3)},
\end{aligned} \tag{12}$$

where

$$\mu_{1,2} = \frac{-\lambda-{\rm i}\omega_0-{\rm i}\omega_2\pm\sqrt{(\lambda+{\rm i}\omega_0-{\rm i}\omega_2)^2-2\lambda\Omega_2^2}}{2}, \tag{13}$$

$$\mu_{3,4} = \frac{-\lambda-{\rm i}\omega_0-{\rm i}\omega_1\pm\sqrt{(\lambda+{\rm i}\omega_0-{\rm i}\omega_1)^2-2\lambda\Omega_1^2}}{2}, \tag{14}$$

and ν_1, ν_2, ν_3 are the roots of the following equation with respect to ν:

$$\begin{aligned}
&(\nu+{\rm i}\omega_1)(\nu+{\rm i}\omega_2)(\nu+{\rm i}\omega_0+\lambda) \\
&+\frac{1}{2}\lambda\Omega_1^2(\nu+{\rm i}\omega_2)+\frac{1}{2}\lambda\Omega_2^2(\nu+{\rm i}\omega_1)=0.
\end{aligned} \tag{15}$$

The inverse Laplace transforms of Eq. (12) produce the following compact expressions for $\alpha_{ij}(t)$:

$$\alpha_{ij}(t)=A_1{\rm e}^{\nu_1 t}+A_2{\rm e}^{\nu_2 t}+A_3{\rm e}^{\nu_3 t}, \tag{16}$$

with

$$\begin{aligned}
A_1 &= \frac{(\mu_1-\nu_1)(\mu_2-\nu_1)}{(\nu_1-\nu_2)(\nu_1-\nu_3)}, \\
A_2 &= \frac{(\mu_1-\nu_2)(\mu_2-\nu_2)}{(\nu_2-\nu_1)(\nu_2-\nu_3)}, \\
A_3 &= \frac{(\mu_1-\nu_3)(\mu_2-\nu_3)}{(\nu_3-\nu_1)(\nu_3-\nu_2)},
\end{aligned}$$

for $\alpha_{11}(t)$;

$$\begin{aligned}
A_1 &= \frac{(\mu_3-\nu_1)(\mu_4-\nu_1)}{(\nu_1-\nu_2)(\nu_1-\nu_3)}, \\
A_2 &= \frac{(\mu_3-\nu_2)(\mu_4-\nu_2)}{(\nu_2-\nu_1)(\nu_2-\nu_3)}, \\
A_3 &= \frac{(\mu_3-\nu_3)(\mu_4-\nu_3)}{(\nu_3-\nu_1)(\nu_3-\nu_2)},
\end{aligned}$$

for $\alpha_{22}(t)$; and

$$\begin{aligned}
A_1 &= \frac{-\lambda\Omega_1\Omega_2}{(\nu_1-\nu_2)(\nu_1-\nu_3)}, \\
A_2 &= \frac{-\lambda\Omega_1\Omega_2}{(\nu_2-\nu_1)(\nu_2-\nu_3)}, \\
A_3 &= \frac{-\lambda\Omega_1\Omega_2}{(\nu_3-\nu_1)(\nu_3-\nu_2)},
\end{aligned}$$

for $\alpha_{12}(t)$ and $\alpha_{21}(t)$.

Secondly, let $\tilde{\alpha}_{ij}(t)={\rm e}^{{\rm i}\omega_i t}\alpha_{ij}(t)$ and $\tilde{\beta}_{jk}(t)={\rm e}^{{\rm i}\omega_k t}\beta_{jk}(t)$, then equation (5) becomes

$${\rm i}\frac{{\rm d}}{{\rm d}t}\tilde{\beta}_{jk}(t)=\sum_i g_{ik}{\rm e}^{{\rm i}(\omega_k-\omega_i)t}\tilde{\alpha}_{ij}(t). \tag{17}$$

Integrating this equation with the initial condition $\beta_{jk}(0)=0$, we have

$${\rm i}{\rm e}^{{\rm i}\omega_k t}\beta_{jk}(t)=\sum_i g_{ik}\int_0^t{\rm e}^{{\rm i}\omega_k t'}\alpha_{ij}(t'){\rm d}t'. \tag{18}$$

Multiplying this equation by its complex conjugation and summing over k, we obtain

$$\begin{aligned}
&\sum_k\beta_{jk}(t)\beta_{j'k}^*(t) \\
&=\sum_{i,i'}\int_0^t{\rm d}t'\int_0^t{\rm d}t''\sum_k g_{ik}g_{i'k}{\rm e}^{{\rm i}\omega_k(t'-t'')}\alpha_{ij}(t')\alpha_{i'j'}^*(t'').
\end{aligned} \tag{19}$$

Performing the continuum limit $\sum_k g_{ik}g_{i'k}\to\int{\rm d}\omega J_{ii'}(\omega)$ and employing Eqs. (7) and (8), we finally arrive at

$$\begin{aligned}
&\sum_k\beta_{jk}(t)\beta_{j'k}^*(t) \\
&=\sum_{i,i'}\frac{\lambda\Omega_i\Omega_{i'}}{2}\int_0^t{\rm d}t'\int_0^t{\rm d}t''\exp[{\rm i}\omega_0(t'-t'') \\
&-\lambda|t'-t''|]\alpha_{ij}(t')\alpha_{i'j'}^*(t'').
\end{aligned} \tag{20}$$

This expression applies in principle to the system of arbitrary high-dimensional V-type atoms, and for the case of three-level V-type atoms, we should let $i,i',j,j'=1,2$. Now, all the elements in density matrix (11) can be evaluated directly through Eqs. (16) and (20).

From Eq. (11), we can then study the evolution of entanglement for atom A with its ancillary atom B. We will employ the entanglement negativity[42,43] defined by

$$N(\rho_{\rm AB}) = \sum_k |\eta_k^{\rm T_A(-)}| = \frac{\sum_k |\eta_k^{\rm T_A}| - 1}{2}, \tag{21}$$

where $\eta_k^{\rm T_A(-)}$ and $\eta_k^{\rm T_A}$ are respectively the negative and all eigenvalues of the partial transpose of $\rho_{\rm AB}$ with respect to subsystem A. Through a tedious but straightforward calculation, we find $\eta_1^{\rm T_A} = 1/3$ and $\eta_{2,3}^{\rm T_A} = \pm|\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}|/3$, $\eta_{4,5}^{\rm T_A} = (\sum_{i,j=1}^{2}|\alpha_{ij}|^2 \pm \sqrt{B})/6$ with $B = (|\alpha_{11}|^2 - |\alpha_{22}|^2)^2 + 4|\alpha_{12}|^2(|\alpha_{11}|^2 + |\alpha_{22}|^2) + 8{\rm Re}(\alpha_{11}\alpha_{22}\alpha_{12}^*\alpha_{21}^*)$, and the remaining four roots are defined by the equation

$$\begin{vmatrix} \sum_k |\beta_{1k}|^2 - 3\eta & \sum_k \beta_{1k}\beta_{2k}^* & \alpha_{11} & \alpha_{21} \\ \sum_k \beta_{1k}^*\beta_{2k} & \sum_k |\beta_{2k}|^2 - 3\eta & \alpha_{12} & \alpha_{22} \\ \alpha_{11}^* & \alpha_{12}^* & -3\eta & 0 \\ \alpha_{21}^* & \alpha_{22}^* & 0 & -3\eta \end{vmatrix} = 0. \tag{22}$$

For simplifying the expressions of $\eta_{4,5}^{\rm T_A}$, we have used the relation $\alpha_{12} = \alpha_{21}$.

In Fig. 2, we show the time evolution of entanglement negativity for different spectral widths. For all the numerical simulations involved in this paper, we always set $\Omega_1^2 = \Omega_2^2 \equiv \Omega$, which has the frequency dimension. The dimensionless time is defined by $\tau = \Omega t$. It is shown for both non-degenerate ($\omega_1 \neq \omega_2$, Fig. 2(a)) and degenerate ($\omega_1 = \omega_2$, Fig. 2(b)) transitions that the less the λ is, the severer the curve oscillates and the slower the negativity decreases. The former suggests that narrower spectral width can increase the non-Markovianity of open quantum process, and the latter suggests that stronger non-Markovianity can effectively prevent the decay of entanglement. These properties are similar to those observed in open two-level quantum systems.[26]

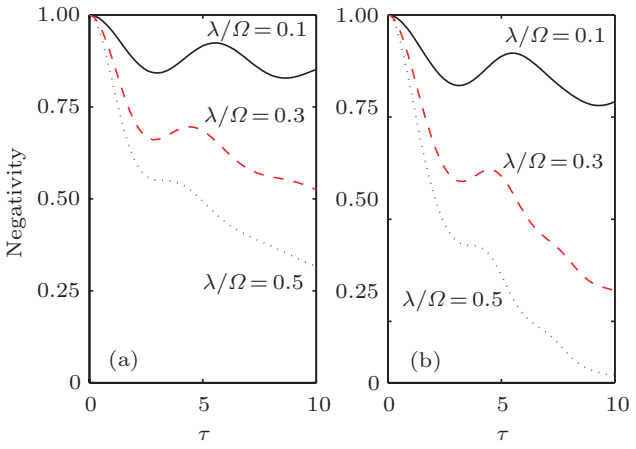


Fig. 2. (color online) Time evolution of negativity for (a) non-degenerate case with $\omega_1 = 5\Omega$, $\omega_2 = 7\Omega$, and $\omega_0 = 6\Omega$, and (b) degenerate case with $\omega_1 = \omega_2 = \omega_0 = 6\Omega$.

Figure 3 shows the time evolution of entanglement negativity in the case of degenerate transition and for different detuning between the transition and the bath frequencies, where we choose $\omega_1 = \omega_2 = 6\Omega$, $\lambda = 0.4\Omega$, and the dimensionless detuning is defined by $\Delta = (\omega_0 - \omega_1)/\Omega$. We see that in the resonant regime ($\omega_0 \simeq \omega_1$), the entanglement negativity decreases very quickly (Fig. 3(a)), because the resonance is beneficial for the dissipation of the open quantum system. In order to see clearly the oscillation, we present a plane graph in Fig. 3(b). It is shown that larger detuning between the transition and the bath frequencies can not only slow the decay of the entanglement negativity, but also improve the non-Markovianity of the quantum process. Similar results also have been obtained in the open two-level quantum systems.[11,26]

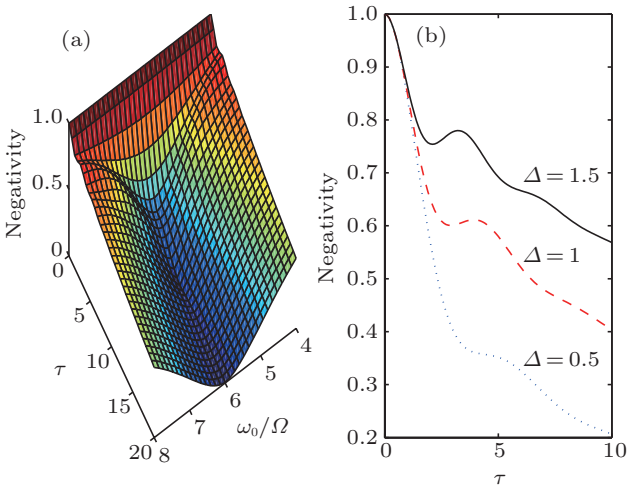


Fig. 3. (color online) (a) Time evolution of negativity for degenerate transition and for different detuning from the bath frequency, where $\omega_1 = \omega_2 = 6\Omega$, $\lambda = 0.4\Omega$, and the dimensionless detuning is defined by $\Delta = (\omega_0 - \omega_1)/\Omega$. (b) A plane view.

In Fig. 4, we show the time evolution of entanglement negativity in the case of non-degenerate transition and for different detuning between the transition and the bath frequencies, where the parameters are set as $\omega_1 = 5\Omega$, $\omega_2 = 7\Omega$, and $\lambda = 0.4\Omega$. It behaves similarly, i.e., the negativity in the two resonant regimes decays faster and the non-Markovianity in the non-resonant regimes becomes larger.

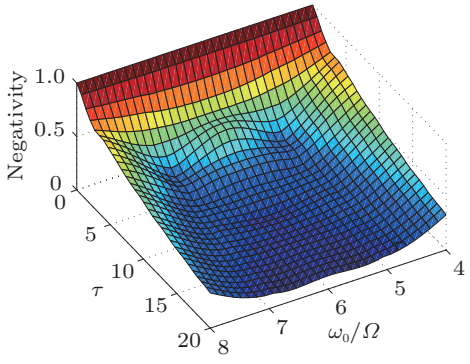


Fig. 4. (color online) Time evolution of negativity for non-degenerate transition and for different detuning from the bath frequency, where $\omega_1 = 5\Omega$, $\omega_2 = 7\Omega$, and $\lambda = 0.4\Omega$.

4. Interference of non-Markovianity

We now discuss the interference between non-Markovianities induced by different transition channels. We will use the non-Markovian measure[12] based on the entanglement with an ancillary particle. Suppose a system of interest is initially prepared in a maximally entangled state with an ancillary particle, where only the system is influenced by a noise environment and the ancillary particle is noise-free. Then the quantity for measuring the non-Markovianity of the quantum process is defined as[12]

$$\mathscr{I} = \int_{\dot{E}(t)>0} \dot{E}(t)\,\mathrm{d}t, \tag{23}$$

where $\dot{E}(t)$ denotes the time derivative of some kind of entanglement between the system and the ancillary particle, and the integration is extended over all time intervals in which $\dot{E}(t)$ is positive. For two-level systems, one usually measures the entanglement $E(t)$ by concurrence, while for higher-dimensional systems, the entanglement negativity may be a suitable choice.

The main and ancillary systems in our problem are respectively atoms A and B, which are initially in the maximally entangled state given by Eq. (2). For the system of three-level V-type atom, the state at any time t is given by Eq. (11) and its entanglement negativity Eq. (21) has already been worked out in the above section. Thus the non-Markovianity of Eq. (23) can be calculated numerically. In Fig. 5, we plot the non-Markovianity as a function of the bath frequency for different transition frequencies, where $\lambda = 0.4\Omega$. We find that the non-Markovianity (the bar charts) in the resonance regimes ($\omega_i \simeq \omega_0$) is smaller and becomes larger when away from the resonance regimes, which agrees with the analysis of the above section.

In order to see the interference effect of non-Markovianity, we also plot the non-Markovianities that only the first transition channel ω_1 or only the second channel ω_2 is opened, i.e., corresponding to a two-level system (The red and green lines in Figs. 5(a) and 5(b) respectively correspond to the non-Markovianity induced by channels ω_1 and ω_2. In the case of resonance in Fig. 5(c), the two lines overlap completely and thus only the red line is shown). The black lines are the sum of the red and green lines. It clearly shows that when the bath frequency is between the two transitions, the non-Markovianity causes constructive interference, and outside this frequency domain, it causes destructive interference. When the two transition frequencies are close to each other, the frequency domain of constructive interference becomes narrower and narrower, and finally disappears for degenerate transitions.

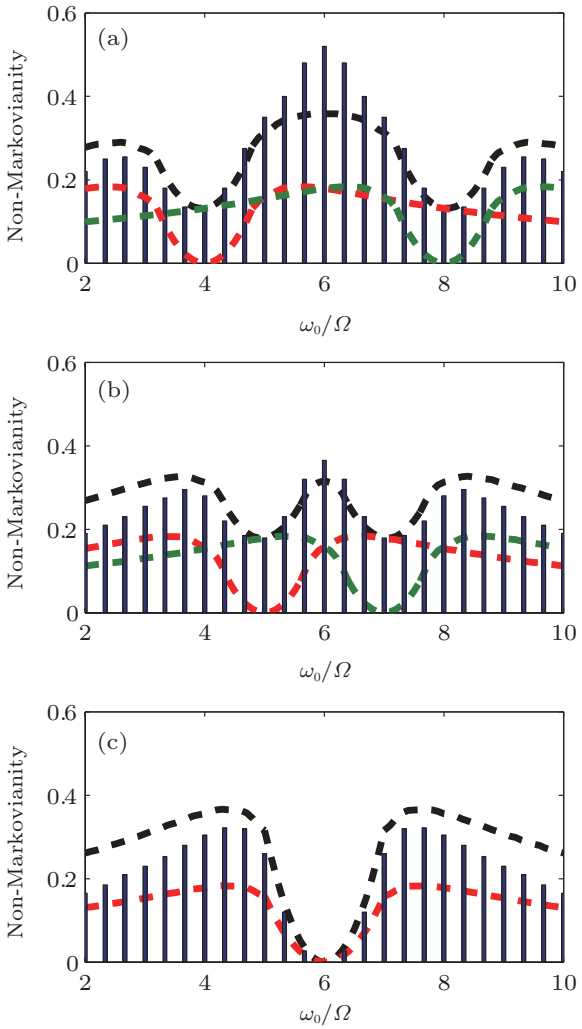


Fig. 5. (color online) Non-Markovianity as a function of the bath frequency with $\lambda = 0.4\Omega$: (a) $\omega_1 = 4\Omega$, $\omega_2 = 8\Omega$; (b) $\omega_1 = 5\Omega$, $\omega_2 = 7\Omega$; and (c) $\omega_1 = \omega_2 = 6\Omega$. The red (green) lines correspond to the non-Markovianity that only the first (second) transition exists and the black lines are the their sum. Note that the red and green lines in panel (c) overlap completely and the green line is omitted. The bar charts correspond to the non-Markovianity of the real three-level atom.

5. Conclusion

We have presented a paradigmatic method for exactly studying non-Markovian dynamics of a multi-level V-type atom interacting with a zero-temperature bosonic bath. For the special case of a three-level V-type atom, the exact analytical expressions for entanglement negativity have been obtained and the dynamical non-Markovianity has been investigated. We have found that the entanglement negativity decays faster and non-Markovianity is smaller in the resonant or quasi-resonant regions compared with that in the non-resonant case. In addition, narrower spectral width is beneficial for improving the dynamical non-Markovianity and preventing the decay of entanglement. These properties are similar to those in the open two-level quantum systems. A distinctive phenomenon observed in the case of three-level V-type atoms is the quantum interference between the non-Markovianities induced by different transition channels. We have demonstrated

the constructive and destructive interferences and the corresponding frequency domains for the two kinds of interferences.

Quantum correlation and interference are very important phenomena in quantum physics. Our method for exactly solving dynamics applies in principle to the case of more higher-dimensional open quantum systems, particularly suitable for the numerical simulation of high-dimensional open systems. We expect that non-Markovian dynamics of high-dimensional open quantum systems will show richer features.

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