Reduction of entropic uncertainty in entangled qubits system by local $\mathscr{P}\mathscr{T}$-symmetric operation*

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We investigate the quantum-memory-assisted entropic uncertainty for an entangled two-qubit system in a local quantum noise channel with $\mathscr{P}\mathscr{T}$-symmetric operation performing on one of the two particles. Our results show that the quantum-memory-assisted entropic uncertainty in the qubits system can be reduced effectively by the local $\mathscr{P}\mathscr{T}$-symmetric operation. Physical explanations for the behavior of the quantum-memory-assisted entropic uncertainty are given based on the property of entanglement of the qubits system and the non-locality induced by the re-normalization procedure for the non-Hermitian $\mathscr{P}\mathscr{T}$-symmetric operation.

Keywords: entropic uncertainty relation, $\mathscr{P}\mathscr{T}$-symmetric operation, quantum entanglement

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1. Introduction

Conventional quantum mechanics requires the Hermiticity of the Hamiltonian of a physical system to ensure that the energy of the system is real and that the time evolution is unitary. In 1998, Bender *et al.* proposed a class of Hamiltonian, the so called parity–time ($\mathscr{P}\mathscr{T}$) symmetric Hamiltonian, which is non-Hermitian while still keeping the spectrum real.[1] Here, $\mathscr{P}$ is the parity reflection operator and $\mathscr{T}$ is the time reversal operator. Four years later, they constructed a new inner product structure by introducing a linear operator $\mathscr{C}$ and they then guaranteed that the time-evolution operator was unitary.[2]

The $\mathscr{P}\mathscr{T}$-symmetric Hamiltonian for a qubit system can be written as[3]

$$H_{\mathscr{P}\mathscr{T}} = s\begin{pmatrix} \mathrm{i}\sin\alpha & 1 \\ 1 & -\mathrm{i}\sin\alpha \end{pmatrix}, \tag{1}$$

where s is a general scaling constant of the matrix, angle $\alpha \in (-\pi/2, \pi/2)$ characterizes the non-Hermiticity of the Hamiltonian, and s and α are real numbers. If $\alpha = 0$, then $H_{\mathscr{P}\mathscr{T}}$ is Hermitian, whereas $\alpha \to \pm\pi/2$, $H_{\mathscr{P}\mathscr{T}}$ is strongly non-Hermitian. The time-evolution operator for $H_{\mathscr{P}\mathscr{T}}$ is

$$U_{\mathscr{P}\mathscr{T}}(t) = \mathrm{e}^{-\mathrm{i}H_{\mathscr{P}\mathscr{T}}t} = \frac{1}{\cos\alpha}\begin{pmatrix} \cos(t'-\alpha) & -\mathrm{i}\sin t' \\ -\mathrm{i}\sin t' & \cos(t'+\alpha) \end{pmatrix}, \tag{2}$$

where $t' = \Delta E t/2$, $\Delta E = E_+ - E_-$, and $E_\pm = \pm s\cos\alpha$ are the eigenvalues of $H_{\mathscr{P}\mathscr{T}}$. The $\mathscr{P}\mathscr{T}$-symmetric Hamiltonians have recently been experimentally realized.[4–10] One of the most surprising results is that the evolution time from an initial state $|\phi_\mathrm{i}\rangle$ to a final state $|\phi_\mathrm{f}\rangle$ can be arbitrarily short among $\mathscr{P}\mathscr{T}$-symmetric Hamiltonians under the same energy constraints, which will be very useful in quantum computing.[11]

On the other hand, as one of the fundamental ideas of quantum theory, the uncertainty principle reveals that the characteristics of quantum mechanics distinctively differ from those of the classical world. The principle constrains the uncertainty about the outcomes of two incompatible measurements performed on a particle. A good way to understand the uncertainty principle is through the uncertainty game between two players, Alice and Bob. They begin the game by agreeing on two measurements Q and R. Bob then prepares a particle A and sends it to Alice. Alice performs one of the two measurements randomly on the particle A and announces her choice but not the outcomes to Bob. The question is whether Bob can predict Alice's measurement outcomes, this uncertainty can be quantified by the entropic uncertainty relation[12–14]

$$H(Q) + H(R) \geq \log_2 \frac{1}{c}, \tag{3}$$

where $H(X)$ denotes the Shannon entropy of the probability distribution of the measurement outcomes of observable X, which can be diagonalized by eigenstates $\{\psi_k^X\}$. The term $1/c$ quantifies the complementarity of Q and R with $c \equiv \max_{j,k}|\langle\psi_j^Q|\psi_k^R\rangle|$. The left-hand side of the relation captures Bob's uncertainty about Alice's measurement outcomes, while the right-hand side provides a lower bound for it. For two incompatible observables, the uncertainty is consistently greater than zero, which means that even with a complete description of the quantum state of particle A, in the case where all of the information that Bob holds about the particle is classical, it is impossible to simultaneously predict the outcomes

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of two incompatible observables perfectly.

However, when the particle A is initially prepared in an entangled state with another particle B, then the particle B can provide quantum information as an aid to minimize the uncertainty about all of the possible measurements on the particle A, and hence Bob can beat the bound. The particle B can be seen as a quantum memory. If A and B are maximally entangled, then the uncertainty of the two measurements vanishes and Bob can guess both Q and R perfectly. The improved entropic uncertainty relation depending on the amount of entanglement is[15]

$$S(Q|B)+S(R|B)\geq \log_2\frac{1}{c}+S(A|B), \tag{4}$$

where $S(A|B)$ is the conditional von Neumann entropy.[16] $S(X|B)$ is the conditional von Neumann entropy of the post-measurement state. Mathematically, a lower bound could be obtained in Eq. (4) by a negative conditional entropy $S(A|B)$ for the entangled system. Equation (4) is named the quantum-memory-assisted entropic uncertainty relation and it has been experimentally proven by Berta *et al.*[17,18]

In effective quantum information processing, however, quantum systems are inevitably exposed to environmental noise.[19] One of the consequences of the interaction between the system and its environment is decoherence, which can be viewed as the loss of information of the system and thus system's entropic uncertainty increases.[20,21] As far as we know, few operations have been discovered to reduce the entropic uncertainty of a quantum system. In this paper, we investigate the behavior of quantum-memory-assisted entropic uncertainty in an entangled qubits system under local $\mathscr{P}\mathscr{T}$-symmetric operation. This paper is organized as follows. In Section 2, we introduce the model of uncertainty game for a two-qubit system, in which one of the two qubits undergoes the quantum noise channel and the $\mathscr{P}\mathscr{T}$-symmetric operation successively, and we study the quantum-memory-assisted entropic uncertainty in the system. It is found that the quantum-memory-assisted entropic uncertainty can be reduced by the local $\mathscr{P}\mathscr{T}$-symmetric operation. In Section 3, we give explanations for the efficiency of the local $\mathscr{P}\mathscr{T}$-symmetric operation in two ways. This paper ends with the conclusions in Section 4.

2. Reduction of entropic uncertainty by local $\mathscr{P}\mathscr{T}$-symmetric operation

We consider the uncertainty game for a two-qubit system. Initially, Bob prepares a pair of qubits in the maximally entangled state

$$\Psi_{\rm i}=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle), \tag{5}$$

and then sends qubit A to Alice. For quantum information processing, qubit A suffers from an amplitude damping noise channel identified by damping rate r, while qubit B is free from environmental noise as a quantum memory kept by Bob. The amplitude damping channel can be described by operation elements as[16]

$$E_0=\begin{pmatrix}1&0\\0&\sqrt{1-r}\end{pmatrix},\quad E_1=\begin{pmatrix}0&\sqrt{r}\\0&0\end{pmatrix}. \tag{6}$$

The state of the qubits system after passing through the local amplitude damping channel can be given in sum-operator representation as

$$\begin{aligned}\rho_{\rm C}&=\varepsilon(\rho_{\rm i})\\&=(E_0\otimes I_B)\rho_{\rm i}(E_0\otimes I_B)^\dagger\\&\quad+(E_1\otimes I_B)\rho_{\rm i}(E_1\otimes I_B)^\dagger\\&=\begin{pmatrix}1/2&0&0&\sqrt{1-r}/2\\0&r/2&0&0\\0&0&0&0\\\sqrt{1-r}/2&0&0&(1-r)/2\end{pmatrix},\end{aligned} \tag{7}$$

where I_B is the identity operator of qubit B (similarly hereinafter), and $\rho_{\rm i}=|\Psi_{\rm i}\rangle\langle\Psi_{\rm i}|$.

After qubit A experiences the noise channel, we immediately perform a local $\mathscr{P}\mathscr{T}$-symmetric operation on it. The experimental scheme is shown in Fig. 1. We set the time that we start the $\mathscr{P}\mathscr{T}$-symmetric operation at $t=0$, then the total non-Hermitian Hamiltonian of the two-qubit system can be written as

$$H_{\rm tot}=H_{\mathscr{P}\mathscr{T},A}\otimes I_B=s\begin{pmatrix}{\rm i}\sin\alpha&1\\1&-{\rm i}\sin\alpha\end{pmatrix}\otimes I_B. \tag{8}$$

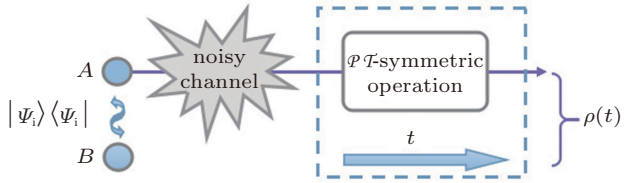


Fig. 1. (color online) Schematic diagram of implementation of two qubits under local $\mathscr{P}\mathscr{T}$-symmetric operation. Qubit A undergoes a local $\mathscr{P}\mathscr{T}$-symmetric operation after passing through the amplitude damping channel, while qubit B remains isolated.

The evolved state of the system can be obtained by using the time-evolution operator introduced in Eq. (2). Then, the final state of the system is

$$\rho(t)=\frac{(U_{\mathscr{P}\mathscr{T},A}(t)\otimes I_B)\rho_{\rm C}(U_{\mathscr{P}\mathscr{T},A}(t)\otimes I_B)^\dagger}{{\rm Tr}[(U_{\mathscr{P}\mathscr{T},A}(t)\otimes I_B)\rho_{\rm C}(U_{\mathscr{P}\mathscr{T},A}(t)\otimes I_B)^\dagger]}. \tag{9}$$

To evaluate the quantum-memory-assisted entropic uncertainty, we employ the Pauli operators for measurements. Without loss of generality, let $Q=\sigma_x$ and $R=\sigma_z$. After qubit A has been measured, the post-measurement state of the system will be

$$\rho_{\sigma_x|B}=\sum_k(|x_k\rangle\langle x_k|\otimes I_B)\rho(t)(|x_k\rangle\langle x_k|\otimes I_B),$$

$$\rho_{\sigma_z|B} = \sum_k (|z_k\rangle\langle z_k| \otimes I_B)\rho(t)(|z_k\rangle\langle z_k| \otimes I_B), \qquad (10)$$

where $\{|x_k\rangle\}$ ($\{|z_k\rangle\}$) are the eigenstates of the observable σ_x (σ_z).

We use E_U to represent the quantum-memory-assisted entropic uncertainty in the system. The dynamics of E_U under the local $\mathscr{PT}$-symmetric operation are plotted in Fig. 2 for three different damping rates: $r = 0.1$ (blue solid line), $r = 0.5$ (red dashed line), and $r = 0.9$ (black dotted line). The E_U of the initial state ρ_i is 0 and inevitably rises as a result of the influence of the amplitude damping noise. The E_U of the system after passing through the local amplitude damping channel is directly proportional to the damping rate r. It can be seen that, for all three of the damping rates considered, E_U oscillates with time, and can be reduced effectively by the local $\mathscr{PT}$-symmetric operation over some time when the damping rate is comparatively small. However, if the channel noise is extremely strong, for example $r = 0.9$, then the $\mathscr{PT}$-symmetric operation has little help to suppress the E_U of measurements in the qubits system.

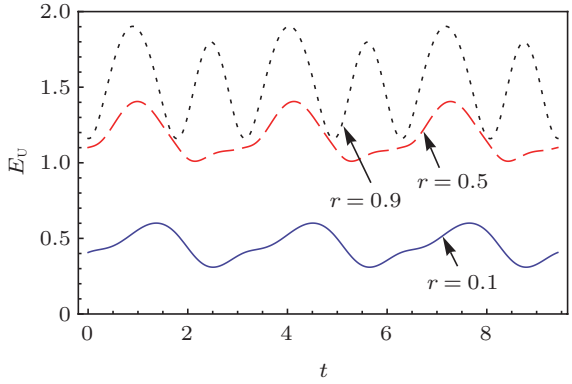


Fig. 2. (color online) The E_U under the local $\mathscr{PT}$-symmetric operation as a function of t (in units of $2/\Delta E$). The blue-solid, red-dashed, and black-dotted curves correspond to cases of $r = 0.1$, $r = 0.5$, and $r = 0.9$ respectively. Here $\alpha = \pi/16$.

To further illustrate the efficiency of the local $\mathscr{PT}$-symmetric operation on reducing E_U in the qubits system, we investigate the dynamics of E_U for different strengths of the $\mathscr{PT}$-symmetric operation, as shown in Fig. 3. For the case of $\alpha = 0$ (black dotted line), the $\mathscr{PT}$-symmetric operation is reduced to a Hermitian operation, there exits small fluctuation around the initial value of E_U. When the strength of the operation is increased to $\alpha = \pm\pi/16$ (blue dashed line and red dashed line, respectively), the efficiency of the local $\mathscr{PT}$-symmetric operation on the suppression of E_U manifests. There is only a phase difference in E_U for the cases of $\alpha = \pi/16$ and $\alpha = -\pi/16$. When the operation strength is increased to $|\alpha| = \pi/2$ (green solid line), the crest value of E_U is enlarged but the minimum value is reduced further. In this case, we may effectively suppress E_U by choosing the time at which E_U reaches its minimum value as the $\mathscr{PT}$-symmetric operation time.

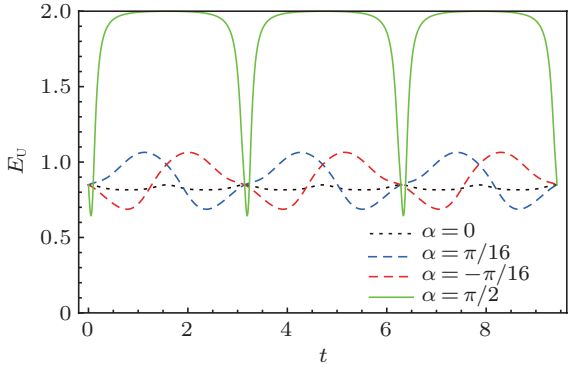


Fig. 3. (color online) The E_U under the local $\mathscr{PT}$-symmetric operation as a function of t (in units of $2/\Delta E$) for the cases of $\alpha = 0$ (black dotted line), $\alpha = \pi/16$ (blue dashed line), $\alpha = -\pi/16$ (red dashed line), and $\alpha = -\pi/2$(green solid line). Here $r = 0.3$.

Another task is to reduce E_U to its lower bound. We use $E_{U,b}$ to represent the lower bound of E_U. The E_U reaches its lower bound $E_{U,b}$ for the maximally entangled state Ψ_i given by Eq. (5). However, after being subjected to the amplitude damping channel, E_U is greater than $E_{U,b}$. By performing the local $\mathscr{PT}$-symmetric operation on qubit A, we can make E_U reduce to its lower bound again in some time. The difference (ΔE_U) between E_U and $E_{U,b}$ for cases of $\alpha = -\pi/2$ (blue solid line), $\alpha = -\pi/4$ (red dashed line), and $\alpha = -\pi/16$ (black dotted line) is plotted in Fig. 4, where we have chosen the damping rate $r = 0.3$. It can be seen that, for $\alpha = -\pi/2$, the $\mathscr{PT}$-symmetric operation is extremely strong, $\Delta E_U = 0$ can be obtained exceedingly quickly; namely, E_U can reach its lower bound $E_{U,b}$ in a very short time. By reducing the $\mathscr{PT}$-symmetric operation strength, the time of ΔE_U reaching the lowest point increases. When $|\alpha|$ is decreased to $\pi/16$, ΔE_U cannot reach the zero point any more but can still be lower than the initial value at some time. As discussed above, if the $\mathscr{PT}$-symmetric operation strength is strong enough, then E_U can be successfully reduced to its lower bound.

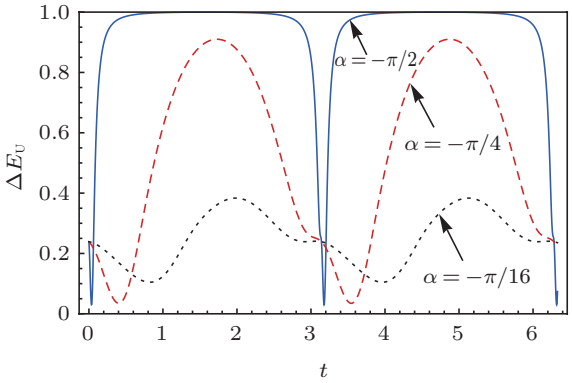


Fig. 4. (color online) The ΔE_U under the local $\mathscr{PT}$-symmetric operation as a function of t (in units of $2/\Delta E$) for the cases of $\alpha = -\pi/2$ (blue solid line), $\alpha = -\pi/4$ (red dashed line), and $\alpha = -\pi/16$ (black dotted line). Here $r = 0.3$.

3. Explanation

We give possible physical explanations for the above behavior of E_U based on the property of entanglement of the

qubits system and the Non-Hermiticity of the $\mathscr{P}\mathscr{T}$-symmetric Hamiltonian, respectively.

As one of the most important resources in quantum information and quantum computation, quantum entanglement[22–24] has attracted much attention in recent years.[25–29] The increase of entanglement can be employed to account for the reduction of $E_{\rm U}$ according to the conception of the quantum-memory-assisted entropic uncertainty relation. We use negativity to measure the degree of entanglement[30]

$$N = \frac{||\rho(t)^{\rm T}|| - 1}{2}, \tag{11}$$

which is equal to the absolute value of the sum of negative eigenvalues of $\rho(t)^{\rm T}$. Here $\rho(t)^{\rm T}$ is the partial transposition of $\rho(t)$. Figure 5 gives N against t when a local $\mathscr{P}\mathscr{T}$-symmetric operation is performed on qubit A for cases of $\alpha = -\pi/2$ (solid line), $\alpha = -\pi/4$ (dashed line), $\alpha = -\pi/16$ (dot dashed line), and $\alpha = 0$ (dotted line). When $\alpha = 0$, the operation is Hermitian and it has no influence on the entanglement of the system. However, it is clearly shown that the entanglement of the system can increase under the local non-Hermitian operation ($|\alpha| > 0$), which violates the property of entanglement monotonicity.[31,32] Moreover, with the increase of the $\mathscr{P}\mathscr{T}$-symmetric operation strength $|\alpha|$, the maximum value of negativity of the system increases and the evolution time of negativity reaching the maximum value is shortened. By comparing Fig. 5 with Fig. 3, we can see that the increase of entanglement is synchronous with the suppression of $E_{\rm U}$ under the same conditions. This stems from the fact that the qubit B as a quantum memory provides information about the outcomes of the measurements on qubit A, and thereby reduces $E_{\rm U}$ as $H(X|B) \leq H(X)$. Therefore, when the entanglement in the system increases, the uncertainty about the outcome of measurement X given information stored in the quantum memory reduces.

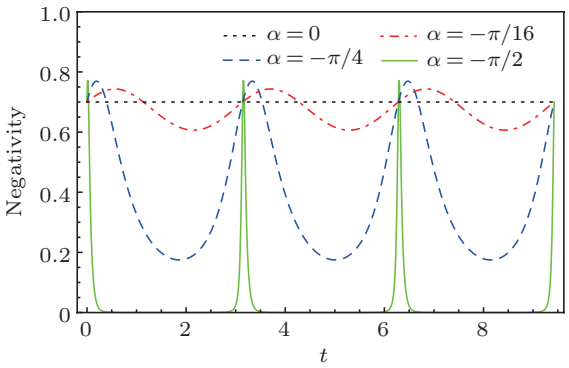


Fig. 5. (color online) Negativity N under the local $\mathscr{P}\mathscr{T}$-symmetric operation as a function of t (in units of $2/\Delta E$) for the cases of $\alpha = -\pi/2$ (solid line), $\alpha = -\pi/4$ (dashed line), $\alpha = -\pi/16$ (dot dashed line), and $\alpha = 0$ (dotted line). Here $r = 0.3$.

Another more essential explanation for the reduction of the entropic uncertainty in the qubits system by the local $\mathscr{P}\mathscr{T}$-symmetric operation is the non-Hermiticity of the operation.[33,34] Because the $\mathscr{P}\mathscr{T}$-symmetric Hamiltonian is non-Hermitian while Bob has to measure qubit B in conventional quantum mechanics, when he deals with the evolved state of the system under the $\mathscr{P}\mathscr{T}$-symmetric operation, a re-normalization procedure of $\rho(t)$ in the way of conventional quantum mechanics is required. The normalization parameter is $\mathrm{Tr}[(U_{\mathscr{P}\mathscr{T},A}(t) \otimes I_B)\rho_{\rm C}(U_{\mathscr{P}\mathscr{T},A}(t) \otimes I_B)^{\dagger}]$ as shown in Eq. (9). For the qubits system initially prepared in the maximally entangled state (see Eq. (5)), after the conventionally local operation has been performed on qubit A, qubit B will remain in the maximally mixed state ($\rho_B = I_B/2$). However, after performing the local $\mathscr{P}\mathscr{T}$-symmetric operation, the reduced density state of subsystem B changes to

$$\rho_B = \begin{pmatrix} \rho_{B,11} & \rho_{B,12} \\ \rho_{B,21} & \rho_{B,22} \end{pmatrix} / T, \tag{12}$$

with

$$\begin{aligned}
\rho_{B,11} &= \cos^2(t-\alpha) + \sin^2 t, \\
\rho_{B,12} &= 2\mathrm{i}\sqrt{1-r}\sin^2 t \sin\alpha, \\
\rho_{B,21} &= -2\mathrm{i}\sqrt{1-r}\sin^2 t \sin\alpha, \\
\rho_{B,22} &= r\cos^2(t-\alpha) + (1-r)\cos^2(t+\alpha) + \sin^2 t, \\
T &= (1+r)\cos^2(t-\alpha) + (1-r)\cos^2(t+\alpha) + 2\sin^2 t.
\end{aligned}$$

This means that the local non-Hermitian $\mathscr{P}\mathscr{T}$-symmetric operation changes the state of qubit B, which exhibits the property of non-locality. The non-locality makes the information exchange between qubits A and B possible. Hence, $E_{\rm U}$ can be reduced with a certain probability once there is information flowing from qubit A to qubit B under the local $\mathscr{P}\mathscr{T}$-symmetric operation. It is worth pointing out that similar efficiency on reducing $E_{\rm U}$ and entanglement of the qubits system can occur under other local non-Hermitian or non-unitary operations which need a re-normalization procedure, such as weak measurements.[35,36]

4. Conclusion

We have investigated the reduction of the entropic uncertainty in an entangled qubits system via the amplitude damping channel by a local $\mathscr{P}\mathscr{T}$-symmetric operation. Our results show that, by choosing a sufficiently strong $\mathscr{P}\mathscr{T}$-symmetric operation, $E_{\rm U}$ of two incompatible measurements of the qubits system can be effectively reduced and it can even reach its lower bound. We give possible physical explanations for the behavior of $E_{\rm U}$ on the basis of the property of entanglement of the qubits system and the non-Hermiticity of the $\mathscr{P}\mathscr{T}$-symmetric Hamiltonian. It can be speculated that some other non-Hermitian or non-unitary operations may also have similar effects on reducing the $E_{\rm U}$ for qubits systems.

References

[1] Bender C M and Boettcher S 1988 *Phys. Rev. Lett.* **80** 5243
[2] Bender C M, Brody D C and Jones H F 2002 *Phys. Rev. Lett.* **89** 270401
[3] Günther U and Samsonov B F 2008 *Phys. Rev. Lett.* **101** 230404
[4] Makris K G, El-Ganainy R, Christodoulides D N and Musslimani Z H 2008 *Phys. Rev. Lett.* **100** 103904
[5] Rüter C E, Makris K G, El-Ganainy R, Christodoulides D N, Segev M and Kip D 2010 *Nat. Phys.* **6** 192
[6] Schindler J, Li A, Zheng M C, Ellis F M and Kottos T 2011 *Phys. Rev. A* **84** 040101(R)
[7] Regensburger A, Bersch C, Miri M A, Onishchukov G, Christodoulides D N and Peschel U 2012 *Nature* **488** 167
[8] Zheng C, Hao L and Long G L 2013 *Philosophical Transactions. Series A, Mathematical, Physical, and Engineering Sciences* **371** 20120053
[9] Liu Y and Cui J X 2014 *Chin. Sci. Bull.* **59** 2298
[10] Peng B, Özdemir S K, Lei F, Monifi F, Gianfreda M, Long G L, Fan S, Nori F, Bender C M and Yang L 2014 *Nat. Phys.* **10** 394
[11] Bender C M, Brody D C, Jones H F and Meister B K 2007 *Phys. Rev. Lett.* **98** 040403
[12] Robertson H P 1929 *Phys. Rev.* **34** 163
[13] Bialynicki-Birula I 2007 *AIP Conf. Proc.* **889** 52
[14] Maassen H and Uffink J B M 1988 *Phys. Rev. Lett.* **60** 1103
[15] Renes J M and Boileau J C 2009 *Phys. Rev. Lett.* **103** 020402
[16] Nielson M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[17] Bertal M, Christandl M, Colbeck R, Renes J M and Renner R 2010 *Nat. Phys.* **6** 659
[18] Li C F, Xu J S, Xu X Y, Li K and Guo G C 2011 *Nat. Phys.* **7** 752
[19] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
[20] Xu Z Y, Yang W L and Feng M 2012 *Phys. Rev. A* **86** 012113
[21] Yao C M, Chen Z H, Ma Z H, Severini S and Serafini A 2014 *Sci. China-Phys. Mech. Astron.* **57** 1703
[22] Schrödinger E 1935 *Naturwissenschaften* **23** 807
[23] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[24] Modi K, Paterek T, Son W, Vedral V and Williamson M 2010 *Phys. Rev. Lett.* **104** 080501
[25] Su X L 2014 *Chin. Sci. Bull.* **59** 1083
[26] Zou Y and Li Y P 2009 *Chin. Phys. B* **18** 2794
[27] Feng H R, Meng X J, Li P and Zheng Y J 2014 *Chin. Phys. B* **23** 073301
[28] Fan Z L, Tian J and Zeng H S 2014 *Chin. Phys. B* **23** 060303
[29] Wang G F, Lü J M and Wang L C 2014 *Chin. Phys. B* **23** 050311
[30] Vidal G and Werner R F 2002 *Phys. Rev. A* **65** 032314
[31] Bennett C H 1996 *Phys. Rev. Lett.* **76** 722
[32] Bennett C H, Bernstein H J, Popescu S and Schumacher B 1996 *Phys. Rev. A* **53** 2046
[33] Lee Y C, Hsieh M H, Flammia S T and Lee R K 2014 *Phys. Rev. Lett.* **112** 130404
[34] Chen S L, Chen G Y and Chen Y N 2014 *Phys. Rev. A* **90** 054301
[35] Wang S C, Yu Z W, Zou W J and Wang X B 2014 *Phys. Rev. A* **89** 022318
[36] Liao X P, Fang M F, Fang J S and Zhu Q Q 2014 *Chin. Phys. B* **23** 020304