Evolution of dark solitons in the presence of Raman gain and self-steepening effect*

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Based on the equation satisfied by optical pulse that is a slowly varying function, the higher-order nonlinear Schrödinger equation (NLSE) including Raman gain and self-steepening effect is deduced in detail, and a new Raman gain function is defined. By using the split-step Fourier method, the influence of the combined effect between Raman gain and self-steepening on the propagation characteristic of dark solitons is simulated in the isotropic fiber. The results show that gray solitons can be symmetrically formed by high order dark soliton, however self-steepening effect will inhibit the formation mechanism through the phenomenon that gray solitons are produced only in the trailing edge of the central black soliton. Meanwhile, the Raman gain changes the propagation characteristic of optical soliton and inhibits the self-steepening effect, resulting in the broadening of pulse width and the decreasing of pulse offset.

Keywords: split-step Fourier method, self-steepening effect, dark soliton propagation characteristic

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1. Introduction

Existing in the regime of normal dispersion, dark soliton has attracted more attention in the fields of nonlinear optics[1–3] and Bose–Einstein condensation.[4–7] When the dark soliton propagates in fiber, higher-order nonlinear effects will play a vital role if its input power is relatively large and pulse width is shorter than 1 ps. Self-steepening (SS), as one of higher-order nonlinear effects, leads to an asymmetry in the self-phase modulation (SPM)-broadened spectrum of ultrashort pulses.[8,9] The distortion and aberration in dark soliton pulses propagating in fiber are due to self-steepening, so that propagation characteristic[10–12] of dark soliton pulses can be widely affected by SS. Generated by the interaction between optical solitons and phonons, the Raman gain needs to be considered if their pulse wavelengths become shorter and their input powers larger. At this moment, SS and Raman gain may not be ignored, so the study of how the combined effect of higher-order effects including Raman gain and SS affects the optical solitons is really necessary. Some researches[13,14] on SS affecting propagation characteristic of optical solitons have been carried out since 1980s. Govindaraji[13] studied dark soliton switching in nonlinear fiber couplers with gain, however, there were less relevant papers considering both Raman gain and SS effect. The Raman gain and SS effect will become intense if the optical soliton wavelength is relatively short, and they will interact and influence each other in the fiber. Therefore, research which considers the combined effect between Raman gain and self-steepening effect remains to be further conducted. The objective of this paper is to investigate the combined case in the single-mode fiber. The evolution process of picosecond dark soliton pulse propagating in the isotropic medium is simulated and the influence on propagation characteristic is analyzed by using the split-step Fourier method[10] to solve the nonlinear Schrödinger equation including SS and Raman gain.

2. Theoretic model

When the optical pulse propagates in the fiber, its slowly varying amplitude $Q(z,t)$ satisfies[12,15]

$$\frac{\partial Q(z,t)}{\partial z}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 Q(z,t)}{\partial t^2}=\mathrm{i}\left[1+\mathrm{i}sT_0\cdot\frac{\partial}{\partial t}\right]\Delta\beta_0\cdot Q(z,t), \quad (1)$$

where β_2 represents the group velocity dispersion (GVD), and the self-steepening parameter s is defined as $s=1/(\omega_0 T_0)$, where T_0 is the pulse width. And the term of $\Delta\beta_0$ on the right-hand side of Eq. (1) contains nonlinear effects of isotropic fiber, it is defined as

$$\Delta\beta_0=\frac{2\pi\omega_\mathrm{p}^2[\chi_{xxxx}^{\mathrm{RN}}+\chi_{xxxx}^{\mathrm{R}}]}{c^2k_\mathrm{p}A_\mathrm{P}}\cdot|Q(z,t)|^2, \quad (2)$$

where the effective mode area of fiber is introduced as

$$A_\mathrm{P}=\frac{\left[\iint_{-\infty}^{\infty}|F(x,y)|^2\,\mathrm{d}x\mathrm{d}y\right]^2}{\iint_{-\infty}^{\infty}|F(x,y)|^4\,\mathrm{d}x\mathrm{d}y},$$

$\chi_{xxxx}^{\mathrm{RN}}$ and χ_{xxxx}^{R} is the electronic and molecular polarizability, respectively.

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Substitution of Eq. (2) into Eq. (1) yields

$$\frac{\partial Q(z,t)}{\partial z}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 Q(z,t)}{\partial t^2} = \mathrm{i}\left[1+\mathrm{i}sT_0\cdot\frac{\partial}{\partial t}\right]\frac{2\pi\omega_\mathrm{p}^2[\chi^{\mathrm{RN}}_{xxxx}+\chi^{\mathrm{R}}_{xxxx}]}{c^2k_\mathrm{p}A_\mathrm{P}} \times |Q(z,t)|^2\cdot Q(z,t). \quad (3)$$

Let the nonlinear coefficient of electron and parallel Raman gain be defined as $\gamma=2\pi\omega_\mathrm{p}^2\chi^{\mathrm{RN}}_{xxxx}/c^2k_\mathrm{p}A_\mathrm{P}$ and $g_0''(\Omega)=\mathrm{Im}(16\mathrm{i}\pi\omega_\mathrm{p}^2\chi^{\mathrm{R}}_{xxxx}/c^2k_\mathrm{p}A_\mathrm{P})$ respectively, then equation (3) will become[16–23]

$$\frac{\partial Q(z,t)}{\partial z}+\frac{\mathrm{i}}{2}\beta_2\frac{\partial^2 Q(z,t)}{\partial t^2} = \mathrm{i}\left[1+\mathrm{i}sT_0\cdot\frac{\partial}{\partial t}\right]\left[\gamma-\frac{\mathrm{i}g_0''(\Omega)}{8}\right]|Q(z,t)|^2\cdot Q(z,t), \quad (4)$$

where $[\gamma-\mathrm{i}g_0''(\Omega)/8]$ is the nonlinear coefficient considering the interactions of incident pulse between electron and phonon, with Ω denoting the frequency difference between pump wave and Stokes wave (or anti-Stokes wave): $\Omega=(\omega_0-\omega_\mathrm{s})>0$ for Stokes wave, while $\Omega=(\omega_0-\omega_\mathrm{s})<0$ for anti-Stokes wave.

Let τ and U represent normalized time scale and normalized amplitude and be defined as

$$\tau=\frac{T}{T_0}=\frac{t-z/v_\mathrm{g}}{T_0}, \quad (5)$$

$$Q(z,t)=\sqrt{P_0}\exp[(-\alpha z)/2]I(z,\tau), \quad (6)$$

where P_0 is the peak power of initial input pulse, and α is the coefficient of fiber loss. Considering both self-steepening effect and Raman gain, the normalized amplitude U is found to satisfy the nonlinear Schrödinger equation such that

$$\frac{\partial I}{\partial z}=-\frac{\mathrm{i}\beta_2}{2T_0^2}\cdot\frac{\partial^2 I}{\partial\tau^2}+\mathrm{i}P_0\left[\gamma-\frac{\mathrm{i}g_0''(\Omega)}{8}\right] \times\left[1+\mathrm{i}s\cdot\frac{\partial}{\partial\tau}\right]|I|^2 I, \quad (7)$$

where β_2 is the second-order dispersion coefficient. In the normal region ($\beta_2>0$), equation (8) has dark solution. The parallel Raman gain $g_0''(\Omega)$ using Lorentzian model[24,25] is used to conduct numerical simulation.

3. Discussion and analysis of numerical simulation

The initial pulse is the dark soliton given by[1]

$$I(0,\tau)=N/\tanh(\tau)\exp(\mathrm{i}\xi), \quad (8)$$

where I is the amplitude of initial pulse, ξ is the phase of dark soliton, N is the order of dark soliton, for $N=1,2,3$, respectively, dark solitons are called fundamental dark soliton, second-order dark soliton, and third-order dark soliton. In this paper, for dark soliton, pulse duration $T_0=10$ fs, step size $\Delta z=1/10000$, $\beta_2=1$ ps^2/km, $\gamma=3$ W^{-1}/km, $P_0=1$ kW, and soliton period[10]

$$z_0=\frac{\pi}{2}\frac{T_0^2}{|\beta_2|},$$

the evolution process of optical solitons propagating in the fibers can be simulated by using the method of split-step Fourier transform.

3.1. Fundamental dark soliton $N=1$

Propagating in the optical fibers, dark solitons neither form a bound state nor follow a periodic evolution pattern in the case of bright solitons. By contract an input pulse with "tanh" amplitude exhibits an intensity "hole" at the center. Figure 1 shows the fundamental dark soliton ($N=1$) evolution in time–distance space in a range of 0–3z_0 under four different conditions. As seen clearly in Fig. 1(a), fundamental dark soliton pulse would maintain its shape in the normal dispersion region under the condition of considering neither Raman gain ($g_0''(\Omega)=0$) nor SS ($s=0$). Propagating without distortion, dark solitons have attracted considerable attention in the optical communication systems.[10] However, if the Raman gain is considered as clearly shown in Fig. 2(b), the central concave part ("hole") is slightly wider and it exhibits a symmetrically oscillatory structure in the wings, meanwhile this oscillation becomes fierce with increasing distance. We can understand it like this, Raman gain $g_0''(\Omega)$ destroyed the balance between GVD and SPM, therefore its width broadens, and peak power will be slightly larger. If the self-steepening effect is considered ($s\neq0$), it would lead to an asymmetry in the spectrum of ultrashort pulse, then producing spectral and temporal shifts of the dark soliton. It is illustrated in Fig. 2(c), which shows pulse shapes at z_0, $2z_0$, and $3z_0$ for a fundamental dark soliton in the presence of self-steepening ($s=0.2$) while in absence of Raman gain ($g_0''(\Omega)=0$). Peak power changes on both sides of the central concave part are asymmetric. In the leading edge, it increases with increasing propagation distance, so that the whole displays an upward convex, while in the trailing edge it decreases, and overall displays a downward concave compared with the scenario in Fig. 2(a). And self-steepening effect makes the fundamental dark soliton pulses shift toward the trailing edge, resulting in tilt sag and the pulse width broadening. As the peak shifts slower than the wings for self-steepening effect changing group velocity, it is delayed and the shift toward the trailing side occurs. In the meanwhile, if both the Raman gain and self-steepening effect are considered (shown in Fig. 2(d)), the temporal shifts decrease and the asymmetry weakens compared with the scenario in Fig. 2(c), indicating that the Raman gain not only destroys the balance between GVD and SPM, but also inhibits the self-steepening effect.

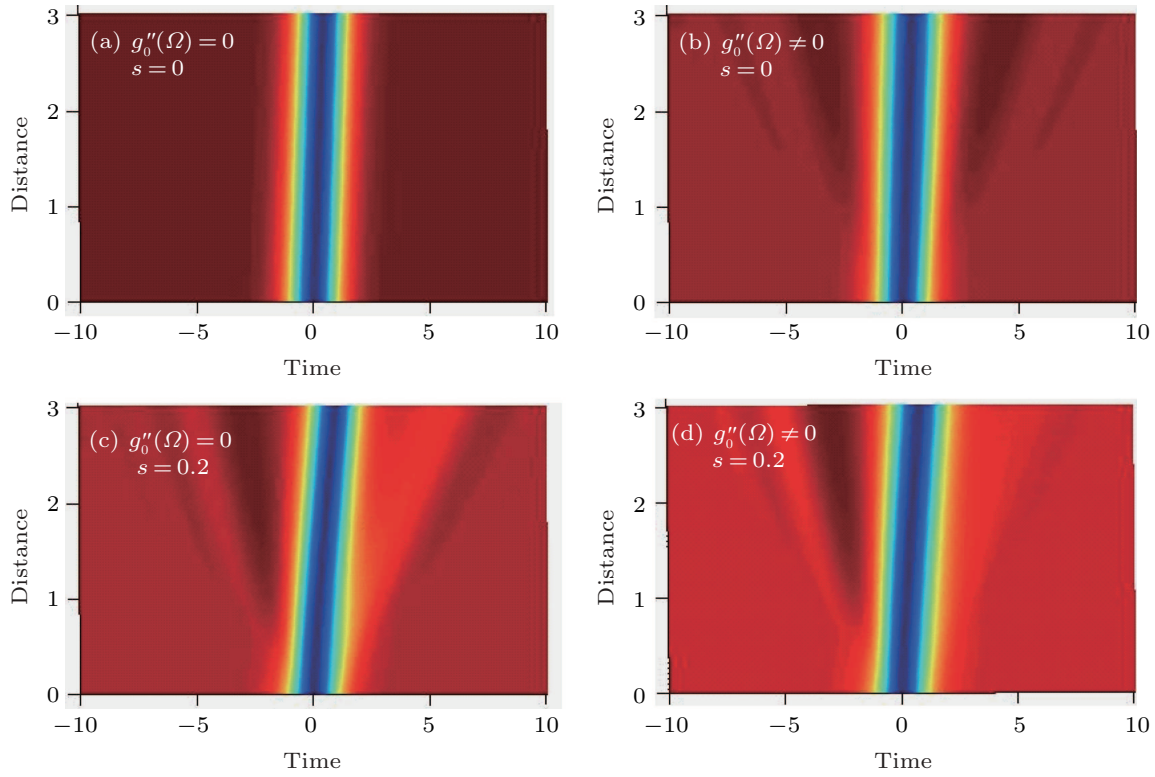


Fig. 1. (color online) Temporal evolution of fundamental dark soliton ($N=1$) induced under four different conditions over 3 soliton periods.

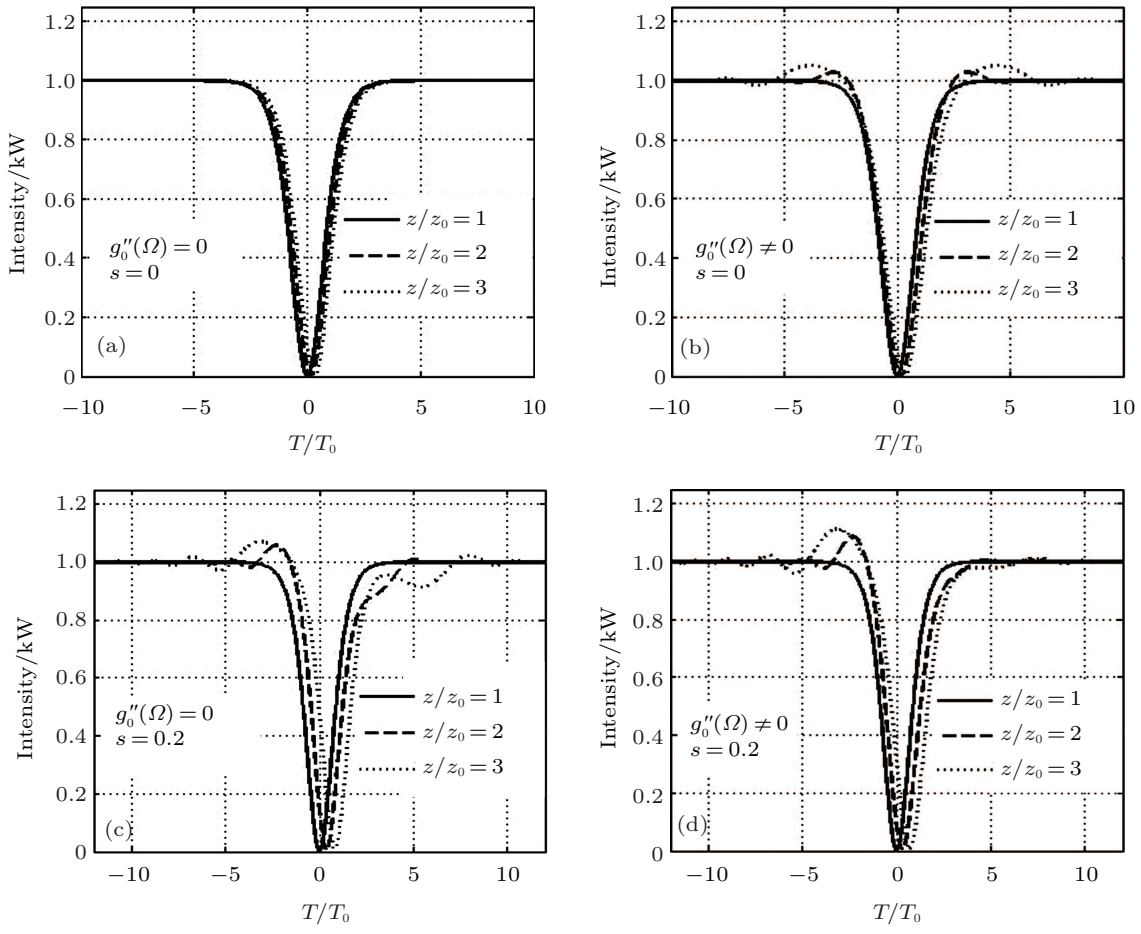


Fig. 2. Temporal evolutions of the fundamental dark soliton are visible under different pulse shapes for soliton periods at $z/z_0=1$, 2, 3, respectively.

3.2. Second-order dark soliton ($N = 2$)

Gray solitons[1] can be symmetrically formed by high-order dark soliton with the propagation distance increasing. Figure 3 shows the temporal evolutions of second-order dark soliton ($N = 2$) over three soliton periods in time-distance space. To be clearer, temporal evolutions are presented in Fig. 4, showing the pulse shapes for soliton periods at z_0, $2z_0$, $3z_0$, respectively. It is evident from the figure that two gray solitons are symmetrically formed and move away from the central black soliton[1] as the propagation distance increases. At the same time, the width of the black soliton decreases. This behavior can be understood by noting that a second-order dark soliton pulse can form a fundamental black soliton of amplitude $2\tanh(2\tau)$[10] provided that its width decreases to half of the original. It sheds part of its energy in the process, and appears in the form of gray solitons. These gray solitons move away from the central black soliton because of their different group velocities and the number of gray solitons is two. However, as illustrated in Fig. 4(b) ($g_0''(\Omega) \neq 0$, $s = 0$), the degree of depression is reduced, symmetric oscillations appear in the wings and the width of black soliton pulse slightly increases compared with the case in Fig. 4(a). It means that Raman gain destroys the balance between GVD and SPM originally built in the second-order dark soliton, resulting in the width of black soliton increasing while the depression degree of two gray solitons decreases. Self-steepening effect inhibits the formation mechanism through the phenomenon that two gray solitons are produced only in the trailing edge of the central black soliton (Fig. 4(c)). In the case of $s \neq 0$, either the gray solitons or the black solitons are forced to shift toward the trailing edge, and the temporal shift of gray solitons is longer than the black solitons. Meanwhile, the depression degree of two gray solitons becomes larger; resulting in a newborn gray soliton being generated in the tail of the pulse. Physically, the group velocity of the pulse is intensity-dependent so that the central black soliton moves at a lower speed than the gray solitons. If the Raman gain is considered, as seen in Fig. 4(d), either the temporal shifts of the whole pulse or the depression degree of gray soltions decrease compared with the scenario in Fig. 4(c). Both of these features can be understood qualitatively from the changes in the dark soliton pulse shape induced by self-steepening. Self-steepening leads to such an asymmetry in the SPM-broadened spectrum of dark soliton pulses that it destroys the symmetrical formalism. The phenomenon that two gray solitons in the trailing edge are separate from each other with different group velocities and their distance increases linearly with the propagation distance increasing, also results from self-steepening. After taking into account the Raman gain, it inhibits the self-steepening effect, leading to the pulse broadening, the temporal shifts becoming small, and a longer distance required for the shaping of newborn gray soliton.

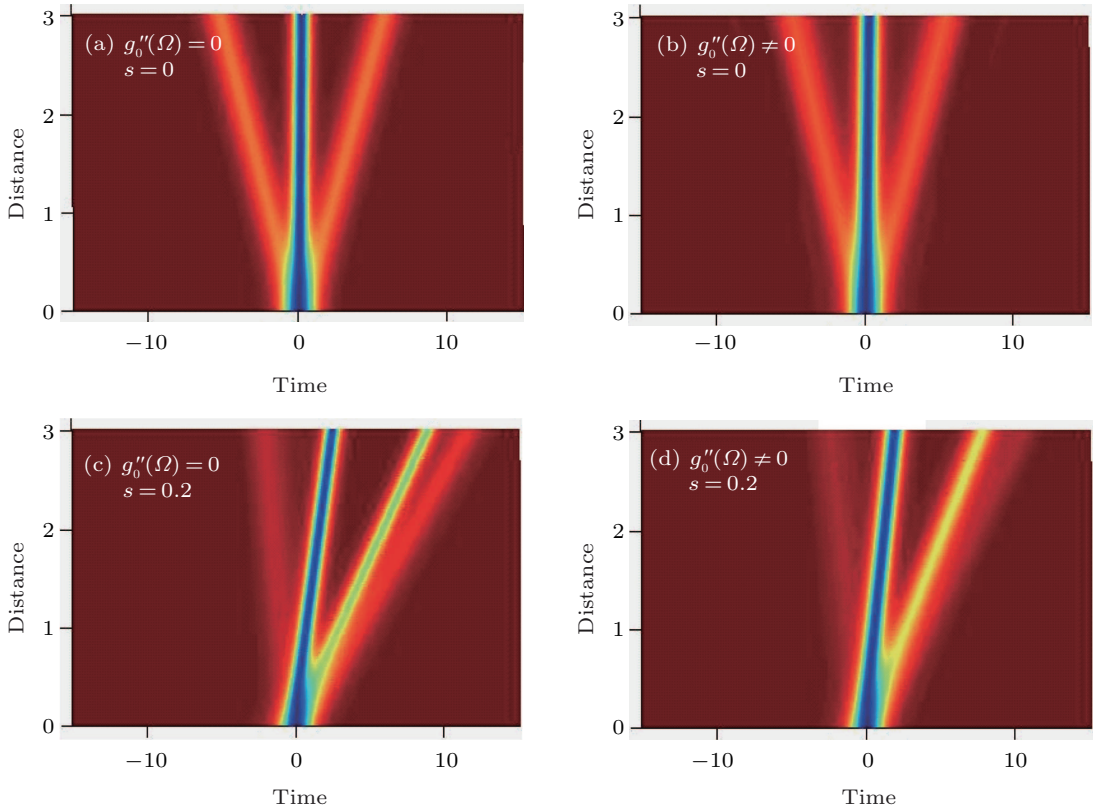


Fig. 3. (color online) Temporal evolution of second-order dark soliton ($N = 2$) induced under four different conditions over 3 soliton periods.

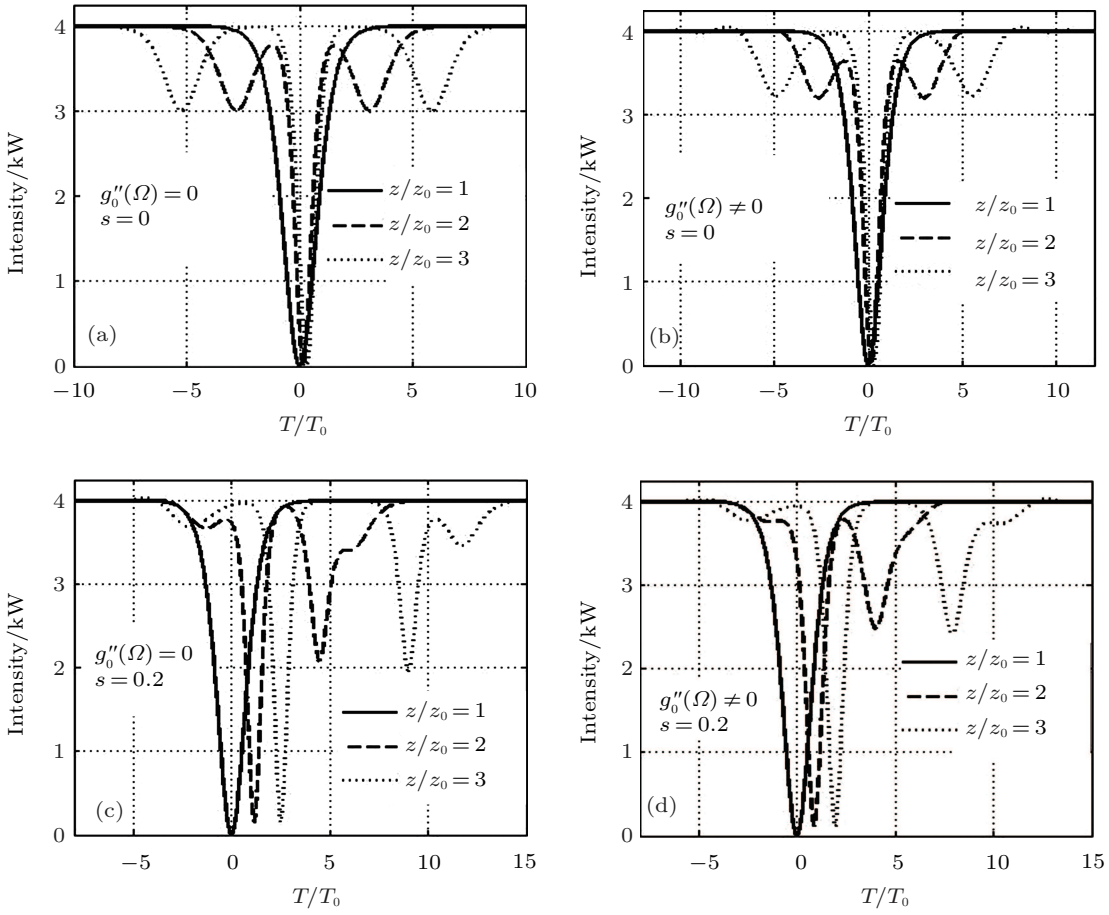


Fig. 4. Temporal evolutions of the second-order dark soliton, which are visible under different pulse shapes for soliton periods at $z/z_0 = 1, 2, 3$, respectively.

3.3. Third-order soliton ($N = 3$)

The above analysis indicates the manner in which a pair of gray solitons can be formed from a second-order dark soliton. In particular, the N-th order ($N \geq 3$) solitons follow a similar manner, and the number of gray soliton pairs is $N-1$.[5] Figure 5 shows temporal evolutions of third-order dark soliton ($N = 3$) in time–distance space over three soliton periods. To be clearer, temporal evolutions are presented in Fig. 6, showing the pulse shapes for soliton periods at z_0, $2z_0$, $3z_0$, respectively. It is evident from the figure that two pairs of gray solitons are symmetrically formed and move away from central black soliton as the propagation distance increases. At the same time, the width of the black soliton decreases. This behavior can be understood by noting that a third-order dark soliton pulse can form a fundamental black soliton of amplitude $3\tanh(3\tau)$ provided that its width decreases by a factor of 3. It sheds part of its energy in the process, and appears in the form of gray solitons. These gray solitons move away from the central black soliton with different group velocities. However, as illustrated in Fig. 6(b) ($g_0''(\Omega) \neq 0, s = 0$), the degree of depression is reduced, symmetric oscillations appear in the wings and the width of black soliton pulse slightly increases compared with the case in Fig. 6(a). For third-order dark solitons, Raman gain also destroys the balance between GVD and SPM, so that the widths of black solitons become broaden while the depression degrees of two pair of gray solitons decrease. Meanwhile, self-steepening effect inhibits the formation mechanism through the phenomenon that two pairs of gray solitons are produced only in the trailing edge of the central black soliton (seen in Fig. 6(c)). In addition, it also produces the temporal shifts of black solitons and gray solitons, resulting in pulse shifting toward the trailing edge and overall pulse tilt sag. At the same time, self-steepening effect changes the group velocity, causing the relative time displacement between the gray solitons, and its size increases linearly with the increase of propagation distance. The temporal shift toward the trailing edge of the central black and gray solitons is smaller than that in the case that the Raman gain ignored, and the newborn soliton has not formed until the dark soliton propagates at $3z_0$. It is because Raman gain destroys the balance built between GVD and SPM in dark soliton, which makes the decay speed of the peak power accelerated. Meanwhile, these features including the shift of third-order solitons decreasing and requiring longer propagation distance for forming newborn gray solitons are due to Raman gain inhibiting the self-steepening effect.

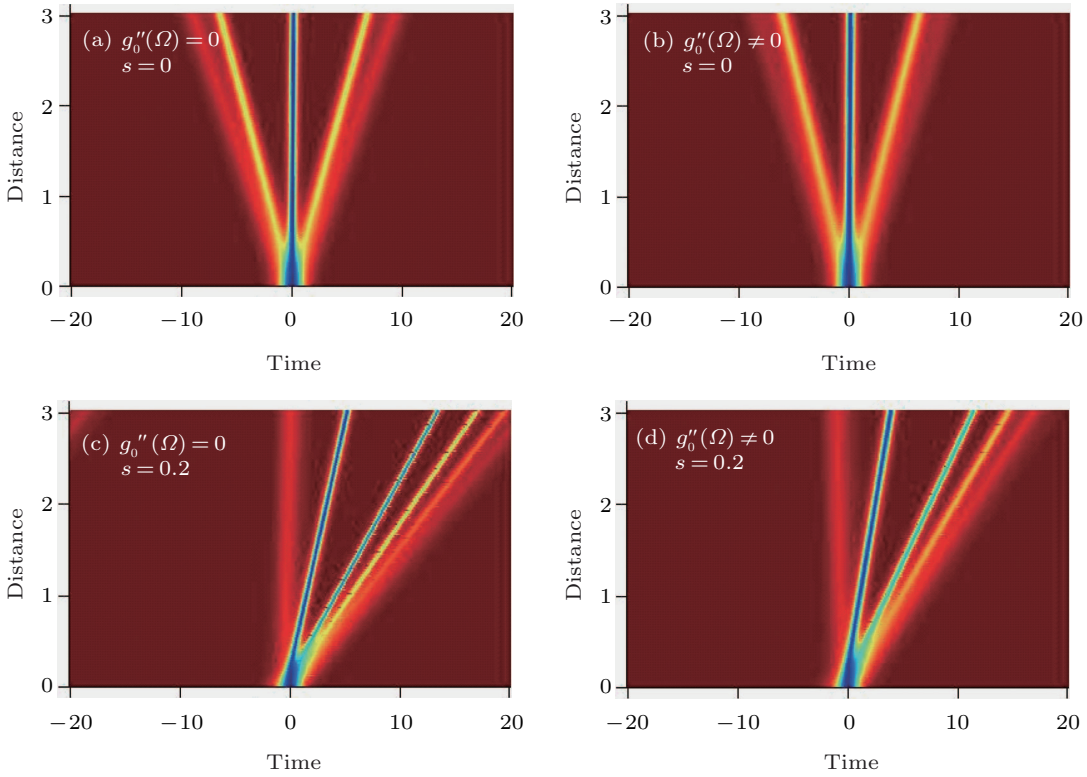


Fig. 5. (color online) Temporal evolutions of third-order dark soliton ($N=3$) induced under four different conditions over 3 soliton periods.

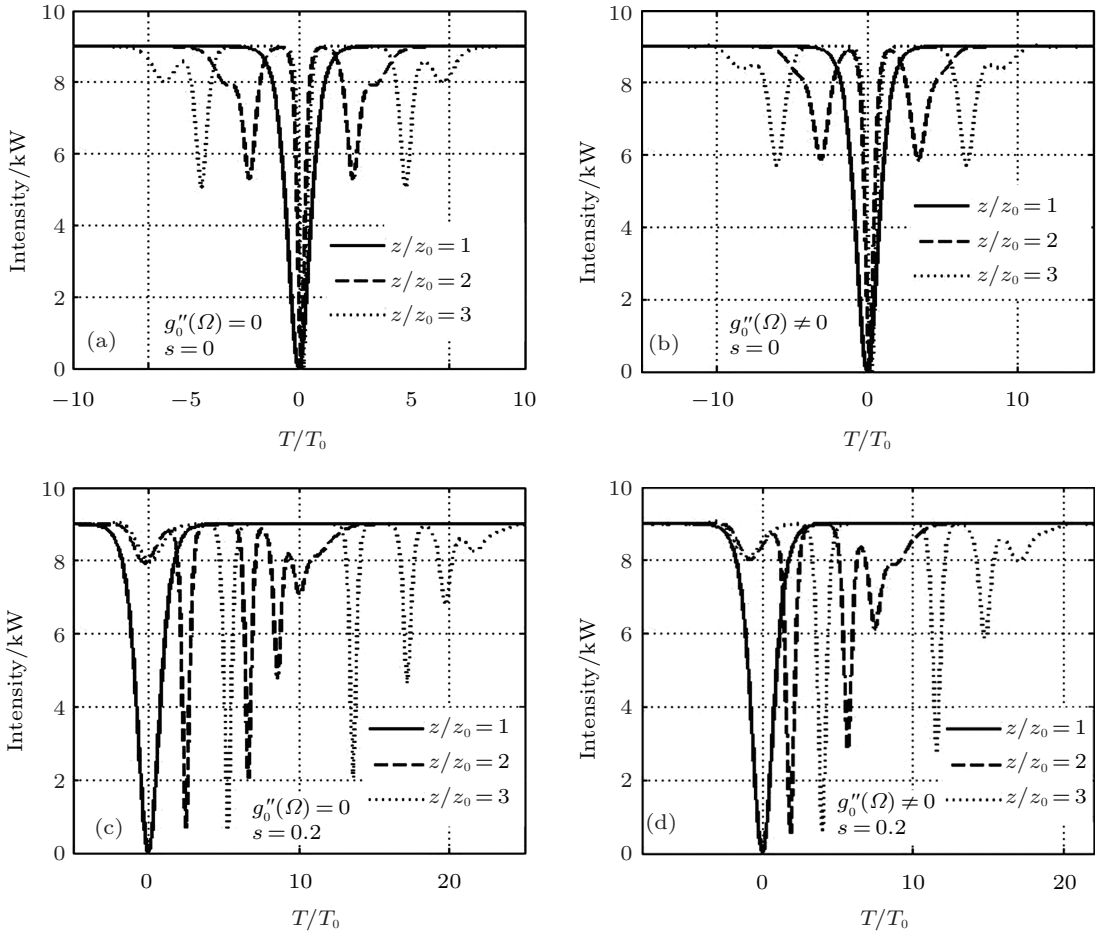


Fig. 6. Temporal evolution of the third-order dark soliton is visible under different conditions where pulse shapes for soliton periods at $z/z_0 = 1$, 2, 3, respectively.

4. Conclusions

Optical soliton pulses are shaped by the balance between GVD and SPM. In the nonlinear Schrödinger equation which governs the normalized amplitude U under condition of both self-steepening effect and Raman gain, $[\gamma - \mathrm{i}g_0''(\Omega)/8]$ is the nonlinear coefficient considering the interactions of incident pulse between electron and phonon. Therefore, the Raman gain certainly affects the balance and SS. It is concluded that gray solitons can be symmetrically formed by high order dark soliton, however self-steepening effect will inhibit the formation mechanism through the phenomenon that gray solitons are produced only in the trailing edge of the central black soliton. Meanwhile, the Raman gain changes the propagation characteristic of optical soliton and inhibits the self-steepening effect, resulting in the broadening of pulse width and the decreasing of pulse offset. As a result, the required propagation distance of newborn gray soliton produced by lateral gray is farther than that in the case that Raman gain is ignored. However, compared with bright solitons this influence is so tiny that dark solitons have potential applications in optical communications. Practically, an appropriate Raman gain should be chosen to reduce the self-steepening effect, so as to ensure that the dark solitons can propagate in the soliton communication systems with high capacity and long distance.

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