Hawking radiation of stationary and non-stationary Kerr–de Sitter black holes

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Hawking radiation of the stationary Kerr–de Sitter black hole is investigated using the relativistic Hamilton–Jacobi method. Meanwhile, extending this work to a non-stationary black hole using Dirac equations and generalized tortoise coordinate transformation, we derived the locations, the temperature of the thermal radiation as well as the maximum energy of the non-thermal radiation. It is found that the surface gravity and the Hawking temperature depend on both time and different angles. An extra coupling effect is obtained in the thermal radiation spectrum of Dirac particles which is absent from thermal radiation of scalar particles. Further, the chemical potential derived from the thermal radiation spectrum of scalar particle has been found to be equal to the highest energy of the negative energy state of the scalar particle in the non-thermal radiation for the Kerr–de Sitter black hole. It is also shown that for stationary black hole space time, these two different methods give the same Hawking radiation temperature.

Keywords: Hamilton–Jacobi method, non-stationary Kerr–de Sitter black hole, Hawking radiation, generalized tortoise coordinate transformation

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1. Introduction

Hawking discovered the thermal radiation effect of a collapsing black hole by applying the technique of quantum field theory from a given classical black hole, which gives rise to a famous paradox – the information loss paradox in a black hole physics.[1,2] From the black hole thermodynamics the area of event horizon is taken as entropy and surface gravity may be regarded as temperature.[3–5] Hawking radiation is a quantum thermal radiation triggered by a vacuum fluctuation near the black hole event horizon. When a virtual pair of particles is created simultaneously just inside the black hole event horizon, the positive energy particle can cross the energy barrier and tunnels to infinity and materializes as a true particle, however the negative energy particle will be absorbed by the black hole and form the Hawking radiation.

Kraus and Wilczek[6,7] proposed the Hawking radiation as a quantum tunneling process and reinterpreted by Parikh and Wilczek.[8] According to this method, the potential barrier is created by the outgoing particle itself and the imaginary part of the action can be derived by applying the semiclassical WKB approximation. Following this method, Hemming and Keski[9] studied the Hawking thermal radiation effect of an anti de Sitter black hole and Medved[10] investigated the radiation from a de Sitter cosmology. However, in all these investigations, the Painleve coordinates are used to calculate the quantum tunneling effect which has many attractive features. Firstly, the components of the metric in the Painleve coordinates are analytic, not diverging at the event horizon. Secondly, the constant time slices are flat Euclidean spaces. Parikh assumes the second one is important because WKB approximation can be applied to calculate the emission rate and it can be derived from the quantum mechanics of flat space time.[11] Thirdly, there exists a time like a Killing vector field which keeps the space time stationary. The Painleve coordinate system should satisfy Landau's coordinate clock synchronization condition.[12]

Zhang and Zhao[13] extended the Parikh method from a static black hole to the massive and charged particles of a non-spherical black hole space time and had a great deal of success.[14–16] Different from the Parikh method, Angheben *et al*.[17] proposed the Hawking radiation effect for an extremal and rotating black hole using the relativistic Hamilton–Jacobi equation and WKB approximation without considering the back reaction effect of the emitted particles. Damour and Ruffini[18] and Sannan[19] proposed tortoise coordinate transformation to study the Hawking radiation applying quantum mechanics in curved space time and made much progress.[20–29]

The organization of this paper is as follows. In Section 2, we study the Hawking radiation of the stationary Kerr–de Sitter black hole in Painleve coordinates using the Hamilton–Jacobi equation. In Section 3, we derive the horizons equation of the non-stationary Kerr–de Sitter black hole utilizing the null hypersurface condition and the generalized coordinate transformation. In Section 4, the Dirac equations are reduced to the first-order form and the second-order form near the horizons using the generalized coordinate transformation and the L'Hospital rule. The two second-order forms of Dirac equations are transformed to a single form of equation near the

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horizons. Adjusting the constant parameters, the Hawking radiation effect of a non-stationary black hole is obtained. Separating the wave equation, the chemical potential and Hawking thermal radiation spectra are derived. In Section 5, the thermal radiation spectrum of Dirac particles in a non-stationary black hole is discussed. We derive the expression of the new extra coupling effect and study of the relationship between the thermal and non-thermal black hole as in the scalar particle. Some conclusions are given in the last section.

2. Hawking radiation of stationary Kerr–de Sitter black hole

The line element, describing the Kerr–de Sitter solution with cosmological constant Λ in Boyer–Lindguist coordinates[30] is given by

$$\begin{aligned} \mathrm{d}s^2 &= R^2(\Delta^{-1}\mathrm{d}r^2+\Delta_\theta^{-1}\mathrm{d}\theta^2) \\ &\quad +R^{-2}\Delta_\theta A[a\sin\theta\,\mathrm{d}t-(r^2+a^2)\,\mathrm{d}\phi]^2 \\ &\quad -\Delta A R^{-2}(\mathrm{d}t-a\sin^2\theta\,\mathrm{d}\phi)^2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} &R^2=r^2+a^2\cos^2\theta,\ \ A=\left(1+\frac{1}{3}\Lambda a^2\right)^{-2}, \\ &\Delta_\theta=1+\frac{1}{3}\Lambda a^2\cos^2\theta, \\ &\Delta=(r^2+a^2)\left(1-\frac{1}{3}\Lambda r^2\right)-2Mr. \end{aligned} \tag{2}$$

Here, Λ is the cosmological constant, while M and a are the mass of the black hole and angular momentum per unit mass. The horizons are determined by

$$\Delta=(r^2+a^2)\left(1-\frac{1}{3}\Lambda r^2\right)-2Mr=0. \tag{3}$$

Equation (3) has four roots. We factorize Δ into the following form:

$$\Delta=(r-r_j)\left(r_j-M-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda a r_j}{3}\right),\ \ j=1,2,3,4. \tag{4}$$

Painleve coordinate For studying the Hawking thermal radiation effect of stationary space time, we perform the following coordinate transformation[31]

$$\mathrm{d}t=\mathrm{d}T+F(r,\theta)\,\mathrm{d}r+G(r,\theta)\,\mathrm{d}\theta, \tag{5}$$

where $F(r,\theta)$ and $G(r,\theta)$ are the two functions to be determined and must satisfy the following integrability condition:

$$\frac{\partial F(r,\theta)}{\partial\theta}=\frac{\partial G(r,\theta)}{\partial r}, \tag{6}$$

where $F(r,\theta)$ satisfies

$$g_{22}+F^2(r,\theta)\hat{g}_{11}=1, \tag{7}$$

we obtain a new metric (namely Painleve KdS one) as

$$\begin{aligned} \mathrm{d}s^2 &= \hat{g}_{11}\,\mathrm{d}T^2+2\sqrt{\hat{g}_{11}\left(1-\frac{R^2}{\Delta}\right)}\,\mathrm{d}T\,\mathrm{d}r+\mathrm{d}r^2 \\ &\quad +\left(\frac{R^2}{\Delta_\theta}+G^2\hat{g}_{11}\right)\mathrm{d}\theta^2 \\ &\quad +2\hat{g}_{11}\,\mathrm{d}T\,\mathrm{d}\theta+2G\sqrt{\hat{g}_{11}\left(1-\frac{R^2}{\Delta}\right)}\,\mathrm{d}r\mathrm{d}\theta, \end{aligned} \tag{8}$$

so we obtain

$$\begin{aligned} g^{11} &= \frac{1}{\hat{g}_{11}R^2}[\Delta+\Delta_\theta G^2\hat{g}_{11}], \\ g^{23} &= g^{32}=0,\ \ g^{22}=\frac{\Delta}{R^2}, \\ g^{12} &= -\frac{\Delta}{\hat{g}_{11}R^2}\sqrt{\hat{g}_{11}\left(1-\frac{R^2}{\Delta}\right)}, \\ g^{31} &= -\frac{G\Delta_\theta}{R^2},\ \ g^{33}=\frac{\Delta_\theta}{R^2}. \end{aligned} \tag{9}$$

The action I of the outgoing particle satisfies the relativistic Hamilton–Jacobi equation

$$g^{ab}\partial_a I\partial_b I+m^2=0, \tag{10}$$

due to the symmetry of the black hole space time, the separation of variables can be written as follows:

$$I=-\omega T+W(r,\theta)+\mathrm{j}\phi, \tag{11}$$

then equation (11) can be written as

$$\begin{aligned} I &= -\omega T+\int\sqrt{\frac{KR^2}{A}}(\omega-\mathrm{j}\Omega) \\ &\quad \times\frac{\sqrt{(R^2-\Delta)}+\sqrt{(R^2-\Delta)-\dfrac{AR^2D\Delta_\theta\Delta}{K(\omega-\mathrm{j}\Omega)^2}}}{R^2\Delta\sqrt{\Delta_\theta}}\mathrm{d}r, \end{aligned} \tag{12}$$

where

$$\begin{aligned} D &= 2G\Delta_\theta(\omega-\mathrm{j}\Omega)\frac{\partial W}{\partial\theta}+\Delta_\theta(\frac{\partial W}{\partial\theta})^2 \\ &\quad +m^2R^2+\left(\Delta_\theta G^2-\frac{K}{A\Delta_\theta R^2}\right)(\omega-\mathrm{j}\Omega)^2, \end{aligned}$$

and $K=\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta$. The imaginary part of the action at the horizons $r=r_j$ can be written as

$$\mathrm{Im}I=\frac{2\pi(r_j^2+a^2)(\omega-\mathrm{j}\Omega_j)}{\sqrt{A}\left(r_j-M-\dfrac{2\Lambda r_j^3}{3}-\dfrac{\Lambda a r_j}{3}\right)}, \tag{13}$$

where $\Omega_j=a/(r_j^2+a^2)$. The Hawking radiation temperature can be recovered as

$$T=\frac{1}{2\pi}\frac{\sqrt{A}}{(r_j^2+a^2)}\left(r_j-M-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda}{3}ar_j\right). \tag{14}$$

3. Hawking radiation of non-stationary Kerr–de Sitter black hole

The line element, describing the non-stationary Kerr–de Sitter black hole in the retarded time coordinates (u,r,θ,ϕ)[32] is

$$\begin{aligned} \mathrm{d}s^2 &= A\frac{1}{\Sigma}[\Delta-\Delta_\theta a^2\sin^2\theta]\mathrm{d}u^2+2\sqrt{A}[\mathrm{d}u-a\sin^2\theta\,\mathrm{d}\phi]\mathrm{d}r \\ &\quad -\frac{\Sigma}{\Delta_\theta}\mathrm{d}\theta^2+A\frac{2a}{\Sigma}[\Delta_\theta(r^2+a^2)-\Delta]\sin^2\theta\,\mathrm{d}u\mathrm{d}\phi \\ &\quad -\frac{A}{\Sigma}[\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta]\sin^2\theta\,\mathrm{d}\phi^2. \end{aligned} \tag{15}$$

Here, $\Xi\bar{\Xi}=\Delta_\theta$ and $M(u)$ is the mass of the black hole and it is a function of retarded time coordinate u. The null hypersurface condition gives the position of horizons $r=r_j$ of stationary or non-stationary black holes[33]

$$g^{ij}\partial_i F\partial_j F=0, \tag{16}$$

and $F(u,r,\theta,\phi)=0$. The tortoise coordinate transformation technique gives the location and the temperature of the horizon in a non-stationary black hole. The Dirac equations can be transformed to a standard form of wave equation by introducing traditional tortoise coordinate transformation $r_*=r+(2\kappa)^{-1}\ln(r-r_j)$ in a non-stationary or stationary spacetime (where κ is the surface gravity to be found out) and we assume that the location of the horizons $r=r_j$ is a function of retarded time coordinate u. According to Yang *et al.*[34] and Ibungochouba,[35] the generalized tortoise coordinate transformation is defined by

$$\begin{aligned} r_* &= r+\frac{1}{2\kappa(u_0,\theta_0,\phi_0)}\ln\left\{\frac{r-r_j(u,\theta,\phi)}{r_j(u,\theta,\phi)}\right\}, \\ u_* &= u-u_0,\ \ \theta_*=\theta-\theta_0,\ \ \phi_*=\phi-\phi_0, \end{aligned} \tag{17}$$

where $r=r_j(u,\theta,\phi_0)$ are the location of horizons. All parameters u_0, θ_0, ϕ_0 are constant under the generalized coordinate transformation and assumed to be the initial state of the hole. Applying the generalized tortoise coordinate transformation to the null surface condition (16) and taking the limit as $r\longrightarrow r_j(u_0,\theta_0,\phi_0)$, $u\longrightarrow u_0$, $\theta\longrightarrow\theta_0$, and $\phi\longrightarrow\phi_0$, we obtain

$$\begin{aligned} &\Delta_j+\frac{2}{\sqrt{A}}(r_j^2+a^2)r_{j,u}+\frac{a^2}{A\Delta_\theta}\sin^2\theta_0 r_{j,u}^2 \\ &+\Delta_\theta r_{j,\theta}^2+\frac{2ar_{j,\phi}}{\sqrt{A}}+\frac{2ar_{j,u}r_{j,\phi}}{A\Delta_\theta} \\ &+\frac{r_{j,\phi}^2}{A\Delta_\theta\sin^2\theta_0}=0, \end{aligned} \tag{18}$$

where

$$\Delta_j=r_j^2+a^2-2M(u_0)r_j-\frac{1}{3}\Lambda r_j^2(r_j^2+a^2).$$

The quantities $r_{j,u}=\partial r_j/\partial u$, $r_{j,\theta}=\partial r_j/\partial\theta$, and $r_{j,\phi}=\partial r_j/\partial\phi$ are the rates of horizons varying with the time u and different angles θ and ϕ. Equation (18) shows that the shapes of horizons depend on the retarded time u and different angles θ and ϕ due to generalized tortoise coordinate transformation.

4. Dirac equations

The four-coupled Dirac equations[36,37] in Newman–Penrose formalism are given by

$$\begin{aligned} &(D+\varepsilon-\rho)F_1+(\bar{\delta}+\pi-\alpha)F_2=\mathrm{i}\mu_0 G_1, \\ &(\nabla+\mu-\gamma)F_2+(\delta+\beta-\tau)F_1=\mathrm{i}\mu_0 G_2, \\ &(\nabla+\bar{\mu}-\bar{\gamma})G_1-(\bar{\delta}+\bar{\beta}-\bar{\tau})G_2=\mathrm{i}\mu_0 F_1, \\ &(D+\bar{\varepsilon}-\bar{\rho})G_2-(\delta+\bar{\pi}-\bar{\alpha})G_1=\mathrm{i}\mu_0 F_2. \end{aligned} \tag{19}$$

By substituting the various spin-coefficients in Eq. (19), we have

$$\begin{aligned} &\left(\partial_r+\frac{1}{\bar{R}}\right)F_1+\frac{1}{\sqrt{2}\bar{R}}L^-F_2=\mathrm{i}\mu_0 G_1, \\ &-\frac{\Delta}{2R^2}D^+F_2+\frac{1}{\sqrt{2}R}\left(L^++\frac{\mathrm{i}a\Xi\sin\theta}{\bar{R}}\right)F_1=\mathrm{i}\mu_0 G_2, \\ &-\frac{\Delta}{2R^2}D^+G_1-\frac{1}{\sqrt{2}\bar{R}}\left(L^-+\frac{\mathrm{i}a\bar{\Xi}\sin\theta}{R}\right)G_2=\mathrm{i}\mu_0 F_1, \\ &-\frac{1}{\sqrt{2}R}L^+G_1+\left(\partial_r+\frac{1}{R}\right)G_2=\mathrm{i}\mu_0 F_2, \end{aligned} \tag{20}$$

where operators are given by

$$\begin{aligned} L^+ &= \Xi\,\partial_\theta+\frac{\mathrm{i}a\sin\theta}{\sqrt{A}\bar{\Xi}}\,\partial_u+\frac{\mathrm{i}}{\sqrt{A}\bar{\Xi}\sin\theta}\,\partial_\phi \\ &\quad +\frac{\Xi\cot\theta}{2}+\frac{\mathrm{i}a\sin\theta(\Lambda/3)^{1/2}}{2}, \\ L^- &= \bar{\Xi}\partial_\theta-\mathrm{i}a\frac{\sin\theta}{\sqrt{A}\Xi}\partial_u-\frac{\mathrm{i}}{\sqrt{A}\bar{\Xi}\sin\theta}\,\partial_\phi \\ &\quad +\frac{\bar{\Xi}\cot\theta}{2}-\frac{\mathrm{i}a\sin\theta(\Lambda/3)^{1/2}}{2}, \\ D^+ &= \partial_r-\Delta^{-1}\left\{\frac{2}{\sqrt{A}}(r^2+a^2)\,\partial_u+\frac{2a}{\sqrt{A}}\,\partial_\phi\right. \\ &\quad \left.-\left(r-M-\frac{2\Lambda r^3}{3}-\frac{\Lambda a^2 r}{3}\right)\right\}. \end{aligned} \tag{21}$$

We define F_1, F_2, G_1, and G_2 by

$$\begin{aligned} F_1=\frac{f_1}{\sqrt{2}\bar{R}},\quad G_1=\frac{g_1}{\sqrt{2}}, \\ F_2=f_2,\quad G_2=\frac{g_2}{2R}, \end{aligned} \tag{22}$$

then equation (20) becomes

$$\begin{aligned} &\frac{\partial f_1}{\partial r}+L^-f_2=\mathrm{i}\mu_0\bar{R}g_1, \\ &L^+f_1-\Delta D^+f_2=\mathrm{i}\mu_0\bar{R}g_2, \\ &-L^+g_1+\frac{\partial g_2}{\partial r}=2\mathrm{i}\mu_0 R f_2, \\ &\Delta D^+g_1+L^-g_2=-2\mathrm{i}\mu_0 R f_1. \end{aligned} \tag{23}$$

Then, after taking the limit as $r \longrightarrow r_j(u_0,\theta_0,\phi_0)$, $u \longrightarrow u_0$, $\theta \longrightarrow \theta_0$, and $\phi \longrightarrow \phi_0$, the first-order forms of Dirac equations near the horizon are as follows:

$$\begin{cases} \dfrac{\partial f_1}{\partial r_*} - \left[\bar{\Xi} r_{j,\theta} - \dfrac{\mathrm{i}a\sin\theta_0 r_{j,u}}{\sqrt{A}\Xi} - \dfrac{\mathrm{i}r_{j,\phi}}{\sqrt{A}\Xi\sin\theta}\right]\dfrac{\partial f_2}{\partial r_*} = 0, \\ \left\{\Delta_j + \dfrac{2(r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \dfrac{2ar_{j,\phi}}{\sqrt{A}}\right\}\dfrac{\partial f_1}{\partial r_*} \\ -\left\{\sigma r_{j,\theta} + \dfrac{\mathrm{i}a\sin\theta_0 r_{j,u}}{\sqrt{A}\bar{\Xi}} + \dfrac{\mathrm{i}r_{j,\phi}}{\bar{\Xi}\sqrt{A}\sin\theta_0}\right\}\dfrac{\partial f_2}{\partial r_*} = 0, \end{cases} \tag{24}$$

$$\begin{cases} \left[\Xi r_{j,\theta} + \dfrac{r_{j,u}\mathrm{i}a\sin\theta_0}{\sqrt{A}\bar{\Xi}} + \dfrac{\mathrm{i}r_{j,\phi}}{\sqrt{A}\bar{\Xi}\sin\theta_0}\right]\dfrac{\partial g_1}{\partial r_*} + \dfrac{\partial g_2}{\partial r_*} = 0, \\ \left\{\Delta_j + \dfrac{2(r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \dfrac{2ar_{j,\phi}}{\sqrt{A}}\right\}\dfrac{\partial g_1}{\partial r_*} \\ -\left\{\bar{\Xi} r_{j,\theta} - \dfrac{\mathrm{i}a\sin\theta_0 r_{j,u}}{\Xi\sqrt{A}} - \dfrac{\mathrm{i}r_{j,\phi}}{\Xi\sqrt{A}\sin\theta_0}\right\}\dfrac{\partial g_2}{\partial r_*} = 0. \end{cases} \tag{25}$$

The four derivatives $\partial f_1/\partial r_*$, $\partial f_2/\partial r_*$, $\partial g_1/\partial r_*$, and $\partial g_2/\partial r_*$ in Eqs. (24) and (25) are non-zero, then the determinant of their co-efficients is zero. That gives an equation like the null surface condition (18). The second-order forms of Dirac equations near the horizons are given by

$$\begin{aligned} &\left\{2\Delta_j + \frac{2(r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \frac{2ar_{j,\phi}}{\sqrt{A}}\right. \\ &+ \frac{1}{\kappa r_j}\left[r_j\left(r_j - M(u_0) - \frac{2\Lambda r_j^3}{3} - \frac{\Lambda}{3}a^2 r_j\right)\right. \\ &+ \Delta_\theta r_{j,\theta}^2 + \frac{(3r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \frac{a^2\sin^2\theta_0 r_{j,u}^2}{A\Delta_\theta} \\ &\left.\left. + \frac{2ar_{j,u}r_{j,\phi}}{A\Delta_\theta} + \frac{ar_{j,\phi}}{\sqrt{A}} + \frac{r_{j,\phi}^2}{A\Delta_\theta\sin^2\theta_0}\right]\right\}\frac{\partial^2 f_1}{\partial r_*^2} \\ &- 2\Delta_\theta r_{j,\theta}\frac{\partial^2 f_1}{\partial r_*\partial\theta_*} - \frac{2}{A\Delta_\theta\sin^2\theta_0} \\ &\times[(r_{j,u}+\sqrt{A}\Delta_\theta)a\sin^2\theta_0 + r_{j,\phi}]\frac{\partial^2 f_1}{\partial r_*\partial\phi_*} \\ &- \frac{2}{A\Delta_\theta}\left[\Delta_\theta\sqrt{A}(r_j^2+a^2) + a^2\sin^2\theta_0 r_{j,u} + ar_{j,\phi}\right] \\ &\times\frac{\partial^2 f_1}{\partial r_*\partial u_*} + \left[r_j - M(u_0) - \frac{2\Lambda r_j^3}{3}\right. \\ &- \frac{\Lambda}{3}a^2 r_j - \frac{4r_j r_{j,u}}{\sqrt{A}} - \frac{2a^2\sin^2\theta_0(\Lambda/3)^{1/2}r_{j,u}}{\sqrt{A}\Delta_\theta} \\ &- \Delta_\theta r_{j,\theta}\cot\theta_0 - \frac{2a(\Lambda/3)^{1/2}r_{j,\phi}}{\sqrt{A}\Delta_\theta} - \Delta_\theta r_{j,\theta\theta} \\ &- \frac{2ar_{j,u\phi}}{A\Delta_\theta} - \frac{a^2\sin^2\theta_0 r_{j,uu}}{A\Delta_\theta} - \frac{r_{j,\phi\phi}}{A\Delta_\theta\sin^2\theta_0} \\ &- \frac{2ar_j M_{,u}(u_0)}{A\Delta_\theta B}(a\sin^2\theta_0 r_{j,u} + r_{j,\phi}) \\ &- \frac{\mathrm{i}}{\sqrt{A}}\left(a\cos\theta_0 r_{j,u} - \frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}\right. \\ &- \frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta} - \sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta} \\ &\left.\left.\left. + \frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}\right)\right]\frac{\partial f_2}{\partial r_*} = 0, \end{aligned} \tag{26}$$

$$\begin{aligned} &\left\{2\Delta_j + \frac{2(r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \frac{2ar_{j,\phi}}{\sqrt{A}}\right. \\ &+ \frac{1}{\kappa r_j}\left[r_j\left(r_j - M(u_0) - \frac{2\Lambda r_j^3}{3} - \frac{\Lambda}{3}a^2 r_j\right)\right. \\ &+ \Delta_\theta r_{j,\theta}^2 + \frac{(3r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \frac{a^2\sin^2\theta_0 r_{j,u}^2}{A\Delta_\theta} \\ &\left.\left. + \frac{2ar_{j,u}r_{j,\phi}}{A\Delta_\theta} + \frac{ar_{j,\phi}}{\sqrt{A}} + \frac{r_{j,\phi}^2}{A\Delta_\theta\sin^2\theta_0}\right]\right\}\frac{\partial^2 f_2}{\partial r_*^2} \\ &- 2\Delta_\theta r_{j,\theta}\frac{\partial^2 f_2}{\partial r_*\partial\theta_*} - \frac{2}{A\Delta_\theta\sin^2\theta_0} \\ &\times[(r_{j,u}+\sqrt{A}\Delta_\theta)a\sin^2\theta_0 + r_{j,\phi}]\frac{\partial^2 f_2}{\partial r_*\partial\phi_*} \\ &- \frac{2}{A\Delta_\theta}\left[\Delta_\theta\sqrt{A}(r_j^2+a^2) + a^2\sin^2\theta_0 r_{j,u} + ar_{j,\phi}\right] \\ &\times\frac{\partial^2 f_1}{\partial r_*\partial u_*} + \left[-r_j + M(u_0) + \frac{2\Lambda r_j^3}{3}\right. \\ &+ \frac{\Lambda}{3}a^2 r_j - \frac{4r_j r_{j,u}}{\sqrt{A}} - \frac{2a^2\sin^2\theta_0(\Lambda/3)^{1/2}r_{j,u}}{\sqrt{A}\Delta_\theta} \\ &- \Delta_\theta r_{j,\theta}\cot\theta_0 - \frac{2a(\Lambda/3)^{1/2}r_{j,\phi}}{\sqrt{A}\Delta_\theta} \\ &- \Delta_\theta r_{j,\theta\theta} - \frac{2ar_{j,u\phi}}{A\Delta_\theta} - \frac{a^2\sin^2\theta_0 r_{j,uu}}{A\Delta_\theta} \\ &- \frac{r_{j,\phi\phi}}{A\Delta_\theta\sin^2\theta_0} + \frac{\mathrm{i}}{\sqrt{A}}\left(a\cos\theta_0 r_{j,u} - \frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}\right. \\ &- \frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta} - \sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta} \\ &\left.\left.\left. + \frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}\right)\right]\frac{\partial f_2}{\partial r_*} = 0, \end{aligned} \tag{27}$$

where

$$B = \Delta_j + \frac{2}{\sqrt{A}}(r^2+a^2)r_{j,u} + \frac{2ar_{j,\phi}}{\sqrt{A}}.$$

In order to obtain the Hawking radiation temperature, the co-efficient of $\partial^2 f_i/\partial r_*^2$, $i = 1,2$ should satisfy the condition

$$\begin{aligned} &2\Delta_j + \frac{2(r_j^2+a^2)r_{j,u}}{\sqrt{A}} + \frac{2ar_{j,\phi}}{\sqrt{A}} \\ &+ \frac{1}{\kappa r_j}\left\{r_j\left(r_j - M(u_0) - \frac{2\Lambda r_j^3}{3} - \frac{\Lambda}{3}a^2 r_j\right)\right. \\ &+ \Delta_\theta r_{j\theta}^2 + \frac{3r_j^2 r_{j,u} + r_{j,u}a^2}{\sqrt{A}} + \frac{a^2\sin^2\theta_0 r_{j,u}^2}{A\Delta_\theta} \\ &\left. + \frac{2ar_{j,u}r_{j,\phi}}{A\Delta_\theta} + \frac{ar_{j,\phi}}{\sqrt{A}} + \frac{r_{j,\phi}^2}{A\Delta_\theta\sin^2\theta_0}\right\} \\ &= \frac{1}{A\Delta_\theta}[\Delta_\theta\sqrt{A}(r_j^2+a^2) + a^2\sin^2\theta_0 r_{j,u} + ar_{j,\phi}]. \end{aligned} \tag{28}$$

Then, the surface gravity can be written as

$$\kappa = \frac{r_j\left(1+\frac{2r_{j,u}}{\sqrt{A}}\right)-M(u_0)-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda}{3}ar_j}{\left(\frac{r_j^2+a^2}{\sqrt{A}}+\frac{a^2\sin^2\theta_0 r_{j,u}}{B\Delta_\theta}\right)(1+2r_{j,u})+Z} - \frac{\Delta_j+\frac{r_{j,u}}{\sqrt{A}}(r_j^2+a^2)+\frac{ar_{j,\phi}}{\sqrt{A}}}{r_j\left[\left(\frac{r_j^2+a^2}{\sqrt{A}}+\frac{a^2\sin^2\theta_0 r_{j,u}}{A\Delta_\theta}\right)(1+2r_{j,u})+Z\right]}, \quad (29)$$

where

$$Z = 2\Delta_\theta r_{j,\theta}^2 + r_{j,\phi}\left(\frac{4ar_{j,u}}{A\Delta_\theta}+\frac{2r_{j,\phi}}{A\Delta_\theta\sin^2\theta}+\frac{2a}{\sqrt{A}}+\frac{a}{A\Delta_\theta}\right). \quad (30)$$

Equations (26) and (28) are transformed into a combined form of wave equation as

$$\frac{\partial^2\Psi}{\partial r_*^2}-2\frac{\partial^2\Psi}{\partial r_*\partial u_*}+C_1\frac{\partial^2\Psi}{\partial r_*\partial\theta_*}+C_2\frac{\partial^2\Psi}{\partial r_*\partial\phi_*}+C_3\frac{\partial\Psi}{\partial r_*}=0, \quad (31)$$

with

$$\begin{aligned} C_1 &= -\frac{2\Delta_\theta^2 A r_{j,\theta}}{\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}}, \\ C_2 &= -\frac{2[(r_{j,u}+\sqrt{A}\Delta_\theta)a\sin^2\theta_0+r_{j,\phi}]}{\sin^2\theta_0[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}, \\ C_3 &= D_1+\mathrm{i}2D_2. \end{aligned} \quad (32)$$

(i) For Dirac particles $\Psi = f_1$,

$$\begin{aligned} D_1 &= \frac{A\Delta_\theta}{\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}} \\ &\times\left[r_j-M(u_0)-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda r_j a^2}{3}\right. \\ &-\frac{4r_j r_{j,u}}{\sqrt{A}}-\frac{2a^2\sin^2\theta_0(\Lambda/3)^{1/2}}{\sqrt{A}\Delta_\theta} \\ &-\Delta_\theta r_{j,\theta}\cot\theta_0-\frac{2a(\Lambda/3)^{1/2}r_{j,\phi}}{\sqrt{A}\Delta_\theta} \\ &-\Delta_\theta r_{j,\theta\theta}-\frac{2ar_{j,u\phi}}{A\Delta_\theta}-\frac{a^2\sin^2\theta_0 r_{j,uu}}{A\Delta_\theta}-\frac{r_{j,\phi\phi}}{A\Delta_\theta\sin^2\theta_0} \\ &\left.-\frac{2ar_j M_{,u}(u_0)}{A\Delta_\theta B}(a\sin^2\theta_0 r_{j,u}+r_{j,\phi})\right], \\ D_2 &= -\frac{\sqrt{A}\Delta_\theta}{2[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}} \\ &\times\left[a\cos\theta_0 r_{j,u}-\frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}\right. \\ &-\frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta}-\sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta} \\ &\left.+\frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}-\frac{2a\sin\theta_0 r_j M_{,u}(u_0)r_{j,\theta}}{B}\right]. \end{aligned} \quad (33)$$

(ii) For $\Psi = f_2$,

$$\begin{aligned} D_1 &= \frac{A\Delta_\theta}{\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}} \\ &\times\left[-r_j+M(u_0)+\frac{2\Lambda r_j^3}{3}+\frac{\Lambda r_j a^2}{3}\right. \\ &-\frac{4r_j r_{j,u}}{\sqrt{A}}-\frac{2a^2\sin^2\theta_0(\Lambda/3)^{1/2}}{\sqrt{A}\Delta_\theta} \\ &-\Delta_\theta r_{j,\theta}\cot\theta_0-\frac{2a(\Lambda/3)^{1/2}r_{j,\phi}}{\sqrt{A}\Delta_\theta} \\ &\left.-\Delta_\theta r_{j,\theta\theta}-\frac{2ar_{j,u\phi}}{A\Delta_\theta}-\frac{a^2\sin^2\theta_0 r_{j,uu}}{A\Delta_\theta}-\frac{r_{j,\phi\phi}}{A\Delta_\theta\sin^2\theta_0}\right], \\ D_2 &= \frac{\sqrt{A}\Delta_\theta}{2[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]} \\ &\times\left[a\cos\theta_0 r_{j,u}-\frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}\right. \\ &-\frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta}-\sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta} \\ &\left.+\frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}\right]. \end{aligned} \quad (34)$$

Following Wu and Cai[20,21] and Hua and Huang,[22,28] we may separate the variables in Eq. (31) for the analysis of the field equations as

$$\Psi = R(r_*,u_*)\Theta(u_*,\theta_*,\phi_*)\,\mathrm{e}^{\mathrm{i}\omega u_*+\mathrm{i}K_\theta+\mathrm{i}K_\phi}, \quad (35)$$

where $\Theta(u_*,\theta_*,\phi_*)$ is an arbitrary real function and ω is the energy of Dirac particles which depend on the tortoise coordinate transformation; K_θ and K_ϕ are components of generalized momentum of Dirac particles. Substituting Eq. (35) into Eq. (31) and after separating the variables, we find the radial and angular parts to be given by

$$\begin{aligned} &\frac{\partial^2 P}{\partial r_*^2}-\frac{\partial P}{\partial r_*}(2\mathrm{i}\omega-C_1\mathrm{i}K_\theta-C_2\mathrm{i}K_\phi-C_3-\alpha)=0, \\ &\frac{2\partial T}{\partial u_*}-(\zeta(u_*)-\alpha)T=0, \end{aligned} \quad (36)$$

where α is a constant in variable of separation and $\zeta(u_*)$ is a function of retarded time u_* in variable of separation, where

$$\zeta(u_*) = 2\frac{\frac{\partial\Theta}{\partial u_*}}{\Theta}+C_1\frac{\frac{\partial\Theta}{\partial\theta_*}}{\Theta}-C_2\frac{\frac{\partial\Theta}{\partial\phi_*}}{\Theta},$$

and $R(r_*,u_*) = P(r_*)T(u_*)$. The two linearly independent solutions of Eq. (36) near the horizons are given by

$$\begin{aligned} \Psi_\omega^{\mathrm{in}} &\sim \mathrm{e}^{\mathrm{i}\omega+\mathrm{i}K_\theta+\mathrm{i}K_\phi}, \\ \Psi_\omega^{\mathrm{out}} &\sim \mathrm{e}^{\mathrm{i}\omega+\mathrm{i}K_\theta+\mathrm{i}K_\phi} \\ &\times\mathrm{e}^{\left[2\mathrm{i}(\omega-\frac{K_\theta C_1}{2}-\frac{K_\phi C_2}{2}-\frac{\mathrm{Im}(C_3)}{2})r_*\right]} \\ &\times\mathrm{e}^{-[\alpha+\mathrm{Re}(C_3)]r_*}, \quad r>r_j, \end{aligned} \quad (37)$$

where $\Psi_\omega^{\rm in}$ represents an incoming wave which is analytic at $r=r_j$; $\Psi_\omega^{\rm out}$ denotes an outgoing wave having logarithmic singularity on the horizons. Following Damour and Ruffini[18] and Sannan,[19] it can continue analytically from outside of the horizons $r=r_j$ into inside by rotating $-\pi$ through the lower half of the complex plane as

$$\begin{aligned}\tilde{\Psi}_\omega^{\rm out} \sim\ & {\rm e}^{{\rm i}\omega+{\rm i}K_\theta+{\rm i}K_\phi}\\ &\times {\rm e}^{\left[\frac{\pi}{\kappa}\left(\omega-\frac{K_\theta C_1}{2}-\frac{K_\phi C_2}{2}-\frac{{\rm Im}(C_3)}{2}\right)\right]}\\ &\times {\rm e}^{\left\{\frac{-{\rm i}\pi}{2\kappa}[\alpha+{\rm Re}(C_3)]\right\}}\\ &\times\left(\frac{r_j-r}{r_j}\right)^{\frac{\rm i}{\kappa}\left[\omega-\frac{K_\theta C_1}{2}-\frac{K_\phi C_2}{2}-\frac{{\rm Im}(C_3)}{2}\right]}\\ &\times\left(\frac{r_j-r}{r_j}\right)^{\frac{-1}{2\kappa}[\alpha+{\rm Re}(C_3)]}, \quad r<r_j. \end{aligned} \tag{38}$$

The relative scattering probabilities at $r=r_j$ are given by

$$\left|\frac{\Psi_\omega^{\rm out}(r>r_j)}{\tilde{\Psi}_\omega^{\rm out}(r<r_j)}\right|^2 = {\rm e}^{-\frac{2\pi}{\kappa}(\omega-\omega_0)}, \tag{39}$$

where

$$\omega_0 = K_\theta\frac{C_1}{2}+K_\phi\frac{C_2}{2}+\frac{{\rm Im}C_3}{2}. \tag{40}$$

According to the suggestion of Damour and Ruffini[18] and extended by Sannan,[19] the thermal radiation spectrum is given by

$$N_\omega = \frac{1}{{\rm e}^{\frac{\omega-\omega_0}{\kappa_{\rm B}T}}\pm 1}, \tag{41}$$

where $\kappa_{\rm B}$ is the Boltzmann constant and the upper positive symbol stands for the Fermi–Dirac distribution and the lower negative symbol corresponds to the Bose–Einstein statistics. Equation (41) shows that the black hole radiates like a black body. The chemical potential is

$$\begin{aligned}\omega_0 =& -\left[\frac{\Delta_\theta^2 A r_{j,\theta}K_\theta}{[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right.\\ &\left.+\frac{K_\phi(r_{j,u}+\sqrt{A}\Delta_\theta a\sin^2\theta_0+r_{j,\phi})}{\sin^2\theta_0[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right]\\ &+D_2. \end{aligned} \tag{42}$$

We observe that the constant coefficient D_2 appears in the expression of the chemical potential. It may represent a particular energy term for Dirac particles which is absent in the thermal radiation of the other scalar particles. Hawking temperature is given by

$$\begin{aligned}T =& \frac{1}{2\pi}\left\{\frac{r_j\left(1+\frac{2r_{j,u}}{\sqrt{A}}\right)-M(u_0)-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda}{3}ar_j}{\left(\frac{r_j^2+a^2}{\sqrt{A}}+\frac{a^2\sin^2\theta_0 r_{j,u}}{A\Delta_\theta}\right)(1+2r_{j,u})+Z}\right.\\ &\left.-\frac{\Delta_j+\frac{r_{j,u}}{\sqrt{A}}(r_j^2+a^2)+\frac{ar_{j\phi}}{\sqrt{A}}}{r_j\left[\left(\frac{r_j^2+a^2}{\sqrt{A}}+\frac{a^2\sin^2\theta_0 r_{j,u}}{A\Delta_\theta}\right)(1+2r_{j,u})+Z\right]}\right\}. \end{aligned} \tag{43}$$

From the above, T determines the temperature of the thermal radiation near the horizons $r=r_j$ due to the Dirac equations in the non-stationary Kerr–de Sitter black hole. For a stationary black hole, equation (43) can be expressed as

$$T = \frac{1}{2\pi}\frac{\sqrt{A}}{(r_j^2+a^2)}\left[r_j-M-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda}{3}ar_j\right], \tag{44}$$

which is equal to the Hawking radiation temperature derived from the relativistic Hamilton–Jacobi equation.

5. Discussion

The Klein–Gordon equation of a scalar particle with mass μ in curve space time is given by

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial}{\partial x^a}\sqrt{-g}g^{ab}\frac{\partial}{\partial x^b}\right]\Phi-\mu^2\Phi=0. \tag{45}$$

Using generalized tortoise coordinate transformation and applying the same method in Section 4 to Eq. (45) after a tedious calculation, the chemical potential of the scalar particle can be obtained as follows:

$$\begin{aligned}\omega_k =& -\left[\frac{\Delta_\theta^2 A r_{j,\theta}K_\theta}{[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right.\\ &\left.+\frac{K_\phi(r_{j,u}+\sqrt{A}\Delta_\theta a\sin^2\theta_0+r_{j,\phi})}{\sin^2\theta_0[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right]. \end{aligned} \tag{46}$$

Again, utilizing generalized tortoise coordinate transformation in Eq. (10) and defining $\partial\Phi/\partial u_*=-\omega$, $\partial\Phi/\partial\theta_*=K_\theta$, and $\partial\Phi/\partial\phi_*=K_\phi$, for real $\partial\Phi/\partial r_*$, the maximum energy of the non-thermal radiation[38–40] can be written as

$$\begin{aligned}\omega_{\rm h} =& \left[\frac{\Delta_\theta^2 A r_{j,\theta}K_\theta}{[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right.\\ &\left.+\frac{K_\phi(r_{j,u}+\sqrt{A}\Delta_\theta a\sin^2\theta_0+r_{j,\phi})}{\sin^2\theta_0[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right]. \end{aligned} \tag{47}$$

The thermal radiation spectrum (41) implies that the total interaction energy of Dirac particles in a non-stationary Kerr–de Sitter black hole is

$$\begin{aligned}\omega_0 =& -\left\{\frac{\Delta_\theta^2 A r_{j,\theta}K_\theta}{[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right.\\ &\left.+\frac{K_\phi(r_{j,u}+\sqrt{A}\Delta_\theta a\sin^2\theta_0+r_{j,\phi})}{\sin^2\theta_0[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right\}\\ &+D_2, \end{aligned} \tag{48}$$

where

$$D_2 = -\frac{\sqrt{A}\Delta_\theta}{2[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}$$

$$\times\left[a\cos\theta_0 r_{j,u}-\frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}\right.$$
$$-\frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta}-\sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta}$$
$$\left.+\frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}-\frac{2a\sin\theta_0 r_j M_{,u}(u)r_{j,\theta}}{B}\right] \quad (49)$$

for $\Psi=f_1$, and

$$D_2=\frac{\sqrt{A}\Delta_\theta}{2[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}$$
$$\times[a\cos\theta_0 r_{j,u}-\frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}$$
$$-\frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta}-\sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta}$$
$$\left.+\frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}\right] \quad (50)$$

for $\Psi=f_2$. The chemical potential ω_0 is composed of two parts. The first part

$$-\left[\frac{\Delta_\theta^2 A r_{j,\theta}K_\theta}{[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right.$$
$$\left.+\frac{K_\phi(r_{j,u}+\sqrt{A}\Delta_\theta a\sin^2\theta_0+r_{j,\phi})}{\sin^2\theta_0[\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}]}\right]$$

is the rotational energy arising from the coupling between different components of generalized momentum of Dirac particles and different rotations of black hole. The second term D_2 indicates the expression of the new extra spin rotation coupling effect and the spin-acceleration coupling effect which can be written in combined form as follows:

$$D_2=\frac{\sqrt{A}\Delta_\theta}{\Delta_\theta\sqrt{A}(r_j^2+a^2)+a^2\sin^2\theta_0 r_{j,u}+ar_{j,\phi}}$$
$$\times\left[p\left(a\cos\theta_0 r_{j,u}-\frac{\cos\theta_0 r_{j,\phi}}{\sin^2\theta_0}\right.\right.$$
$$-\frac{a^3\sin^2\theta_0\cos\theta_0(\Lambda/3)r_{j,u}}{\Delta_\theta}-\sqrt{A}a\sin\theta_0(\Lambda/3)^{1/2}r_{j,\theta}$$
$$\left.+\frac{a^2\cos\theta_0(\Lambda/3)r_{j,\phi}}{\Delta_\theta}\right)$$
$$\left.-(p-s)\frac{a\sin\theta_0 r_j M_{,u}(u)r_{j,\theta}}{B}\right]. \quad (51)$$

Here,

$$\Psi_p=\begin{pmatrix} f_1 & : & p=-1/2, & s=1/2 \\ f_2 & : & p=+1/2, & s=1/2 \end{pmatrix}.$$

The value of D_2 will vanish for the stationary black hole and other scalar particles. When $D_2=0$, equations (47) and (48) indicate that the chemical potential derived from the thermal radiation spectrum of Dirac particles is equal to the highest energy of the negative energy state of scalar particles in non-thermal radiation of Kerr–de Sitter black hole ($\omega_0=-\omega_{\rm h}$). This brings out a clear relationship between the two kinds of radiation processes of black holes.

If we use the traditional tortoise coordinate transformation given in Refs. [20]–[22], [24], and [28], a similar conclusion can be established as Eqs. (42), (46), and (47) but the Hawking radiation temperature effect will be given by

$$T=\frac{1}{2\pi}\left\{\frac{r_j\left(1+\frac{2r_{j,u}}{\sqrt{A}}\right)-M(u_0)-\frac{2\Lambda r_j^3}{3}-\frac{\Lambda}{3}ar_j}{\left(\frac{r_j^2+a^2}{\sqrt{A}}+\frac{a^2\sin^2\theta_0 r_{j,u}}{A\Delta_\theta}\right)(1+2r_{j,u})+Z}\right\}, \quad (52)$$

which indicates that the different tortoise coordinate transformations correspond to the different Hawking radiation temperatures in non-stationary black hole space time.[26]

6. Conclusions

In this paper, we have studied the Hawking thermal radiation effect of the stationary Kerr–de Sitter black hole using the relativistic Hamilton–Jacobi equation and the non-stationary Kerr–de Sitter black hole by applying generalized tortoise coordinate transformation. For the non-stationary space time, the location of horizons, Eq. (18), and the temperature of the black hole, Eq. (43), depend on the retarded time coordinate u and different angles θ and ϕ. The expression of D_2 gives the intersection between the intrinsic spin of the particles and generalized momentum of the non-stationary Kerr–de Sitter black hole. However, the value of D_2 will vanish for the stationary black hole space time and scalar particles. It reveals that the chemical potential derived from scalar particles is equal to the highest energy of the negative energy state of non-thermal radiation of the Kerr–de Sitter black hole ($\omega_k=-\omega_{\rm h}$). It is also shown that the Hawking radiation temperature derived from the Hamilton–Jacobi equation is equal to radiation temperature derived from the Dirac equations for the stationary space time near the horizons $r=r_j$.

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