Time evolution of a squeezed chaotic field in an amplitude damping channel when used as a generating field for a squeezed number state*

Xu Xing-Lei(徐兴磊)[a)b)†], Li Hong-Qi(李洪奇)[a)b)], and Fan Hong-Yi(范洪义)[c)]

[a)] Department of Physics and Electronic Engineering, Heze University, Heze 274015, China
[b)] Key Laboratory of Quantum Communication and Calculation, Heze University, Heze 274015, China
[c)] Department of Material Science and Engineering, University of Science and Technology of China, Hefei 230026, China

(Received 10 November 2014; revised manuscript received 5 February 2015; published online 18 May 2015)

We investigate how an optical squeezed chaotic field (SCF) evolves in an amplitude dissipation channel. We have used the integration within ordered product of operators technique to derive its evolution law. We also show that the density operator of SCF can be viewed as a generating field of the squeezed number state.

Keywords: squeezed chaotic field, amplitude damping channel, squeezed number state, evolution law

PACS: 03.65.–w, 63.20.–e **DOI:** 10.1088/1674-1056/24/7/070306

1. Introduction

Quantum noise exists in every natural system.[1] Squeezed states exist where the noise in one of the chosen pair of observables is reduced below the vacuum or ground-state noise level, at the expense of increased noise in the other observable. Indeed, the squeezing effect improves both the interferometric and spectroscopic measurements. Therefore, in the context of interferometric detection and of gravitational waves, the squeezed state is very useful.[2] On the other hand, since every system is not absolutely isolated, dissipation or dephasing usually happens when a system is immersed in a thermal environment. A quantum state that passes through an amplitude damping channel is described by the master equation for density operator ρ [3,4]

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = \kappa\left(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a\right), \tag{1}$$

where κ is the rate of decay, $a^{\dagger}$ and a are the creation operator and the annihilation operator, respectively, and $\left[a, a^{\dagger}\right] = 1$. In this present work we investigate how an initial squeezed chaotic field (SCF), described by $\rho_0 = \left(1-\mathrm{e}^{k}\right) S^{\dagger}\mathrm{e}^{ka^{\dagger}a}S$, evolves in an amplitude dissipative channel (ADC), where $\left(1-\mathrm{e}^{k}\right)\mathrm{e}^{ka^{\dagger}a}$ is the density operator of chaotic field, and

$$k = -\frac{\omega\hbar}{KT}, \quad (\mathrm{e}^{-k}-1)^{-1} = (\mathrm{e}^{\frac{\omega\hbar}{KT}}-1)^{-1} = n,$$

where K is the Boltzman constant, $S(r) = \exp\left[\mathrm{i}r\left(QP+PQ\right)/2\right]$ is the single-mode squeezing operator, with $[Q,P] = \mathrm{i}$, $\hbar = 1$, and r is the squeezing parameter. Using the thermo entangled state representation,[5] we have neatly derived the solution to Eq. (1), which takes the infinitive operator-sum representation[6,7]

$$\rho(t) = \sum_{n=0}^{\infty}\frac{T'^{n}}{n!}\mathrm{e}^{-\kappa t a^{\dagger}a}a^{n}\rho_0 a^{\dagger n}\mathrm{e}^{-\kappa t a^{\dagger}a}, \quad T' = 1-\mathrm{e}^{-2\kappa t}. \tag{2}$$

(Although this solution can also be deduced with the use of the superoperator method, these superoperators have no real representations.) Our task in this paper is to investigate how a SCF evolves in an amplitude dissipation channel, when an optical chaotic field gets squeezed in a cavity and later dissipates in an thermo-enviroment, we are also curious about how it evolves with time. At first glance, one might think that it would be easy to substitute $\rho_0 = \left(1-\mathrm{e}^{k}\right) S^{\dagger}\mathrm{e}^{ka^{\dagger}a}S$ into Eq. (2) to obtain the result, actually, it is hard to do so, because the summation over n is impeded due to the existence of ρ_0 sandwiching between a^{n} and $a^{\dagger n}$. We shall employ the technique of integration (summation) within ordered product of operators[8] to overcome this obstacle and derive its evolution law. We also show that the density operator of SCF can be viewed as a generating field of the squeezed number state.

2. Squeezed number state as an Hermite excitation on the squeezed vacuum state

Before substituting $\rho_0 = \left(1-\mathrm{e}^{k}\right) S^{\dagger}\mathrm{e}^{ka^{\dagger}a}S$ into Eq. (2) to make a summation, we should first convert ρ_0 into its normal ordering and then transform the result in anti-normal ordering so that the summation as Eq. (1) indicated can be proceeded, because the creation operator and the annihilation operators can be permuted within the anti-normal ordering. For this purpose, we write ρ_0 in the Fock representation

$$\rho_0 = \left(1-\mathrm{e}^{k}\right)\sum_{n=0}\mathrm{e}^{kn}S^{-1}\left|n\right\rangle\left\langle n\right|S, \tag{3}$$

*Project supported by the National Natural Science Foundation of China (Grant No. 10574647), the Natural Science Foundation of Shandong Province, China (Grant No. Y2008A16), and the University Experimental Technology Foundation of Shandong Province of China (Grant No. S04W138).

†Corresponding author. E-mail: xxlwlx@126.com

© 2015 Chinese Physical Society and IOP Publishing Ltd

where $|n\rangle = a^{\dagger n}|0\rangle/\sqrt{n!}$ is the Fock state, and $S^{-1}|n\rangle$ is the squeezed number state. Let us recall that the coordinate representation of the squeezing operator is[9]

$$S(\lambda) = \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{\sqrt{\mu}} \left|\frac{q}{\mu}\right\rangle \langle q|, \quad \mu = \mathrm{e}^{\lambda}, \tag{4}$$

where $|q\rangle$ is the coordinate eigenstate

$$|q\rangle = \pi^{-1/4} \exp\left[-\frac{q^2}{2} + \sqrt{2}qa^{\dagger} - \frac{a^{\dagger 2}}{2}\right]|0\rangle, \tag{5}$$

with the overlap

$$\langle q|\, n\rangle = \frac{1}{\sqrt{\sqrt{\pi}2^n n!}} \mathrm{e}^{-\frac{q^2}{2}} \mathrm{H}_n(q).$$

Here, $\mathrm{H}_n(q)$ is the Hermite polynomial, thus

$$\begin{aligned} S(\lambda)|n\rangle &= \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{\sqrt{\mu}} \left|\frac{q}{\mu}\right\rangle \langle q|\, n\rangle \\ &= \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{\sqrt{\mu\pi 2^n n!}} \exp\left[-\frac{q^2}{2}\left(1+\frac{1}{\mu^2}\right)\right. \\ &\quad \left. + \sqrt{2}\frac{q}{\mu}a^{\dagger} - \frac{a^{\dagger 2}}{2}\right] \mathrm{H}_n(q)|0\rangle \\ &= \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{\sqrt{\mu\pi 2^n n!}} \exp\left\{-\frac{\mu^2+1}{2\mu^2}\left[\left(q - \frac{\sqrt{2}\mu a^{\dagger}}{\mu^2+1}\right)^2\right]\right. \\ &\quad \left. + \frac{1-\mu^2}{2(\mu^2+1)}a^{\dagger 2}\right\} |0\mathrm{H}_n(q)\rangle \\ &= \frac{\sqrt{\mathrm{sech}\,\lambda}}{\sqrt{2^n n!}} (-\tanh\lambda)^{n/2} \mathrm{H}_n\left(\frac{a^{\dagger}}{\sqrt{-2\sinh(2\lambda)}}\right) \\ &\quad \times \exp\left(-\frac{\tanh\lambda}{2}a^{\dagger 2}\right)|0\rangle, \end{aligned} \tag{6}$$

where in the last step we have used the integration formula

$$\begin{aligned} &\int_{-\infty}^{\infty} \mathrm{d}x \mathrm{e}^{-\frac{(x-y)^2}{2f}} \mathrm{H}_n(x) \\ &= \sqrt{2\pi f}(1-2f)^{n/2} \mathrm{H}_n\left[y(1-2f)^{-1/2}\right], \end{aligned} \tag{7}$$

or we have

$$\begin{aligned} S^{-1}(\lambda)|n\rangle &= \frac{\mathrm{sech}^{\frac{1}{2}}\lambda}{\sqrt{2^n n!}} (\tanh\lambda)^{n/2} \mathrm{H}_n\left(\frac{a^{\dagger}}{\sqrt{\sinh(2\lambda)}}\right) \\ &\quad \times \exp\left(\frac{a^{\dagger 2}}{2}\tanh\lambda\right)|0\rangle. \end{aligned} \tag{8}$$

Therefore, a squeezed number state can be considered as an Hermite excitation on the squeezed vacuum state.

3. Squeezed chaotic field as a generating field of the squeezed number state

By substituting Eq. (8) into Eq. (3) and using

$$|0\rangle\langle 0| =: \mathrm{e}^{-a^{\dagger}a}: \tag{9}$$

as well as the generating formula of the Hermite polynomials

$$\begin{aligned} &\sum_{n=0} \frac{t^n}{2^n n!} \mathrm{H}_n(x)\mathrm{H}_n(y) \\ &= \frac{1}{\sqrt{1-t^2}} \exp\left[\frac{2ytx - t^2(x^2+y^2)}{1-t^2}\right], \end{aligned} \tag{10}$$

we have

$$\begin{aligned} \rho_0 &= \left(1-\mathrm{e}^{k}\right) \sum_{n=0}^{\infty} \frac{\left(\mathrm{e}^{k}\tanh\lambda\right)^n}{2^n n!\cosh\lambda} : \mathrm{H}_n\left(\frac{a^{\dagger}}{\sqrt{\sinh(2\lambda)}}\right) \mathrm{H}_n\left(\frac{a}{\sqrt{\sinh(2\lambda)}}\right) \mathrm{e}^{\frac{\left(a^{\dagger 2}+a^2\right)\tanh\lambda}{2} - a^{\dagger}a} : \\ &= \frac{\left(1-\mathrm{e}^{k}\right)\mathrm{sech}\lambda}{\sqrt{1-\mathrm{e}^{2k}\tanh^2\lambda}} : \exp\left[\frac{2a^{\dagger}a\mathrm{e}^{k}\tanh\lambda - \mathrm{e}^{2k}\tanh^2\lambda\left(a^{\dagger 2}+a^2\right)}{\left(1-\mathrm{e}^{2k}\tanh^2\lambda\right)\sinh(2\lambda)}\right] \mathrm{e}^{\frac{\left(a^{\dagger 2}+a^2\right)\tanh\lambda}{2} - a^{\dagger}a} : \\ &= \frac{1-\mathrm{e}^{k}}{\sqrt{\cosh^2\lambda - \mathrm{e}^{2k}\sinh^2\lambda}} \exp\left[\frac{-a^{\dagger 2}\left(\mathrm{e}^{2k}-1\right)\tanh\lambda}{2\left(1-\mathrm{e}^{2k}\tanh^2\lambda\right)}\right] \\ &\quad \times : \exp\left[a^{\dagger}a\left(\frac{\mathrm{e}^{k}}{\cosh^2\lambda - \mathrm{e}^{2k}\sinh^2\lambda} - 1\right)\right] : \exp\left[\frac{-a^2\left(\mathrm{e}^{2k}-1\right)\tanh\lambda}{2\left(1-\mathrm{e}^{2k}\tanh^2\lambda\right)}\right], \end{aligned} \tag{11}$$

which is the normally ordered expansion of the squeezed chaotic field.

4. ρ_0 as a bivatriate distribution operator in phase space

Due to $k = -\omega\hbar/KT$,

$$\left(\mathrm{e}^{-k}-1\right)^{-1} = \left(\mathrm{e}^{\frac{\omega\hbar}{KT}}-1\right)^{-1} \equiv n, \tag{12}$$

and

$$\frac{\mathrm{e}^{k}-1}{\mathrm{e}^{k}+1} = -\frac{1}{2n+1}, \tag{13}$$

we let

$$(2n+1)\,\mathrm{e}^{2\lambda} + 1 = 2\tau_1^2, \quad (2n+1)\,\mathrm{e}^{-2\lambda} + 1 = 2\tau_2^2. \tag{14}$$

It follows that

$$\tau_1^2\tau_2^2 = \frac{\cosh^2\lambda - \mathrm{e}^{2k}\sinh^2\lambda}{(1-\mathrm{e}^k)^2}, \tag{15}$$

$$\tau_1^2 - \tau_2^2 = (2n+1)\sinh(2\lambda), \tag{16}$$

$$\tau_1^2 + \tau_2^2 = (2n+1)\cosh(2\lambda)+1, \tag{17}$$

$$-\frac{1}{2\tau_1^2} - \frac{1}{2\tau_2^2} = -1 + \frac{\mathrm{e}^k}{\cosh^2\lambda - \mathrm{e}^{2k}\sinh^2\lambda}, \tag{18}$$

$$\frac{\tau_1^2-\tau_2^2}{4\tau_1^2\tau_2^2} = \frac{\left(1-\mathrm{e}^{2k}\right)\tanh\lambda}{2\left[1-\mathrm{e}^{2k}\tanh^2\lambda\right]}, \tag{19}$$

thus

$$\begin{aligned}\rho_0 &= \left(1-\mathrm{e}^k\right)\sum_{n=0}\mathrm{e}^{kn}S^{-1}|n\rangle\langle n|S \\ &= \frac{1}{\tau_1\tau_2}:\exp\left\{-\frac{Q^2}{2\tau_1^2}-\frac{P^2}{2\tau_2^2}\right\}:,\end{aligned} \tag{20}$$

where

$$Q = \frac{1}{\sqrt{2}}\left(a+a^\dagger\right), \quad P = \frac{1}{\sqrt{2}\mathrm{i}}\left(a-a^\dagger\right), \tag{21}$$

the density operator of the SCF is a normally ordered bivatriate distribution operator in phase space. One can check the completeness that

$$\frac{1}{\tau_1\tau_2}\int\frac{\mathrm{d}\tau_1\,\mathrm{d}\tau_2}{2\pi}:\exp\left\{-\frac{P^2}{2\tau_1^2}-\frac{Q^2}{2\tau_2^2}\right\}:=1. \tag{22}$$

5. The anti-normally ordered form of ρ_0

Further, let

$$f = \frac{1}{2\tau_1^2}, \quad g = \frac{1}{2\tau_2^2}, \tag{23}$$

then

$$\begin{aligned}\rho_0 &= 2\sqrt{fg}:\exp\left(-fQ^2-gP^2\right): \\ &= 2\sqrt{fg}:\exp\left[\frac{1}{2}(g-f)\left(a^2+a^{\dagger2}\right)-(f+g)aa^\dagger\right]:.\end{aligned} \tag{24}$$

Next, we derive the anti-normal ordering form of ρ_0 by using[10]

$$\rho_0 = \int\frac{\mathrm{d}^2\beta}{\pi}\vdots\langle-\beta|\rho_0|\beta\rangle\exp\left(|\beta|^2+\beta^*a-\beta a^\dagger+a^\dagger a\right)\vdots, \tag{25}$$

where $\vdots\ \ \vdots$ stands for the anti-normal ordering and $|\beta\rangle$ is a coherent state

$$|\beta\rangle = \exp\left(\frac{-|\beta|^2}{2}+\beta a^\dagger\right)|0\rangle,$$

$$\langle-\beta|\beta\rangle = \exp\left(-2|\beta|^2\right).$$

By substituting Eq. (20) into Eq. (25) and using the integral formula[10,11]

$$\int\frac{\mathrm{d}^2z}{\pi}\exp\left(\zeta|z|^2+\xi z+\eta z^*+f'z^2+g'z^{*2}\right)$$

$$= \frac{1}{\sqrt{\zeta^2-4f'g'}}\exp\left[\frac{-\zeta\xi\eta+\xi^2g'+\eta^2f'}{\zeta^2-4f'g'}\right], \tag{26}$$

whose convergent condition is

$$\mathrm{Re}\left(\xi+f'+g'\right)<0, \quad \mathrm{Re}\left(\frac{\zeta^2-4f'g'}{\xi+f'+g'}\right)<0, \tag{27}$$

or

$$\mathrm{Re}\left(\xi-f'-g'\right)<0, \quad \mathrm{Re}\left(\frac{\zeta^2-4f'g'}{\xi-f'-g'}\right)<0, \tag{28}$$

we have the anti-normal ordering form of ρ_0

$$\begin{aligned}\rho_0 &= 2\sqrt{fg}\int\frac{\mathrm{d}^2\beta}{\pi}:\exp\left[-|\beta|^2-\frac{f}{2}(\beta-\beta^*)^2\right. \\ &\quad\left.+\frac{g}{2}(\beta+\beta^*)^2+\beta^*a-\beta a^\dagger+a^\dagger a\right]: \\ &= 2\sqrt{fg}\int\frac{\mathrm{d}^2\beta}{\pi}:\exp\left[(f+g-1)|\beta|^2-a^\dagger\beta+a\beta^*\right. \\ &\quad\left.+\frac{1}{2}(g-f)\left(\beta^2+\beta^{*2}\right)+a^\dagger a\right]: \\ &= 2\sqrt{\frac{fg}{D}}:\exp\left[\frac{g-f}{2D}\left(a^{\dagger2}+a^2\right)\right. \\ &\quad\left.+\frac{4fg-f-g}{D}a^\dagger a\right]:.\end{aligned} \tag{29}$$

We note that the operator formula

$$\mathrm{e}^{\alpha a^\dagger a} = \mathrm{e}^{-\alpha}:\exp\left[\left(1-\mathrm{e}^{-\alpha}\right)aa^\dagger\right]:, \tag{30}$$

so

$$\begin{aligned}&:\exp\left[\frac{4fg-f-g}{D}a^\dagger a\right]: \\ &= :\exp\left[\left(1-\frac{1-f-g}{D}\right)a^\dagger a\right]: \\ &= \frac{D}{1-f-g}\exp\left[a^\dagger a\ln\frac{D}{1-f-g}\right],\end{aligned} \tag{31}$$

and equation (29) becomes

$$\begin{aligned}\rho_0 &= \frac{2\sqrt{fgD}}{1-f-g}\exp\left[\frac{g-f}{2D}a^2\right]\exp\left[a^\dagger a\ln\frac{D}{1-f-g}\right] \\ &\quad\times\exp\left[\frac{g-f}{2D}a^{\dagger2}\right].\end{aligned} \tag{32}$$

6. Evolution law of a squeezed chaotic field in an amplitude damping channel

By substituting Eq. (32) into Eq. (2) and using

$$\mathrm{e}^{-\alpha a^\dagger a}f\left(a,a^\dagger\right)\mathrm{e}^{\alpha a^\dagger a} = f\left(a\mathrm{e}^\alpha, a^\dagger\mathrm{e}^{-\alpha}\right), \tag{33}$$

equation (32) becomes

$$\rho(t) = \frac{2\sqrt{fgD}}{1-f-g}\sum_{n=0}^{\infty}\frac{T'^n}{n!}\mathrm{e}^{-\kappa t a^\dagger a}a^n\mathrm{e}^{\frac{g-f}{2D}a^2}$$

$$
\begin{aligned}
&\times \exp\left[a^{\dagger}a \ln \frac{D}{1-f-g}\right] \mathrm{e}^{\frac{g-f}{2D}a^{\dagger 2}} a^{\dagger n} \mathrm{e}^{-\kappa t a^{\dagger}a} \\
&= \frac{2D}{1-f-g}\sqrt{\frac{fg}{D}} \sum_{n=0}^{\infty} \frac{T'^n}{n!} a^n \mathrm{e}^{2n\kappa t} \mathrm{e}^{\frac{g-f}{2D}a^2 \mathrm{e}^{2\kappa t}} \\
&\times \exp\left[a^{\dagger}a \ln \frac{D\mathrm{e}^{-2\kappa t}}{1-f-g}\right] \mathrm{e}^{\frac{g-f}{2D}a^{\dagger 2}\mathrm{e}^{2\kappa t}} a^{\dagger n} \\
&= \frac{2D}{1-f-g}\frac{1-f-g}{D\mathrm{e}^{-2\kappa t}}\sqrt{\frac{fg}{D}} \sum_{n=0}^{\infty} \frac{T'^n}{n!} a^n \mathrm{e}^{2n\kappa t} \mathrm{e}^{\frac{g-f}{2D}a^2 \mathrm{e}^{2\kappa t}} \\
&\times :\exp\left[\left(1-\frac{1-f-g}{D\mathrm{e}^{-2\kappa t}}\right)a^{\dagger}a\right]: \mathrm{e}^{\frac{g-f}{2D}a^{\dagger 2}\mathrm{e}^{2\kappa t}} a^{\dagger n} \\
&= 2\mathrm{e}^{2\kappa t}\sqrt{\frac{fg}{D}} :\mathrm{e}^{\frac{g-f}{2D}a^2 \mathrm{e}^{2\kappa t}} \\
&\times \exp\left[\left(1-\frac{1-f-g}{D\mathrm{e}^{-2\kappa t}}+T'\mathrm{e}^{2\kappa t}\right)a^{\dagger}a\right] \mathrm{e}^{\frac{g-f}{2D}a^{\dagger 2}\mathrm{e}^{2\kappa t}}: \\
&= 2\mathrm{e}^{2\kappa t}\sqrt{\frac{fg}{D}} :\mathrm{e}^{\frac{g-f}{2D}a^2 \mathrm{e}^{2\kappa t}} \\
&\times \exp\left[\mathrm{e}^{2\kappa t}\frac{4fg-f-g}{D}a^{\dagger}a\right] \mathrm{e}^{\frac{g-f}{2D}a^{\dagger 2}\mathrm{e}^{2\kappa t}}:, \qquad (34)
\end{aligned}
$$

which is the antinormally ordered form of $\rho(t)$.

Using the P-representation theory in the coherent state basis and $\int \frac{\mathrm{d}^2 z}{\pi} |z\rangle\langle z| = 1$, as well as the normally ordered vacuum projector $|0\rangle\langle 0| =: \exp\left(-a^{\dagger}a\right):$, we can transform Eq. (34) into its normally ordered form

$$
\begin{aligned}
\rho(t) &= \int \frac{\mathrm{d}^2 z}{\pi}\rho(t)|z\rangle\langle z| = 2\mathrm{e}^{2\kappa t}\sqrt{\frac{fg}{D}}\int \frac{\mathrm{d}^2 z}{\pi} :\mathrm{e}^{\frac{g-f}{2D}a^2\mathrm{e}^{2\kappa t}} \exp\left[\mathrm{e}^{2\kappa t}\left(1-\frac{1-f-g}{D}\right)a^{\dagger}a\right] \mathrm{e}^{\frac{g-f}{2D}a^{\dagger 2}\mathrm{e}^{2\kappa t}}: |z\rangle\langle z| \\
&= 2\mathrm{e}^{2\kappa t}\sqrt{\frac{fg}{D}}\int \frac{\mathrm{d}^2 z}{\pi} :\exp\left\{\frac{g-f}{2D}\left(z^2+z^{*2}\right)\mathrm{e}^{2\kappa t} + \left[\mathrm{e}^{2\kappa t}\frac{4fg-f-g}{D}-1\right]|z|^2 + za^{\dagger} + z^{*}a - a^{\dagger}a\right\}: \\
&= \frac{2\sqrt{fgD}}{\sqrt{\left(4fg-f-g-D\mathrm{e}^{-2\kappa t}\right)^2-(g-f)^2}} \\
&\quad \times :\exp\left\{\frac{\left[D^2\mathrm{e}^{-4\kappa t}-D\mathrm{e}^{-2\kappa t}(4fg-f-g)\right]a^{\dagger}a + \frac{g-f}{2}D\mathrm{e}^{-2\kappa t}\left(a^{\dagger 2}+a^2\right)}{\left(4fg-f-g-D\mathrm{e}^{-2\kappa t}\right)^2-(g-f)^2} - a^{\dagger}a\right\}: . \qquad (35)
\end{aligned}
$$

We can check that when $t = 0$, $\mathrm{e}^{-2\kappa t} = 1$, due to

$$
\begin{aligned}
&(4fg-f-g-D)^2-(g-f)^2 = D, \\
&D-(4fg-f-g) = 1-f-g, \qquad (36)
\end{aligned}
$$

then,

$$
\begin{aligned}
&\rho(t) \to \rho_0 \\
&= 2\sqrt{fg}:\exp\left[\frac{1}{2}(g-f)\left(a^2+a^{\dagger 2}\right)-(f+g)a^{\dagger}a\right]:,
\end{aligned}
$$

as expected.

By comparing Eq. (35) with Eq. (24) we can see clearly the evolution law of a SCF in an amplitude damping channel.

7. Summary

In summary, we have investigated how a SCF evolves in an amplitude dissipation channel. By using the integration within ordered product method,[12–16] we derived its evolution law. We also show that the density operator of the SCF can be viewed as a generating field of the squeezed number state.

References

[1] Gardiner C W and Zoller P 2000 *Quantum Noise* (Berlin: Springer)
[2] Loudon R and Knight P L 1987 *J. Mod. Opt.* **34** 709
[3] Carmichael H J 1999 *Statistical Methods in Quantum Optics* 1*: Master Equations and Fokker–Planck Equations* (Berlin: Springer)
[4] Preskill J 1998 *Lecture Notes for Physics* 229*: Quantum Information and Computation* CIT
[5] Fan H Y and Klauder J R 1994 *Phys. Rev. A* **49** 704
[6] Fan H Y and Hu L Y 2008 *Mod. Phys. Lett. B* **22** 2435
[7] Chen F, Chen J H and Fan H Y 2013 *Int. J. Theor. Phys.* **52** 2407
[8] Fan H Y, Lu H L and Fan Y 2006 *Ann. Phys.* **321** 480
[9] Fan H Y, Zaidi H R and Klauder J R 1987 *Phys. Rev. D* **35** 1831
[10] Fan H Y 1991 *Phys. Lett. A* **161** 1
[11] Wang J S, Meng X G and Fan H Y 2015 *Chin. Phys. B* **24** 014203
[12] Fan H Y and Wang Z 2014 *Int. J. Theor. Phys.* **53** 964
[13] Liu T K, Shan C J, Liu J B and Fan H Y 2014 *Chin. Phys. B* **23** 030303
[14] Xu X L, Li H Q and Fan H Y 2014 *Chin. Phys. B* **23** 030305
[15] Lv C H, Fan H Y and Li D W 2015 *Chin. Phys. B* **24** 020301
[16] Meng X G, Wang J S and Liang B L 2013 *Chin. Phys. B* **22** 030307