Explicit solution of diffusion master equation under the action of linear resonance force via the thermal entangled state representation*

Yao Fei(姚 飞), Wang Ji-Suo(王继锁)†, and Xu Tian-Niu(徐天牛)

Shandong Provincial Key Laboratory of Laser Polarization and Information Technology, College of Physics and Engineering, Qufu Normal University, Qufu 273165, China

(Received 11 January 2015; revised manuscript received 31 January 2015; published online 18 May 2015)

Using the well-behaved features of the thermal entangled state representation, we solve the diffusion master equation under the action of a linear resonance force, and then obtain the infinitive operator-sum representation of the density operator. This approach may also be effective for treating other master equations. Moreover, we find that the initial pure coherent state evolves into a mixed thermal state after passing through the diffusion process under the action of the linear resonance force.

Keywords: diffusion process, linear resonance force, thermal state representation, infinitive operator-sum representation

PACS: 03.65.Ud, 42.50.Dv **DOI:** 10.1088/1674-1056/24/7/070304

1. Introduction

In quantum optics, solving master equations describing interactions between fields and external environments is gradually becoming a hot topic, since quantum dissipation is unavoidable in the transmission of optical fields through media and this process can destroy nonclassicality of the fields. As shown in the literature,[1–3] solving the master equations usually employs either the Fokker–Planck equation or the Langevin equation through inserting the density operators into the P-representation or the Wigner representation.[4,5] Later, it was pointed out in Refs. [6]–[9] that the thermal entangled state representation can help one tackle these master equations conveniently because of some well-behaved properties of the entangled states. In this paper, we adopt the entangled state approach to treat the diffusion master equation under the action of a linear resonance force. Thus we establish a fresh picture of the time evolution of density operators by virtue of the thermal entangled state representation instead of the P-representation or the Wigner function.

The paper is arranged as follows. In Section 2, based on the well-behaved properties of the entangled state representation, we concentrate on solving the diffusion master equation under the action of a linear resonance force and give the infinitive operator-sum representation of the density operators. In Section 3, we prove the normalization of the infinitive operator-sum representation. In Section 4, we investigate the time-evolution rule of coherent states in a diffusion process acted upon by a linear resonance force.

2. Infinitive operator-sum representation for describing the diffusion process under the action of linear resonance force

A diffusion process under the action of a linear resonance force can be described by the master equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \mathrm{i}\lambda\left[a^{\dagger}+a,\rho\right] - k(a^{\dagger}a\rho - a^{\dagger}\rho a - a\rho a^{\dagger} + \rho a a^{\dagger}), \quad (1)$$

where parameters λ and k denote the resonance force strength and the decay rate, respectively. Usually, the master equation (1) is used to describe decoherence. In particular, the system is only acted upon by the linear resonance force when $k=0$. Also, for $\lambda=0$, equation (1) describes the pure diffusion process of a light field. To solve the master equation (1), we introduce the thermal entangled state representation,[10] which is defined in the enlarged Fock space

$$|\eta\rangle = \mathrm{e}^{-|\eta|^2/2+\eta a^{\dagger}-\eta^{*}\tilde{a}^{\dagger}+a^{\dagger}\tilde{a}^{\dagger}}\left|0\tilde{0}\right\rangle = D(\eta)\,|\eta=0\rangle\,, \quad (2)$$

where

$$D(\eta) = \mathrm{e}^{\eta a^{\dagger}-\eta^{*}\tilde{a}^{\dagger}} \quad (3)$$

is the displacement operator, and

$$|\eta=0\rangle = \exp\left[a^{\dagger}\tilde{a}^{\dagger}\right]\left|0\tilde{0}\right\rangle. \quad (4)$$

Here we need to point out that $\tilde{a}^{\dagger}$ is the operator of a fictitious mode corresponding to the real photon creation operator $a^{\dagger}$, $\tilde{a}$ can annihilate $\left|\tilde{0}\right\rangle$ and has the relations $[\tilde{a},\tilde{a}^{\dagger}]=1$ and $[a,\tilde{a}^{\dagger}]=0$. Moreover, the state $|\eta\rangle$ is the common eigenvector of $\left(a-\tilde{a}^{\dagger}\right)$ and $\left(a^{\dagger}-\tilde{a}\right)$, i.e.,

$$\left(a-\tilde{a}^{\dagger}\right)|\eta\rangle = \eta\,|\eta\rangle\,, \quad \left(a^{\dagger}-\tilde{a}\right)|\eta\rangle = \eta^{*}\,|\eta\rangle\,, \quad (5)$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11347026, 11147009, and 11244005), the Natural Science Foundation of Shandong Province, China (Grant Nos. ZR2013AM012 and ZR2012AM004), and the Scientific Research Project of Liaocheng, China.

†Corresponding author. E-mail: jswang@mail.qfnu.edu.cn

© 2015 Chinese Physical Society and IOP Publishing Ltd

and the set of $|\eta\rangle$ forms a complete entangled state representation

$$\int \frac{\mathrm{d}^2\eta}{\pi} |\eta\rangle\langle\eta| = 1. \tag{6}$$

On the other hand, one can easily find that $|\eta = 0\rangle$ possesses the following properties:

$$a|\eta = 0\rangle = \tilde{a}^\dagger |\eta = 0\rangle,\ a^\dagger |\eta = 0\rangle = \tilde{a}|\eta = 0\rangle, \tag{7}$$

$$a^\dagger a|\eta = 0\rangle = \tilde{a}^\dagger \tilde{a}|\eta = 0\rangle. \tag{8}$$

Using the above well-behaved properties of $|\eta = 0\rangle$, master equations for the diffusion process under the action of the linear resonance force can be concisely solved. In the following, we will provide a detailed derivation. Denoting $|\rho\rangle = \rho|\eta = 0\rangle = \rho|I\rangle$ and with both sides of Eq. (1) acting on state $|\eta = 0\rangle$, we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}|\rho\rangle &= \mathrm{i}\lambda\left[a^\dagger + a, \rho\right]|\eta = 0\rangle \\ &\quad - k(a^\dagger a\rho - a^\dagger \rho a - a\rho a^\dagger + \rho a a^\dagger)|\eta = 0\rangle \\ &= \left[\mathrm{i}\lambda(a^\dagger + a - \tilde{a} - \tilde{a}^\dagger) - k\left(a^\dagger - \tilde{a}\right)\left(a - \tilde{a}^\dagger\right)\right]|\rho\rangle. \end{aligned} \tag{9}$$

Thus, its formal solution is

$$|\rho\rangle = \mathrm{e}^{\mathrm{i}\lambda\left(a-\tilde{a}^\dagger\right)t}\,\mathrm{e}^{\mathrm{i}\lambda\left(a^\dagger-\tilde{a}\right)t}\,\mathrm{e}^{-k\left(a^\dagger-\tilde{a}\right)\left(a-\tilde{a}^\dagger\right)t}|\rho_0\rangle, \tag{10}$$

where ρ_0 is the initial density operator, $|\rho_0\rangle = \rho_0|I\rangle$. Now inserting the completeness relation of $|\eta\rangle$ into the right side of Eq. (10) and employing the eigen-vector equation (5), we obtain

$$\begin{aligned}|\rho\rangle &= \int \frac{\mathrm{d}^2\eta}{\pi}\mathrm{e}^{\mathrm{i}\lambda t(\eta+\eta^*)-kt|\eta|^2}|\eta\rangle\langle\eta|\rho_0\rangle = \int \frac{\mathrm{d}^2\eta}{\pi} : \mathrm{e}^{-(1+kt)|\eta|^2+\eta(a^\dagger-\tilde{a}+\mathrm{i}\lambda t)+\eta^*\left(a-\tilde{a}^\dagger+\mathrm{i}\lambda t\right)-\left(a-\tilde{a}^\dagger\right)\left(a^\dagger-\tilde{a}\right)} : |\rho_0\rangle \\ &= \frac{1}{1+kt} : \mathrm{e}^{\frac{1}{1+kt}(a^\dagger-\tilde{a}+\mathrm{i}\lambda t)\left(a-\tilde{a}^\dagger+\mathrm{i}\lambda t\right)-\left(a^\dagger-\tilde{a}\right)\left(a-\tilde{a}^\dagger\right)} : |\rho_0\rangle = \frac{1}{1+kt} : \mathrm{e}^{\frac{-kt}{1+kt}\left(a^\dagger-\tilde{a}\right)\left(a-\tilde{a}^\dagger\right)+\frac{\mathrm{i}\lambda t}{1+kt}\left(a-\tilde{a}^\dagger\right)+\frac{\mathrm{i}\lambda t}{1+kt}\left(a^\dagger-\tilde{a}\right)-\frac{\lambda^2 t^2}{1+kt}} : |\rho_0\rangle \\ &= \mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\,\mathrm{e}^{\frac{kt}{1+kt}a\dagger\tilde{a}^\dagger+\frac{\mathrm{i}\lambda t}{1+kt}\left(a^\dagger-\tilde{a}^\dagger\right)}\left(\frac{1}{1+kt}\right)^{a^\dagger a+\tilde{a}^\dagger\tilde{a}+1}\mathrm{e}^{\frac{kt}{1+kt}\tilde{a}a+\frac{\mathrm{i}\lambda t}{1+kt}(a-\tilde{a})}|\rho_0\rangle, \end{aligned} \tag{11}$$

where we have used the normally ordered form of the vacuum projector[11] (the mark : : denotes normal ordering)

$$|0\rangle\langle 0| =: \mathrm{e}^{-a^\dagger a} : \tag{12}$$

and the operator identity[11]

$$\mathrm{e}^{\lambda a^\dagger a} =: \exp[(\mathrm{e}^\lambda - 1)a^\dagger a] : . \tag{13}$$

Further, using the relations (7) and (8), we can rewrite Eq. (11) as

$$\begin{aligned}|\rho\rangle &= \frac{1}{1+kt}\mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\sum_{m,n}\frac{1}{m!n!} \\ &\quad\times\left(\frac{kt}{1+kt}\right)^m a^{\dagger m}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}\left(\frac{1}{1+kt}\right)^{a^\dagger a} \\ &\quad\times\left(\frac{kt}{1+kt}\right)^n a^n\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}\rho_0\,\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n} \\ &\quad\times\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}a^m|I\rangle. \end{aligned} \tag{14}$$

Then we obtain the solution to Eq. (1)

$$\begin{aligned}\rho(t) &= \frac{1}{1+kt}\mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\sum_{m,n}\frac{1}{m!n!}\left(\frac{kt}{1+kt}\right)^{m+n}a^{\dagger m}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger} \\ &\quad\times\left(\frac{1}{1+kt}\right)^{a^\dagger a}a^n\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}\rho_0\,\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n} \\ &\quad\times\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}a^m, \end{aligned} \tag{15}$$

which is just an infinitive operator-sum form of the density operator $\rho(t)$. From Eq. (15), we can clearly see that the infinitive operator-sum form is very useful, since for any given initial state ρ_0, the evolution density operator $\rho(t)$ can be directly calculated. By defining the Kraus operator $M_{m,n}$ corresponding to $\rho(t)$

$$\begin{aligned}M_{m,n} &= \frac{1}{\sqrt{m!n!}}\mathrm{e}^{-\frac{\lambda^2 t^2}{2(1+kt)}}\left(\frac{kt}{1+kt}\right)^{(m+n)/2}a^{\dagger m}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger} \\ &\quad\times\left(\frac{1}{1+kt}\right)^{a^\dagger a+1/2}a^n\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}, \end{aligned} \tag{16}$$

we can further rewrite Eq. (15) as

$$\rho(t) = \sum_{m,n} M_{m,n}\rho_0 M_{m,n}^\dagger. \tag{17}$$

3. Normalization of the infinitive operator-sum representation

To show that equation (15) is qualified to be an infinitive operator-sum representation, we must check the normalization condition

$$\begin{aligned}&\sum_{m,n} M_{m,n}^\dagger M_{m,n} \\ &= \frac{1}{1+kt}\mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\sum_{m,n}\frac{1}{m!n!}\left(\frac{kt}{1+kt}\right)^{m+n}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger} \\ &\quad\times a^{\dagger n}\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}a^m a^{\dagger m}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger} \end{aligned}$$

$$\times\left(\frac{1}{1+kt}\right)^{a^\dagger a} a^n \mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}$$
$$= \frac{1}{1+kt}\mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n \mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n}\left(\frac{1}{1+kt}\right)^{a^\dagger a}$$
$$\times \mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}{:}\mathrm{e}^{\frac{kta^\dagger a}{1+kt}}{:}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}\left(\frac{1}{1+kt}\right)^{a^\dagger a} a^n \mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}. \quad (18)$$

By using the operator identity[11]

$$\mathrm{e}^{\lambda a^\dagger a} = \mathrm{e}^{-\lambda}{:}\exp\left[\left(1-\mathrm{e}^{-\lambda}\right)aa^\dagger\right]{:}, \quad (19)$$

where the symbol $:\ :$ represents the anti-normal ordering, equation (18) becomes

$$\sum_{m,n} M_{m,n}^\dagger M_{m,n}$$
$$= \mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n \mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n}\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}$$
$$\times\left(\frac{1}{1+kt}\right)^{-a^\dagger a}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}\left(\frac{1}{1+kt}\right)^{a^\dagger a} a^n \mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}. \quad (20)$$

Noticing the operator formula

$$\mathrm{e}^{\beta a^\dagger a} f\left(a, a^\dagger\right)\mathrm{e}^{-\beta a^\dagger a} = f(a\mathrm{e}^{-\beta}, a^\dagger \mathrm{e}^{\beta}), \quad (21)$$

we have

$$\sum_{m,n} M_{m,n}^\dagger M_{m,n}$$
$$= \mathrm{e}^{-\frac{\lambda^2 t^2}{1+kt}}\sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n \mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n}$$
$$\times\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}\mathrm{e}^{\mathrm{i}\lambda t a^\dagger} a^n \mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a}. \quad (22)$$

Using the relation $\mathrm{e}^A\mathrm{e}^B = \mathrm{e}^B\mathrm{e}^A\mathrm{e}^{[A,B]}$ yields the condition $[A,B]\neq 0$, therefore

$$\sum_{m,n} M_{m,n}^\dagger M_{m,n}$$
$$= \sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n \mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n}\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{\mathrm{i}\lambda t a^\dagger} a^n. \quad (23)$$

At last, using Eqs. (12) and (20), we obtain

$$\sum_{m,n} M_{m,n}^\dagger M_{m,n}$$
$$= \sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n \mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}a^{\dagger n}\left(\frac{1}{1+kt}\right)^{a^\dagger a}$$
$$\times \mathrm{e}^{\mathrm{i}\lambda t a^\dagger}\left(\frac{1}{1+kt}\right)^{-a^\dagger a}\left(\frac{1}{1+kt}\right)^{a^\dagger a} a^n$$
$$= \sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n a^{\dagger n}\left(\frac{1}{1+kt}\right)^{a^\dagger a} a^n$$
$$= \sum_n \frac{1}{n!}\left(\frac{kt}{1+kt}\right)^n a^{\dagger n}{:}\ \mathrm{e}^{-\frac{kt}{1+kt}a^\dagger a}{:}\, a^n$$
$$= {:}\ \mathrm{e}^{\frac{kt}{1+kt}a^\dagger a}\mathrm{e}^{-\frac{kt}{1+kt}a^\dagger a}{:}\ = 1, \quad (24)$$

so the normalization of $M_{m,n}$ holds, and thus

$$\mathrm{Tr}\rho(t) = \mathrm{Tr}(\sum_{m,n}\rho_0 M_{m,n}^\dagger M_{m,n}) = \mathrm{Tr}\rho_0 = 1, \quad (25)$$

which exhibits that $M_{m,n}$ is a trace-preserving quantum operation and the density operator $\rho(t)$ is qualified to be an infinitive operator-sum representation.

4. Evolution of coherent states governed by Eq. (15)

Now we want to investigate how the coherent state $\rho_0 = |\alpha\rangle\langle\alpha|$[12] as an initial state evolves in the diffusion process acted upon by a linear resonance force. From Eq. (15), we obtain

$$\rho_{|\alpha\rangle}(t) = \frac{1}{1+kt}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}(\alpha-\alpha^*)-\frac{\lambda^2+kt}{1+kt}}\sum_{m,n}\frac{|\alpha|^{2n}}{m!n!}\left(\frac{kt}{1+kt}\right)^{m+n}$$
$$\times a^{\dagger m}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}\left(\frac{1}{1+kt}\right)^{a^\dagger a}|\alpha\rangle\langle\alpha|$$
$$\times\left(\frac{1}{1+kt}\right)^{a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}a^m. \quad (26)$$

Inserting the operator identity

$$\left(\frac{1}{1+kt}\right)^{a^\dagger a}|\alpha\rangle$$
$$= \mathrm{e}^{a^\dagger a\ln\frac{1}{1+kt}}|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}+\frac{\alpha a^\dagger}{1+kt}\right)|0\rangle \quad (27)$$

and the vacuum state projector (12) into Eq. (26) leads to the normal ordering form of $\rho_{|\alpha\rangle}(t)$

$$\rho_{|\alpha\rangle}(t) = \frac{1}{1+kt}\mathrm{e}^{-|\alpha|^2+\frac{\mathrm{i}\lambda t}{1+kt}(\alpha-\alpha^*)-\frac{\lambda^2+kt}{1+kt}}\sum_{m,n}^{\infty}\frac{|\alpha|^{2n}}{m!n!}\left(\frac{kt}{1+kt}\right)^{m+n}{:}\ a^{\dagger m}\mathrm{e}^{\frac{\mathrm{i}\lambda t}{1+kt}a^\dagger}\mathrm{e}^{\frac{\alpha a^\dagger}{1+kt}+\frac{\alpha^* a}{1+kt}-a^\dagger a}\mathrm{e}^{-\frac{\mathrm{i}\lambda t}{1+kt}a}a^m{:}$$
$$= \frac{1}{1+kt}\mathrm{e}^{-|\alpha|^2+\frac{\mathrm{i}\lambda t}{1+kt}(\alpha-\alpha^*)-\frac{\lambda^2+kt}{1+kt}}\mathrm{e}^{\frac{\mathrm{i}\lambda t+\alpha}{1+kt}a^\dagger}{:}\,\mathrm{e}^{-\frac{1}{1+kt}a^\dagger a}{:}\,\mathrm{e}^{\frac{-\mathrm{i}\lambda t+\alpha^*}{1+kt}a}. \quad (28)$$

Using Eq. (13), we obtain

$$\rho_{|\alpha\rangle}(t) = \frac{1}{1+kt}\mathrm{e}^{-|\alpha|^2+\frac{\mathrm{i}\lambda t}{1+kt}(\alpha-\alpha^*)-\frac{\lambda^2+kt}{1+kt}}\mathrm{e}^{\frac{\mathrm{i}\lambda t+\alpha}{1+kt}a^\dagger}\mathrm{e}^{a^\dagger a\ln\frac{kt}{1+kt}}\mathrm{e}^{\frac{-\mathrm{i}\lambda t+\alpha^*}{1+kt}a}, \quad (29)$$

which is just a superposition of the photon-added mixed state. Therefore, after undergoing the diffusion process under the action of the linear resonance force, the pure coherent state evolves into a mixed state owing to the existence of quantum noise.

5. Summary

we have obtained the explicit solution of the diffusion master equation under the action of a linear resonance force and found the infinite-sum representations for the density operators of this process. For some other master equations of density operators, we also employ this method and obtain the explicit infinite-sum representations for the density operators. Finally, we find that the initial pure coherent state evolves into a mixed thermal state as a result of diffusion and linear resonance force.

References

[1] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: Wiley) pp. 390–404
[2] Carmichael H J 1999 *Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations* (Berlin: Springer-Verlag) pp. 89–94
[3] Arnoldus H F 1996 *J. Opt. Soc. Am. B* **13** 1099
[4] Agarwal G S and Wolf E 1970 *Phys. Rev. D* **2** 2161
[5] Schleich W P 2001 *Quantum Optics in Phase Space* (Berlin: Wiley-VCH) pp. 66–98
[6] Fan H Y and Hu L Y 2008 *Mod. Phys. Lett. B* **22** 2435
[7] Fan H Y and Hu L Y 2008 *Opt. Commun.* **281** 5571
[8] Liu T K, Shan C J, Liu J B and Fan H Y 2014 *Chin. Phys. B* **23** 030303
[9] Fan H Y and Hu L Y 2009 *Chin. Phys. B* **18** 1061
[10] Fan H Y and Fan Y 1998 *Phys. Lett. A* **246** 242
[11] Fan H Y 2003 *J. Opt. B: Quantum. Semiclass. Opt.* **5** R147
[12] Glauber R 1963 *Phys. Rev.* **131** 2766