Quantum nonlocality of generic family of four-qubit entangled pure states*

Ding Dong(丁 东)[a)b)], He Ying-Qiu(何英秋)[a)], Yan Feng-Li(闫凤利)[a)†], and Gao Ting(高 亭)[c)‡]

[a)] College of Physics Science and Information Engineering, Hebei Normal University, Shijiazhuang 050024, China
[b)] Department of Basic Curriculum, North China Institute of Science and Technology, Beijing 101601, China
[c)] College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang 050024, China

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We directly introduce a Bell-type inequality for four-qubit systems. Using the inequality we investigate quantum nonlocality of a generic family of states $|G_{abcd}\rangle$ [*Phys. Rev. A* **65** 052112 (2002)] and several canonical four-qubit entangled states. It has been demonstrated that the inequality is maximally violated by the so called "four-qubit the maximally entangled state $|G_{\rm m}\rangle$" and it is also violated by four-qubit W state and a special family of states $|G_{ab00}\rangle$. Moreover, a useful entanglement-nonlocality relationship for the family of states $|G_{ab00}\rangle$ is derived. Finally, we present a scheme of preparation of the state $|G_{\rm m}\rangle$ with linear optics and cross-Kerr nonlinearities.

Keywords: quantum nonlocality, Bell-type inequality, quantum entanglement

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1. Introduction

Entanglement is an inherent consequence of multipartite quantum systems and as a physical resource, quantum entanglement has many important applications in quantum computation and quantum information.[1] It is worth noting that a fundamental question in the theory of entanglement is how to characterize and quantify entanglement.[2–9] In recent years, the characterization for multipartite entangled states has been considerably developed. The classification of a bipartite system under local unitary operators can be carried out by singular value decomposition.[1] Under stochastic local operations and classical communication (SLOCC), a three-qubit entangled pure state can be converted either to the GHZ-state or to the W-state.[10,11] In the case of a four-qubit entangled pure state, it has been shown[12] that there are nine classes of states. Recently, for the pure multipartite entangled states, Liu *et al.*[13] proposed a method for local unitary classification using the symmetry and tensor decomposition. For quantification of the entanglement, as is well-known, one can take the concurrence[14] as a measure of bipartite entanglement. In 2000, Coffman *et al.*[15] proposed the three-tangle which is a measure of genuine tripartite entanglement. Next the expressions of four-tangle[16–18] and n-tangle[19,20] have been derived.

Quantum theory allows correlations between spatially separated systems, while local hidden variable (LHV) theory gives an upper bound that quantifies the degree of nonlocality for a given inequality. A famous inequality as a way of verifying predictions of quantum mechanics versus those of all possible LHV theories was first proposed by Bell.[21] In 1969, Clauser *et al.* derived another correlation inequality, the Clauser–Horne–Shimony–Holt (CHSH) inequality,[22] which also provides a way of experimentally testing LHV theory as an independent hypothesis separated from the quantum formalism. It has been proved that[23,24] for the standard projective measurements all pure entangled states of two qubits violate the correlation CHSH inequality. In the case of three parties, Acín *et al.*[25] considered a Bell inequality and showed that all three-partite pure entangled states violate it. For the N-qubit case, the best-known inequality is the Mermin–Ardehali–Belinskiĭ–Klyshko (MABK) inequality,[26–28] which is included in the Werner–Wolf–Żukowski–Brukner (WWZB) inequality[29,30] as a special case. It has been proved that quantum violation of this inequality increases with the number of particles. Several Bell-type inequalities have been established by using an algebraical method,[31–33] and by finding out the stabilizer group of quantum states.[34,35] By determining facets of the correction polytope, Pitowsky *et al.*[36] and Wu *et al.*[37] obtained the new Bell inequalities.

In this paper, we discuss mainly the nonlocality for several canonical four-partite entangled states. In the Hilbert space $(\mathbb{C}^2)^{\otimes 4}$, a general four-qubit pure state may be written as

$$|\Psi^{ABCD}\rangle = \sum_{ijkl} a_{ijkl}|ijkl\rangle, \quad i,j,k,l = 0,1, \tag{1}$$

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†Corresponding author. E-mail: flyan@hebtu.edu.cn
‡Corresponding author. E-mail: gaoting@hebtu.edu.cn

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where a_{ijkl} are the complex coefficients with the basis vectors $|ijkl\rangle$, and A, B, C, and D are the locations of qubits, respectively. In nine classes of four-qubit entangled pure states,[12] we here consider the generic family

$$\begin{aligned}|G_{abcd}\rangle = & \frac{a+d}{2}(|0000\rangle+|1111\rangle)+\frac{a-d}{2}(|0011\rangle+|1100\rangle)\\ & +\frac{b+c}{2}(|0101\rangle+|1010\rangle)\\ & +\frac{b-c}{2}(|0110\rangle+|1001\rangle),\end{aligned} \tag{2}$$

where the complex parameters a, b, c, and d satisfy the normalization condition $|a|^2+|b|^2+|c|^2+|d|^2=1$. For the sake of simplicity throughout this paper we concern ourselves only with the cases where all a, b, c, and d are real. Obviously, the generic family contains four-qubit GHZ state $|\mathrm{GHZ}\rangle=(|0000\rangle+|1111\rangle)/\sqrt{2}$, cluster state $|\mathrm{cluster}\rangle=(|0000\rangle+|0011\rangle+|1100\rangle-|1111\rangle)/2$, two-EPR-pair state $|G_{1000}\rangle=(|0000\rangle+|0011\rangle+|1100\rangle+|1111\rangle)/2$, and the maximally entangled state

$$\begin{aligned}|G_{\mathrm{m}}\rangle := & \frac{1}{2\sqrt{2}}(|0000\rangle+|1111\rangle+|0011\rangle+|1100\rangle\\ & +|0101\rangle+|1010\rangle+|0110\rangle+|1001\rangle).\end{aligned} \tag{3}$$

We note that these states do not violate the previous four-qubit Bell inequalities[34,35] with the specified experimental settings. Next we introduce a Bell-type inequality that shows different degrees of violation when evaluated for different entangled states. In particular, the maximal violation is obtained with the four-qubit maximally entangled state $|G_{\mathrm{m}}\rangle$. At last, we present a scheme for preparing the state $|G_{\mathrm{m}}\rangle$ with linear optics and cross-Kerr nonlinearities.

2. Quantum nonlocality of generic family $|G_{abcd}\rangle$

2.1. Method

Firstly, we consider the four-qubit maximally entangled state $|G_{\mathrm{m}}\rangle$. Quantum mechanically, there are 16 operators $\{S_i, i=1,\ldots,16\}$ satisfying the eigenvalue equations $S_i|G_{\mathrm{m}}\rangle=\pm|G_{\mathrm{m}}\rangle$, namely, $\sigma_0\sigma_0\sigma_0\sigma_0$, $\sigma_x\sigma_x\sigma_x\sigma_x$, $\sigma_y\sigma_y\sigma_y\sigma_y$, $\sigma_z\sigma_z\sigma_z\sigma_z$, $\sigma_x\sigma_x\sigma_0\sigma_0$, $\sigma_x\sigma_0\sigma_x\sigma_0$, $\sigma_0\sigma_x\sigma_x\sigma_0$, $\sigma_x\sigma_0\sigma_0\sigma_x$, $\sigma_0\sigma_x\sigma_0\sigma_x$, $\sigma_0\sigma_0\sigma_x\sigma_x$, $\sigma_y\sigma_y\sigma_z\sigma_z$, $\sigma_y\sigma_z\sigma_y\sigma_z$, $\sigma_z\sigma_y\sigma_y\sigma_z$, $\sigma_y\sigma_z\sigma_z\sigma_y$, $\sigma_z\sigma_y\sigma_z\sigma_y$, and $\sigma_z\sigma_z\sigma_y\sigma_y$, where σ_0 is the identity operator and σ_i $(i=x,y,z)$ is Pauli operators. From the above 16 operators, we choose four operators and combine them as

$$\sigma_z\sigma_z\sigma_z\sigma_z-\sigma_z\sigma_y\sigma_y\sigma_z-\sigma_y\sigma_y\sigma_z\sigma_z-\sigma_y\sigma_z\sigma_y\sigma_z. \tag{4}$$

Whereupon, we can derive the expectation value of the above operator for the four-qubit maximally entangled state $|G_{\mathrm{m}}\rangle$, that is,

$$\begin{aligned}&\langle G_{\mathrm{m}}|\sigma_z\sigma_z\sigma_z\sigma_z-\sigma_z\sigma_y\sigma_y\sigma_z\\ &-\sigma_y\sigma_y\sigma_z\sigma_z-\sigma_y\sigma_z\sigma_y\sigma_z|G_{\mathrm{m}}\rangle=4.\end{aligned} \tag{5}$$

Based on the assumption of local realism, where all four qubits are locally but realistically correlated, we can introduce a linear Bell-type inequality. Suppose that every observer is allowed to choose between two dichotomic observables. More precisely, there is an ensemble of four spatially separated qubits and the measurements $A_1=\boldsymbol{a}_1\cdot\hat{\boldsymbol{\sigma}}$ or $A_2=\boldsymbol{a}_2\cdot\hat{\boldsymbol{\sigma}}$ are performed on qubit 1, $B_1=\boldsymbol{b}_1\cdot\hat{\boldsymbol{\sigma}}$ or $B_2=\boldsymbol{b}_2\cdot\hat{\boldsymbol{\sigma}}$ are performed on qubit 2, $C_1=\boldsymbol{c}_1\cdot\hat{\boldsymbol{\sigma}}$ or $C_2=\boldsymbol{c}_2\cdot\hat{\boldsymbol{\sigma}}$ are performed on qubit 3, and $D_1=\boldsymbol{d}_1\cdot\hat{\boldsymbol{\sigma}}$ or $D_2=\boldsymbol{d}_2\cdot\hat{\boldsymbol{\sigma}}$ are performed on qubit 4, where $\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{c}_1,\boldsymbol{c}_2,\boldsymbol{d}_1$, and $\boldsymbol{d}_2$ are unit vectors, $\hat{\boldsymbol{\sigma}}$ is Pauli operator. It is noteworthy that in terms of absolute local realism each outcome X_i ($X=A,B,C,D$ and $i=1,2$) can take one of the values $+1$ or -1.

Define the Bell-type operator

$$\begin{aligned}\mathcal{B} := & A_1B_1C_1D_2-A_1B_2C_2D_2\\ & -A_2B_2C_1D_1-A_2B_1C_2D_1.\end{aligned} \tag{6}$$

Clearly, under the experimental setting

$$\begin{aligned}&\boldsymbol{a}_1=\boldsymbol{b}_1=\boldsymbol{c}_1=\boldsymbol{d}_1=\boldsymbol{d}_2=(0,0,1),\\ &\boldsymbol{a}_2=\boldsymbol{b}_2=\boldsymbol{c}_2=(0,1,0),\end{aligned} \tag{7}$$

the present Bell-type operator is equal to the expression (4).

2.2. Bell-type inequality

Suppose that $p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)$ is the probability that, before the measurements are performed, the system is in a state, where $A_1=a_1$, $A_2=a_2$, $B_1=b_1$, $B_2=b_2$, $C_1=c_1$, $C_2=c_2$, $D_1=d_1$, and $D_2=d_2$. Let correlation function $\langle\mathcal{B}\rangle$ denote the average over many runs of the experiment. We then have

$$\begin{aligned}|\langle\mathcal{B}\rangle_{\mathrm{LHV}}| = & \Big|\sum_{a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2}p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)\\ & \times(a_1b_1c_1d_2-a_1b_2c_2d_2-a_2b_2c_1d_1-a_2b_1c_2d_1)\Big|\\ \le & \sum_{a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2}p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)2\\ = & \ 2.\end{aligned} \tag{8}$$

In addition,

$$\begin{aligned}|\langle\mathcal{B}\rangle_{\mathrm{LHV}}| = & \ |\langle A_1B_1C_1D_2-A_1B_2C_2D_2-A_2B_2C_1D_1-A_2B_1C_2D_1\rangle|\\ = & \Big|\sum_{a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2}p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)a_1b_1c_1d_2-\sum_{a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2}p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)a_1b_2c_2d_2\end{aligned}$$

$$- \sum_{a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2} p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)a_2b_2c_1d_1 - \sum_{a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2} p(a_1,a_2,b_1,b_2,c_1,c_2,d_1,d_2)a_2b_1c_2d_1|$$
$$= |\langle A_1B_1C_1D_2\rangle - \langle A_1B_2C_2D_2\rangle - \langle A_2B_2C_1D_1\rangle - \langle A_2B_1C_2D_1\rangle|. \quad (9)$$

Combining inequality (8) and Eq. (9), we obtain Bell-type inequality

$$|\langle\mathcal{B}\rangle_{\rm LHV}| = |\langle A_1B_1C_1D_2\rangle - \langle A_1B_2C_2D_2\rangle - \langle A_2B_2C_1D_1\rangle - \langle A_2B_1C_2D_1\rangle| \le 2, \quad (10)$$

which holds for LHV theories. It should be pointed out that one can obtain the above Bell inequality by using Wu's method.[37] As a matter of fact, LHV models can be associated with correlation polytope, whose facets define Bell inequalities. It is easy to prove that $\mathcal{B}/2 = 1$ is a facet of polytope for a four-qubit system.

2.3. Quantum nonlocality of the generic family

We next study the quantum nonlocality of the generic family $|G_{abcd}\rangle$ using the above operator $\mathcal{B}$ in the special setting stated by Eq. (7). It turns out that

$$\begin{aligned}&\langle G_{abcd}|\mathcal{B}|G_{abcd}\rangle\\ &= \frac{1}{2}[(a+d)^2+(a-d)^2+(b+c)^2+(b-c)^2]\\ &\quad +(b-c)(a+d)+(b+c)(a-d)\\ &\quad +(a-d)(a+d)+(b+c)(b-c)\\ &\quad +(b+c)(a+d)+(b-c)(a-d)\\ &= 2(a+b)^2. \end{aligned} \quad (11)$$

Under the constraint condition, $a^2+b^2 \le 1$, we plot in Fig. 1 the quantum prediction $\langle G_{abcd}|\mathcal{B}|G_{abcd}\rangle$ versus a and b. Obviously, the numerical value of the violation of the inequality depends on a and b.

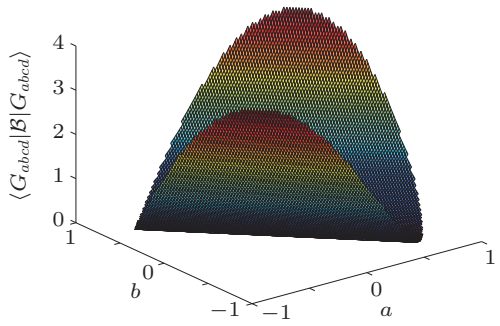


Fig. 1. (color online) Plot of the quantum prediction $\langle G_{abcd}|\mathcal{B}|G_{abcd}\rangle$ versus a and b with $a^2+b^2 \le 1$.

In particular, for the four-qubit maximally entangled state $|G_{\rm m}\rangle$, based on Eq. (5) one can conclude that this is indeed a violation of the inequality (10) where the LHV bound is 2. Under the optimal setting given above, the maximal violation of the Bell-inequality reaches its maximum value of 4 for the four-qubit maximally entangled state $|G_{\rm m}\rangle$. The inequality is therefore, in a sense, optimal. This is also the reason why we call $|G_{\rm m}\rangle$ the maximally entangled state.

3. Four-tangle versus nonlocality of the states $|G_{ab00}\rangle$

A remarkable feature of the present inequality is that there exists a family of the four-qubit entangled states where all states violate it. The family reads

$$|G_{ab00}\rangle = \frac{a}{2}(|0000\rangle+|1111\rangle+|0011\rangle+|1100\rangle) + \frac{b}{2}(|0101\rangle+|1010\rangle+|0110\rangle+|1001\rangle), \quad (12)$$

where a and b satisfy $ab > 0$ and $a^2+b^2 = 1$. Consider genuine four-tangle $\tau_{(4,8)}$,[17,18] which is regarded as a measure of four-qubit entanglement and constructed by means of a four-qubit invariant of degree 8 expressed in terms of three-qubit invariants, we have

$$\begin{aligned}&\tau_{(4,8)}(|G_{abcd}\rangle)\\ &= 4|\{[(a^2-b^2)(d^2-c^2)-(a^2-d^2)(b^2-c^2)]^2\\ &\quad +(a^2-b^2)(d^2-c^2)(a^2-d^2)(b^2-c^2)\}^{1/2}|. \end{aligned} \quad (13)$$

Then, for the state $|G_{ab00}\rangle$, four-tangle is $\tau_{(4,8)} = 4a^2b^2$. In the light of the result in Eq. (11) we obtain a useful relationship between four-partite entanglement (four-tangle) and nonlocality for the family of the four-qubit entangled states $|G_{ab00}\rangle$

$$\langle G_{ab00}|\mathcal{B}|G_{ab00}\rangle = 2(1+\sqrt{\tau_{(4,8)}}). \quad (14)$$

Figure 2 shows the quantum nonlocality versus four-tangle for states $|G_{ab00}\rangle$. It has been shown that quantum violation varies with four-tangle $\tau_{(4,8)}$ and when $\tau_{(4,8)} = 1$ (i.e., $a = b = 1/\sqrt{2}$) the maximal violation is obtained.

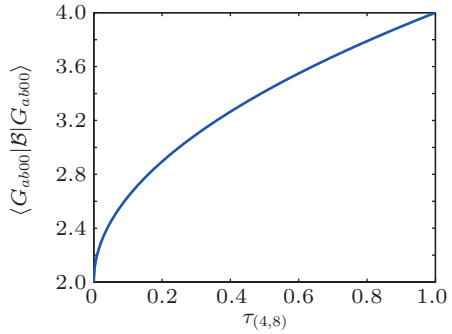


Fig. 2. (color online) Numerical results of quantum nonlocality versus four-tangle for the family of the four-qubit entangled states $|G_{ab00}\rangle$, which violate the present inequality.

4. Preparation for the maximally entangled state $|G_{\rm m}\rangle$

We now discuss a possible experimental realization of the maximally entangled state $|G_{\rm m}\rangle$ with linear optics and cross-Kerr nonlinearities.[38,39] The setup for creating the desired state is shown in Fig. 3. Initial state is fed by $[(|H\rangle + |V\rangle)/\sqrt{2}]^{\otimes 4}$, where the qubits are encoded with the polarization modes $|H\rangle \equiv |0\rangle$ and $|V\rangle \equiv |1\rangle$. In order to project the input state into expected subspaces, we here introduce an auxiliary single-photon state $|H\rangle_{\rm T}$ as target photon to construct four controlled-NOT gates. Note that there exists the scheme for constructing near deterministic linear optical controlled-NOT gate with linear optics and cross-Kerr nonlinearities.[38] In the present scheme, therefore, we suggest that these optical controlled-NOT gates can be realized by using several cross-Kerr media as well as linear optics. Then, a bit-flip operation ($|H\rangle \rightleftharpoons |V\rangle$) occurs on the auxiliary photon when any of these four signal photons is horizontal polarization. At last, one makes a single-photon detection on the auxiliary photon in a basis $\{|H\rangle, |V\rangle\}$. When detector D_H clicks, we obtain the maximally entangled state $|G_{\rm m}\rangle$. While for D_V clicks, alternatively, one can also get the desired state by performing a bit-flip operation on any one of the four photons.

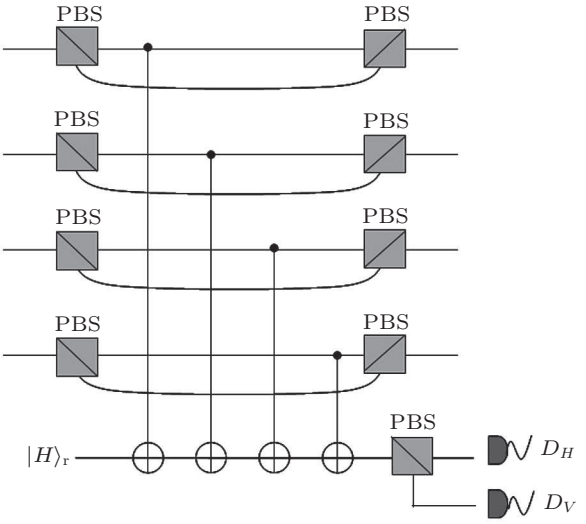


Fig. 3. The schematic diagram of preparation of the maximally entangled state $|G_{\rm m}\rangle$. Each polarizing beam splitter (PBS) is used to transmit $|H\rangle$ polarization photons and reflect $|V\rangle$ polarization photons. Four optical controlled-NOT gates are realized by means of cross-Kerr nonlinearities.

5. Discussion

Finally, let us discuss several specific examples for the candidates of four-qubit maximally entangled state using the introduced Bell-type inequality. Under the specified experimental setting, we calculate expectation values of operator $\mathcal{B}$ for four-qubit GHZ state, cluster state, W state, two-EPR-pair state,

$$|\chi\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle + |1111\rangle - |0011\rangle + |1100\rangle - |0101\rangle + |1010\rangle + |0110\rangle + |1001\rangle),$$

presented by Yeo and Chua,[40]

$$|\mathrm{HS}\rangle = \frac{1}{\sqrt{6}}\left[|0011\rangle + |1100\rangle + \exp\left(\mathrm{i}\frac{2\pi}{3}\right)(|0101\rangle + |1010\rangle) + \exp\left(\mathrm{i}\frac{4\pi}{3}\right)(|0110\rangle + |1001\rangle)\right],$$

introduced by Higuchi and Sudbery,[41] and

$$|\Phi\rangle = \frac{1}{2}(|0000\rangle + |1101\rangle) + \frac{1}{2\sqrt{2}}(|0011\rangle + |0110\rangle + |1011\rangle - |1110\rangle),$$

investigated by Brown *et al.*[42] The result is that the inequality is not violated by these states except the W state, $|\langle \mathrm{W}|\mathcal{B}|\mathrm{W}\rangle| = 2.5$. Hence, the present inequality can act as a strong entanglement witness for the entangled state $|G_{\rm m}\rangle$.

6. Summary

In summary, we have introduced a four-qubit Bell-type inequality with the method of the stabilizer group of quantum states. Under the given experimental setting, the inequality is maximally violated by the four-qubit maximally entangled state $|G_{\rm m}\rangle$ which does not violate previous four-qubit Bell-type inequality.[34,35] We have studied the quantum nonlocality of the generic family $|G_{abcd}\rangle$ and several four-qubit states using the inequality and shown that the inequality is also violated by the four-qubit W state and a family of four-qubit entangled states $|G_{ab00}\rangle$. Also, we have derived a useful relationship between four-partite entanglement and nonlocality for the family of four-qubit entangled states $|G_{ab00}\rangle$. Therefore, the results derived from the inequality indicate that it can act as a strong entanglement witness for the family $|G_{ab00}\rangle$, especially for $|G_{\rm m}\rangle$. Experimentally, we notice that the present inequality is easier to evaluate because it contains only four terms. In addition, we have proposed a scheme of experimental realization of the state $|G_{\rm m}\rangle$ with linear optics and cross-Kerr nonlinearities. It is straightforward to show in principle that the desired state can be prepared after a simple single-photon detection and post-selection. Finally, it is worth noting that the discovery of the entanglement-nonlocality relationship, as well as the results reported in Refs. [43]–[46] will greatly facilitate the development of entanglement and their application in quantum information processing.[47–52]

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865
[3] Yan F L, Gao T and Chitambar E 2011 *Phys. Rev. A* **83** 022319
[4] Gao T, Yan F L and van Enk S J 2014 *Phys. Rev. Lett.* **112** 180501

[5] Bai Y K, Xu Y F and Wang Z D 2014 *Phys. Rev. Lett.* **113** 100503
[6] Ma Z H, Chen Z H, Chen J L, Spengler C, Gabriel A and Huber M 2011 *Phys. Rev. A* **83** 062325
[7] Hong Y, Gao T and Yan F L 2012 *Phys. Rev. A* **86** 062323
[8] Gao T and Hong Y 2011 *Eur. Phys. J. D* **61** 765
[9] Gao T and Hong Y 2010 *Phys. Rev. A* **82** 062113
[10] Dür W, Vidal G and Cirac J I 2000 *Phys. Rev. A* **62** 062314
[11] Acín A, Andrianov A, Costa L, Jané E, Latorre J I and Tarrach R 2000 *Phys. Rev. Lett.* **85** 1560
[12] Verstraete F, Dehaene J, De Moor B and Verschelde H 2002 *Phys. Rev. A* **65** 052112
[13] Liu B, Li J L, Li X K and Qiao C F 2012 *Phys. Rev. Lett.* **108** 050501
[14] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[15] Coffman V, Kundu J and Wootters W K 2000 *Phys. Rev. A* **61** 052306
[16] Sharma S S and Sharma N K 2010 *Phys. Rev. A* **82** 012340
[17] Sharma S S and Sharma N K 2013 *Phys. Rev. A* **87** 022335
[18] Sharma S S and Sharma N K arXiv:quant-ph/1309.2713
[19] Wong A and Christensen N 2001 *Phys. Rev. A* **63** 044301
[20] Li D F, Li X R, Huang H T and Li X X 2009 *J. Math. Phys.* **50** 012104
[21] Bell J S 1987 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)
[22] Clauser J F, Horne M A, Shimony A and Holt R A 1969 *Phys. Rev. Lett.* **23** 880
[23] Gisin N 1991 *Phys. Lett. A* **154** 201
[24] Gisin N and Peres A 1992 *Phys. Lett. A* **162** 15
[25] Acín A, Chen J L, Gisin N, Kaszlikowski D, Kwek L C, Oh C H and Żukowski M 2004 *Phys. Rev. Lett.* **92** 250404
[26] Mermin N D 1990 *Phys. Rev. Lett.* **65** 1838
[27] Ardehali M 1992 *Phys. Rev. A* **46** 5375
[28] Belinskiĭ A V and Klyshko D N 1993 *Phys. Usp.* **36** 653
[29] Werner R F and Wolf M M 2001 *Phys. Rev. A* **64** 032112
[30] Żukowski M and Brukner Č 2002 *Phys. Rev. Lett.* **88** 210401
[31] Svetlichny G 1987 *Phys. Rev. D* **35** 3066
[32] Li M and Fei S M 2012 *Phys. Rev. A* **86** 052119
[33] Grandjean B, Liang Y C, Bancal J D, Brunner N and Gisin N 2012 *Phys. Rev. A* **85** 052113
[34] Scarani V, Acín A, Schenck E and Aspelmeyer M 2005 *Phys. Rev. A* **71** 042325
[35] Wu C F, Yeo Y, Kwek L C and Oh C H 2007 *Phys. Rev. A* **75** 032332
[36] Pitowsky I and Svozil K 2001 *Phys. Rev. A* **64** 014102
[37] Wu Y C, Żukowski M, Chen J L and Guo G C 2013 *Phys. Rev. A* **88** 022126
[38] Nemoto K and Munro W J 2004 *Phys. Rev. Lett.* **93** 250502
[39] Ding D, Yan F L and Gao T 2013 *J. Opt. Soc. Am. B* **30** 003075
[40] Yeo Y and Chua W K 2006 *Phys. Rev. Lett.* **96** 060502
[41] Higuchi A and Sudbery A 2000 *Phys. Lett. A* **273** 213
[42] Brown I D K, Stepney S, Sudbery A and Braunstein S L 2005 *J. Phys. A* **38** 1119
[43] Ghose S, Sinclair N, Debnath S, Rungta P and Stock R 2009 *Phys. Rev. Lett.* **102** 250404
[44] Ajoy A and Rungta P 2010 *Phys. Rev. A* **81** 052334
[45] Ghose S, Debnath S, Sinclair N, Kabra A and Stock R 2010 *J. Phys. A* **43** 445301
[46] Zhao J Q, Cao L Z, Lu H X and Wang X Q 2013 *Acta Phys. Sin.* **62** 120301 (in Chinese)
[47] Deng F G, Long G L and Liu X S 2003 *Phys. Rev. A* **68** 042317
[48] Gao T, Yan F L and Wang Z X 2004 *J. Phys. A* **37** 3211
[49] Gao T 2004 *Commun. Theor. Phys.* **42** 223
[50] Li X H, Duan X J, Sheng Y B, Zhou H Y and Deng F G 2009 *Chin. Phys. B* **18** 3710
[51] Ding D and Yan F L 2013 *Phys. Lett. A* **377** 1088
[52] Liu Y, Chen T Y, Wang L J, Liang H, Shentu G L, Wang J, Cui K, Yin H L, Liu N L, Li L, Ma X F, Pelc J S, Fejer M M, Peng C Z, Zhang Q and Pan J W 2013 *Phys. Rev. Lett.* **111** 130502