Disorder-enhanced nuclear spin relaxation at Landau level filling factor one*

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(Received 17 March 2015; revised manuscript received 25 March 2015; published online 10 April 2015)

The nuclear spin relaxation rate ($1/T_1$) is measured for GaAs two-dimensional (2D) electron systems in the quantum Hall regime with an all-electrical technique for agitating and probing the nuclear spins. A "tilted plateau" feature is observed near the Landau level filling factor $\nu = 1$ in $1/T_1$ versus ν. Both the width and magnitude of the plateau increase with decreasing electron density. At low temperatures, $1/T_1$ exhibits an Arrhenius temperature dependence within the tilted plateau regime. The extracted energy gaps are up to two orders of magnitude smaller than the corresponding charge transport gaps. These results point to a nontrivial mechanism for the disorder-enhanced nuclear spin relaxation, in which microscopic inhomogeneities play a key role for the low energy spin excitations related to skyrmions.

Keywords: nuclear spin relaxation, Landau level filling factor one, skyrmions

PACS: 73.40.–c, 73.20.–r, 73.63.Hs **DOI:** 10.1088/1674-1056/24/6/067302

1. Introduction

Because the relaxation of nuclear spins can proceed via the hyperfine interaction with electrons, measurement of the nuclear spin relaxation rate provides a powerful means of probing complex and rich spin-related electronic structures that exist in various systems. This is particularly true for the two-dimensional electron system (2DES) exposed to a perpendicular magnetic field B (for reviews see Refs. [1] and [2]). In the case of a sufficiently strong magnetic field, the energy spectrum is discretized and the B-field also causes a large mismatch between the Zeeman energies of electrons and nuclei. It suppresses the nuclear spin relaxation via the hyperfine interaction, since the electron–nuclear spin flip–flop processes are forbidden by the law of energy conservation. Nuclear spin relaxation via the hyperfine interaction is however possible, if the Zeeman energy mismatch can be compensated in some other features in the energy spectrum. Examples include the existence of an overlap between the disorder broadened Landau levels of opposite spins,[3] differently populated edge channels of opposite spin orientations,[4,5] the presence of interaction-induced gapless low energy excitations involving spin flips as they occur in a Skyrme crystal,[6] and the coexistence of domains with opposite spin originating from a spin-related phase transition in the electronic system as a result of disorder.[7–9] Nuclear spin relaxation measurements have played an important role in unveiling the nature of complex low energy excitations and spin-related phase transitions in the 2D systems.[1,2]

Numerous experiments have reported a strong enhancement of the nuclear spin relaxation rate near but not at Landau level filling factor $\nu \equiv nh/(eB) = 1$, where n is the 2D electron density, h is the Planck constant, e is electron charge, and B is the perpendicular magnetic flux density (i.e. $\boldsymbol{B}$-field component perpendicular to the 2DES plane).[6–8,10–14] When the strength of disorder is weak, the ground state at $\nu = 1$ is a fully spin-polarized quantum Hall ferromagnet.[11] When the filling deviates from exact $\nu = 1$, skyrmions, vortex-like spin textures possessing topological charge, emerge as a consequence of lowering the exchange energy between the electrons.[15–20] In the presence of a sufficient number of skyrmions the nuclear spin relaxation rate ($1/T_1$) is enhanced. This enhancement has frequently been attributed to a gapless Goldstone mode of a Skyrme crystal that forms at low temperatures.[21–23] Even though such low energy excitations of skyrmions provide a natural explanation of the enhanced nuclear spin relaxation, controversy remains in view of discrepancies in the observed temperature dependence of $1/T_1$ [6,12–14] and in the predicted temperature for the melting of the Skyrme crystal.[23–26] It

*Project supported by the National Basic Research Program of China (Grant Nos. 2012CB921703 and 2015CB921102), the National Natural Science Foundation of China (Grant Nos. 91121003, 11374337, and 61425015), the Chinese Academy of Sciences, the BMBF, and the German–Israel Foundation.

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has also been pointed out theoretically that quantum fluctuations may lead to a liquid ground state near $\nu = 1$ at zero temperature.[27] When disorder effects are included, the fully spin-polarized quantum Hall ferromagnet from which it all starts may be an oversimplified description of the $\nu = 1$ ground state even at zero temperature, and other electronic phases, such as paramagnetic states, partially polarized ferromagnetic states, and quantum Hall spin glasses, have been predicted.[26,28–31] Previous experiments on nuclear spin relaxation at or near $\nu = 1$ have, however, not been performed such that the disorder effects on the electron spin structures can be explored in detail, in particular at temperatures below 40 mK.

In this work, we shed more light on the nuclear spin relaxation issue near and at $\nu = 1$ by extending existing studies to samples with different levels of disorder and to temperatures less than 15 mK. This temperature regime has become accessible by using all-electrical nuclear spin relaxation experiments. The role of disorder is studied either by varying the electron density with a back gate or by comparing samples of different quality. The nuclear spin relaxation rate exhibits a tilted plateau structure around $\nu = 1$. This plateau vanishes with improved sample quality indicating an important role of disorder. Previously invoked mechanisms[3,32,33] to account for disorder-enhanced nuclear spin relaxation near odd filling factors are not compatible with our data.

2. Experiments

Our studies reported here were mainly performed on two GaAs/AlGaAs quantum well wafers. Both wafers were patterned into 400-μm Hall bar shaped samples equipped with a backgate. In sample A, the 2DES resides in a 16-nm wide quantum well and the electron density n varies with the backgate voltage V_G in units of volts according to $n = (1.79 + 0.89V_G) \times 10^{11}$ cm^{-2}. Sample B has a 30-nm wide quantum well and $n = (1.79 + 0.69V_G) \times 10^{11}$ cm^{-2}. The electron mobilities at $V_G = 0$ are 0.8×10^6 cm^2/V·s and 1.3×10^7 cm^2/V·s for samples A and B, respectively. The data shown below were taken on sample A unless otherwise specified.

The all-electrical method to detect the nuclear spin relaxation rate at an arbitrary filling factor $\nu_{\rm rest}$ only requires quasi-dc electron transport.[34] The nuclear spin system is first driven out of equilibrium to elicit a time-dependent response, for instance by imposing an electrical current through the sample. Subsequently, the recovery to equilibrium is monitored. At the B-fields relevant here, the Fermi sea of composite fermions at $\nu = 1/2$ is partially polarized. The two spin populations have a continuous and finite density of states. This enables Korringa-like hyperfine coupling, so that the nuclear spin system can efficiently interact with the electronic spin system.[1] This property is exploited to drive the nuclear spin system out of equilibrium. The nuclear spin polarization is estimated to be approximately 10% if the system is left to equilibrate sufficiently long at base temperature. A small ac current is imposed in order to heat up the electronic system. Entropy is transferred via the hyperfine coupling and the existing thermal nuclear spin polarization is partially annihilated. The longitudinal resistance R_{xx} at $\nu = 1/2$ depends on the electron Zeeman energy as long as the composite fermion sea is partially spin-polarized. This is demonstrated in Fig. 1(a) where R_{xx} at $\nu = 1/2$ is plotted as a function of the total magnetic field $B_{\rm tot}$ for three different values of the perpendicular B-field. The value of R_{xx} increases with increasing the total field, i.e. Zeeman energy, and then saturates when the composite fermion sea is fully spin-polarized (marked by arrows in Fig. 1(a)). A detailed description of the observed behavior can be found in Refs. [35] and [36]. The degree of nuclear spin polarization modifies the composite fermion Zeeman energy. Therefore, changes in the nuclear spin polarization can be monitored by recording the evolution of R_{xx} at $\nu = 1/2$.

The measurement sequence to detect the nuclear spin relaxation rate at filling $\nu_{\rm rest}$ is illustrated in Fig. 1(b). Initially, a small low frequency ac current at $\nu = 1/2$ is imposed to increase the electron temperature and partially depolarize the nuclear spin system.[34] Simultaneously, R_{xx} at $\nu = 1/2$ can be measured. The current is switched off and the gate voltage is then set to reach $\nu_{\rm rest}$ where the system is left for some time $t_{\rm rest}$, during which the nuclear spin polarization relaxes towards the equilibrium value for $\nu_{\rm rest}$. Subsequently, the filling is set back to 1/2 for a sufficiently long time ($t_{\rm meas}$). The ac current is turned on as before in order to measure R_{xx} and partially depolarize the nuclear spin system to prepare for the next sequence. As illustrated in Fig. 2(b), the trace of R_{xx} at $\nu = 1/2$ versus t is fitted to the exponential decay function:

$$R_{xx}(t) = R_0 + \Delta R_{xx} \exp(-t/\tau). \tag{1}$$

Note that only ΔR_{xx} is of interest, since $1/\tau$ does not reflect the nuclear spin relaxation rate for $\nu_{\rm rest}$ but for $\nu = 1/2$. In the examples of Fig. 1(b), $\Delta R_{xx} < 0$. The measurement sequence is repeated for different $t_{\rm rest}$ and $\nu_{\rm rest}$. The (ΔR_{xx}, $t_{\rm rest}$) data is then fitted to a similar exponential function:

$$\Delta R_{xx}(t_{\rm rest}) = \Delta R_\infty + \Delta R \exp(-t_{\rm rest}/\tau_1). \tag{2}$$

The extracted time constant τ_1 yields the relaxation rate at $\nu_{\rm rest}$ (Fig. 1(c)). If ΔR_{xx} is not too large, τ_1 is close to the nuclear spin relaxation time T_1.[1] If not, the required correction can be easily carried out as described in Ref. [34]. This all-electrical method is not limited to the use of $\nu = 1/2$. Also, the spin transition at $\nu \approx 2/3$ can be similarly used for the all-electrical spin relaxation measurements.[8,34,37]

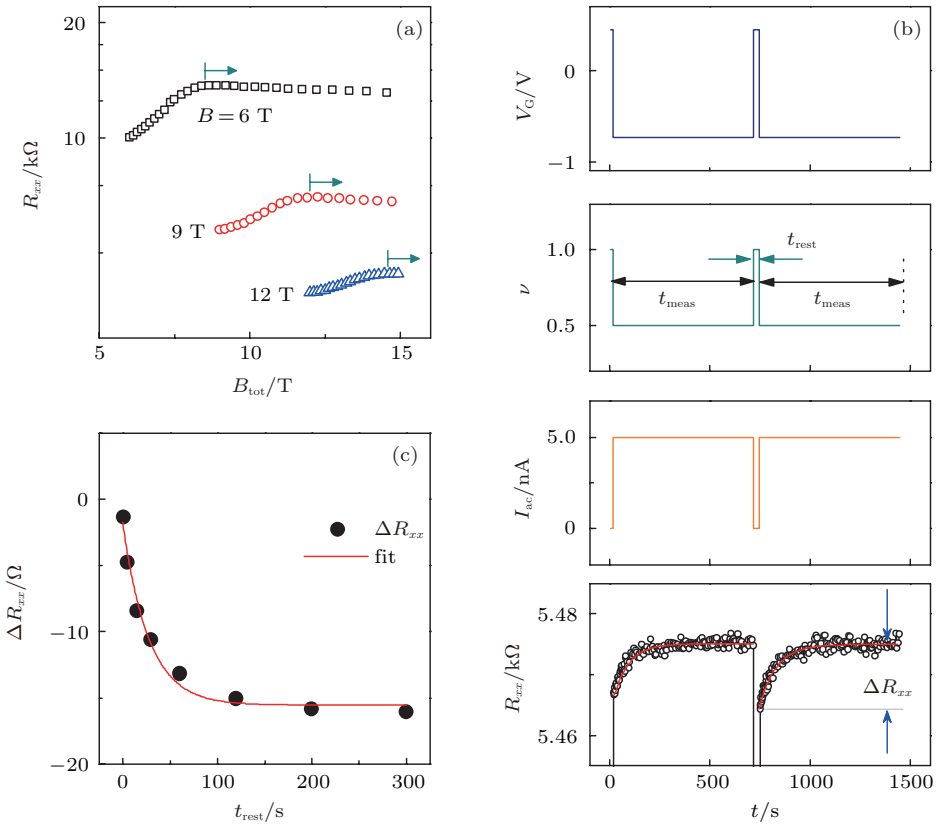


Fig. 1. (color online) (a) R_{xx} at $\nu = 1/2$ as a function of the total magnetic field, $B_{\rm tot}$, at the base temperature for perpendicular fields $B = 6$, 9, and 12 T. Increasing of $B_{\rm tot}$ leads to transitions from partial to full spin polarization, which are marked by short vertical lines. The arrows denote regions of full spin polarization. (b) A fraction of the measurement sequence for the nuclear spin relaxation measurements. The bottom panel shows an example of an R_{xx}–t trace (circles) and the exponential decay fit to Eq. (1) (line). (c) An example of how to extract time constant τ_1 with a fit of the ΔR_{xx} data extracted from the procedure illustrated in panel (b) to Eq. (2).

The nuclear spin relaxation measurements reported here were carried out in two top-loading dilution refrigerators with superconducting magnets. Extensive low pass filtering in the electrical wirings have been implemented in order to obtain low electron temperatures. The temperature was monitored with calibrated ruthenium oxide sensors mounted near the location of the samples. Measurements of fractional quantum Hall states with small activation gaps (such as $\nu = 5/2$ states[38]) were used to confirm that electrons in the quantum Hall regime can be cooled down to at least 15 mK in both dilution refrigerator systems.

3. Results and discussion

Figure 2(a) displays the filling factor dependence of the relaxation rate measured with $B = 9$ T at the base temperature. The $1/T_1$ data exhibit pronounced peaks at $|\nu - 1| \approx 0.18$. These peaks are qualitatively similar to those observed previously by Barrett *et al.* with optically pumped nuclear magnetic resonance (NMR) at 2.1 K[6] and by resistive detection at $\nu \approx 2/3$.[7,8] They were attributed to the gapless Goldstone mode of the Skyrme crystal. According to the theory of Côté *et al.*, the enhancement of the nuclear spin relaxation rate is proportional to both $|\nu - 1|$ and the temperature.[23] We will comment on this later. The drop of $1/T_1$ beyond the peaks (i.e., $|\nu - 1| > 0.18$) may just reflect that skyrmionic excitations are no longer favorable at high densities (larger $|\nu - 1|$) and the system may give way to quantum liquids as $|\nu - 1|$ increases.[27]

An unusual feature of the $1/T_1$ versus $\nu_{\rm rest}$ data is the "tilted plateau" structure, which exists between the two $1/T_1$ peaks and around $\nu = 1$. As shown in Fig. 2(a), $1/T_1$ could be much larger than that expected for a perfect Skyrme crystal within the plateau. Such a structure is robust against increasing temperature. Figure 2(b) shows that a similar plateau shows up at temperatures up to 84 mK for $B = 9$ T despite much larger values of $1/T_1$ at higher T. Similar measurements were also carried out with different magenetic fields for sample A and also for sample B. For the sake of identifying features, R_{xx} is shown in Fig. 2(c) as a function of the filling factor for $B = 6$, 9, 12 T for sample A and 4.1 T for sample B. The corresponding filing factor dependences of $1/T_1$ are depicted in Fig. 2(d), which shows that the width of the tilted plateau ($\Delta\nu_{\rm TP}$) decreases significantly as B is increased from 6 T to 12 T. At exact $\nu_{\rm rest} = 1$, the nuclear spin relaxation rate is unexpectedly larger than that expected for a perfect quantum Hall ferromagnet.[23]

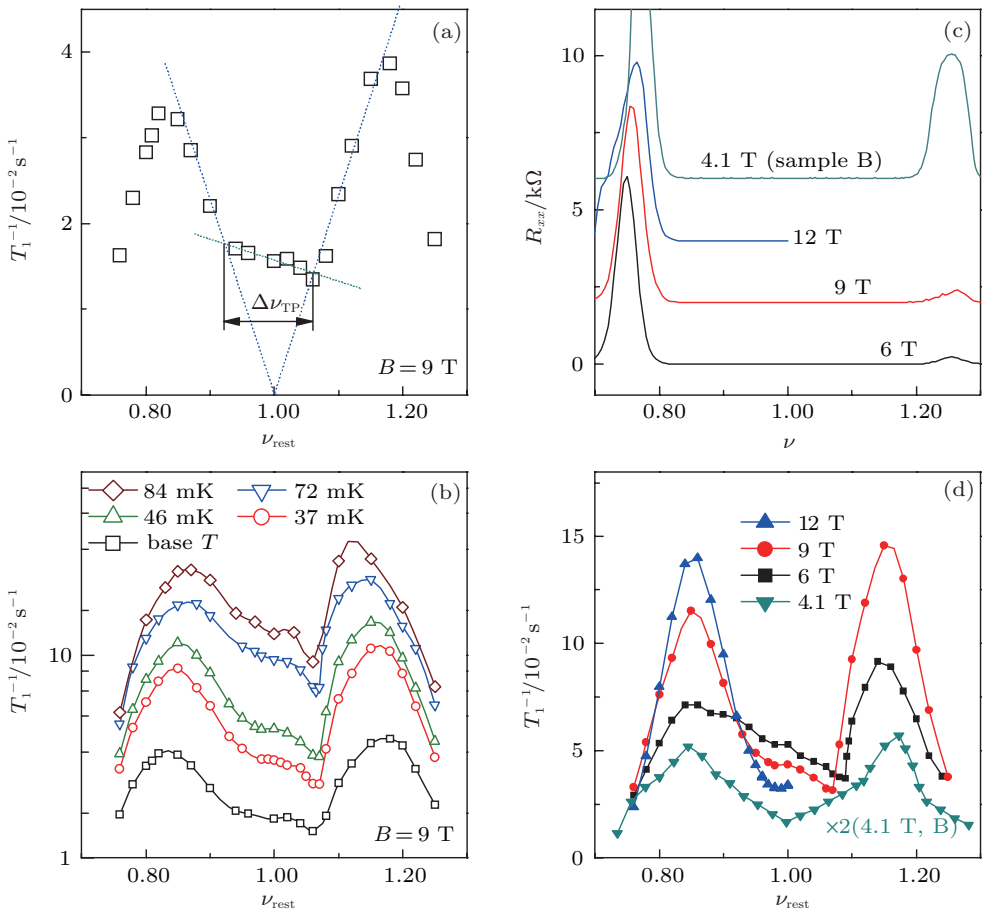


Fig. 2. (color online) (a) Filling factor dependence of the nuclear spin relaxation rate, $1/T_1$, measured at $B = 9$ T at the base temperature for sample A. The experimental data (symbols) are compared with the model of Côté *et al.*[23] proposed for disorder-free Skyrme crystals (dotted lines following $1/T_1 \propto |\nu - 1|$). The width of the tilted plateau is labeled as $\Delta\nu_{\mathrm{TP}}$. (b) Filling factor dependences of $1/T_1$ for sample A with $B = 9$ T at $T = 10$, 37, 46, 72, and 84 mK (from bottom to top). (c) and (d) Filling factor dependences of the longitudinal resistance R_{xx} (c) and $1/T_1$ (d) at $B = 6$, 9, 12 T for sample A and 4.1 T for sample B. The measurements were carried out at $T = 46$ mK and the base temperature for samples A and B, respectively. In panel (c), the R_{xx}–ν curves are offset by 2 kΩ intervals for clarity.

For a homogeneous disorder free quantum Hall ferromagnet at $\nu = 1$, the lowest energy charged excitations involving electronic spin flips are skyrmions, and the lowest energy neutral excitations are spin waves or magnons.[10] Both types of excitations require an energy that exceeds by orders of magnitude the energy involved in flipping a nuclear spin.[19,21] Nuclear spin relaxation assisted by the hyperfine coupling should therefore be strongly suppressed and $1/T_1$ is expected to be vanishingly small at the temperatures encountered in this work. Contrary to these expectations, the observed $1/T_1$ at $\nu_{\mathrm{rest}} = 1$ in Fig. 2 on sample A is anomalously large. The nuclear spin relaxation rate at $\nu_{\mathrm{rest}} = 1$ is largest at low fields and drops at higher fields as the tilted plateau becomes narrower and less pronounced. On sample B with a much higher mobility the tilted plateau has vanished all together. Based on these observations, we conjecture that the tilted plateau structure is associated with the level of disorder. As we tune the density in sample A to higher values, the bare disorder potential, i.e., the amplitude of the density variations Δn, remains approximately the same,[39] but the spatial variations in the filling factor become smaller as ν varies with $\Delta n/n$.

In fact, disorder-enhanced nuclear spin relaxation was studied long ago.[3,32,33] In the experimental work by Berg *et al.*,[3] $1/T_1$ was found to be correlated with σ_{xx} (or R_{xx}) near $\nu = 3$ at $T = 1.3$ K, and the data could be fairly well explained by considering the exchanged enhanced Zeeman splitting and disorder-induced Landau level broadening. A non-vanishing overlap between the densities of states for the spin-up and spin-down electronic states allows for Korringa-like nuclear spin relaxation. This simple, phenomenological model, however, does not fit our data. As shown in Fig. 2, the filling factor dependence of $1/T_1$ is uncorrelated with the dependence of R_{xx}. Figure 3(a) illustrates that $1/T_1$ at $\nu = 1$ is significantly higher than at $\nu = 0.76$ for $T > 50$ mK. In contrast, σ_{xx}, which can be calculated from R_{xx} in Fig. 3(b) is vanishingly small at $\nu = 1$ due to the large thermal activation gap (about 50 K), whereas σ_{xx} is quite large at $\nu = 0.76$. At $\nu = 0.85$, where the nuclear spins relax most rapidly at $T > 30$ mK, σ_{xx} is also nearly zero (See Figs. 3(b) and 4(a)). Hence, the disor-

der model of Berg *et al.* fails to explain the anomalously large relaxation rate at $\nu = 1$ as well as the tilted plateau feature.

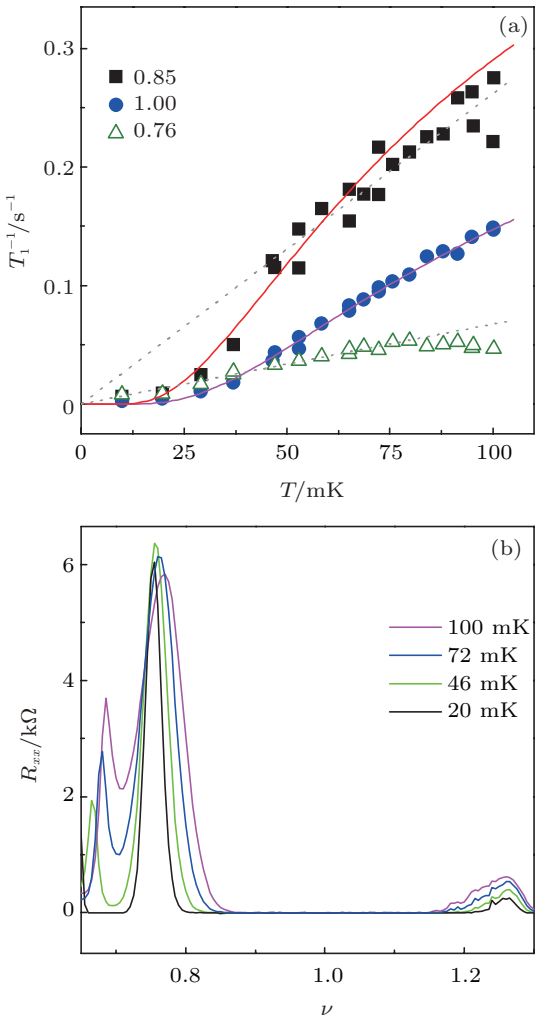


Fig. 3. (color online) (a) Temperature dependence of the nuclear spin relaxation rate $1/T_1$ at $\nu = 0.85$ (squares), 1.00 (circles), and 0.76 (triangles) with $B = 9$ T. The solid lines are Arrhenius fits to the data. The gray dotted lines illustrate linear temperature dependence expected from the Korringa law. (b) Filling factor dependence of R_{xx} at four temperatures between 20 mK and 100 mK.

We note that not only the relaxation rate measurements shown above but also previous measurements of the filling factor dependence of the electron spin polarization[20,40–42] exhibited tilted plateau features around $\nu = 1$. For a disorder free quantum Hall ferromagnet, a quick depolarization from full spin polarization is expected as one moves away from exact filling factor 1, i.e. from exact matching between the number of flux quanta and that of electrons. For every unmatched flux quantum or electron, an anti-skyrmion or skyrmion is generated involving more than one electron spin flip.[16] The measured electron spin polarization at $\nu = 1$ is, however, far away from the expected full polarization and only outside of the tilted plateau, the electron spin polarization follows the quick depolarization.[20,41,42] Local compressibility measurements with a single electron transistor have revealed that electrons attempt to screen, i.e. flatten, the bare disorder potential by setting up a spatially dependent density profile $\Delta n(x,y)$.[39] However, as one approaches exact integer quantum Hall fillings, electrons fail to screen the bare disorder potential in some parts of the sample because the local density of states has been exhausted. As a result, a landscape of anti-dots or dots surrounded by an incompressible lake with the nearby integer filling emerges. If disorder is sufficiently strong, dots and anti-dots may coexist. In these anti-dot regions, electrons are missing and hence anti-skyrmions form. In dot regions, skyrmions are present. These skyrmion and anti-skyrmion puddles will significantly lower the average electron spin polarization. They are bound to be responsible for the observed reduced spin polarization.[20,41,42] As we deviate from average filling factor 1, anti-dots shrink, while dots expand or *vice versa*. This compensation effect may account for the tilted plateau. This puddle picture of skyrmions and anti-skyrmions may also play an important role for the observed plateau in the nuclear spin relaxation rate in view of the close resemblance with the behavior of the electron spin polarization. With increased B-field, the same landscape of dots and anti-dots recurs at higher average density.[39] Since the bare disorder potential does not vary with the field, the spatial density profile $\delta n(x,y)$ set up by the electrons remains the same. As a consequence, the filling factor variation ($\Delta\nu \sim \Delta n/n$) drops with increasing B-field. The tilted plateau width, $\Delta\nu_{\rm TP}$, which likely reflects the spatial variation in filling factor, will therefore shrink with increasing magnetic field, in qualitative agreement with the experimental data in Fig. 2(d).

The temperature dependence of $1/T_1$ may also shed light on the underlying relaxation mechanisms. At $\nu = 1$, the nuclear spin relaxation rate in Fig. 3(a) exhibits thermally activated behavior within the investigated temperature range. An activation gap $\Delta_s \approx 0.11$ K is extracted from an Arrhenius fit to the data. Apparently an energy barrier prevents the nuclear spin relaxation in the limit of zero temperature. It is tempting to argue that the incompressible region separating the skyrmion and anti-skyrmion puddles is the origin of this energy barrier. Yet, a microscopic picture is lacking for what kind of low energy electronic spin excitations would emerge in such a disordered spin system and how these excitations would enhance nuclear spin flips while propagating between these puddles.

Figure 3(a) also contains the temperature dependence of $1/T_1$ at other filling factors. It is noteworthy that near $\nu = 0.85$, the relaxation rate is close to the largest among the filling factors investigated in this work and in several previously reported theoretical and experimental studies.[12–14,23,26] The nuclear spin relaxation rate varies approximately linearly with temperature down to about 45 mK. Below this temperature, $1/T_1$ drops more rapidly in a nonlinear fashion. The Arrhenius

fit yields an energy gap $\Delta_s \approx 0.08$ K, while the gap for charge transport at this filling factor equals $\Delta_c \approx 0.54$ K (Fig. 4(a)). A summary of the charge transport gap Δ_c and the thermal activation energy Δ_s for the nuclear spin relaxation is displayed in Fig. 4(b). For the sake of completeness, we point out that the linear temperature dependence observed at $\nu = 0.85$ down to 45 mK is consistent with previous resistively detected NMR data by Tracy *et al.*[13] Lower temperatures were not accessible in their nuclear magnetic resonance (NMR)-based experiment. Resistively detected NMR data in Ref. [12] suggested, however, an opposite behavior, namely increasing nuclear spin relaxation rate with decreasing temperature.

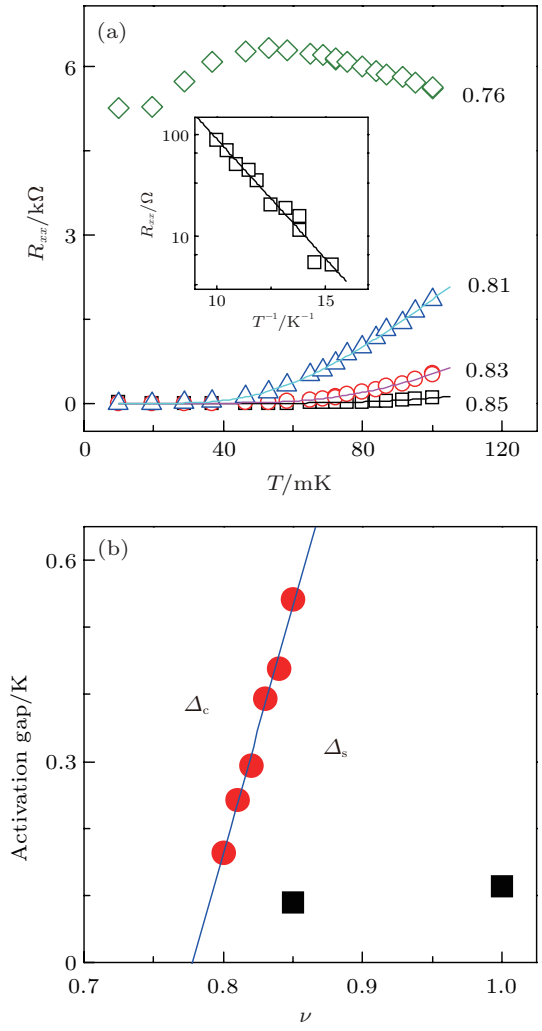


Fig. 4. (color online) (a) Temperature dependence of R_{xx} at $\nu = 0.76$ (diamonds), 0.81 (triangles), 0.83 (circles), and 0.85 (squares) and their fits to the Arrhenius law (lines) at $\nu = 0.81$–0.85. The inset shows the $\log R_{xx}$ versus $1/T$ plot at $\nu = 0.85$. (b) Thermal activation gaps Δ_c (circles) and Δ_s (squares), at different filling factors, extracted from the Arrhenius fits of the T-dependences of R_{xx} and $1/T_1$, respectively.

Côté *et al.*[23] have argued that by moving away sufficiently from $\nu = 1$ skyrmions may form a crystalline lattice, and this lattice possesses a gapless Goldstone magnon mode that couples strongly to the nuclear spins because it is gapless. It provides a channel for rapid relaxation of nuclear spins. It would give rise to a strongly enhanced relaxation rate with a Korringa-like, i.e. linear T-dependence of $1/T_1$. Even when the skyrmionic crystal melts into a liquid state, the Goldstone mode would presumably still be present as an overdamped mode. While the linear temperature dependence above 45 mK observed here and in Ref. [13] is consistent with this picture, the strongly suppressed relaxation at lower temperature is not. It should be noted that disorder is absent in the considerations of Ref. [23]. Moreover, Bayot *et al.* observed a huge anomalous heat capacity peak at $T \approx 40$ mK in a multiple-quantum-well sample.[43,44] They attributed it to the liquid-to-solid transition of the Skyrme system near $T = 40$ mK. Within the model of Ref. [23], the suppressed spin relaxation observed in this work, however, does not favor a transition from a liquid to a crystal as T drops below 40 mK.

Even though the microscopic picture presented above can capture the main features of our experimental observations, it is still at a crude, qualitative level. Theoretical effort must be exerted on low energy spin excitations emerging from disordered skyrmion systems in order to fully understand the related spin phenomena. Unfortunately, theories based on Hartree–Fock calculations[28,29] or more rigorous quantum mechanical treatments[30,31] have not considered the filling factor and temperature dependences of $1/T_1$. Nevertheless, these theories and a recent experiment on charge transport near $\nu = 1$[45] point to different ground states (e.g. quantum Hall spin glass) other than the simple quantum Hall ferromagnet when disorder is sufficiently strong in comparison to the electron–electron interactions.

4. Conclusion

In summary, we have shown that the nuclear spin relaxation measurements can be extended to temperatures well below 40 mK by using the electrical methods for agitating and probing nuclear spins. The nuclear spin relaxation is found to be enhanced by disorder and exhibits thermally activated behavior. The energy gaps extracted from the nuclear spin relaxation measurements are 1–2 orders of magnitude smaller than those from the charge transport. We conclude that none of the existing models can quantitatively describe the temperature dependence and the filling factor dependence of the nuclear spin relaxation rate observed in this work. These features point to a nontrivial ground state and low energy spin excitations near filling factor $\nu = 1$.

Acknowledgment

We are grateful to Shahal Ilani, Wan Xin, Yang Kun and Zhang Guang-Ming for valuable discussions. We acknowledge technical assistance from Yang Wen-Min.

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