Analysis of field coupling to transmission lines with random rotation over the ground*

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(Received 2 December 2014; revised manuscript received 22 January 2015; published online 10 April 2015)

In this paper we analyze plane wave coupling to transmission lines rotating randomly over an infinite and perfectly conducting ground and present an efficient method to calculate average voltage. Under the assumption of small rotation quantity, the factors affecting the induced voltage and their effects are analyzed and then an efficient method to calculate the average voltage is presented when the distribution of the random rotation angles is uniform in $[-\pi, \pi]$. The results show that voltage variation is mainly due to the change of the source term. The effects of the source term increase linearly with the magnitude of the incident wave, change periodically with the rotation angle, and are larger in the high frequency range than in the low frequency range. The results show that the average voltages obtained by the proposed method agree well with those via the Monte Carlo method and the proposed method is much more efficient. The results also imply that the effect of random rotation is more important than that of random translation.

Keywords: electromagnetic interference, induced voltage, randomness, statistical analysis, transmission line

PACS: 05.10.–a, 41.20.–q, 84.40.Az **DOI:** 10.1088/1674-1056/24/6/060501

1. Introduction

Transmission lines, such as power lines, control lines, and data lines, are the most important coupling path through which an external electromagnetic pulse (EMP) interacts with electronic systems.[1,2] Transmission lines have been extensively investigated.[1–15] Most of them focused on transmission lines with exact positions. However, many transmission lines are hard to fix exactly and have randomness in real life. The induced interferences generated by external fields on these random transmission lines are random, too. Consequently, the analysis of fields coupling to a random transmission line should be studied to help the evaluation of the effects on electronic systems induced by external fields.

The Monte Carlo (MC) method combined with transmission line theory or numerical simulation has been used to obtain the statistical results of electromagnetic fields coupling with transmission lines,[13–16] where the MC method was employed to model random transmission lines and then transmission line theory or numerical simulation was used to calculate the interferences on each sample. However, this method is not efficient. Because nonuniform transmission lines can be approximated as a series of cascaded segments of a uniform transmission line, the study of a uniform transmission line with random movement is the basis and important. A plane wave coupled multiconductor transmission line with random translation over ground has been analyzed in Ref. [17] where an efficient method was proposed for evaluating the average induced voltages and the probability distribution of the voltage variation. However, multiconductor transmission lines with random rotation have not been studied yet.

In this paper, a plane wave coupled transmission line with random rotation over an infinite and perfectly conducting ground is studied. Based on the analysis of the influence factors of the random voltages generated by the plane wave, a simple and efficient calculation method is proposed for the average induced voltages. The results are in good agreement with those via the MC method but the proposed method is more efficient. The study also implies that the effect of random rotation is important and should be considered compared with that of random translation.

2. Influence factors for included voltages

For a uniform transmission line rotating randomly over an infinite and perfectly conducting ground under the excitation of an incident electromagnetic field as shown in Fig. 1, the transmission line equation can be written from Agrawal's model as[1,3]

$$\begin{aligned} &\frac{\mathrm{d}\boldsymbol{V}^{\mathrm{sca}}(\alpha,z)}{\mathrm{d}z}+\mathrm{j}\omega\boldsymbol{L}(\alpha)\boldsymbol{I}(\alpha,z)=\boldsymbol{V}_{\mathrm{F2}}(\alpha,z),\\ &\frac{\mathrm{d}\boldsymbol{I}(\alpha,z)}{\mathrm{d}z}+\mathrm{j}\omega\boldsymbol{C}(\alpha)\boldsymbol{V}^{\mathrm{sca}}(\alpha,z)=\boldsymbol{0}. \end{aligned} \tag{1}$$

Here α is the rotation angle of the line shown in Fig. 1. $\boldsymbol{V}^{\mathrm{sca}}$ and $\boldsymbol{I}$ are the scattered line voltage vector and the line current vector, respectively. $\boldsymbol{L}$ and $\boldsymbol{C}$ are the per-unit-length (p.u.l.) inductance and capacitance matrices of the line. $\boldsymbol{V}_{\mathrm{F2}}$ is a distributed voltage source vector which denotes the effect of the

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61231003 and 61201090).

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external fields along the line. The boundary condition is given by

$$\begin{aligned}\boldsymbol{V}^{\rm sca}(\alpha,0) &= -\boldsymbol{Z}_{\rm S}\boldsymbol{I}(\alpha,0)-\boldsymbol{V}^{\rm ex}(\alpha,0),\\ \boldsymbol{V}^{\rm sca}(\alpha,l) &= \boldsymbol{Z}_{\rm L}\boldsymbol{I}(\alpha,l)-\boldsymbol{V}^{\rm ex}(\alpha,l),\end{aligned} \tag{2}$$

where $Z_{\rm S}$ and $\boldsymbol{Z}_{\rm L}$ are the impedance matrices at the near and far ends, $\boldsymbol{V}^{\rm ex}$ is the lumped voltage source at the line end due to the excitation field.[3] The diagram for the Agrawal model of a transmission line is shown in Fig. 2.

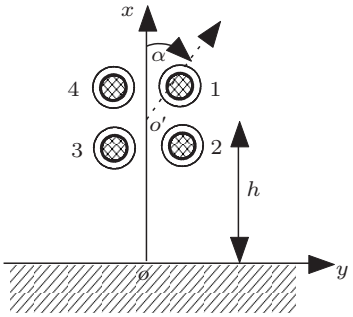


Fig. 1. A multiconductor transmission line over the ground, excited by an incident field.

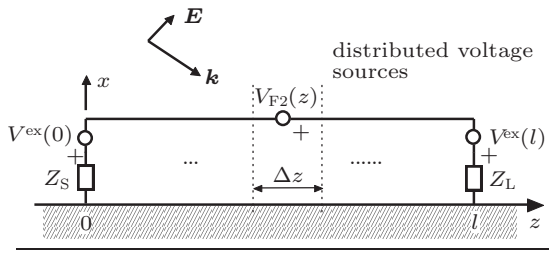


Fig. 2. Diagram for the Agrawal model of field coupling with transmission lines.

When the transmission line rotates randomly over the ground, the matrices $\boldsymbol{L}$, $\boldsymbol{C}$, and the voltage source vector $\boldsymbol{V}_{\rm F2}$ change randomly, which leads to the random voltage and current vectors.

The voltage vector $\boldsymbol{V}(\alpha,z)$ is continuous and differentiable. Thus according to Ref. [17], when the rotation angle α is a small quantity, then the averages of the induced voltages and currents can be approximated by

$$\begin{aligned}\bar{\boldsymbol{V}}(\alpha) &\approx \boldsymbol{V}(\bar{\alpha}) \equiv \widehat{\boldsymbol{V}},\\ \bar{\boldsymbol{I}}(\alpha) &\approx \boldsymbol{I}(\bar{\alpha}) \equiv \widehat{\boldsymbol{I}},\end{aligned} \tag{3}$$

where the symbol "$\widehat{\boldsymbol{V}}$" denotes the value at the average rotation angle $\bar{\alpha}$. $\widehat{\boldsymbol{V}}$ and $\widehat{\boldsymbol{I}}$ conform to the equation and the boundary condition as

$$\begin{aligned}\frac{{\rm d}\widehat{\boldsymbol{V}}^{\rm sca}(z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{L}}\widehat{\boldsymbol{I}}(z) &= \widehat{\boldsymbol{V}}_{\rm F2}(z),\\ \frac{{\rm d}\widehat{\boldsymbol{I}}(z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{C}}\widehat{\boldsymbol{V}}^{\rm sca}(z) &= \boldsymbol{0},\end{aligned} \tag{4}$$

and

$$\begin{aligned}\widehat{\boldsymbol{V}}^{\rm sca}(0) &= -\boldsymbol{Z}_{\boldsymbol{S}}\widehat{\boldsymbol{I}}(0)-\widehat{\boldsymbol{V}}^{\rm ex}(0),\\ \widehat{\boldsymbol{V}}^{\rm sca}(l) &= \boldsymbol{Z}_{\boldsymbol{L}}\widehat{\boldsymbol{I}}(l)-\widehat{\boldsymbol{V}}^{\rm ex}(l).\end{aligned} \tag{5}$$

As a result, the voltage variation and current variation can be written as

$$\begin{aligned}\Delta\boldsymbol{V}^{\rm sca}(\alpha,z) &\equiv \boldsymbol{V}^{\rm sca}(\alpha,z)-\bar{\boldsymbol{V}}^{\rm sca}(z) \approx \boldsymbol{V}^{\rm sca}(\alpha,z)-\widehat{\boldsymbol{V}}^{\rm sca}(z),\\ \Delta\boldsymbol{I}(\alpha,z) &\equiv \boldsymbol{I}(\alpha,z)-\bar{\boldsymbol{I}}(z) \approx \boldsymbol{I}(\alpha,z)-\widehat{\boldsymbol{I}}(z).\end{aligned} \tag{6}$$

Substituting $\widehat{\boldsymbol{V}}^{\rm sca}(z)$ and $\widehat{\boldsymbol{I}}(z)$ into Eq. (1), subtracting the result from Eq. (1), and ignoring the second-order small quantities, the voltage variation and current variation satisfy the equation and the boundary condition as

$$\begin{aligned}\frac{{\rm d}\Delta\boldsymbol{V}^{\rm sca}(\alpha,z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{L}}\Delta\boldsymbol{I}(\alpha,z) &= \Delta\boldsymbol{V}_{\rm F2}(\alpha,z)-{\rm j}\omega\Delta\boldsymbol{L}\widehat{\boldsymbol{I}}(z),\\ \frac{{\rm d}\Delta\boldsymbol{I}(\alpha,z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{C}}\Delta\boldsymbol{V}^{\rm sca}(\alpha,z) &= -{\rm j}\omega\Delta\boldsymbol{C}\widehat{\boldsymbol{V}}^{\rm sca}(z),\end{aligned} \tag{7}$$

and

$$\begin{aligned}\Delta\boldsymbol{V}^{\rm sca}(\alpha,0) &= -\boldsymbol{Z}_{\boldsymbol{S}}\Delta\boldsymbol{I}(\alpha,0)-\Delta\boldsymbol{V}^{\rm ex}(\alpha,0),\\ \Delta\boldsymbol{V}^{\rm sca}(\alpha,l) &= \boldsymbol{Z}_{\boldsymbol{L}}\Delta\boldsymbol{I}(\alpha,l)-\Delta\boldsymbol{V}^{\rm ex}(\alpha,l).\end{aligned} \tag{8}$$

When the transmission line rotates on the ground, with a plane wave coupled, there are two factors that affect the induced voltages. One is the changes of the line parameters, such as p.u.l. inductance and capacitance matrices, and the other is the change of the source term $\boldsymbol{V}_{\rm F2}$. As a result, the voltage and current variations $\Delta\boldsymbol{V}^{\rm sca}$ and $\Delta\boldsymbol{I}$ are composed of two components , $\Delta\boldsymbol{I}_{\rm p}$ and $\Delta{\rm V}_{\rm ex}^{\rm sca}$, $\Delta\boldsymbol{I}_{\rm ex}$, which denote the voltage and current variations due to the changes of line parameters and source term, respectively. According to Eqs. (7) and (8), $\Delta\boldsymbol{V}_{\rm p}^{\rm sca}$, $\Delta\boldsymbol{I}_{\rm p}$ and $\Delta\boldsymbol{V}_{\rm ex}^{\rm sca}$, $\Delta\boldsymbol{I}_{\rm ex}$ satisfy the following equations:

$$\begin{aligned}\frac{{\rm d}\Delta\boldsymbol{V}_{\rm p}^{\rm sca}(\alpha,z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{L}}\Delta\boldsymbol{I}_{\rm p}(\alpha,z) &= -{\rm j}\omega\Delta\boldsymbol{L}\widehat{\boldsymbol{I}}(z),\\ \frac{{\rm d}\Delta\boldsymbol{I}_{\rm p}(\alpha,z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{C}}\Delta\boldsymbol{V}_{\rm p}^{\rm sca}(\alpha,z) &= -{\rm j}\omega\Delta\boldsymbol{C}\widehat{\boldsymbol{V}}^{\rm sca}(z),\end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{{\rm d}\Delta\boldsymbol{V}_{\rm ex}^{\rm sca}(\alpha,z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{L}}\Delta\boldsymbol{I}_{\rm ex}(\alpha,z) &= \Delta\boldsymbol{V}_{\rm F2}(\alpha,z),\\ \frac{{\rm d}\Delta\boldsymbol{I}_{\rm ex}(\alpha,z)}{{\rm d}z}+{\rm j}\omega\widehat{\boldsymbol{C}}\Delta\boldsymbol{V}_{\rm ex}^{\rm sca}(\alpha,z) &= \boldsymbol{0},\end{aligned} \tag{10}$$

respectively. The corresponding boundary conditions for , $\Delta\boldsymbol{I}_{\rm p}$ and $\Delta{\rm V}_{\rm ex}^{\rm sca}$, $\Delta\boldsymbol{I}_{\rm ex}$ are described by

$$\begin{aligned}\Delta\boldsymbol{V}_{\rm p}^{\rm sca}(\alpha,0) &= -\boldsymbol{Z}_1\Delta\boldsymbol{I}_{\rm p}(\alpha,0),\\ \Delta\boldsymbol{V}_{\rm p}^{\rm sca}(\alpha,l) &= \boldsymbol{Z}_2\Delta\boldsymbol{I}_{\rm p}(\alpha,l),\end{aligned} \tag{11}$$

and

$$\begin{aligned}\Delta\boldsymbol{V}_{\rm ex}^{\rm sca}(\alpha,0) &= -\boldsymbol{Z}_{\rm S}\Delta\boldsymbol{I}_{\rm ex}(\alpha,0)-\Delta\boldsymbol{V}^{\rm ex}(\alpha,0),\\ \Delta\boldsymbol{V}_{\rm ex}^{\rm sca}(\alpha,l) &= \boldsymbol{Z}_{\rm L}\Delta\boldsymbol{I}_{\rm ex}(\alpha,l)-\Delta\boldsymbol{V}^{\rm ex}(\alpha,l).\end{aligned} \tag{12}$$

3. Analysis of induced voltage variation

As described above, there are two factors affecting the induced voltage and current. The effects of these two factors are analyzed below.

Comparing Eq. (9) with the transmission line equation for the inner transmission line system of shielded cable in Ref. [18], it can be concluded that equation (9) is similar to the transmission line equation of the inner system of shielded cable. Thus $\Delta \boldsymbol{V}_{\rm p}^{\rm sca}$ and $\Delta \boldsymbol{I}_{\rm p}$ have the same relationship between $\widehat{\boldsymbol{V}}^{\rm sca}(z)$ and $\widehat{\boldsymbol{I}}(z)$ as that between the voltage $V_{\rm in}$ and current $I_{\rm in}$ of the inner system and the voltage $V_{\rm out}$ and current $I_{\rm out}$ of the outer system of shielded cable. $\Delta \boldsymbol{L}$ and $\Delta \boldsymbol{C}$ are equivalent to the transfer inductance $L_{\rm t}$ and capacitance $C_{\rm t}$ of shielded cable. The results in Ref. [18] show that the voltage $V_{\rm in}$ and current $I_{\rm in}$ can be about four and three orders of magnitude smaller than the voltage $V_{\rm out}$ and current $I_{\rm out}$ in the low and high frequency ranges, respectively. The transfer inductance $L_{\rm t}$ and capacitance $C_{\rm t}$ are about 1 nH and 0.1 pF, respectively. The inductance variation $\Delta \boldsymbol{L}$ has the same order of magnitude as $L_{\rm t}$, and the capacitance variation $\Delta \boldsymbol{C}$ is one order of magnitude smaller than $C_{\rm t}$ when the rotation quantity $\delta\alpha$ is 0.0873 rad, which will be illustrated in the numerical validation in Section 5. As a result, the voltage variation $\Delta \boldsymbol{V}_{\rm p}^{\rm sca}$ due to the parameter change being about four and three orders of magnitude smaller than $\widehat{\boldsymbol{V}}^{\rm sca}$ in the low frequency range (below 1 MHz) and in the high frequency range (1 MHz to 1GHz), respectively. Thus the values of $|[\Delta \boldsymbol{V}_{\rm p}^{\rm sca}]_i/([\widehat{V}^{\rm sca}]_i \cdot \delta\alpha)|$ are smaller than 0.001 and 0.01 in the low and high frequency ranges, respectively.

Comparing Eq. (10) with Eq. (4), equation (10) has the same form as Eq. (4), except the variables and the source term. When the incident wave is a plane wave, the source terms for $\widehat{\boldsymbol{V}}(z)$ and $\widehat{\boldsymbol{I}}(z)$ can be derived from Ref. [2] and are given by

$$\begin{aligned}
[\widehat{\boldsymbol{V}}_{\rm F2}(z)]_i &= -2{\rm j}\hat{E}_0 e_z \sin(\beta_x \widehat{x}_i)\, {\rm e}^{-{\rm j}\beta_y \widehat{y}_i}\, {\rm e}^{-{\rm j}\beta_z z},\\
[\widehat{\boldsymbol{V}}^{\rm ex}(0)]_i &\approx -2\hat{E}_0 e_x \frac{\sin(\beta_x \widehat{x}_i)}{\beta_x}\, {\rm e}^{-{\rm j}\beta_y \widehat{y}_i},\\
[\widehat{\boldsymbol{V}}^{\rm ex}(l)]_i &\approx -2\hat{E}_0 e_x \frac{\sin(\beta_x \widehat{x}_i)}{\beta_x}\, {\rm e}^{-{\rm j}\beta_y \widehat{y}_i}\, {\rm e}^{-{\rm j}\beta_z l}.
\end{aligned} \tag{13}$$

When the rotation quantity $\delta\alpha$ is a small quantity, the source terms for $\Delta \boldsymbol{V}_{\rm ex}$ and $\Delta \boldsymbol{I}_{\rm ex}$ can be written as

$$\begin{aligned}
[\Delta \boldsymbol{V}_{\rm F2}(z)]_i &\approx [\widehat{\boldsymbol{V}}_{\rm F2}(z)]_i \cdot \Delta \boldsymbol{B}_i,\\
[\Delta \boldsymbol{V}^{\rm ex}(0)]_i &\approx [\widehat{\boldsymbol{V}}^{\rm ex}(0)]_i \cdot \Delta \boldsymbol{B}_i,\\
[\Delta \boldsymbol{V}^{\rm ex}(l)]_i &\approx [\widehat{\boldsymbol{V}}^{\rm ex}(l)]_i \cdot \Delta \boldsymbol{B}_i,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\Delta \boldsymbol{B}_i &= -[{\rm j}\beta_y(\widehat{x}_i - h) + \beta_x \widehat{y}_i c\tan(\beta_x \widehat{x}_i)] \cdot \delta\alpha\\
&\approx -[{\rm j}\beta_y(\widehat{x}_i - h) + \widehat{y}_i/\widehat{x}_i] \cdot \delta\alpha.
\end{aligned} \tag{15}$$

It can be derived from Eq. (15) that

$$|[\Delta \boldsymbol{V}_{\rm ex}^{\rm sca}]_i/([\widehat{V}^{\rm sca}]_i \cdot \delta\alpha)| \approx |{\rm j}\beta_y(\widehat{x}_i - h) + \widehat{y}_i/\widehat{x}_i| \tag{16}$$

is much larger than $|[\Delta \boldsymbol{V}_{\rm p}^{\rm sca}]_i/([\widehat{V}^{\rm sca}]_i \cdot \delta\alpha)|$. This means that the source term has a larger influence than the parameters.

To indicate the influence of the random rotation, we define the sensitivity and relative sensitivity of the induced voltage to the rotation angle as $S_\alpha = \lim\limits_{\delta\alpha\to 0} \Delta V/\delta\alpha$ and $S_{{\rm r}\alpha} = S_\alpha/\bar{V}$, respectively. Then the voltage variation due to random rotation can be expressed as $\Delta \boldsymbol{V} \approx \boldsymbol{S}_\alpha \cdot \delta\alpha$ or $[\Delta \boldsymbol{V}]_i \approx [\boldsymbol{S}_\alpha]_i \cdot [\bar{\boldsymbol{V}}]_i \cdot \delta\alpha$. The components of the S_α and $S_{{\rm r}\alpha}$ are denoted by $S_{\alpha{\rm p}}$, $S_{{\rm r}\alpha{\rm p}}$ and $S_{\alpha{\rm ex}}$, $S_{{\rm r}\alpha{\rm ex}}$, which denote the influences of the line parameter and the source term, respectively. As discussed above, $S_{\alpha{\rm ex}}$ and $S_{{\rm r}\alpha{\rm ex}}$ are much larger than $S_{\alpha{\rm p}}$ and $S_{{\rm r}\alpha{\rm p}}$. With Eqs. (7), (9), (10), and the definitions of the sensitivity and relative sensitivity, the sensitivity, the relative sensitivity, and their components can be calculated.

4. Efficient evaluation for average induced voltage

When the rotation quantity $\delta\alpha$ is a small quantity, the average voltage can be computed through Eqs. (4) and (5) by using the Baum–Liu–Tesche (BLT) equation [1] or the SPICE model.[2] However, in reality it is very common that the rotation quantity may be large. The wire bundle rotating in the interval $[-\pi, \pi]$ uniformly is the most common case. Because the effect of the parameter change is much weaker than that of the excitation field change and the voltage variation is mainly due to the change of the excitation field no matter what the angle α is, the effect of the parameter change can be ignored and the average voltage and current, when the wire rotates uniformly in the interval $[-\pi, \pi]$, can be approximately solved by

$$\begin{aligned}
&\frac{{\rm d}\bar{\boldsymbol{V}}^{\rm sca}(z)}{{\rm d}z} + {\rm j}\omega \bar{\boldsymbol{L}} \bar{\boldsymbol{I}}(z) = \bar{\boldsymbol{V}}_{\rm F2}(z),\\
&\frac{{\rm d}\bar{\boldsymbol{I}}(z)}{{\rm d}z} + {\rm j}\omega \bar{\boldsymbol{C}} \bar{V}^{\rm sca}(z) = \boldsymbol{0},
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\bar{\boldsymbol{V}}^{\rm sca}(0) &= -\boldsymbol{Z}_{\boldsymbol{S}} \bar{\boldsymbol{I}}(0) - \bar{\boldsymbol{V}}^{\rm ex}(0),\\
\bar{\boldsymbol{V}}^{\rm sca}(l) &= \boldsymbol{Z}_{\boldsymbol{L}} \bar{\boldsymbol{I}}(l) - \bar{\boldsymbol{V}}^{\rm ex}(l).
\end{aligned} \tag{18}$$

Here, $\bar{\boldsymbol{L}}$ and $\bar{\boldsymbol{C}}$ are the inductance and capacitance matrix computed by using the average parameter, that is, the average position of the wire, and are given by

$$\begin{aligned}
&\bar{\boldsymbol{L}} = \mu \bar{\boldsymbol{F}}, \quad \bar{\boldsymbol{C}} = \varepsilon \bar{\boldsymbol{F}}^{-1},\\
&[\bar{\boldsymbol{F}}]_{ii} = \ln\left(\frac{2h}{a_i}\right),\\
&[\bar{\boldsymbol{F}}]_{ij} = \frac{1}{2}\ln\left(1 + \frac{4h^2}{{\rm d}_{ij}^2}\right),
\end{aligned} \tag{19}$$

where μ and ε are the permeability and permittivity of the medium around the wire, h is the height of the line, as shown in Fig. 1, a_i is the radius of the i-th wire, and d_{ij} is the distance between the i-th wire and the j-th wire. $\bar{V}_{\mathrm{F2}}(z)$, $\bar{V}^{\mathrm{ex}}(0)$, and $\bar{V}^{\mathrm{ex}}(l)$ are the average source terms when the angle α distributes uniformly in the interval $[-\pi, \pi]$ and they can be approximated by

$$
\begin{aligned}
[\widehat{V}_{\mathrm{F2}}(z)]_i &= -2\mathrm{j}\hat{E}_0 e_z \sin(\beta_x h)\,\mathrm{e}^{-\mathrm{j}\beta_z z},\\
[\widehat{V}^{\mathrm{ex}}(0)]_i &\approx -2\hat{E}_0 e_x \frac{\sin(\beta_x h)}{\beta_x},\\
[\widehat{V}^{\mathrm{ex}}(l)]_i &\approx -2\hat{E}_0 e_x \frac{\sin(\beta_x h)}{\beta_x}\,\mathrm{e}^{-\mathrm{j}\beta_z l},
\end{aligned}
\tag{20}
$$

when $\beta r_i \ll 1$.

5. Numerical validation and discussion

Figure 1 shows a four-wire transmission line with a length of 2 m over a perfectly and infinitely conducting ground. The distance r_0 between the wire and the center o' is 2 mm. The height h of the center o' over the ground is 15 mm. The radii of the wires are all 0.68 mm. The initial angles between the x axis and the wire 1, wire 2, wire 3, and wire 4 are $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$, respectively. The near-end impedances Z_{S1}, Z_{S2}, Z_{S3}, and Z_{S4} of the wires are 50 Ω, 60 Ω, 80 Ω, and 100 Ω, respectively. Their far-end impedances Z_{L1}, Z_{L2}, Z_{L3}, and Z_{L4} have the same values as Z_{S1}, Z_{S2}, Z_{S3}, and Z_{S4}, respectively. The transmission line is 2-m long. The incident field is a sine wave with unit magnitude and a frequency of 100 MHz. The incident and polarization angles are $\theta_{\mathrm{p}} = \pi/4$, $\phi_{\mathrm{p}} = -\pi/3$, and $\theta_E = \pi/2$, where the definition of the incident and polarization angles can be found in Fig. 3.

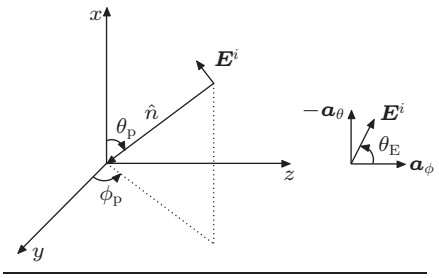


Fig. 3. Definition of the incident and polarization angles.

To illustrate the weight of the effects of changes in the parameters and the source term on voltage variation, the variations of the inductance and capacitance are given first and then the values of $S_{\alpha\mathrm{p}}$ and $S_{\alpha\mathrm{ex}}$ are given.

When the rotation quantity $\delta\alpha$ is 0.0873 rad (or 5°), the $\boldsymbol{L}$ and $\boldsymbol{C}$ matrices at the new position can be computed by using the line parameter. Then $\Delta\boldsymbol{L}$ and $\Delta\boldsymbol{C}$ can be obtained by subtracting the old values of the $\boldsymbol{L}$ and $\boldsymbol{C}$ matrices from the new ones, and they are given by

$$
\begin{aligned}
\Delta\boldsymbol{L} &= \begin{bmatrix} -1.574 & -1.643 & 0.153 & -0.071 \\ -1.643 & -1.743 & 0.070 & -0.153 \\ 0.153 & 0.070 & 1.885 & 1.643 \\ -0.071 & -0.153 & 1.643 & 1.431 \end{bmatrix} \text{ nH},\\
\Delta\boldsymbol{C} &= \begin{bmatrix} 2.077 & 2.353 & -0.107 & 0.083 \\ 2.353 & 3.181 & -0.127 & 0.107 \\ -0.107 & -0.127 & -3.394 & -2.310 \\ 0.083 & 0.107 & -2.310 & -1.861 \end{bmatrix} \times 10^{-14} \text{ F}.
\end{aligned}
\tag{21}
$$

$\Delta\boldsymbol{L}$ has the same order of magnitude as the transfer inductance L_{t} of shielded cable, and $\Delta\boldsymbol{C}$ is one order of magnitude smaller than the transfer capacitance C_{t} of shielded cable. As discussed above, the values of $|[\Delta\boldsymbol{V}_{\mathrm{p}}^{\mathrm{sca}}]_i/([\widehat{\boldsymbol{V}}^{\mathrm{sca}}]_i \cdot \delta\alpha)|$ are smaller than 0.001 and 0.01 in the low frequency range (below 1 MHz) and the high frequency range (1 MHz to 1 GHz), respectively. According to Eq. (16), $|[\Delta\boldsymbol{V}_{\mathrm{ex}}^{\mathrm{sca}}]_i/([\widehat{\boldsymbol{V}}^{\mathrm{sca}}]_i \cdot \delta\alpha)|$ equals 0.09 approximately. Thus $S_{\alpha\mathrm{ex}}$ is much larger than $S_{\alpha\mathrm{p}}$. From Eqs. (9)–(12) and the definition of the sensitivity, S_α, $S_{\alpha\mathrm{p}}$, and $S_{\alpha ex}$ can be calculated. The voltage sensitivities and their components for the loads Z_{S1}, Z_{S2}, Z_{L1}, and Z_{L2} at the average height are given in Table 1. It can be seen from the results that $S_{\alpha\mathrm{ex}}$ is much larger than $S_{\alpha\mathrm{p}}$ and $S_{\alpha\mathrm{ex}}$ has the opposite polarity to $S_{\alpha\mathrm{p}}$, which implies that the effects of the source term and the parameter are opposite.

Table 1. Sensitivities obtained by the proposed method.

	$S_{\alpha\mathrm{p}}$/(mV/rad)	$S_{\alpha\mathrm{ex}}$/(mV/rad)	S_α/(mV/rad)
Z_{S1}	–0.031–0.064i	0.236+0.358i	0.205+0.295i
Z_{S2}	–0.028-0.096i	0.206+0.429i	0.177+0.334i
Z_{S3}	0.025+0.135i	–0.320–0.623i	–0.295–0.488i
Z_{S4}	0.016+0.121i	–0.251–0.694i	–0.235–0.573i
Z_{L1}	–0.032–0.080i	0.243+0.456i	0.210+0.376i
Z_{L2}	–0.032–0.121i	0.215+0.546i	0.183+0.425i
Z_{L3}	0.030+0.168i	–0.332–0.783i	–0.301–0.614i
Z_{L4}	0.022+0.151i	–0.277–0.731i	–0.256–0.731i

To validate that the effect of the source term plays a main role, the sensitivity, the relative sensitivity, and their components versus the frequency, the magnitude of incident wave, and the rotation angle are also studied.

Figure 4 shows the sensitivity, the relative sensitivity, and their components versus the magnitude of the incident field. The results imply that the sensitivity is linearly and directly proportional to the magnitude of the incident field but the relative sensitivity does not depend on the magnitude of the incident field and the dominance of $S_{\alpha\mathrm{ex}}$ does not change with the magnitude of the incident wave. It can also be concluded that larger voltage variation will be observed if the external field is larger but the influence does not change with external field because the induced voltage also increases.

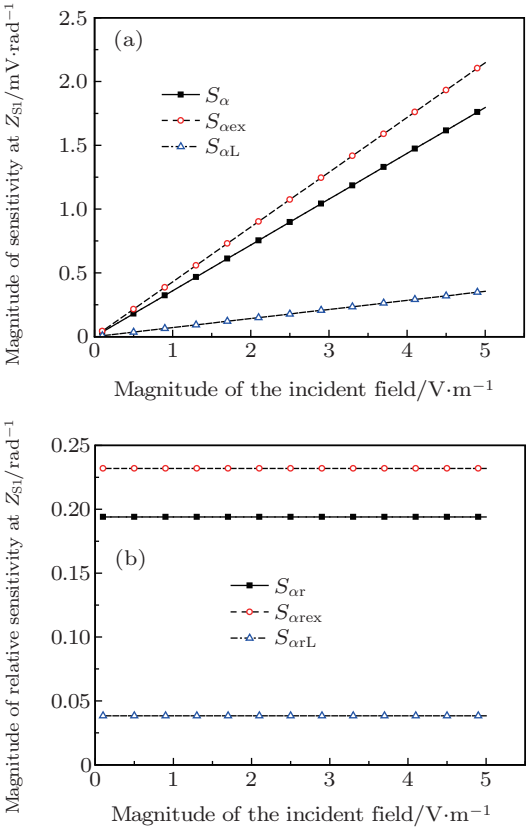


Fig. 4. (color online) Variations of (a) sensitivity and (b) relative sensitivity at the load Z_{S1} with the magnitude of the incident field.

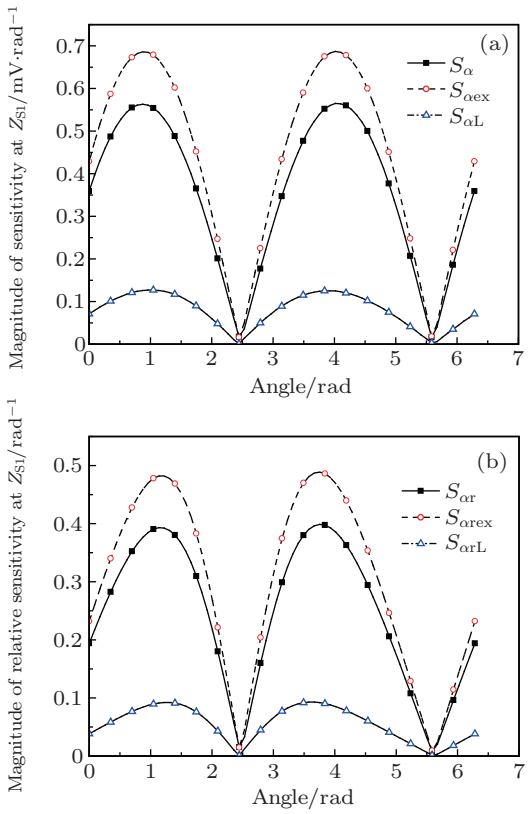


Fig. 5. (color online) Variations of (a) sensitivity and (b) relative sensitivity at the load Z_{S1} with rotation angle.

Figure 5 shows the changes of the sensitivity and the relative sensitivity at the load Z_{S1} with rotation angle. The results show that the sensitivity, the relative sensitivity, and their components change with the rotation angle periodically and they have the same trends. The results also show that the dominance of $S_{\alpha ex}$ does not change with the rotation angle.

Figures 6(a) and 6(b) show the variations of the sensitivity and the relative sensitivity at the load Z_{S1} with frequency. Results show that $S_{\alpha p}$ and $S_{\alpha rp}$ are far smaller than $S_{\alpha ex}$ and $S_{\alpha rex}$ in the low frequency range, respectively, but the distinction between them becomes small in the high frequency range. The results also show that $S_{\alpha ex}$ and $S_{\alpha rex}$ dominate in the frequency range under study.

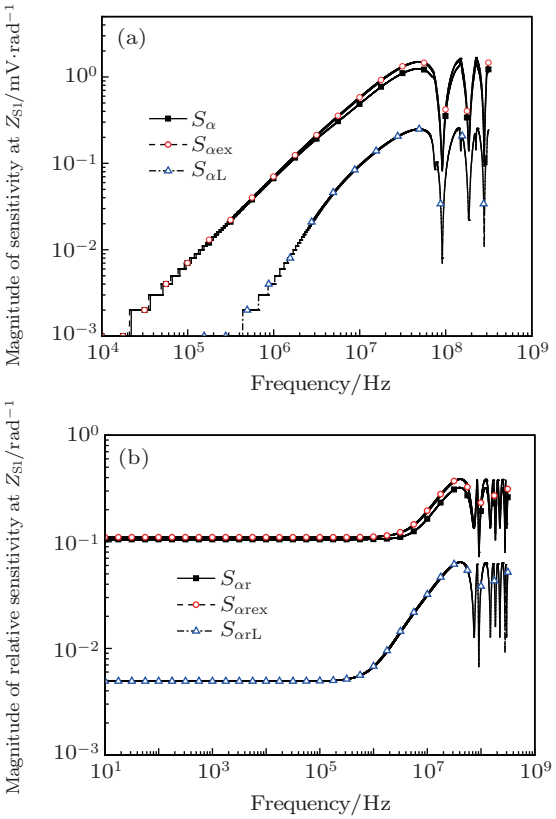


Fig. 6. (color online) Variations of (a) sensitivity and (b) relative sensitivity at the load Z_{S1} with frequency.

As illustrated above, the effects of source term dominate in the voltage variation.

When the rotation angles are distributed uniformly in the interval $[-\pi, \pi]$, the average voltages induced by external fields computed via Eqs. (17)–(20) are given in Table 2, which are in good agreement with those obtained by the MC method. In the MC method, the number of samples is 8000 and the BLT equation is adopted to calculate the voltage of each sample. Thus the time taken by the proposed method is about 1/8000 times that taken by the MC method. The average voltages obtained by the proposed method and the MC method, when the distance r_0 between the wire and the center o' is 10 mm, are given in Table 3, where the results of the two methods agree well with each other.

Table 2. Average voltages obtained by the proposed method and the MC method when r_0 is 2 mm.

	Proposed method	Monte Carlo
$V_{Z_{S1}}$/mV	–1.43–0.40i	–1.43–0.39i
$V_{Z_{S2}}$/mV	–1.47–0.52i	–1.47–0.51i
$V_{Z_{S3}}$/mV	–1.59–0.85i	–1.59–0.84i
$V_{Z_{S4}}$/mV	–1.53–1.00i	–1.54–1.00i
$V_{Z_{L1}}$/mV	–1.37–0.58i	–1.37–0.58i
$V_{Z_{L2}}$/mV	–1.41–0.74i	–1.41–0.72i
$V_{Z_{L3}}$/mV	–1.54–1.15i	–1.54–1.13i
$V_{Z_{L4}}$/mV	–1.50–1.34i	–1.51–1.35i

Table 3. Average voltages obtained by the proposed method and the MC method when r_0 is 10 mm.

	Proposed method	Monte Carlo
$V_{Z_{S1}}$/mV	–2.04–0.86i	–2.02–0.81i
$V_{Z_{S2}}$/mV	–2.23–1.14i	–2.20–1.08i
$V_{Z_{S3}}$/mV	–2.55–1.74i	–2.54–1.65i
$V_{Z_{S4}}$/mV	–2.58–2.23i	–2.60–2.16i
$V_{Z_{L1}}$/mV	–1.97–1.17i	–1.94–1.12i
$V_{Z_{L2}}$/mV	–2.15–1.52i	–2.11–1.45i
$V_{Z_{L3}}$/mV	–2.47–2.24i	–2.46–2.14i
$V_{Z_{L4}}$/mV	–2.52–2.86i	–2.54–2.76i

Now, we compare the effects of the random translation and random rotation for this line structure. According to Ref. [17], the relative sensitivity S_{hr}, when this line translates over the ground, is about 66.6 m^{-1} ($\approx 1/h$) and figure 5(b) shows the maximum of $S_{\alpha r}$ is about 0.5 rad^{-1}. The value of S_{hr} is much larger then that of $S_{\alpha r}$. However, it is impossible for a line to translate randomly with large quantity, and the translation quantity is usually small, say, 10 mm. But the line may rotate in its cross section with large angles and the polarity of the voltage may change with the rotation angle as shown in Fig. 5. As a result, the effect of random rotation has a larger effect on the voltage of the line than that of random translation in reality and the influence of the random rotation should be considered.

6. Conclusions

In this study, we analyze a plane wave coupled transmission line rotating randomly over an infinite and perfectly conducting ground and propose an efficient method to calculate the average voltage. The voltage variations induced by the changes of the line parameters and excitation field are investigated. The voltage variation is mainly due to the change of the source term. The results show that the sensitivity of the voltage to the rotation increases linearly with the magnitude of the incident wave and changes periodically with the rotation angle. The results also show that the random rotation has a larger influence on the induced voltages in the high frequency range than in the low frequency range. The average voltages obtained by the proposed method are in good agreement with those obtained by the MC method and the proposed method is much more efficient. The results also show that the effect of random rotation is more important than that of random translation.

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