REVIEW

Low-frequency oscillations in Hall thrusters*

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In this paper, we summarize the research development of low-frequency oscillations in the last few decades. The findings of physical mechanism, characteristics and stabilizing methods of low-frequency oscillations are discussed. It shows that it is unreasonable and incomplete to model an ionization region separately to analyze the physical mechanism of low-frequency oscillations. Electro-dynamics as well as the formation conditions of ionization distribution play an important role in characteristics and stabilizing of low-frequency oscillations. Understanding the physical mechanism and characteristics of low- frequency oscillations thoroughly and developing a feasible method stabilizing this instability are still important research subjects.

Keywords: low-frequency oscillations, plasma discharge, Hall thrusters

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1. Introduction

A Hall thruster is an annular device in which a propellant, usually xenon, is ionized and then accelerated by electrostatic force to create a propulsive thrust.[1] Magnetic-confined plasma discharge is the main principle of Hall thrusters. However, the magnetic-confined plasmas could not be in the ideal thermodynamic equilibrium state. This means that these plasmas have comparatively high free energy. Thus, microscopic or macroscopic motion of plasmas coming from a higher-energy state to a lower-energy state exists inherently. Research in the past few years shows that whichever magnetic topology is used, rich and complex wave and noise characteristics over a wide frequency spectrum are inherent in the magnetic-confined plasma discharge. Therefore, It has been a central issue of plasma physics to study all kinds of plasma instability, to clarify the physical mechanism, to understand their characteristics, and to seek the stabilization method.[2–8]

Hall thrusters, as typical magnetized plasma discharge devices, exhibit varied oscillations with different lengths and time scales including many kinds of physical phenomena. The spectrum of plasma instabilities ranges from kHz to GHz and these instabilities play a major role in the process of ionization, diffusion and acceleration of particles. Low frequency discharge current oscillations, also named as breathing mode oscillations in some references, are one of the major research topics of Hall thrusters. Lots of previous research has evidenced the existence of low-frequency oscillations with large amplitudes in the range of 10–100 kHz in Hall thrusters.[9,10] These oscillations might affect the power processing unit, lower the efficiency and specific impulse and shorten the operating life of Hall thrusters, and therefore, its suppression is an essential challenge in the practical applications of Hall thrusters, not only to determine the effects of such an oscillation on the thruster operation, reliability and lifetime but also to control electromagnetic compatibility with other equipment installed on satellites.[11,12] Researchers had paid attention to the low-frequency oscillation phenomena with Hall thrusters' presence since the early 1970's.[13–15] Even now, the study on this subject is still underway. In this paper, we focus on the subject of low-frequency oscillations and attempt to summarize/discuss the progress of study on low-frequency oscillations in recent years.

The organization of the rest of this paper is as follows. In Section 2, the research groups concerning low-frequency oscillations are outlined. Section 3 addresses the research development of the physical mechanism of low-frequency oscillations. In Section 4 we discuss the influencing factors and characteristics of low-frequency oscillations. The research development of low-frequency oscillation stabilizing is explored in Section 5, and then the conclusion and open questions are given in Section 6.

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2. Research group

Internationally, there are many electric propulsion research laboratories, such as the Princeton Plasma Physics Laboratory (PPPL) in USA, the Laboratory on Plasma and Conversion of Energy (LAPLACE) in France, the Komurasaki–Koizumi Laboratory in Japan, the Moscow Institute of Radioengineering in Russia and some other laboratories in Asia and Europe. Different groups focus on different research objectives. For low-frequency oscillations, some work has been done in Russia in the early and mid-1970's, while the main research groups continuing to study this subject are located in USA, France, Poland, Japan, and China. They researched this subject from different angles and helped us to fully understand the low-frequency oscillations in Hall thrusters.

In USA, the researches on low-frequency oscillations are mainly in some universities. The Princeton Plasma Physics Laboratory (PPPL), where N. J. Fisch is the project head and Y. Raitses is the lead physicist, established the Hall Thruster Experiment (HTX) in 1999. Their research objectives include reduced beam divergence, scaling of Hall thrusters, physics involved in Hall thrusters as well as plasma instabilities and their control.[16–19] They studied low-frequency oscillations mainly through experiments and established some excellent plasma diagnostics devices, such as high-speed positioning system for plasma measurements inside Hall Thrusters, a new segmented probe and a movable radial probe.[20–23] Another research group in Princeton is the Electric Propulsion and Plasma Dynamics Lab (EPPDyL), which is presently directed by E. Y. Choueiri. EPPDyL has continuously researched in the physics and applications of plasma thrusters for spacecraft propulsion for more than thirty years and is currently involved in active space experiments. A review on plasma oscillations in Hall thrusters published in "Physics of Plasmas" is their typical oscillation research work.[24] A. D. Gallimore, graduated as a Ph.D. from EPPDyL and founded and directs the Plasma dynamics and Electric Propulsion Laboratory (PEPL) at the University of Michigan. A. Gallimore is one of the world's leading electric propulsion researchers. PEPL has operated all thruster types including electrothermal propulsion, electrostatic thrusters, and electromagnetic thrusters. Early research efforts in the 1990s considered arcjets and MPDs. A majority of the research presently performed in the lab is focused on Hall thrusters and ion thrusters.[25] In addition, the Stanford Plasma Physics Lab headed by M. A. Cappelli and the Space Propulsion Laboratory (SPL) at the Massachusetts Institute of Technology headed by Manuel Martinez-Sanchez are also advancing theoretical and experimental research to understand plasmas in space propulsion including the study of low-frequency oscillations.

In Europe, research groups concerning low-frequency oscillations in Hall thrusters are mainly in France and Poland. In France, Hall thrusters have been studied in the frame of a Research Group on Plasma Propulsion consisting of about 10 academic labs, which are specialized in all the fields of Hall thrusters' physics (plasma physics, optical diagnostics, magnetism, ceramics, numerical simulation, etc.). Laboratory on Plasma and Conversion of Energy (LAPLACE) in the University of Toulouse headed by J. P. Boeuf and the Research Group on Energy of Ionized (GREMI) in the Orléans University headed by A. Bouchoule are two of the most important research groups. These two groups performed a lot of theoretical and experimental studies on discharge instabilities in Hall thrusters ranging from low to medium-high frequencies.[26] S. Barral in the Dynamics of Ionized Media group in the Institute of Plasma Physics and Laser Microfusion in Poland has been one of the greatest activity researchers in Hall thrusters discharge instability for the past few years.[10,27–30] He mainly focused on computer modeling, theoretical study of discharge instabilities, and thruster/power supply interactions. In the modeling and controlling of low-frequency oscillations, S. Barral has done a lot of research and gave some new perspectives in low-frequency oscillations. Moreover, Plasma propulsion equipment especially (EP2) in Spain headed by E. Ahedo, who has also done some theoretical studies on the subject of discharge oscillations in Hall thrusters. EP2 have built research relationships with SPL, PPPL, and Dynamics of Ionized Media group to concern the fundamental physics in Hall thrusters.[28,30–33]

Another research center of low-frequency oscillations in Hall thrusters is in Asia. The works of the Komurasaki–Koizumi Laboratory in the University of Tokyo headed by K. Komurasaki includes model developing and improving, characteristics and suppression of low-frequency oscillations.[9,34,35] The work of Gas Discharge Physics Lab in Korea Advanced Institute of Science and Technology headed by Wonho Choe mainly focus on the cylindrical Hall thrusters and the plasma diagnostics.[36–38] Harbin Institute of Technology Plasma Propulsion Laboratory (HPPL) in China headed by Yu Daren is a new research group, which began to build their experiments facilities and study Hall thrusters in 2004. They have done some exploratory researches on low-frequency oscillations, including the influencing factors, physical mechanism, and stabilizing.[5,12,39–44]

In addition to all of the above, some researches in Japan (Nagoya University, Gifu University, etc.),[45] in Israel (such as in Technion Israel Institute of Technology and Holon Institute of Technology),[46,47] or in USA (such as Air Force Institute of Technology),[48] etc. also perform some research work on the subject of low-frequency oscillations in Hall thrusters.

3. Physical mechanism of low-frequency oscillations

The previous analysis assumed that Hall thrusters essentially operate in a steady-state mode, that is why Hall thrusters are also named stationary plasma thrusters. However, the plasmas in Hall thrusters are not in the steady-state, especially, the low-frequency oscillations in the 10–100 kHz band with a bigger amplitude play a direct role in setting the performance and stability. This oscillation mode was identified as a "loop" or "circuit" oscillation as it is sensitive to the circuit of the system in the 1970's in Refs. [24] and [25]. Researchers proposed numerous models to study the mechanism of low-frequency oscillations. The most representative models are Fife's predator-prey model and Boeuf's breathing type model. Fife *et al.* proposed a two-dimensional numerical model in which the electron temperature is predicted through a detailed electron–insulator interaction model and concluded that the low-frequency oscillations seem to be related to an ionization fluctuation which can be modeled in a predator-prey fashion. The model emphasizes the interaction between neutral replenishment and ionization avalanche, and shows the relationship between neutrals and ions in the course of low-frequency oscillations lively.[50] The frequency characteristics of low-frequency oscillations analyzed by Fife's predator-prey model were also verified through experiments by Cappelli *et al.*[51] A numerical model built by Boeuf and Garrigues explained the oscillation as a movement of the ionization zone.[4] This is caused by the large magnetic field near the exit of a thruster, which causes a large degree of ionization depleting the neutrals in the ionization zone. The ionization zone moves upstream where the smaller magnetic field has a reduced rate of ionization. This reduction then starves the ionization zone of electrons, allowing the ionization zone to move downstream near the exit. Then the cycle repeats itself. The simulations also predict the existence of discharge regimes with large amplitude of low-frequency current oscillations. These oscillations are associated with a "breathing" of the production zone location and the front of neutral atoms.

Due to lack of uniformity in the initial theoretical study, researchers struggle for seeking more meticulous methods based on dynamics of charged particles. Noguchi *et al.* proposed a one-dimensional linear perturbation model and found that the low-frequency oscillations have several characteristics of an ionization cycle.[33] Yamamoto *et al.* studied the low-frequency oscillations in the linear type and anode layer type thruster. To study the oscillation mechanism, the plasma behavior was measured with a high-speed camera in the acceleration channel. The emission intensity and the number density of excited xenon ions oscillates with the same period as the discharge current oscillation, which indicates that low-frequency oscillations are caused by the ionization instability.[52] Further experimental results showed that the amplitude/frequency of low-frequency oscillations is affected by the magnetic field variations greatly but not by the change of electric field. Since there are electric fields at the end of the ionization zone,[53–55] Yamamoto pointed out that the models of Fife[50] and Baranov[56] have shortness and concluded that electron dynamics traveling in the magnetic field pays an important role in low-frequency oscillations. Based on the experiments and theoretical analysis, an expanding model reflecting the interaction between neutral atoms and electrons was built and the unstable operation zone was deduced.[9,34,35,57] Chable *et al.* measured axial profiles of the plasma parameters through laser induced fluorescence methods, which show different characteristics of ionization zone and acceleration zone.[58] After that, they linearized a stationary quasineutral hybrid model and commented that the main reason for low-frequency oscillations is the strong coupling between electric field and ion current. These instabilities also appear in electrons-drift quasineutral plasma and are so-called Buneman's instabilities.[59,60] Yu *et al*. also considered the interaction between the ionization zone and acceleration zone and recognized the limitation of the assumptions that acceleration zone is quasi-neutrality. The classical predator-prey type model was improved and reasonable boundary conditions were given in Ref. [44].

In recent years, Barral *et al*. have been doing a great deal of research to improve the low-frequency oscillation model, analyze its physical mechanism and stabilize theory. They began their study with a one-dimensional (1D) model through the linear-stability analysis of the stationary solutions. The dispersion relation was used to analyze the profiles of different perturbation magnitudes. A time scale separation procedure was applied to a quasi-stationary model, time-dependent terms of neutrals transport were treated individually with electron/ion process. After building the relationship of whole equations with the discharge current growth rate and atom density, the oscillation frequency expression was given through the simplification of short-wave and long-wave solutions.[61,62] Barral *et al.* then used a non-linear model to deduce a simplified model, where only the dynamic of electrons and neutrals were described. The growth rate of electrons was given to clarify the great dependent relationship between ionization and the derivative of electric field. The relationship between electron multiplication ratio and discharge current indicates that the breathing mode oscillations result from the interplay between current instability and the axial motion of ionization front.[63] Through further analysis, the effects of discharge voltage on the amplitude/frequency of low-frequency oscillations were given in Ref. [28]. Along with these, the frequency expressions of a predator-prey cycle and a convective cycle were analyzed. It showed that the

predator-prey cycle gives a better description of the instability mechanism.[64] For further analysis of predator-prey dynamics, the ionization zone was separated into ionization region and avalanche region, then a simple predator-prey model was derived. Theoretical analysis indicated that the low-frequency oscillations result from a delayed feedback between ionization and avalanche electron multiplication, which were causes by the transit of neutrals ionizing upstream of the avalanche region.[27,65]

Given closer analysis, we can see that it is unreasonable and incomplete to model ionization zone separately to analyze the low-frequency oscillation physical mechanism. Actually, the ionization zone and the acceleration zone are coupled to each other. The ions move from ionization zone to acceleration zone and bring the status of ionization zone to the acceleration zone. The electrons come from the acceleration zone into the ionization zone and reflect the influence of the acceleration zone on the ionization zone. It is known that the electric field equation is the electron momentum equation. The electric field reflects the influence of the acceleration zone on the ionization zone. The ion momentum causes the redistribution of electric fields, which influences the movement of ions and electrons, and therefore the ionization process itself. It is considered that the electric field feedback, together with the processes of neutral replenishment and ionization avalanche, lead to low-frequency oscillations of Hall thrusters. In addition, it should be pointed out that the quasi-neutral hypothesis has its limitations in the acceleration zone modeling since the velocity of ions is much higher than that of electrons in the acceleration zone. Therefore, the effects and the non-quasi-neutral characteristics of acceleration zone should be taken into account in the study of low-frequency oscillation mechanism.

4. Influencing factors and characteristics of low-frequency oscillations

Experimental and numerical observations/measurements showed that the amplitude and frequency of low-frequency oscillations are strongly dependent on operating conditions, such as mass flow rate, propellant type, discharge voltage, magnetic field, geometry, wall materials, filter unit, as well as the dynamics of discharge voltage or the magnetic field. Using different types of Hall thrusters, much work has been done on the influencing factors and characteristics of low-frequency oscillations. In general, these researches can be divided into two parts: the effects of operating parameters (mainly includes mass flow rate, discharge voltage, magnetic field) and the effects of components (mainly includes cathode, wall materials, filter, etc.). In addition, these researches might get involved in the relationship between low-frequency oscillations and the plasma characteristics or plume, performance, wall erosion, and so on.

As to the effects of operating parameters on the low-frequency oscillations, much work has been done with various types of Hall thrusters, different operating parameters, and different diagnostic methods.[66–69] For example, Fisch *et al.* studied the effects of discharge voltage and mass flow rate on the low-frequency oscillations in a cylindrical Hall thruster. Experimental results showed that the discharge current oscillations become larger in amplitude and complexity as the voltage increases and are reduced sharply with mass flow rate increasing. The frequency of low-frequency oscillations does not increase with discharge voltage as that in some traditional Hall thrusters, but exhibits complex nonlinear oscillation modes.[21,70–72] Komurasaki *et al.* commented that the low-frequency oscillations are sensitive to the amplitude of magnetic field in an anode layer type Hall thruster and the structure of anode would dramatically affect the discharge stability.[73] Gallimore *et al.* measured near-field and far-field plume-plasmas through adjusting discharge voltage using an electrostatic high-speed dual Langmuir probe (HDLP) system, and concluded that the frequency of low-frequency oscillations increases with the increase in discharge voltage and the variation of plume-plasma parameter is roughly proportional to the amplitude of low-frequency oscillations.[25] Internal measurements indicated that as the anode mass flow rate increases, the peak ion density, electron temperature, and electric field move downstream, which is attributed to the increase of electron–neutral collision frequency.[74] Their experimental results firstly provided high-spatial and high-temporal resolution measurements to low-frequency oscillations. Low-frequency oscillations were observed in all thrusters (also in clustered Hall thruster) at all spatial locations and in all measured plasma properties (discharge current, electron density, electron temperature, and plasma potential), and the phase between discharge current and plasma parameter were also observed in low-frequency oscillation timescale.[75] The same conclusion was also provided in Ref. [76]. Yu *et al.* measured the low-frequency oscillations of plasma density, electron temperature, and different energy ions in plume and showed that these oscillations exhibit the same frequency as the low-frequency oscillations and are in the same phase in the circumferential direction at the timescale of low-frequency oscillations. The time delay in the axial direction is caused by the propagation of plasma density.[76] Another great contribution of Gallimore *et al.* is that they firstly achieved the estimation of the thrust fluctuation induced by low-frequency oscillations, and therefore provided an approach to study the relationship between oscillations and performance of Hall thrusters. Boeuf *et al.* adopted different electron conductivities to a 1D quasineutral hybrid model and provided time-averaged plasma properties. The amplitude/frequency of low-frequency oscillations under different magnetic field intensity/spatial distri-

butions have been simulated and compared with experimental results.[77,78] Then, they pointed out that a configuration with a zero magnetic field and a smaller region with a large magnetic field tend to decrease wall erosion and low-frequency oscillations.[79,80] Gascon and Bouchoule *et al*. divided the low-frequency oscillations into different fluctuation modes with the variation of discharge voltage.[81,82] An electrical efficiency corresponding to the ratio between the numbers of ions ejected and the electrons circulated was defined and discussed in the range of low-frequency oscillation timescale.[83] The group headed by Cappelli performed a lot of research work on the plasma instability in a wide frequency range.[84–87] They paid much attention to various waves in discharge plasma instead of the discharge current oscillation. For example, they concluded that the low-frequency azimuthal drift waves would be overtaken by lower frequency disturbances with discharge voltage increasing. The lower frequency disturbances are confirmed to be ionization instabilities, and their frequency results agree with the results of the "predator-prey" model.[51]

The effects of components on low-frequency oscillations, such as cathode, wall material, anode orifice or size, filter unit (which also called matching network in some references), have all been studied in the past few years. Fisch *et al*. commented that the enhancement of cathode electron emission above its self-sustained level will suppress low-frequency oscillation amplitude and Hall thrusters with various configurations have the same trends.[22,88] Gallimore *et al.* studied the effects of cathode operating modes on the low-frequency oscillations and concluded that the increase of neutral density near the cathode exit can mitigate the cathode induced plasma potential oscillations and then stabilize the ionization instabilities under the low discharge voltage condition.[89] Gascon together with Cappelli and Barral *et al.* studied the effects of different wall materials on low-frequency oscillations. The wall materials investigated included alumina, boron nitride, boron nitride–silica mixture, silicon carbide, graphite, and plate diamond. Experimental results indicated that the wall materials have effects on the location of the ionization zone, as well as the characteristics of low-frequency oscillations. Their experiments using plate diamond and boron nitride as wall materials in a linear-geometry Hall thruster also showed that there is a new "pulsed" oscillation mode in the case of diamond walls. The different characteristics in low-frequency oscillations are responsible for the differences of the secondary electron emission characteristics,[90–92] and backscattering of electrons by the walls was also believed to play an equally fundamental role.[93] Using a 1-kW class anode layer type Hall thruster, the amplitude of low-frequency oscillations was measured under various hollow anode widths and axial positions. The experimental results showed that there are stable and unstable operation modes and a plasma density threshold for stable discharge. Meanwhile, the conclusion was verified through fully kinetic particle-in-cell (PIC) and direct simulation Monte Carlo (DSMC) methods.[73,94] Aside from the effects of wall materials, the effects of the anode orifice configuration were investigated for various orifice diameters by using a simplified hybrid-PIC code. The effects of neutral distribution on the ionization process were confirmed. The higher neutral density for a smaller orifice results in concentrated ionization and therefore along with the increase of the low-frequency oscillation amplitude.[95] The influences of low-frequency oscillations on plume divergence and erosion rate were also investigated. Because different low-frequency oscillation amplitude was achieved through adjusting the magnetic field, necessary relations between oscillations and plume divergence/erosion rates were not confirmed.[96] In the past ten years, some excellent signal processing methods and measuring methods, such as EMD (empirical mode decomposition)[97–99] and laser-induced fluorescence diagnostics[100–104] have been introduced to study the low-frequency oscillations by Bouchoule and Mazouffre et *al*. The fluctuation of plasma parameters in the low-frequency oscillation timescale was measured and their phase relationship was given based on the EMD method. It was verified that there are time delays between anode current and electron density or discharge voltage, and there are relations in high-frequency instabilities and low-frequency dynamics in Hall thrusters.[98,99] Experiments with laser-induced fluorescence diagnostics indicated that the ionization zone and the acceleration zone overlap with each other. The ion energy distribution widens and the acceleration zone moves toward the exit under the high magnetic field condition. With the increase of discharge voltage or the decrease of mass flow rate, the electric field shifts toward the anode. The existence of very slow and very fast ions is confirmed and attributable to the electric field oscillation in space and time.[100,101] The temporal and steady-state characteristics of the Xe^+ ion axial velocity distribution function monitored through the laser induced fluorescence diagnostic tool also provided useful information for further research of low-frequency oscillations.[102,103] Recently, the experimental results showed that the increase in dielectric wall temperature can result in an increase in low-frequency oscillation amplitude and a slight decrease in its frequency.[105] Research on the relationship between filter unit and low-frequency oscillations was also performed by experiments and numerical simulations. A filter unit was known to have a sizable impact on the onset and magnitude of these oscillations.[30] Because the effects of the filter unit is strongly related to the control of low-frequency oscillations, it will be discussed in Section 5.

According to the arguments above, the low-frequency oscillations are strongly dependent on operating conditions. The influencing regularities of discharge voltage and mass flow

rate on low-frequency oscillations have been confirmed by much research work. Experimental results showed that the amplitude of low-frequency oscillations becomes larger as the voltage is increased, and is reduced with the mass flow rate increasing. The frequency of low-frequency oscillations increases with the discharge voltage in predominantly traditional Hall thrusters. However, the effects of magnetic field on the low-frequency oscillations lack uniformity. The lack of consensus reflects the complexity of magnetic-confined plasma on the one hand and also reflects the necessity of this study on the other hand. Tamida *et al*. proposed an expression to combine discharge parameters with the oscillation strength and oscillation mode.[106] Wang *et al*. studied the relationship between magnetic field strength and low-frequency oscillations under different mass flow rates and discussed the different low-frequency oscillation modes.[39,42] Along with the variation of operating conditions, the low-frequency oscillations exhibit different oscillation modes.[39,81,82,106] This is the reason for inconsistencies in different experiments. Meanwhile, the existence of different oscillation modes also tell us that a simple relationship between low-frequency oscillation amplitude (reflected by a root-mean-square or a peak-to-peak value) and operating parameters are not sufficiently scientific. Yu *et al.* tried to explain the characteristics of low-frequency oscillations with different ionization distributions[39]and then the effects of operating parameters on the ionization distribution were given experimentally. It was pointed out that the variation of operating parameters change the distribution of "matter" or "energy", in the discharge channel, and then the ionization distribution is affected and the characteristics of low-frequency oscillations are changed.[107] These results provided some useful information in the study of low-frequency oscillations. Besides, much research work indicated that the electron dynamics plays an important role in low-frequency oscillations. Because many influencing factors such as magnetic field, cathode or wall materials are related to electron dynamics, a thorough understanding of electro-dynamics would help us to understand low-frequency oscillations deeply.

5. Stabilizing of low-frequency oscillations

Because the low-frequency oscillations might affect the power processing unit or even affect performances of Hall thrusters, researchers tried many methods to eliminate it. Theoretically, Yamamoto *et al*. built up a set of state equation of atoms and electrons in the ionization zone, and gave a model similar to the predator-prey model. Along with dispersion analysis, they found that the model has conditional stability. Then the stability conditions of low-frequency oscillations based on discharge voltage and magnetic field strength were given. Thereafter, Yamamoto *et al*. improved the proposed model. The fluctuation characteristics of the escaped ions from the ionization zone were considered.[108,109] The numerical simulation results were verified through their experiments.[57] They also tried to modify the width or position of the anode and concluded that the low-frequency oscillation characteristics changing with the discharge voltage is essentially the same in numerical simulations and experiments.[9,110] Actually, the explicit expression of stability conditions was not given out in their early study, but the stable boundary through substituting the experimental parameter values was shown. Since it is difficult to obtain the characteristic of the whole system from the complex expression of low-frequency oscillations, it is acceptable to simplify the discriminate to achieve stability conditions according to experimental data.[111,112] Then, Yamamoto *et al.* gave the physical interpretation of stability conditions as follows: when the transfer rate of electrons in the ionization zone is bigger than the ionization rate, the low-frequency oscillations will not be motivated. Conversely, the low-frequency oscillations will be active. Recently, the control action of a filter unit in the discharge circuit was discussed. A filter unit was a main component applied to reduce the low-frequency oscillations, the design of which originates from the 1970's. While the low pass filtering characteristics of a filter unit is known to protect the power supply against AC currents, their role in the active control of oscillations is usually overlooked.[30]

Previous studies indicated that the low-frequency oscillations are sensitive to parameters of the filter and can be mitigated to an acceptable level with proper values.[113] Barral *et al.* built a model to consider two types of controllers: the external RLC network (a filter unit) and a PID controller acting uniformly on the magnetic field generated by a coil current as a function of the discharge current (this magnetic field production by means of electromagnetic coils connected in a series with a discharge circuit was also named the self-excited mode in some references[43,114]). Their simulations showed that both methods have a very significant impact on the low-frequency oscillation amplitude. It was pointed out that the inductance plays the leading role in the filter. Comparatively, the self-excited mode has been shown to be an effective way to mitigate low-frequency oscillations. A simple equivalent circuit modeling the ac behavior of a thruster was also derived to help to design a robust filter.[115] Yu *et al.* also studied the effects of filter parameters on low-frequency oscillations based on the one-dimensional quasineutral hydrodynamic model. The results indicated that the amplitude of low-frequency oscillations decreases with increasing the inductance and the resistance in the filter; an eigenvalue study of the linear stability has been performed and the stability conditions were given.[40] Though it has been proved in theory that proper filter parameters can mitigate low-frequency oscillations, a "clean" filter setup does not exist actually. The effects of the inductor frequency characteristic were studied and advantageous fre-

quency characteristics of filter inductors were suggested.[12] The phase compensation function of a resistor in the filter was also discussed.[5]

Apart from the study of the stability conditions in theory and using the stability method through a filter unit, researchers also tried all kinds of possible ways to mitigate low-frequency oscillations. Dorf *et al.* observed that the anode fall can be changed from negative to positive through creating a magnetic field configuration with a zero magnetic field region and concluded that the amplitude of the low-frequency oscillations decreases, when the magnetic field configuration exhibits a zero field inside the channel.[116] This phenomenon was also observed in the ATON type Hall thruster, where the magnetic field topology includes a point of zero magnetic field.[69,117,118] The pre-ionization in the buffer chamber by "quick electrons"[41,119] or "microwave injection"[120] of Hall thrusters was shown to have reductive effects on the amplitude of low-frequency oscillations. Komurasaki *et al.* proposed a novel approach which is azimuthally non-uniform propellant flows to reduce oscillation in an anode layer Hall thruster. Although the thrust efficiency decreased, the trade-off curve between oscillation amplitude and thrust efficiency was improved.[121] Due to the strong influence of the magnetic field on plasmas, many researchers also tried to mitigate the low-frequency oscillations through an automatic control method. Petrenko *et al.* might be the earliest researchers to mitigate low-frequency oscillations with the automatic control system method. The electromagnet current was used as a regulated variable to achieve optimal discharging conditions.[122] Gallimore *et al*. also used a high speed DAQ control system and chose the inner magnet current as the control signal to control the discharge current oscillations of clustered Hall thrusters.[123] Recently theoretical and experimental studies showed that the closed-loop feedback control of the magnetic field can restrict motion of the ionization front into a small range, and then the low-frequency oscillations are notably suppressed.[43]

To the best of our knowledge, it is acceptable that the low-frequency oscillations can be stabilized in theory. However, this conclusion is mainly based on the model analysis method or dispersion analysis. The simplified theoretical mode cannot reflect uncertainties of the complex nonlinear systems: the process of low-frequency oscillations. In fact, plasma flow instabilities induced by the ionization avalanche are inevitable because of weakly damped characteristics of plasmas. The oscillations can be mitigated to an acceptable level but can never be eliminated. The essence of all control methods is to regulate some steady parameters or dynamic parameters so as to achieve the stability of Hall thrusters. It should be emphasized that this kind of stability is relative. The control parameters can be steady parameters or dynamic parameters, including the distribution of propellants, the magnetic field configuration, or the geometry of Hall thrusters or the dynamic electric field realized through filter unit, dynamic magnetic field realized through self-excited mode or building a closed-loop feedback control devices.

The low frequency discharge current oscillations are kinds of macro-instability due to the ionization and supplement of propellants. In the discharge process, the rise of electron density accelerates the ionization, the ionization process would increase exponentially if there is no restraint. Actually, with the ionization of atoms, the decrease of atom density restrains the increase of the process of ionization. With the ionization and supplement dominating alternately, the atom-front moves forward and backward periodically, while the discharge current increases and decreases periodically. The movement of atom-front determines the scope of ionization zone fluctuations which shows as the ionization distribution characteristic. Thus, the ionization distribution characteristic has strong inter-relation with the characteristics of low-frequency oscillations. A wide ionization distribution implies that the atom-front swings in a large range. The discharge current exhibits an oscillation with dominative low frequency components. Meanwhile, this kind of long time interaction provides an opportunity for further development of the instabilities. Thus, the amplitude of low-frequency oscillations is high under this condition. In addition, the narrow ionization distribution has a sharp slant along the axial directions of the channel. This causes the ionization and supplement of propellant in a small region. This rapid alteration suppresses the ionization instabilities in Hall thrusters. Therefore, the low-frequency oscillations with the narrow ionization distribution always exhibit the characteristics of scattering frequency and low amplitude. Essentially, different stabilizing methods for low-frequency oscillations are aimed to condense the ionization distribution. However, the characteristics of ionization distribution reflect the position and strength of the ionization chain reaction. The occurrence of the ionization chain reaction requires two basic conditions: "matter principle" and "energy principle". The "matter principle" is that there should be enough high-density electrons and atoms in the ionization region. The "energy principle" is that electrons should have enough energy. The change of mass flow rate or the distribution of propellants affects the ionization process mainly by means of a "matter principle"; the change of discharge voltage activates through an "energy principle." The variations of the magnetic field strength or topology affect the electron density and also affect the energy of the electron obtained from the electric field.

Using a filter unit to suppress the low-frequency oscillations is very popular in practice. The basic principle of a filter unit can be described as follows: from the view of control theory, the discharge circuit of a Hall thruster can be seen

as a feedback control system which consists of a thruster, a discharge power supply, and a filter. In this system, the Hall thruster is the controlled object, the filter is the controller, and the discharge power supply voltage is the reference signal. The filter regulates the voltage across itself according to the variation of discharge current so as to decrease its fluctuation in the discharge circuit. Proper filter parameters can provide a required fluctuant voltage with an appropriate phase-angle and amplitude, which would increase or decrease the ion mobility to balance the ion production in the discharge channel and then decrease the fluctuation of the plasma density and lower the low-frequency oscillations. However, in our opinions, the control ability of a filter unit is limited. The main function of proper filter parameters is to expand its stability boundary. Therefore, the filter parameters are always given through the trial-and-error method in practice. Up to now, there has not been a complete design theory for the filter design in Hall thrusters. The proposition of a filter design theory aimed at the low-frequency oscillation control may need more detailed experimental data and theoretical models of plasma flow characteristics in the ionization process.

Because the control ability of a filter unit on low-frequency oscillations is restricted, a kind of dynamically magnetic-field control method is proposed. The electromagnets are driven in series by the discharge current, which means that the magnetic-field intensity increases with plasma density increasing. When the electron density increases, the increasing magnetic-field intensity restrains the motion of electrons and then limits the ionization development. If the electron density decreases, the magnetic field intensity decreases, its capability for bounding electrons becomes weak. Consequently, the electrons can move longer and obtain more energy from the electric field, which is favorable for ionization. Thus, we see that the amplitude of low-frequency oscillations is less in self-excited mode Hall thrusters. Because of the strong control action of magnetic on the plasmas, the electromagnets driven in series by the discharge current has good suppressive effects on low-frequency oscillations for thrusters of different types or discharge parameters. Low-frequency oscillations in Hall thrusters are a multi-parameter coupling and complicate system. Much remains to be done in suppressing this instability.

6. Conclusions and open questions

The study on low-frequency oscillations is an important subject in the Hall thruster research area. Researchers have done a lot of work on the mechanism, characteristics, and methods of instability control. In this paper, we try our best to give out the main thread and relevancy, and discuss the performed work and the fundamental conclusion. Accordingly, we clarify gaps, conflict, and development directions of low-frequency oscillations, and also build the relationship among numerous research findings. These may help us to understand this subject deeply. Though the low-frequency oscillations have been studied for many years, some questions are still open, such as the physical mechanism of burst mode in low-frequency oscillations, the modulation of low-frequency oscillations on high-frequency plasma instabilities, the consensus problem in ground-experiments and flight-type condition, and the effects of time-evolving geometry. Moreover, the control of low-frequency oscillations is stuck on the layer of recognition and the lack of a scientific and complete theoretical framework. Much remains to be done.

References

[1] Zhurin V V, Kaufman H R and Robinson R S 1999 *Plasma Sources Sci. Technol.* **8** R1
[2] Singh S, Malik H K and Nishida Y 2013 *Phys. Plasmas* **20** 102109
[3] Malik H K and Singh S 2011 *Phys. Rev. E* **83** 036406
[4] Boeuf J B and Garrigues L 1998 *J. Appl. Phys.* **84** 3541
[5] Wei L Q, Wang C S, Ning Z Z, Liu W W, Zhang C H and Yu D R 2011 *Phys. Plasmas* **18** 063508
[6] Malik H K and Singh S 2013 *Phys. Plasmas* **20** 052115
[7] Singh S and Malik H K 2012 *J. Appl. Phys.* **112** 013307
[8] Singh S and Malik H K 2011 *IEEE Trans. Plasma Sci.* **39** 1910
[9] Yamamoto N, Komurasaki K and Arakawa Y 2005 *J. Propul. Power* **21** 870
[10] Barral S, Kaczmarczyk J, Kurzyna J and Dudeck M 2011 *Phys. Plasmas* **18** 083504
[11] Tamida T, Nakagawa T, Suga I, Osuga H, Ozaki T and Matsui K 2007 *J. Appl. Phys.* **102** 043304
[12] Wei L Q, Ning Z Z, Peng E and Yu D R 2010 *J. Vac. Sci. Technol. A* **28** L9
[13] Morozov A I, Epsinchuck Y, Kapulkin A M and Smirnov V A 1972 *Sov. Phys. Tech. Phys.* **17** 482
[14] Esipchuck Y, Morozov A I, Tilinin G and Trofimov A 1974 *Sov. Phys. Tech. Phys.* **18** 928
[15] Esipchuck Y and Tilinin G 1976 *Sov. Phys. Tech. Phys.* **21** 417
[16] Raitses Y, Smirnov A and Fisch N J 2007 *Appl. Phys. Lett.* **90** 221502
[17] Granstedt E M, Raitses Y and Fisch N J 2008 *J. Appl. Phys.* **104** 103302
[18] Smirnov A, Raitses Y and Fisch N J 2008 *IEEE Trans. Plasma Sci.* **36** 1998
[19] Raitses Y, Smirnov A and Fisch N J 2009 *Phys. Plasmas* **16** 057106
[20] Staack D, Raitses Y and Fisch N J 2004 *Rev. Sci. Instrum.* **75** 393
[21] Polzin K A, Sooby E A, Raitses Y, Merino E and Fisch N J 2009 *Proceedings of the* 31*st International Electric Propulsion Conference*, September 20–24, 2009, Ann Arbor, MI, Paper No. IEPC-2009-122
[22] Raitses Y, Parker J B and Fisch N J 2009 *Proceedings of the* 31*st International Electric Propulsion Conference*, September 20–24, 2009, Ann Arbor, MI, IEPC-2009-123
[23] Parker J B, Raitses Y and Fisch N J 2010 *Appl. Phys. Lett.* **97** 091501
[24] Choueiri E Y 2001 *Phys. Plasmas* **8** 1411
[25] Lobbia R B and Gallimore A D 2009 *Proceedings of the* 31*st International Electric Propulsion Conference*, September 20–24, 2009, Ann Arbor, Michigan, Paper No. IEPC-2009-106
[26] Boeuf J B and Garrigues L 1998 *J. Appl. Phys.* **84** 3541
[27] Barral S and Peradzyński Z 2010 *Phys. Plasmas* **17** 014505
[28] Barral S and Ahedo E 2009 *Phys. Rev. E* **79** 046401
[29] Barral S and Miedzik J 2011 *J. Appl. Phys.* **109** 013302
[30] Barral S, Miedzik J and Ahedo E 2008 *Procedings of the* 44*th AIAA Joint Propulsion Conference*, July 21–23, 2008, Hartford, CT, Paper No. AIAA 2008-4632
[31] Ahedo E 2011 *Plasma Phys. Control. Fusion* **53** 124037
[32] Parra F I, Ahedo E, Fife J M and Martínez-Sánchez M 2006 *J. Appl. Phys.* **100** 023304
[33] Noguchi R, Martinez-Sanchez M and Ahedo E 1999 *Proceedings of the* 26*th International Electric Propulsion Conference*, October 17–21, Kitakyushu, Japan, 1999, p. 617
[34] Yamamoto N, Nakagawa T, Komurasaki K and Arakawa Y 2002 *Vacuum* **65** 375

[35] Yamamoto N, Momo S, Komurasaki K and Arakawa Y 2001 *Adv. Appl. Plasma Sci.* **3** 95

[36] Kim H, Lim Y, Choe W and Seon J 2014 *Appl. Phys. Lett.* **105** 144104

[37] Li F Y, Sheng Z M, Chen M, Wu H C, Liu Y, Meyer-ter-Vehn J, Mori W B and Zhang 2014 *J. Appl. Phys. Lett.* **105** 161102

[38] Lim Y, Kim H, Lee S H, Seon J, Lee H J and Choe W 2014 *Phys. Plasmas* **21** 103502

[39] Wei L Q, Wang C S, Han K and Yu D R 2012 *Phys. Plasmas* **19** 012107

[40] Yu D R, Wang C S, Wei L Q, Gao C and Yu G 2008 *Phys. Plasmas* **15** 113503

[41] Yu D R, Wei L Q, Zhao Z Y, Han K and Yan G J 2008 *Phys. Plasmas* **15** 043502

[42] Wang C S, Wei L Q, Ning Z Z and Yu D R 2011 *Phys. Plasmas* **18** 013507

[43] Wei L Q, Han K, Wang C S, Li H, Zhang C H and Yu D R 2012 *J. Vac. Sci. Technol. A* **30** 061304

[44] Wang C S, Wei L Q and Yu D R 2011 *Contrib. Phys. Plasmas* **51** 981

[45] Miyasaka T, Shibata Y and Asato K 2009 *Vacuum* **83** 61

[46] Kapulkin A and Guelman M M 2008 *IEEE Trans. Plasma Sci.* **36** 2082

[47] Fruchtman A and Cohen-Zur A 2006 *Appl. Phys. Lett.* **89** 111501

[48] Liu D, Huffman R E, Branam R D and Hargus Jr W A 2011 *IEEE Trans. Plasma Sci.* **39** 2926

[49] Tilinin G 1977 *Sov. Phys. Tech. Phys.* **22** 974

[50] Fife J M, Martínez-Sánchez M and Szabo J 1997 *Proceedings of the* 33*rd AIAA Joint Propulsion Conference*, January 6–7, 1997, Reno, NV, Paper No. AIAA 97-3052

[51] Meezan N B, Hargus W A and Cappelli M A 1998 *Proceedings of the* 34*th Joint Propulsion Conference*, July 13–15, 1998, Cleveland, OH, Paper No. AIAA 98-3502

[52] Yamamoto N, Yokota S and Watanabe K 2005 *Trans. Jpn. Soc. Aeronaut Space Sci.* **48** 169

[53] Guerrini G, Michaut C, Dudeck M, Vesselovzorov A N and Bacal M 1997 25*th International Conference on Electric Propulsion*, August 24–28, 1997, Cleveland, OH, p. 326

[54] Haas J M, Hofer R R and Gallimore A D 1999 *Proceedings of the* 35*th Joint Propulsion Conference*, June 20–24, 1999, Los Angeles, CA, Paper No. AIAA99-2426

[55] Yuri M Y, Kent K J, John B M and Edward J B 1999 *Proceedings of the* 26*th International Electric Propulsion Conference*, October 17–21, Kitakyushu, Japan, 1999, p. 572

[56] Baranov V I, Nazarenko Y S, Pelrosov V A, Vasin A I and Yashnov Y M 1996 *Proceedings of the* 32*th Joint Propulsion Conference*, July 1–3, 1996, Lake Buena Vista, FL, Paper No. AIAA1996-3192

[57] Yamamoto N, Momo S, Komurasaki K and Arakawa Y 2000 *Proceedings of the* 22*nd International Symposium on Space Technology and Science*, May 28–June 4, 2000, Morioka, Japan, p. 349

[58] Dorval N, Bonnet J, Marque J P, Chable S, Rogier F and Lasgorceix P 2002 *J. Appl. Phys.* **91** 4811

[59] Chable S and Rogier F 2005 *Phys. Plasmas* **12** 033504

[60] Buneman O 1963 *Phys. Rev. Lett.* **10** 285

[61] Ahedo E, Martinez P and Martinez-Sanchez M 2000 *Proceedings of the* 36*th Joint Propulsion Conference*, July 17–19, 2000, Huntsville, AL, Paper No. AIAA2000-3655

[62] Barral S and Ahedo E 2006 *Procedings of the* 42*nd AIAA Joint Propulsion Conference*, July 9–12, 2006, Sacramento, CA, Paper No. AIAA 2006-5172

[63] Barral S, Lapuerta V, Sancho A and Ahedo E 2005 *Proceedings of the* 29*th International Electric Propulsion Conference*, October 31–November 4, 2005, Princeton, NJ, Paper No. IEPC-2005-120

[64] Barral S and Ahedo E 2008 *AIP Conference Proceedings* **993** 439

[65] Barral S and Peradzynski Z 2009 *Proceedings of the* 31*st International Electric Propulsion Conference*, September 20–24, 2009, Ann Arbor, Michigan, Paper No. IEPC-2009-70

[66] Albar'ede L, Mazouffre S, Bouchoule A and Dudeck M 2006 *Phys. Plasmas* **13** 063505

[67] Kurzyna J, Mazouffre S, Lazurenko A, Albarède L, Bonhomme G, Makowski K, Dudeck M and Peradzyński K 2005 *Phys. Plasmas* **12** 123506

[68] Darnon F, Garrigues L, Boeuf J B, Bouchoule A and Lyszyk M 1999 *IEEE Trans. Plasma Sci.* **27** 98

[69] Touzeau M, Prioul M, Roche S, Gascon N, Perot C, Darnon F, Bechu S, Philippe-Kadlec C, Magne L, Lasgorceix P, Pagnon D, Bouchoule A and Dudeck M 2000 *Plasma Phys. Control. Fusion* **42** B323

[70] Raitses Y and Fisch N J 2001 *Phys. Plasmas* **8** 2579

[71] Smirnov A, Raitses Y and Fisch N J 2002 *J. Appl. Phys.* **92** 5673

[72] Smirnov A, Raitses Y and Fisch N J 2007 *Phys. Plasmas* **14** 057106

[73] Yasui S, Kumakura K, Yamamoto N, Komurasaki K and Arakawa Y 2003 39*th AIAA Joint Propulsion Conference*, July 20–23, 2003, Huntsville, Alabama, Paper No. AIAA-2003-4702

[74] Reid B M 2009 *The Influence of Neutral Flow Rate in the Operation of Hall Thrusters* (Ph. D. dissertation) (Michigan: University of Michigan)

[75] Lobbia R B 2010 *A Time-resolved Investigation of the Hall Thruster Breathing Mode* (Ph.D. dissertation) (Michigan: University of Michigan)

[76] Yu D R, Wei L Q, Wei Z L and E P 2008 *Plasma Sources Sci. Technol.* **17** 035022

[77] Boeuf J P and Garrigues L 1998 *J. Appl. Phys.* **84** 3541

[78] Garrigues L, Pérot C and Gascon N 1999 *Proceedings of the* 26*th International Electric Propulsion Conference*, October 17–21, Kitakyushu, Japan, 1999, p. 595

[79] Garrigues L, Hagelaar G J M, Bareilles J, Boniface C and Boeuf J P 2003 *Phys. Plasmas* **10** 4886

[80] Garrigues L, Boniface C, Bareilles J, Hagelaar G J M and Boeuf J P 2003 *Proceedings of the* 28*th International Electric Propulsion Conference*, March 17–21, 2003, Toulouse, France, Paper No. IEPC-2003-183

[81] Gascon N, Perot C, Bonhomme G, Caron X, Bchu S, Lasgorceix P, Izrar B and Dudeck M 1999 *Proceedings of the* 35*th Joint Propulsion Conference*, June 20–24, 1999, Los Angeles, California, Paper No. AIAA1999-2427

[82] Bouchoule A, Philippe-Kadlec C and Prioul M 2001 *Plasma Sources Sci. Technol.* **10** 364

[83] Darnon F, Lyszyk M and Bouchoule A 1997 *Proceedings of the* 33*rd AIAA Joint Propulsion Conference*, January 6–7, 1997, Reno, NV, Paper No. AIAA 1997-3051

[84] Hargus W A, Meezan N B and Cappelli M A 1997 *Proceedings of the* 33*rd AIAA Joint Propulsion Conference*, January 6–7, 1997, Reno, NV, Paper No. AIAA 97-3050

[85] Gascon N and Cappelli M A 2003 39*th AIAA Joint Propulsion Conference*, July 20–23, 2003, Huntsville, AL, Paper No. AIAA-2003-4857

[86] Knoll A, Thomas C, Gascon N and Cappelli M A 2006 *Procedings of the* 42*nd AIAA Joint Propulsion Conference*, July 9–12, 2006, Sacramento, CA, Paper No. AIAA 2006-5171

[87] Fernandez E, Scharfe M K, Thomas C A, Gascon N and Cappelli M A 2008 *Phys. Plasmas* **15** 012102

[88] Raitses Y, Smirnov A and Fisch N J 2009 *Phys. Plasmas* **16** 057106

[89] Brown D L and Gallimore A D 2009 *Proceedings of the* 31*st International Electric Propulsion Conference*, September 20–24, 2009, Ann Arbor, Michigan, Paper No. IEPC-2009-074

[90] Thomas N C, Hermann W and Cappelli M A 2003 *Proceedings of the* 28*th International Electric Propulsion Conference*, March 17–21, 2003, Toulouse, France, Paper No. IEPC-2003-327

[91] Gascon N, Dudeck M and Barral S 2003 *Phys. Plasmas* **10** 4123

[92] Barral S, Makowski K, Peradzynski Z, Gascon N and Dudeck M 2003 *Phys. Plasmas* **10** 4137

[93] Barral S, Makowski K, Peradzynski Z, Gascon N and Dudeck M 2001 *Proceedings of the* 27*th International Electric Propulsion Conference*, October 14–19, 2001, Pasadena, CA, Paper No. IEPC-2001-027

[94] Komurasaki K, Yokota S, Yasui S and Arakawa Y 2004 *Proceedings of the* 40*th AIAA Joint Propulsion Conference*, July 12–14, 2004, Fort Lauderdale, Fla, Paper No. AIAA 2004-3954

[95] Miyasaka T, Shibata Y and Asato K 2008 *Vacuum* **83** 61

[96] Yamamoto N, Nakagawa T, Komurasaki K and Arakawa Y 2002 *Proceedings of the* 23*rd International Symposium on Space Technology and Science*, May 26–June 2, 2002, Tokyo, Matsue, Japan, p. 319

[97] Albarède L, Mazouffre S, Bouchoule A and Dudeck M 2006 *Phys. Plasmas* **13** 063505

[98] Kurzyna J, Mazouffre S, Lazurenko A, Bonhomme G, Makowski K, Dudeck M and Peradzyński Z 2005 *Phys. Plasmas* **12** 123506

[99] Kurzyna J, Makowski K, Peradzynski Z, Lazurenko A, Mazouffre S and Dudeck M 2008 *AIP Conference Proceedings* **993** 443

[100] Mazouffre S, Gawron D, Kulaev V and Sadeghi N 2008 *IEEE Trans. Plasma Sci.* **36** 1967
[101] Gawron D, Mazouffre S, Sadeghi N and Heron A 2008 *Plasma Sources Sci. Technol.* **17** 025001
[102] Mazouffre S, Kulaev V and Perez-Luna J 2009 *Plasma Sources Sci. Technol.* **18** 034022
[103] Mazouffre S, Gawron D and Sadeghi N 2009 *Phys. Plasmas* **16** 043504
[104] Mazouffre S 2013 *Plasma Sources Sci. Technol.* **22** 013001
[105] Guo N, Wei L Q and Ding Y J 2014 *J. Vac. Sci. Technol. A* **32** 030604
[106] Tamida T, Nakagawa T, Suga I, Osuga H, Ozaki T and Matsui K 2007 *J. Appl. Phys.* **102** 043304
[107] Wei L Q, Wang C S, Zhang C H and Yu D R 2013 *Appl. Phys. Lett.* **102** 173505
[108] Yamamoto N, Nakagawa T, Komurasaki K and Arakawa Y 2001 *Advances in Applied Plasma Science* **3** 95
[109] Yamamoto N, Nakagawa T, Komurasaki K and Arakawa Y 2002 *Vacuum* **65** 375
[110] Yamamoto N, Komurasaki K and Arakawa Y 2004 *Proceedings of the* 40*th AIAA Joint Propulsion Conference*, July 12–14, 2004, Fort Lauderdale, Fla, Paper No. AIAA 2004-4100
[111] Yamamoto N, Nakagawa T, Komurasaki K and Arakawa Y 2002 *Proceedings of the* 38*th AIAA Joint Propulsion Conference*, July 8–12, Indianapolis, Ind, Paper No, AIAA 2002-3953
[112] Yamamoto N, Komurasaki K and Arakawa Y 2003 *Proceedings of the* 28*th International Electric Propulsion Conference*, March 17–21, 2003, Toulouse, France, Paper No. IEPC-2003-086
[113] Randolph T, Fischer G, Kahn J, Kaufman H, Zhurin V V, Kozubsky K and Kim V 1994 *Proceedings of the* 30*th AIAA Joint Propulsion Conference*, June 27–29, 1994, Indianapolis, IN, Paper No. AIAA94-2857
[114] Han K, Wei L Q, Ding Y J, Ji Y C and Yu D R 2012 *J. Propul. Power* **28** 27
[115] Barral S 2007 *Proceedings of the* 30*th International Electric Propulsion Conference*, September 17–20, 2007, Florence, Italy, Paper No. IEPC-2007-261
[116] Dorf L, Raitses Y and Fisch N J 2006 *Phys. Plasmas* **13** 057104
[117] Garrigues L, Hagelaar G J M, Bareilles J, Boniface C and Boeuf J P 2003 *Phys. Plasmas* **10** 4886
[118] Morozov A I, Bugrova A I, Desyatskov A V, Ermakov Y A, Kozintseva M V, Lipatov A S, Pushkin A A, Khartchevnikov V K and Churbanov D V 1997 *Plasma Phys. Rep.* **23** 587
[119] Bishaev A M, Bugrova A I, Desyatskov A V and Kozintseva M V 1999 *Fourth All-Russian Seminar on Problems of Theoretical and Applied Electron Optics*, October 21–22, 1999, Moscow, Russia, p. 79
[120] Kuwano H, Ohno A, Kuninaka H and Nakashima H 2007 *Proceedings of the* 30*th International Electric Propulsion Conference*, September 17–20, 2007, Florence, Italy, Paper No. IEPC-2007-085
[121] Fukushima Y, Yokota S, Takahashi D, Hara K, Cho S, Komurasaki K and Arakawa Y 2009 *Proceedings of the* 31*st International Electric Propulsion Conference*, September 20–24, 2009, Ann Arbor, Michigan, Paper No. IEPC-2009-148
[122] Petrenko O N and Yatcina O N 1997 25*th International Conference on Electric Propulsion*, August 24–28, 1997, Cleveland, OH, p. 821
[123] Lobbia R B and Gallimore A D 2005 *Proceedings of the* 41*st AIAA Joint Propulsion Conference*, July 10–13, 2005, Tucson, Ariz, Paper No. AIAA 2005–3679