Model of hybrid interfacial domain wall in ferromagnetic/antiferromagnetic bilayers*

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A general model of a hybrid interfacial domain wall (HIDW) in ferromagnetic/antiferromagnetic exchange biased bilayers is proposed, where an interfacial domain wall is allowed to extend into either the ferromagnetic or antiferromagnetic layer or across both. The proposition is based on our theoretical investigation on thickness and field dependences of ferromagnetic domain wall (FMDW) and antiferromagnetic domain wall (AFDW), respectively. Good match of the simulation to the hysteresis loops of a series of NiFe/FeMn exchange-biased bilayers confirms the existence of the HIDW, where the AFDW part is found to preferentially occupy the entire antiferromagnetic layer while the FMDW shrinks with the increased magnetic field as expected. The observed asymmetry between the ascending and descending branches of the hysteresis loop is explained naturally as a consequence of different partition ratios between AFDW and FMDW.

Keywords: exchange bias, interfacial domain wall, ferromagnetic/antiferromagnetic bilayer

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1. Introduction

To investigate the mechanism for the exchange coupling and therefore to understand the fundamental properties of exchange biased ferromagnetic/antiferromagnetic systems,[1–8] the knowledge of the magnetic structure at the interface is essential. Several models[9–15] have been proposed, among which a model of an interfacial domain wall, first proposed by Néel,[10] is compatible and may coexist with others.[9] Mauri developed the model further and quantitatively described what occurs during magnetization reversal.[11] In general, the occurrence of interfacial domain wall structure can be easily understood based on the idea of exchange length to reduce the exchange energy of an abrupt local change of spin direction. However, in the early model of Néel and Mauri, only a limiting case that a thin ferromagnetic layer coupled with a very thick antiferromagnetic layer and reasonably the interfacial domain wall in the antiferromagnetic side, has been considered. Later theoretical studies as well as experimental investigations[16–31] suggest that the interfacial domain wall may exist in either the antiferromagnetic or ferromagnetic layer depending on the layer thickness and the magnetic-field intensity.[25] Nevertheless, a systematic study of both the full/partial antiferromagnetic domain wall (AFDW) and ferromagnetic domain wall (FMDW) in the practical exchange-biased bilayers is lacking, particularly with the interfacial exchange coupling involved. Moreover, the field-dependent behavior of the interfacial domains in the magnetization process has not been thoroughly investigated. In this paper, we propose a concept of hybrid interfacial domain wall (HIDW), by which we aim to reproduce the magnetic behavior unambiguously in commonly used exchange-biased bilayers (where the sub-layer thickness is around a few or tens of nanometers and much less than the natural 180° domain wall width) and to offer an approach to solving the problems in the exchange-coupled interfaces including the external magnetic field.

Our theoretical investigations on a normal exchange bias system of a $Ni_{80}Fe_{20}/Fe_{50}Mn_{50}$ bilayer start with the thickness effect and magnetic-field effect on the domain walls in the antiferromagnetic and ferromagnetic thin films, respectively, which reveal a general trend of the HIDW's formation and physically predict its composition varying with the field. Successful simulation for the experimental hysteresis loops of these bilayers demonstrates the existence of HIDW during the magnetization reversal and thereby shows directly that the partition ratio of the AFDW to FMDW changes with magnetic field and magnetization history. This also explains naturally the asymmetry observed in the hysteresis loops. The primary cause of why the FMDW is not easy to observe experimentally[32] has been addressed as well.

2. Experiment

A series of typical exchange bias systems, $Ni_{80}Fe_{20}$ (40 nm)/$Fe_{50}Mn_{50}$ (2 nm–20 nm) bilayers, were prepared by sputtering with a base pressure of $\sim 10^{-8}$ Torr (the unit

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1 Torr = 1.33322×10^2 Pa). Both the antiferromagnetic and the ferromagnetic sublayers are much thinner than their natural 180° domain walls, which implies that the thickness effect cannot be neglected in the studies of the interfacial domain wall structure. An external magnetic field of 250 Oe (1 Oe = 79.5775 A·m^{-1}) was applied in the sputtering process to induce a uniaxial anisotropy in the NiFe layer. The FeMn layer was then grown on the top of the saturated NiFe layer to produce a proper pinning effect.[33] The observed dependences of the coercivity (H_C) and exchange bias field (H_E) on the antiferromagnetic-layer thickness[34] are consistent with those in the previous literature,[35] which allows us to obtain the useful magnetic parameters for the antiferromagnetic layers: the induced uniaxial anisotropy constant $K_{AF} = 2 \times 10^5$ erg/cm^3 and the exchange stiffness $A_{AF} = 3 \times 10^{-7}$ erg/cm. At the same time, the parameters for the ferromagnetic layers are deduced from the magnetometry measurements combined with a perfect fitting of the ferromagnetic resonance results:[34] $K_{FM} = 2 \times 10^3$ erg/cm^3, $A_{FM} = 1.2 \times 10^{-6}$ erg/cm, and the saturated magnetization $M_S = 780$ emu/cm^3.

3. Proposition of hybrid interfacial domain wall

With the above values, the thickness effect and the magnetic-field effect on the domain wall have been calculated by variational principle.[36–39] Here we briefly introduce the axis frame applied to our calculation before showing the important results, which lead to the idea of HIDW.

As shown in Fig. 1, the x axis is set to be perpendicular to the film plane, and the z axis along the uniaxial easy axis. Thus, the y–z plane coincides with the film plane. Assuming that the magnetic moment rotates in the y–z plane as a Bloch-type domain wall, we make the magnetization angle θ change with x and independent of the y and z coordinates.

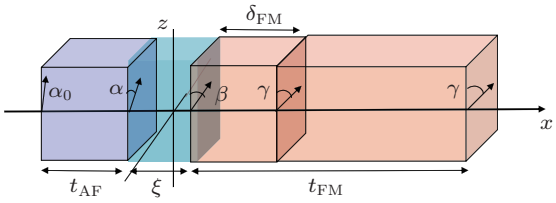


Fig. 1. (color online) Scheme of the HIDW model. The z axis coincides with the uniaxial easy axis for both antiferromagnetic and ferromagnetic layers. The external reversing magnetic field is applied in the $-z$ direction. The magnetic moment directions on both sides of the antiferromagnetic layer are allowed to rotate away from the easy axis by angles α_0 and α. The magnetization in the FMDW rotates from an angle β at the interface to γ, forming the domain wall thickness δ_{FM}. Beyond the FMDW, the magnetization is assumed to rotate uniformly by γ in the ferromagnetic domain. The difference between spin directions across the interface, α–β, corresponds to an inter-space ξ.

We first assume that under zero field, a 180°-AFDW and a 180°-FMDW are, respectively, so compressed that each of them has a thickness t. The calculated thickness dependences of the domain wall energy density (E_{AF} and E_{FM}) are shown in Fig. 2. Both E_{AF} and E_{FM} increase with reduced t, and the increase is abruptly accelerated when t is smaller than 40 nm. Such an increase is mainly due to the enhanced exchange energy in reduced thickness, suggesting that a partial non-180°-domain wall (for simplicity, hereafter we call it a partial domain wall) instead of a full 180°-domain wall is more energetically favored in thin sublayers. It is noteworthy that the relation between E_{AF} and E_{FM} is reversed when t is below 120 nm. That is, E_{FM} becomes larger than E_{AF}, and the increase of E_{FM} with reduced t is much more rapid than that of E_{AF}; in which case, we attribute it to the fact that the exchange energy plays a dominant role in the compressed domain wall in thin film and exceeds the effect of the anisotropy energy. As a result, a partial AFDW has a priority over an FMDW to appear in thin films, due to its smaller exchange stiffness than that of the FMDW.

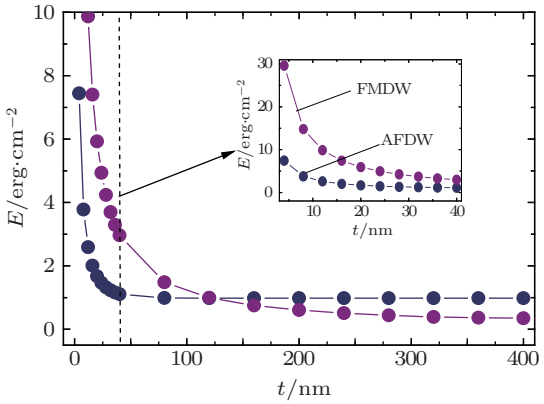


Fig. 2. (color online) Variations of energy density of the 180°-FMDW and 180°-AFDW with layer thickness. Inset shows the enlarged plot for layer thickness smaller than 40 nm.

Besides the thickness effect, the magnetic-field effect has also been studied. It is well known that a magnetic field compresses the FMDW.[37–39] Precisely, it is the steep part of the FMDW with a large angle-variation that is compressed, while the flat part in which the magnetization aligns close to the field direction is broadened. In other words, the FMDW is deformed by the magnetic field. Such a deformation is ascribed to an energy re-distribution after a field has been applied, as illustrated in Fig. 3. During the re-magnetization by an applied magnetic field H in the $-z$ direction, the energy of a 40-nm-thick 180°-FMDW is re-distributed compared with the case under zero field. When $H = 0$, the energy density differential, $dE/d\theta$, is symmetrically distributed across the FMDW and has a maximum value at $\theta = 90°$. When $H \neq 0$, the symmetry is broken: $dE/d\theta$ decreases for the part where $\theta > \theta_{Cr}$, and increases where $\theta < \theta_{Cr}$. Here θ_{Cr} is the angle where the curve $dE/d\theta$ ($H \neq 0$) coincides with $dE/d\theta$ ($H = 0$), and it increases with the augment of magnitude of magnetic field. Such a re-distribution of FMDW energy is obviously caused by the involvement of Zeeman energy, which is determined by

the value of magnetic field and the angle between the magnetization and field, $\theta(M,H)$. Note that $\theta(M,H) = 180°–\theta$ since the magnetic field is applied in the $-z$ direction. During the magnetization reversal, the enhanced-energy part of the FMDW is not energetically favored to exist on the ferromagnet side. It is expected to have a high possibility of being pushed into the antiferromagnet side where no Zeeman energy exists, in order to reduce the total energy. The reduced-energy part, especially the FMDW tail with its magnetization aligned close to the direction of the field, may remain on the ferromagnet side. Thus, an HIDW is inferred as a low energy interfacial spin structure generally.

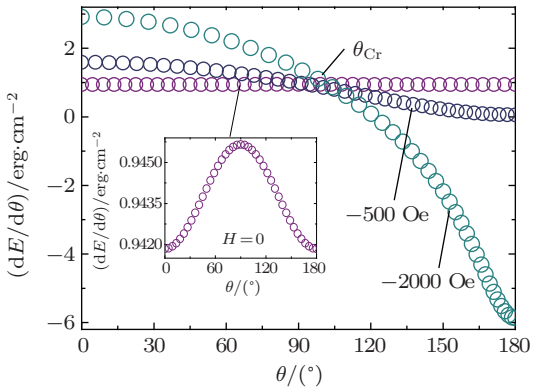


Fig. 3. (color online) Variations of energy density distribution in 40-nm-thick FMDW with magnetization angle θ under $H = 0$, $H = -500$ Oe, and $H = -2000$ Oe, respectively. Inset shows the enlarged plot for $H = 0$.

Following the above theoretical analysis of both the thickness effect and the magnetic-field effect, we propose an HIDW with a reasonable division of the two partial domain walls on both sides of the interface during the magnetization reversal of the antiferromagnetic/ferromagnetic bilayers. The partition ratio of the partial AFDW to the partial FMDW is not simply determined by a balance between their energies. The interfacial exchange energy and any other possible energy should also be considered; to be exact, it depends on the minimization of the total free energy for the whole system instead.

The HIDW model is established in a reference frame shown schematically in Fig. 1. By a partial AFDW, we refer to the case that the magnetic moments on both sides of the thin antiferromagnetic layer are generally allowed to rotate away from the easy axis by angles α_0 and α. The magnetization in the FMDW rotates from an angle β (at the interface) to γ, and the magnetization beyond the FMDW is assumed to rotate uniformly by γ in the ferromagnetic domain. The difference between the spin directions across the interface, α–β, is controlled by the strength of the exchange interaction between the two interfacial atomic planes. Here we should point out that for an extreme case where the antiferromagnetic layer is thinner than the critical thickness t_{Cr}, the exchange bias effect reduces to even zero, which has been discussed frequently.[35] In such a case, the antiferromagnetic anisotropy energy $t_{AF}K_{AF}$ is comparable to or even smaller than the interfacial exchange coupling between the adjacent antiferromagnetic and ferromagnetic spins,[40] so that α_0 is expected to be nonzero and even larger than 90°. In this paper, we focus on a typical case where the antiferromagnetic layer has a thickness above t_{Cr} so that a reasonable pinning effect is produced in the exchange-biased bilayers due to its high anisotropy energy $t_{AF}K_{AF}$. Thus, we choose $\alpha_0 = 0$.

For simplicity, based on Refs. [32] and [33] the total free energy can be expressed as follows:

$$\begin{aligned} F &= 2\sqrt{A_{AF}K_{AF}}(1-\cos\alpha) + A_{12}/\xi[1-\cos(\alpha-\beta)] \\ &\quad + 2\sqrt{A_{FM}K_{FM}}(\cos\beta-\cos\gamma) \\ &\quad + HM\delta_{FM}\left[1-\cos\left(\frac{\beta+\gamma}{2}\right)\right] \\ &\quad + K_{FM}(t_{FM}-\delta_{FM})\sin^2\gamma \\ &\quad + HM(t_{FM}-\delta_{FM})(1-\cos\gamma). \end{aligned} \tag{1}$$

The first term on the right-hand side of Eq. (1) is the AFDW energy. The second one is the interfacial exchange energy, with A_{12} being the exchange stiffness across the interface and ξ the average distance between the adjacent antiferromagnetic and ferromagnetic atomic planes. The third and the fourth terms describe the partial FMDW energy, including an approximate Zeeman energy correction. The fifth term and the sixth one are the anisotropy energy and Zeeman energy, respectively, for the ferromagnetic domain in uniform rotation. Using Lilley's method,[41] the FMDW width δ_{FM} can be approximated by

$$\begin{aligned} \delta_{FM} &\approx (\gamma-\beta)\left.\frac{dx}{d\theta}\right|_{\theta=\theta_0} \\ &= \delta_0(\gamma-\beta)/\sqrt{\sin^2\theta_0 - HM\cos\theta_0/K_{FM} + C}. \end{aligned}$$

Here the coefficient C is determined by the proper boundary condition and $\delta_0 = \sqrt{A_{FM}/K_{FM}}$. Conventionally θ_0 is the angle at the inflexion point in a natural 180°-domain wall.[42] However, in the partial FMDW we find that $\theta_0 = (\beta+\gamma)/2$ is a good approximation because of the small variation in the angle across the partial wall. In addition, $d\theta/dx = 0$ when $\theta = \pi$ under a large reversed field is a suitable boundary condition in our following simulation.

4. Characteristics of hybrid interfacial domain wall

The hysteresis loops of NiFe/FeMn bilayers are calculated by finding the angles α, β, and γ from Eq. (1) based on the principle of energy minimum.[11,43] With the magnetization history considered, the solved angles for each point on the loop are used as the starting angles for finding the next successive point following the magnetization process. The good match of the simulation to the experimental data indicates the validity of this model. The only adjustable parameter, A_{12}/ξ,

is found to vary slightly with samples at around 0.11 erg/cm^2 and the fitted A_{12} is smaller than the exchange stiffness in either FeMn or NiFe. It is reasonable due to the rough uncompensated interface where a number of grains with different orientations cross the interface and defects can be embedded in our polycrystalline samples.[33]

Figure 4 shows the simulation results for a typical sample NiFe (40 nm)/FeMn (6 nm). For comparison, a simulation with only an interfacial AFDW involved is first plotted in Fig. 4(a). Obviously it does not provide a satisfactory fit for both the asymmetry shape and the values of H_E and H_C simultaneously. On the other hand, the theoretical calculation by the HIDW model is in good agreement with the experimental data instead, as shown in Fig. 4(b). Accordingly the representative structures of the HIDW during the magnetization found from our simulation are shown in Tables 1 and 2.

Table 1. HIDW in the descending branch of the hysteresis loop for the NiFe(40 nm)/FeMn(6 nm) exchange-biased bilayer.

H/Oe	α/(°)	β/(°)	γ/(°)	δ_{AF}/nm	δ_{FM}/nm	$(\gamma-\beta)$/(°)	M/M_S
–30	1	6	6	6	~0	0	0.99
–33	3	18	18	6	~0	0	0.95
–35	6	30	31	6	0.8	1	0.86
–37	14	96	104	6	9.0	8	–0.35
–41	10	146	152	6	15	6	–0.87

Table 2. HIDW in the ascending branch of the hysteresis loop for the NiFe(40 nm)/FeMn(6 nm) exchange-biased bilayer.

H/Oe	α/(°)	β/(°)	γ/(°)	δ_{AF}/nm	δ_{FM}/nm	$(\gamma-\beta)$/(°)	M/M_S
–41	8	152	158	6	19	6	–0.91
–37	12	128	136	6	15	8	–0.72
–35	14	112	120	6	11	8	–0.48
–33	12	88	94	6	7.0	6	–0.06
–30	4	18	18	6	~0	0	0.95

From Tables 1 and 2 we see that i) the HIDW does exist with the angles spanning from 0 to α in the antiferromagnetic layer and from β to γ in the ferromagnetic layer respectively, ii) the partition ratio of the AFDW to FMDW varies with the field, and iii) the difference between the composition of HIDW in the descending branch and that in the ascending branch explains the asymmetry between the two branches (as shown in Fig. 4).

In the descending branch, when the sample is remagnetized by a reversal field $H = -30$ Oe, a partial AFDW appears, while the magnetization across the whole ferromagnetic layer remains in uniform rotation. When the magnetic field becomes more negative, the ferromagnetic domain magnetization M rotates further toward the direction of the magnetic field and the angle $\theta(M,H)$ decreases. Consequently, the Zeeman energy is reduced with the gradually reduced magnetic field, which favors a partial FMDW to arise. When $H = -35$ Oe, the FMDW appears as expected. The formation of the partial AFDW prior to that of the FMDW is due to the relatively small exchange stiffness and zero Zeeman energy in the antiferromagnetic layer, as aforementioned in the discussion about Fig. 2. With more and more magnetizations reversed by the decreasing field, the deformation of the FMDW is more distinct by the energy re-distribution as discussed in Fig. 3. The compressed steep part in the FMDW has its energy enhanced, and is consequently pushed more into the antiferromagnetic side, leading to the increase of α. At the same time, the FMDW tail is broadened due to the highly-reduced energy with the reduced field, giving rise to the continuous growth of the partial FMDW thickness as shown in Table 1.

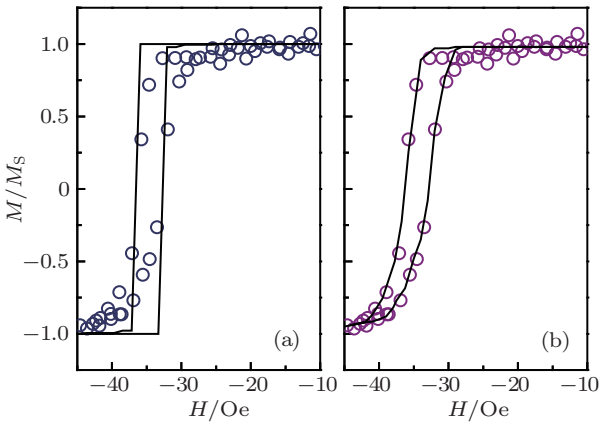


Fig. 4. (color online) Simulations of the hysteresis loop for NiFe (40 nm)/FeMn (6 nm): (a) with only interfacial AFDW considered, and (b) using the HIDW model, respectively. The circles represent the experimental data, and the solid lines are fittings to the loop.

The analysis for the data of the ascending branch in Table 2 is similar to the above for the descending branch in Table 1. The partial FMDW shrinks with the increased magnetic field. This is seen to occur in both branches. Notice that the partial FMDW with relatively large angle-variation, $(\gamma-\beta)$, is not wide, while the broad one has a relatively small angle-variation across the wall. Thus, the FMDW might be outside the resolution of detection and therefore difficult to observe experimentally.

5. Conclusion

Motivated by our systematic investigations on the thickness effect and the magnetic-field effect on the domain walls in ferromagnetic and antiferromagnetic thin films, respectively, we propose a model of a hybrid interfacial domain wall in this paper. The model describes a general interfacial spin structure in practical exchange-biased bilayers, containing a partial AFDW, a partial FMDW, and an abrupt spin flop across the interface, which may be a good candidate for the understanding of all kinds of exchange-coupled interfaces and may coexist with any other existing interfacial mechanism.[44–46] Previous models of the interfacial domain wall existing only

on one side of the interface[10,11] can be regarded as particular cases of HIDWs. The simulation of the hysteresis loops in typical exchange-biased bilayers demonstrates the validity of our model, from which several important insights could be provided. i) During the magnetization reversal, the HIDW functions as a spring to store and supply energy for exchange bias. ii) The partial AFDW first arises in the magnetization reversal, and then the partial FMDW appears with the decrease of field. When the field is further reduced, the higher-energy part of the FMDW is pushed into the antiferromagnetic side and the lower-energy part remains in the ferromagnetic layer. iii) The composition of HIDW varies with the magnetic field and simultaneously, varies at different points in the hysteresis loop, leading to the asymmetry between the ascending and descending branches naturally.

The main results obtained from our calculations on the thickness and magnetic-field effects play an essential role in studying the domain wall structure at the interface. Our theoretical investigation on the ferromagnetic and antiferromagnetic domain walls combined with a numerical simulation to the experimental data provides a sensible approach to solving the problems in exchange-coupled multilayers in the presence of an external field.

References

[1] Radu F, Etzkorn M, Siebrecht R, Schmitte T, Westerholt K and Zabel H 2003 *Phys. Rev. B* **67** 134409
[2] Li Z P, Petracic O, Morales R, Olamit J, Batlle X, Liu K and Schuller I K 2006 *Phys. Rev. Lett.* **96** 217205
[3] Kiwi M, Mejía-López J, Portugal R D and Ramírez R 1999 *Appl. Phys. Lett.* **75** 3995
[4] Stamps R L and Usadel K D 2006 *Europhys. Lett.* **74** 512
[5] Lin J, Shi Z, Zhou S M, Zhang X and Xia Y J 2009 *Chin. Phys. Lett.* **26** 107501
[6] Zhu J R, Xiang M and Hu J G 2012 *Acta Phys. Sin.* **61** 187504 (in Chinese)
[7] Shahzad F, Siddiqi S A, Im M Y, Avallone A, Fischer P, Hussain Z, Siddiqi I and Hellman F 2010 *Chin. Phys. B* **19** 037504
[8] Qi X J, Wang Y G, Zhou G H, Li Z Q and Guo M 2010 *Chin. Phys. B* **19** 037503
[9] Stiles M D and McMichael R D 1999 *Phys. Rev. B* **59** 3722
[10] Néel L 1967 *Ann. Phys.* **2** 61
[11] Mauri D, Siegmann H C, Bagus P S and Kay E 1987 *J. Appl. Phys.* **62** 3047
[12] Molozemoff A P 1987 *Phys. Rev. B* **35** 3679
[13] Slonczewski J C 1995 *J. Magn. Magn. Mater.* **150** 13
[14] Koon N C 1997 *Phys. Rev. Lett.* **78** 4865
[15] Schulthess T C and Butler W H 1998 *Phys. Rev. Lett.* **81** 4516
[16] Scholl A, Liberati M, Arenholz E, Ohldag H and Stöhr J 2004 *Phys. Rev. Lett.* **92** 247201
[17] Geshev J, Pereira L G and Schmidt J E 2001 *Phys. Rev. B* **64** 184411
[18] Gloanec M, Rioual S, Lescop B, Zuberek R, Szymczak R, Aleshkevych P and Rouvellou B 2010 *Phys. Rev. B* **82** 144433
[19] Yang F Y and Chien C L 2000 *Phys. Rev. Lett.* **85** 2597
[20] Ball A R, Leenaers A J G, van der Zaag P J, Shaw K A, Singer B, Lind D M, Fredrikze H and Rekveldt M Th 1996 *Appl. Phys. Lett.* **69** 1489
[21] Miller B H and Dan Dahlberg E 1996 *Appl. Phys. Lett.* **69** 3932
[22] Nagamine L C C M, Schmidt J E, Baibich M N, Saitovitch E B and Geshev J 2006 *Phys. B: Condens. Matter* **384** 132
[23] Mejia-Lopez J and Ramírez R 2002 *J. Magn. Magn. Mater.* **241** 364
[24] Kiwi M, Mejía-López J, Portugal R D and Ramírez R 1999 *Europphys. Lett.* **48** 573
[25] Morosov A I and Sigov A S 2003 *J. Magn. Magn. Mater.* **258** 388
[26] Leighton C, Nogués J, Suhl H and Schuller I K 1999 *Phys. Rev. B* **60** 12837
[27] Romanens F, Pizzini S, Yokaichiya F, Bonfim M, Pennec Y, Camarero J, Vogel J, Sort J, Garcia F, Rodmacq B and Dieny B 2005 *Phys. Rev. B* **72** 134410
[28] Arenholz E, Liu K, Li Z and Schuller I K 2006 *Appl. Phys. Lett.* **88** 072503
[29] Yamato T, Kume T, Kato T, Tsunashima S, Nakamura T, Fujiwara Y and Iwata S 2006 *J. Appl. Phys.* **100** 113907
[30] Paul A, Kentzinger E, Rücker U and Brückel T 2006 *Phys. Rev. B* **74** 054424
[31] Guo Z B, Wu Y H, Qiu J J, Zong B Y and Han G C 2008 *Phys. Rev. B* **78** 184413
[32] Parkin S S P, Deline V R, Hilleke R O and Felcher G P 1990 *Phys. Rev. B* **42** 10583
[33] Tsang C, Heiman N and Lee K 1981 *J. Appl. Phys.* **52** 2471
[34] Zhang W 2005 MSc Thesis (Nanjing: Southeast University)
[35] Mauri D, Kay E, Scholl D and Howard J K 1987 *J. Appl. Phys.* **62** 2929
[36] Zijlstra H 1979 *IEEE. Trans. Magn.* **15** 1246
[37] Shirobokov M Y 1939 *Bulletins of the Russian Academy of Sciences: Physics* **24** 426
[38] Shirobokov M Y 1945 *J. Exp. Theor. Phys.* **15** 57
[39] Aharoni A 1993 *Phys. Rev. B* **47** 8296
[40] Moyerman S, Gannett W, Borchers J, Doucet M, Carey M, Sparks P, Eckert J 2006 *IEEE Trans. Magn.* **42** 2630
[41] Hubert A and Schafer R 1998 *Magnetic Domains — the Analysis of Magnetic Microstructures* (New York: Springer) p. 219 ISBN 978-3-540-64108-7
[42] Lilley B A 1950 *Philos. Mag.* **41** 792
[43] Geshev J 2000 *Phys. Rev. B* **62** 5627
[44] Berkowitz A E and Takano K 1999 *J. Magn. Magn. Mater.* **200** 552
[45] Zhao G P, Zhao M G, Lim H S, Feng Y P and Ong C K 2005 *Appl. Phys. Lett.* **87** 162513
[46] Zhao G P, Deng Y, Zhang H W, Chen L, Feng Y P and Bo N 2010 *J. Appl. Phys.* **108** 093928