High-dimensional quantum state transfer in a noisy network environment*

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We propose and analyze an efficient high-dimensional quantum state transfer protocol in an XX coupling spin network with a hypercube structure or chain structure. Under free spin wave approximation, unitary evolution results in a perfect high-dimensional quantum swap operation requiring neither external manipulation nor weak coupling. Evolution time is independent of either distance between registers or dimensions of sent states, which can improve the computational efficiency. In the low temperature regime and thermodynamic limit, the decoherence caused by a noisy environment is studied with a model of an antiferromagnetic spin bath coupled to quantum channels via an Ising-type interaction. It is found that while the decoherence reduces the fidelity of state transfer, increasing intra-channel coupling can strongly suppress such an effect. These observations demonstrate the robustness of the proposed scheme.

Keywords: quantum state transfer, quantum spin, hypercubes, spin wave

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1. Introduction

Quantum state transfer (QST) between two remote parties is an essential ingredient of scalable quantum information processing. Quantum channels enabling universal operations between two physically separated registers have been proposed in a variety of quantum systems, including superconducting transmission lines,[1] Coulomb coupling trapped ions,[2] and optical photons.[3,4] Multi-mode entangled photon states, which are useful for quantum networks, can be generated efficiently using an integrated photonic circuit.[5] In particular, coupled quantum spin systems have attracted much attention for short distance quantum communication in recent decades.[6–16] Such coupled-spin quantum channels provide an interesting alternative to direct register interactions and interfacing with photonic flying qubits. For this reason, most of the feasible proposals have been proposed for perfect QST, in which the quantum channels commonly rely upon engineered couplings entailing suitable dispersion relations,[17–20] weak couplings undergoing an effective Rabi oscillation or a ballistic regime,[21–24] and dynamical manipulation.[25–28]

In contrast to two-dimensional quantum systems, high-dimensional quantum systems serving as qudits have been extensively studied due to their key advantages in large capacity and high security. While such advantages have been explored in quantum communications ranging from quantum key distribution to quantum teleportation,[29–39] qudit systems are also candidates for quantum computation ranging from universal quantum simulations and asymptotically optimal quantum circuits to one-way quantum computation and fault-tolerant quantum computation.[40–53] Indeed, several high-dimensional QST protocols have been presented in coupled-spin chains.[54–57]

Similar to classical supercomputers involving parallel processing,[58] hypercube topology can potentially allow diverse applications in QST,[17,59–61] quantum random walks[62–65] and quantum search algorithms.[66–69] A hypercube is a generalization from a three-dimensional cube to a high-dimensional configuration, and it possesses many appealing topological features, e.g., node-symmetry, good balance between nodes and edges.[58] Prior state transfer schemes in hypercubes have focused on qubits using either spins,[17] single-photons,[60] or multiphoton states.[61] In this article, we propose an efficient high-dimensional QST in XX coupling spin hypercube networks. The hypercubes are multiple Cartesian products of a chain of either two or three spins, which works as building blocks of these topologies.[70] Upon performing free spin wave approximation, unitary evolution for a specific time results in a perfect high-dimensional swap gate and evolution time is independent of either the distance between two registers or the dimension of the sent state.[61] It require neither weak coupling nor projective measurement nor external modulation nor coupling engineering. In addition, utilizing the Schwinger picture yields a perfect mirror high-

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dimensional swap operation in a coupled-spin chain of arbitrary length. Numerical results confirm that the spin-wave interaction leads to the leakage of quantum information, and under the free spin wave approximation, the perfect high-dimensional QST is achieved.

The proposed efficient high-dimensional QST occurs in a closed quantum channel. However, a quantum channel can rarely be isolated from its surrounding environment, especially the spin bath. Thus it is necessary to discuss the decoherence effects on such protocols. The considered decoherence is characterized by a pure dephasing model for quantum channels coupled to an antiferromagnetic (AFM) spin bath via a typical Ising interaction,[71–76] allowing for the conservation of channel energy. In a low temperature regime, unitary evolution analytically gives a swap gate experiencing decoherence. To be specific, we perform numerical simulations when the spin bath is in a thermal equilibrium state. We find that while increasing either the spin bath temperature or the bath-channel coupling enhances the irreversible leakage of quantum information into the spin bath, strong intrachannel coupling can depress the decoherence effects to ensure the high fidelity of state transfer. Observing these can help us to understand the decoherence effects on quantum communication in coupled quantum spin systems.

The rest of this paper is organized as follows. In Section 2, we calculate the Hamiltonian and the operator evolution under free spin wave approximation. In Section 3, we study the high-dimensional QST in hypercubes and chains, respectively. In Section 4, specifically, the decoherence of the spin bath in the thermal equilibrium state is studied. The last section is devoted to a summary.

2. Model and calculation

The composite system under consideration consists of a quantum spin-S_0 network which acts as a quantum channel, and an AFM spin-S bath whose lattice is divided into two identical sublattices a and b. The network Hamiltonian with spins coupled to their nearest neighbors on a finite lattice of site N_0 is of XX coupling

$$H_{\mathrm{S}} = \sum_{u,v=1}^{N_0} K_{uv} S_{0,u}^{-} S_{0,v}^{+}, \tag{1}$$

where K is an $N_0 \times N_0$ coupling matrix and K_{uv} represents the coupling strength between sites u and v, $S^{\pm} = S^x \pm \mathrm{i}S^y$ are two ladder operators and S^{μ} $(\mu = x, y, z)$ is the μ component of a spin operator $\boldsymbol{S}$. The AFM spin bath Hamiltonian is described by a Heisenberg model

$$H_B = \frac{J}{2} \sum_{\boldsymbol{i},\boldsymbol{\delta}} \boldsymbol{S}_{a,\boldsymbol{i}} \cdot \boldsymbol{S}_{b,\boldsymbol{i}+\boldsymbol{\delta}} + \frac{J}{2} \sum_{\boldsymbol{j},\boldsymbol{\delta}} \boldsymbol{S}_{b,\boldsymbol{j}} \cdot \boldsymbol{S}_{a,\boldsymbol{j}+\boldsymbol{\delta}}, \tag{2}$$

where $J > 0$ is the exchange interaction, $\boldsymbol{\delta}$ is a vector joining a site to its nearest neighbor, $\boldsymbol{S}_a$ and $\boldsymbol{S}_b$ are spin operators in the sublattices a and b, respectively. The interaction Hamiltonian between the network and the spin bath is characterized by a typical Ising model[73]

$$H_{\mathrm{I}} = -\frac{J_0}{\sqrt{N}} \sum_{u=1}^{N_0} S_{0,u}^{z} \otimes \sum_{\boldsymbol{i}} \left(S_{a,\boldsymbol{i}}^{z} + S_{b,\boldsymbol{i}}^{z} \right), \tag{3}$$

with the coupling strength J_0 and the number of spins N in each magnetic sublattice. The dynamics of a composite system is governed by the total Hamiltonian $H = H_{\mathrm{S}} + H_{\mathrm{B}} + H_{\mathrm{I}}$, and upon introducing the Holstein–Primakoff (HP) transformation,[77] we have

$$\begin{aligned} S_{a,i}^{-} &= a_i^{\dagger} \sqrt{2S - a_i^{\dagger} a_i}, \quad S_{a,i}^{z} = S - a_i^{\dagger} a_i, \\ S_{b,j}^{-} &= \sqrt{2S - b_j^{\dagger} b_j}\, b_j, \quad S_{b,j}^{z} = b_j^{\dagger} b_j - S, \\ S_{0,u}^{-} &= c_u^{\dagger} \sqrt{2S_0 - c_u^{\dagger} c_u}, \quad S_{0,u}^{z} = S_0 - c_u^{\dagger} c_u, \end{aligned} \tag{4}$$

the total Hamiltonian can be expressed in terms of boson operators, wherein the conservation of total spin z projection of channel, $\left[\sum_{u=1}^{N_0} S_{0,u}^{z}, H\right] = 0$, becomes conservation of the boson number, $\left[\sum_{u=1}^{N_0} c_u^{\dagger} c_u, H\right] = 0$.

The network is initialized to a simple ferromagnetic order with spins aligning in a parallel way. The low-lying level states of a sender, ranging from $|0\rangle$ to $|d-1\rangle$, are employed to encode quantum information as an input state, and we assume that the dimension d of the sent state is much smaller than $2S_0$, $d \ll 2S_0$. The conservation law ensures that $\langle c_u^{\dagger} c_u \rangle \ll 2S_0$, and the HP transformation is simplified into $S_{0,u}^{-} = (2S_0)^{1/2} c_u^{\dagger}$, which leads to

$$H_{\mathrm{S}} = 2S_0 \sum_{u,v=1}^{N_0} K_{uv} c_u^{\dagger} c_v. \tag{5}$$

The subsequent diagonalization of this tight-binding Hamiltonian is realized through an orthogonal transformation Q, such that $\Lambda = QKQ^{\dagger}$ with $\Lambda_{qq'} = \lambda_q \delta_{qq'}$. This transformation results in $H_{\mathrm{S}} = \sum_{q=1}^{N_0} \varepsilon_q c_q^{\dagger} c_q$, where $c_q = \sum_{u=1}^{N_0} Q_{qu} c_u$ and $\varepsilon_q = 2S_0 \lambda_q$.

The spin bath surrounding the quantum channel is in the low-temperature and low-excitation limit such that the spin operators can be approximated as $S_{a,\boldsymbol{i}}^{-} = (2S)^{1/2} a_{\boldsymbol{i}}^{\dagger}$ and $S_{b,\boldsymbol{j}}^{-} = (2S)^{1/2} b_{\boldsymbol{j}}$.[78] Working with Fourier transformation and Bogoliubov transformation yields

$$H_{\mathrm{B}} = E_0 + \sum_{\boldsymbol{k}} \omega_{\boldsymbol{k}} \left\{ \left(\alpha_{\boldsymbol{k}}^{\dagger} \alpha_{\boldsymbol{k}} + \frac{1}{2} \right) + \left(\beta_{\boldsymbol{k}}^{\dagger} \beta_{\boldsymbol{k}} + \frac{1}{2} \right) \right\}, \tag{6}$$

where $\alpha_{\boldsymbol{k}}$ and $\beta_{\boldsymbol{k}}$ are two degenerate AFM mangnon branches, and $E_0 = -z_0 JNS(S+1)$. The spin-wave elementary excitation spectrum (EES) is

$$\omega_{\boldsymbol{k}} = z_0 JS \sqrt{1 - \gamma_{\boldsymbol{k}}^2}, \tag{7}$$

wherein z_0 is the number of the nearest neighbors, and $\gamma_{\boldsymbol{k}} = z_0^{-1}\sum_{\boldsymbol{\delta}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{\delta}}$ is the structure factor. In the low-energy regime, corresponding to long wavelengths, $k \ll l$, the EES is linear: $\omega_k = (2z_0)^{1/2} JSkl$, with the lattice constant l. The interaction Hamiltonian is likewise transformed to

$$H_{\mathrm{I}} = -\frac{J_0}{\sqrt{N}} \sum_{q=1}^{N_0} (S_0 - c_q^{\dagger} c_q) \otimes \sum_{\boldsymbol{k}} \left(\beta_{\boldsymbol{k}}^{\dagger}\beta_{\boldsymbol{k}} - \alpha_{\boldsymbol{k}}^{\dagger}\alpha_{\boldsymbol{k}}\right). \tag{8}$$

The Heisenberg equation drives an operator evolution in the Heisenberg picture, $c_m^{\dagger}(t) = \mathrm{e}^{\mathrm{i}Ht} c_m^{\dagger}(0)\, \mathrm{e}^{-\mathrm{i}Ht}$. By using this equation and the commutation relation $[H_{\mathrm{I}}, H] = 0$, we have

$$c_m^{\dagger}(t) = \mathrm{e}^{\mathrm{i}Y(t)} \tilde{c}_m^{\dagger}(t). \tag{9}$$

Here, the quantum noise operator $Y(t)$ reads

$$Y(t) = \frac{J_0 t}{\sqrt{N}} \sum_{\boldsymbol{k}} \left(\beta_{\boldsymbol{k}}^{\dagger}\beta_{\boldsymbol{k}} - \alpha_{\boldsymbol{k}}^{\dagger}\alpha_{\boldsymbol{k}}\right), \tag{10}$$

which arises from the AFM spin bath, and $\tilde{c}_m^{\dagger}(t) = \mathrm{e}^{\mathrm{i}H_S t} c_m^{\dagger}(0)\, \mathrm{e}^{-\mathrm{i}H_S t}$ is the free evolution of the network. Owing to $c_q^{\dagger}(t) = \mathrm{e}^{\mathrm{i}\varepsilon_q t} c_q^{\dagger}$, and

$$(\mathrm{e}^{\mathrm{i}2S_0 Kt})_{um} = \sum_{q=1}^{N_0} Q_{uq}^{\dagger}\, \mathrm{e}^{\mathrm{i}\varepsilon_q t} Q_{qm}, \tag{11}$$

$\tilde{c}_m^{\dagger}(t)$ turns out to be

$$\tilde{c}_m^{\dagger}(t) = \sum_{u=1}^{N_0} (\mathrm{e}^{\mathrm{i}2S_0 Kt})_{um} c_u^{\dagger}, \tag{12}$$

and the free evolution of the network is determined by its coupling matrix.

3. Perfect high-dimensional quantum state transfer

In this section, high-dimensional QST protocols with high fidelity in hypercubes and chains are proposed, respectively. We begin with the case of hypercubes, which are multiple Cartesian products of either two-spin chain G_1 or three-spin chain G_2 working as basic building blocks to build such coupled-spin hypercubes. The coupling matrix of a hypercube network G in Eq. (1) is $K = \kappa A(G)$, and κ is the coupling strength between two spins in the network, which is characterized by its adjacency matrix $A(G)$.[70]

After g-fold Cartesian products of either of the two simple chains, $A(G)$ is given by

$$A(G) = \sum_{j=0}^{g-1} I^{\otimes j} \otimes A(G_\theta) \otimes I^{\otimes(g-j-1)}, \tag{13}$$

where I is an identity matrix, and $A(G_\theta)$ is an adjacency matrix of G_θ for $\theta = 1, 2$. As a consequence,

$$\mathrm{e}^{\mathrm{i}2S_0 Kt} = [\mathrm{e}^{\mathrm{i}2S_0 \kappa A(G_\theta) t}]^{\otimes g}, \tag{14}$$

and it results in

$$(\mathrm{e}^{\mathrm{i}2S_0 K\tau_\theta})_{um} = \mathrm{i}^{\theta g}\delta_{u,N+1-m} \tag{15}$$

at the following evolution time

$$\tau_\theta = \pi/(2^{1+1/\theta} S_0 \kappa). \tag{16}$$

Substituting Eq. (15) into Eq. (12) leads to

$$\tilde{c}_m^{\dagger}(\tau_\theta) = \mathrm{i}^{\theta g} c_{N+1-m}^{\dagger}. \tag{17}$$

Upon absorbing a phase factor of $\mathrm{i}^{\theta g}$ into $c_{N+1-m}^{\dagger}$, unitary evolution under H_S for a time τ_θ results in $\tilde{c}_m^{\dagger}(\tau_\theta) = c_{N+1-m}^{\dagger}$, which allows the realization of a swap operation between two spins m and $N+1-m$. It is determined by the natural dynamics of the Hamiltonian and requires neither external modulation nor inter-spin coupling strength engineering. Moreover, the evolution time is independent of the distance between the two remote spins, and the speedup of perfect high-dimensional QST is possible. For simplicity, we take the square and cube networks for example, as shown in Figs. 1(a) and 1(b), which are the two-fold and three-fold Cartesian products of G_1, respectively; the time evolution builds a swap gate between two spins along each main diagonal at the optimal time.

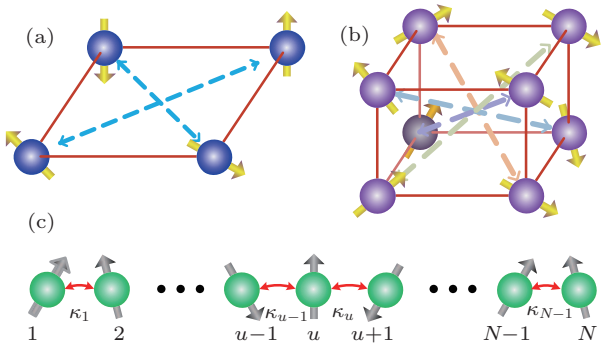


Fig. 1. (color online) Schematic configurations for two-fold (a) and three-fold (b) Cartesian products of a two-spin chain with uniform coupling strength κ. Unitary evolution for a time $\tau_1 = \pi/(4S_0\kappa)$ results in a high-dimensional swap operation between two spins along each main diagonal. In a coupled-spin chain of length N (c), unitary evolution for a time $\tau' = \pi/(2S_0 g_0)$ enables a mirror-like high-dimensional swap operation with respect to its center through engineering inter-spin coupling strengths $\kappa_u = g_0\sqrt{u(N-u)}$.

While the case of a multi-dimensional hypercube has been chosen to focus on, we extend such results to a one-dimensional coupled-spin system. By choosing

$$K_{uv} = \kappa_{u-1}\delta_{u,v+1} + \kappa_u \delta_{u,v-1}, \tag{18}$$

in the Hamiltonian of Eq. (1), such that it provides a Hamiltonian of a coupled-spin chain with coupling strengths κ_u between two spins u and $u+1$ as shown in Fig. 1(c). The coupling matrix is identical to a pseudo Hamiltonian $H' = g_0 L_x$, where L_x is the x component of a fictitious spin $L = (N_0 - 1)/2$, and g_0 is a coupling parameter. In the Schwinger

picture, L_x can be rewritten in terms of two boson operators as[79]

$$L_x = \frac{1}{2}\left(l_1^\dagger l_2 + l_1 l_2^\dagger\right), \tag{19}$$

and hence, H' is the Hamiltonian governing two bosons with coupling strength $g_0/2$. Unitary evolution under $2S_0H'$ for a time $\tau' = \pi/(2S_0 g_0)$ gives $l_1^\dagger(\tau') = \mathrm{i}l_2^\dagger$ and $l_2^\dagger(\tau') = \mathrm{i}l_1^\dagger$, yielding

$$(\mathrm{e}^{\mathrm{i}2S_0K\tau'})_{um} = \mathrm{i}^{N_0-1}\delta_{u,N+1-m}. \tag{20}$$

Therefore, $\tilde{c}_m(\tau')$ is evaluated to be

$$\tilde{c}_m(\tau') = \mathrm{i}^{N_0-1}c_{N_0+1-m}^\dagger. \tag{21}$$

Upon absorbing a factor of i^{N_0-1} into $c_{N_0+1-m}^\dagger$ as desired, it enables a swap operation between spins m and N_0+1-m, $\tilde{c}_m(\tau') = c_{N_0+1-m}^\dagger$, and a mirror inversion of quantum states with respect to the center of the chain is implemented.

To confirm the perfect high-dimensional QST, we perform a numerical calculation, as shown in Fig. 2. The initial state of the network is assumed to be

$$|\Psi\rangle_{\mathrm{ini}} = |\varphi\rangle_m \otimes |0\rangle_{\bar{m}}, \tag{22}$$

where

$$|\varphi\rangle_m = \sum_{v=0}^{d-1} C_v |v\rangle$$

is the sent state at the spin m, and $|0\rangle_{\bar{m}}$ represents that all spins are in the vacuum state apart from just one spin at the site m. The normalized coefficients C_v can be measured by the Hurwitz parametrization with $d-1$ polar angles χ_p and $d-1$ azimuthal angles ϑ_p as[80,81]

$$\left(\cos\vartheta_{d-1},\ \sin\vartheta_{d-1}\cos\vartheta_{d-2}\,\mathrm{e}^{\mathrm{i}\chi_{d-1}}, \sin\vartheta_{d-1}\sin\vartheta_{d-2}\right.$$
$$\left.\cos\vartheta_{d-3}\,\mathrm{e}^{\mathrm{i}\chi_{d-2}}, \cdots, \prod_{i=1}^{d-1}\sin\vartheta_i\,\mathrm{e}^{\mathrm{i}\chi_1}\right), \tag{23}$$

where $0 \le \vartheta_p \le \pi/2$ and $0 \le \chi_p < 2\pi$ for $p = 1, 2, \ldots, d-1$. The average fidelity over the whole manifold of such states is

$$\langle F(t)\rangle = \frac{1}{V_d}\int_{CP^{d-1}} F(t)\,\mathrm{d}V. \tag{24}$$

Here, $F(t)$ is defined as $F(t) = {}_1\langle\varphi|\rho_m(t)|\varphi\rangle_1$ with $\rho_m(t)$ being the reduced density matrix of spin m at the evolution time t. The volume element on the generalized Bloch sphere of Eq. (23) in the complex space CP^{d-1} is

$$\mathrm{d}V = \prod_{p=1}^{d-1}\cos\vartheta_p(\sin\vartheta_p)^{2p-1}d\vartheta_p\,\mathrm{d}\chi_p, \tag{25}$$

and the total volume of the $2(d-1)$-dimensional manifold of pure states is $V_d = \pi^{d-1}/(d-1)$.

The average fidelity varies with spin number S_0 as shown in Fig. 2. The infidelity, $\varepsilon = 1 - \langle F\rangle$, results from the leakage of quantum information into the spin wave interaction found in nonlinear terms of HP transformation. In such a regime $2S_0 \sim d$, the evolution of the quantum channel is dominated by the spin wave interaction suppressing the average fidelity. By ensuring $2S_0/d \ll 1$, the spin wave interaction is negligible and the evolution is governed by the free spin wave found in linear terms of HP transformation, enabling a perfect high-dimensional QST.

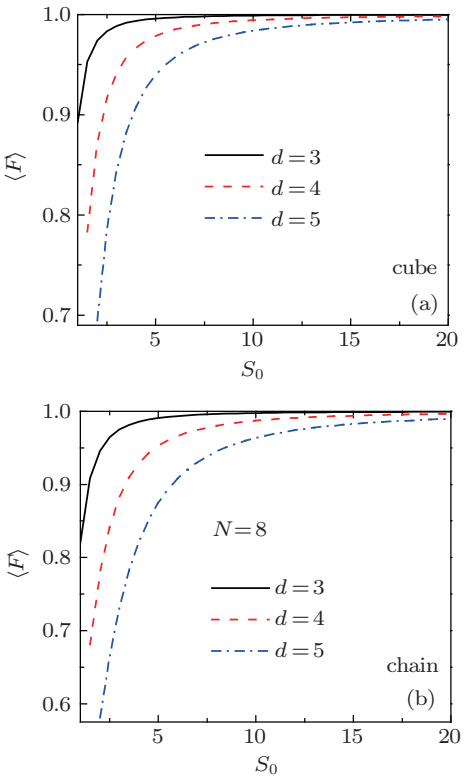


Fig. 2. (color online) Variations of average fidelity $\langle F\rangle$ with quantum spin number for either $d = 3$ (solid lines), $d = 4$ (dashed lines), or $d = 5$ (dotted–dashed lines). (a) Coupled-spin cube; (b) coupled-spin chain of length $N = 8$. Here, the evolution time is the optimal time for the two different configurations.

4. Spin bath in the thermal equilibrium state

To be specific, we consider a spin bath in a thermal equilibrium state, wherein its variables are distributed in an uncorrelated thermal equilibrium mixture of states, and the density matrix satisfies the Boltzmann distribution

$$\rho_{\mathrm{B}} = \frac{1}{Z}\mathrm{e}^{-H_{\mathrm{B}}/T}, \tag{26}$$

where $Z = \mathrm{Tr}\left(\mathrm{e}^{-H_{\mathrm{B}}/T}\right)$ is a partial function and T represents the temperature. The combination of Eq. (26) and Eq. (22) yields the initial state of the composite system, including quantum channel and spin bath, as a direct product

$$\rho(0) = (|\varphi\rangle\langle\varphi|)_m \otimes (|0\rangle\langle 0|)_{\bar{m}} \otimes \rho_{\mathrm{B}}(0). \tag{27}$$

Evolution for time τ' drives the state to

$$\rho(\tau') = \mathrm{e}^{-\mathrm{i}H\tau'}\rho(0)\,\mathrm{e}^{\mathrm{i}H\tau'},$$

and the reduced density matrix of the quantum channel is $\rho_{\mathrm{S}}(\tau') = \mathrm{Tr}_{\mathrm{B}}[\rho(\tau')]$. Without loss of generality, we choose

a couple-spin chain, which is directly analogous to the hypercube quantum channel. Upon utilizing $|\nu\rangle = \left(c^{\dagger}\right)^{\nu}/\sqrt{\nu!}|0\rangle$ and $\mathrm{e}^{-\mathrm{i}H\tau'}c^{\dagger}(0)\,\mathrm{e}^{\mathrm{i}H\tau'} = c^{\dagger}(-\tau')$, the finial state of spin N_0+1-m is calculated as

$$\rho_{N_0+1-m}\left(\tau'\right) = \sum_{\nu,\nu'=0}^{d-1} \rho_{\nu\nu'}\left(\tau'\right) D_{\nu\nu'}\left(T\right), \tag{28}$$

with $\rho_{\nu\nu'}(\tau') = C_{\nu}C^{*}_{\nu'}|\nu\rangle\langle\nu'|$, and a decoherence factor $D_{\nu\nu'}(T)$ given by

$$D_{\nu\nu'}\left(T\right) = \frac{1}{Z}\mathrm{Tr}_{\mathrm{B}}[\mathrm{e}^{-i\left(\nu-\nu'\right)Y\left(\tau'\right)-H_{\mathrm{B}}/T}], \tag{29}$$

which turns out to be $D_{\nu\nu'}(T) = r^{+}_{\nu\nu'}r^{-}_{\nu\nu'}/r_0^2$. Therein,

$$r_0 = \prod_{\boldsymbol{k}} \frac{1}{1-\mathrm{e}^{-\omega_{\boldsymbol{k}}/T}}, \tag{30}$$

and

$$r^{\pm}_{\nu\nu'} = \prod_{\boldsymbol{k}} \frac{1}{1-\mathrm{e}^{\pm\phi_{\nu\nu'}}\mathrm{e}^{-\omega_{\boldsymbol{k}}/T}}, \tag{31}$$

where $\phi_{\nu\nu'} = (\nu-\nu')J_0\tau' N^{-1/2}$. Upon introducing quantities

$$\xi_{\pm} = \frac{f_{\pm}\left(\phi_{\nu\nu'}\right)}{\phi^2_{\nu\nu'}}, \tag{32}$$

and

$$f_{\pm}\left(\phi_{\nu\nu'}\right) = \int_0^{\infty} x^2 \ln \frac{1-\mathrm{e}^{\pm\mathrm{i}\phi_{\nu\nu'}}\mathrm{e}^{-\omega_{\boldsymbol{k}}/T}}{1-\mathrm{e}^{-\omega_{\boldsymbol{k}}/T}}\mathrm{d}x, \tag{33}$$

with $x = kl$, the decoherence factor is transformed to

$$D_{\nu\nu'}\left(T\right) = \mathrm{e}^{-\left(\xi_{+}+\xi_{-}\right)\left(\nu-\nu'\right)^2 J_0^2\tau'^2/\left(2\pi^2\right)}, \tag{34}$$

where the summation over $\boldsymbol{k}$ has been replaced by an integral

$$\sum_{\boldsymbol{k}} = \frac{V}{(2\pi)^3}\int \mathrm{d}k 4\pi k^2, \tag{35}$$

with $N = V/l^3$. In the thermodynamic limit, $N\to\infty$, which leads to $\xi_0 = \xi_{\pm}$, and

$$\xi_0 = \frac{\pi^2 T^3}{12 z_0\sqrt{2z_0}J^3 S^3}. \tag{36}$$

Thus the decoherence factor becomes

$$D_{\nu\nu'}\left(T\right) = \mathrm{e}^{-\left(\nu-\nu'\right)^2\tau'^2/\tau_c^2}, \tag{37}$$

with a decoherence time

$$\tau_c = \frac{\pi}{J_0\sqrt{\xi_0}}. \tag{38}$$

The combination of the fidelities of spins m and N_0+1-m is

$$F\left(\tau'\right) = \sum_{\nu,\nu'=0}^{d-1} |C_{\nu}|^2|C_{\nu'}|^2 D_{\nu\nu'}\left(T\right). \tag{39}$$

To illustrate the decoherence effects of spin bath on the quantum channels, we perform the numerical simulation as shown in Fig. 3.

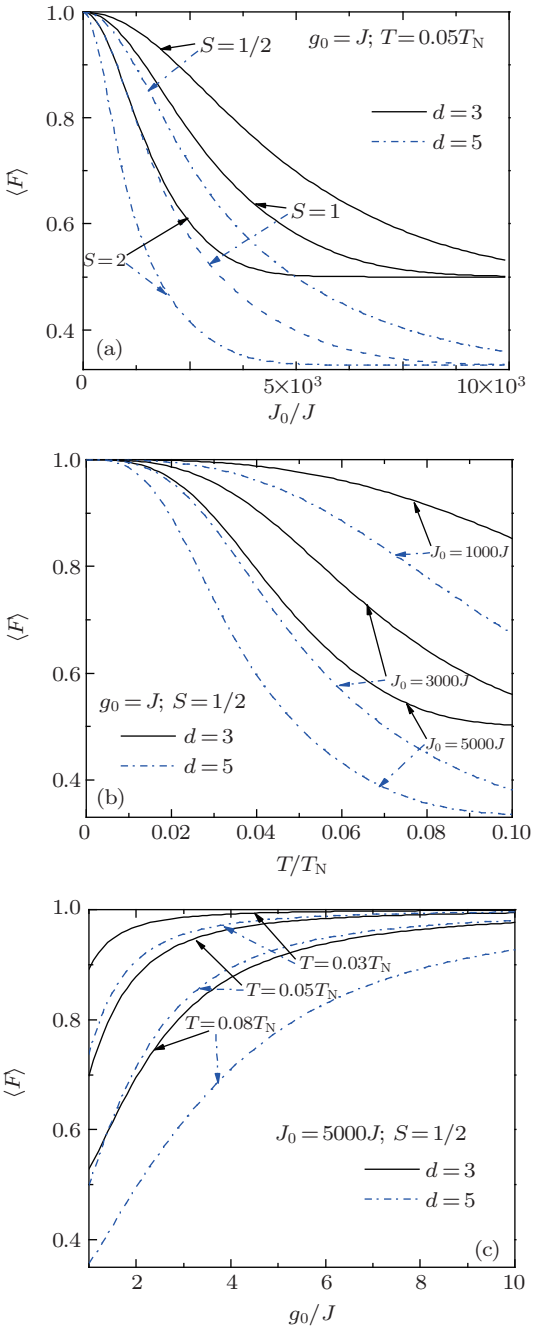


Fig. 3. (color online) Decoherence effects on the high-dimensional QST proposal in a coupled-spin chain with either $d = 3$ (solid lines), or $d = 5$ (dotted–dashed lines). The average fidelity as functions of (a) J_0 with $g_0 = J$ and $T = 0.05T_{\mathrm{N}}$, (b) T with $g_0 = J$ and $S = 1/2$, and (c) g_0 with $J_0 = 5000J$ and $S = 1/2$.

Under the random phase approximation, the Néel temperature of an AFM system reads[78]

$$T_{\mathrm{N}} = \frac{z_0 JS\left(S+1\right)}{3\zeta}, \tag{40}$$

with $\zeta = N^{-1}\sum_{\boldsymbol{k}}\left(1-\gamma_{\boldsymbol{k}}\right)^{-1}$. The summation over $\boldsymbol{k}$ is restricted in the first Brillouin zone of the sublattice, such that it has half the volume of an atomic Brillouin zone, yielding $\zeta = 1.51638$ for a simple cubic lattice and $\zeta = 1.39320$ for a body-center cubic lattice. Owing to the validity of spin wave

theory only in the low temperature regime, the spin bath temperature is restricted below $0.1T_{\rm N}$, $T \leq 0.1T_{\rm N}$. The average fidelity varies with either coupling strength, J_0, or spin bath temperature, T, as shown in Figs. 3(a) and 3(b). The infidelity, ε, results from the leakage of quantum information into the spin bath. Increasing either J_0 or T enhances such an irreversible process. Figure 3(c) shows the variations of average fidelity with coupling parameter, g_0. A strong coupling parameter can partly counteract the thermal effects to prevent the quantum information from leaking into the spin bath and ensure the high fidelity.

5. Summary

In this paper, we study a high-dimensional QST protocol in spin networks of either hypercubes or chains coupled to an antiferromagentic spin bath. Under the free spin wave approximation and in ideal conditions without decoherence, the time evolution presents a perfect high-dimensional QST between two registers of arbitrary distance for both of the two configurations, requiring neither weak coupling nor external modulation nor projective measurement. The essence of our method is that a perfect swap operation is allowable for a chain of either two or three bosons. One of its direct applications is that it can potentially provide the high-dimensional entanglement distribution for quantum computation. Moreover, the state transfer independent of either the distance between two registers or the dimension of sent state enables the enhancement of computational efficiency. In the low temperature regime and thermodynamic limit, the decoherence induced by the AFM spin bath is also investigated in this work. Increasing either the spin bath temperature or the channel-bath coupling, the decoherence becomes more serious. However, strong intrachannel coupling can partly counteract the decoherence effects, and ensure the high fidelity to demonstrate robustness against external noise. Observing these will deepen our understanding of the decoherence effects on quantum communication in the coupled quantum spin systems.

While we have focused on the specific case of a hypercube topology, the conceptual framework can be used in a wide range of topologies by means of the free spin wave approximation to yield the tight-binding Hamiltonian described in Eq. (5). By ensuring $(\mathrm{e}^{\mathrm{i}2S_0K\tau})_{um} = \delta_{u,N_0+1-m}$ for a specific evolution time τ, the achievement of a perfect high-dimensional QST is possible.

References

[1] Sillanpää M A, Park J I and Simmonds R W 2007 *Nature* **449** 438
[2] Schmidt-Kaler F, Häffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 *Nature* **422** 408
[3] Blinov B B, Moehring D L, Duan L M and Monroe C 2004 *Nature* **428** 153
[4] Dong H, Xu D Z, Huang J F and Sun C P 2012 *Light Sci. Appl.* **1** e2
[5] Heilmann R, Gräfe M, Nolte S and Szameit A 2015 *Sci. Bull.* **60** 96
[6] Bose S 2003 *Phys. Rev. Lett.* **91** 207901
[7] Cubitt T S and Cirac J I 2008 *Phys. Rev. Lett.* **100** 180406
[8] Godsil C, Kirkland S, Severini S and Smith J 2012 *Phys. Rev. Lett.* **109** 050502
[9] Yao N Y, Laumann C R, Gorshkov A V, Weimer H, Jiang L, Cirac J I, Zoller P and Lukin M D 2013 *Nat. Commun.* **4** 1585
[10] Ping Y, Lovett B W, Benjamin S C and Gauger E M 2013 *Phys. Rev. Lett.* **110** 100503
[11] Wu N, Nanduri A and Rabitz H 2014 *Phys. Rev. A* **89** 062105
[12] Korzekwa K, Machnikowski P and Horodecki P 2014 *Phys. Rev. A* **89** 062301
[13] Xiang J D, Qin L G and Tian L J 2014 *Chin. Phys. B* **23** 110305
[14] Zhang A P and Li F L 2013 *Chin. Phys. B* **22** 030308
[15] Wang T J, Lu Y and Long G L 2012 *Phys Rev A* **86** 042337
[16] Hong C H, Heo J, Lim J I and Yang H J 2014 *Chin. Phys. B* **23** 090309
[17] Christandl M, Datta N, Ekert A and Landahl A J 2004 *Phys. Rev. Lett.* **92** 187902
[18] Zhang J, Long G L, Zhang W, Deng Z, Liu W and Lu Z 2005 *Phys. Rev. A* **72** 012331
[19] Vinet L and Zhedanov A 2012 *Phys. Rev. A* **85** 012323
[20] Ajoy A and Cappellaro P 2013 *Phys. Rev. Lett.* **110** 220503
[21] Wójcik A, Łuczak T, Kurzyński P, Grudka A, Gdala T and Bednarska M 2005 *Phys. Rev. A* **72** 034303
[22] Banchi L, Apollparo T J G, Cuccoli A, Vaia R and Verrucchi P 2010 *Phys. Rev. A* **82** 052321
[23] Apollparo T J G, Banchi L, Cuccoli A, Vaia R and Verrucchi P 2012 *Phys. Rev. A* **85** 052319
[24] Yao N Y, Jiang L, Gorshkov A V, Gong Z X, Zhai A, Duan L M and Lukin M D 2011 *Phys. Rev. Lett.* **106** 040505
[25] Shi T, Li Y, Song Z and Sun C P 2005 *Phys. Rev. A* **71** 032309
[26] Fitzsimons J and Twamley J 2006 *Phys. Rev. Lett.* **97** 090502
[27] Cappellaro P, Ramanathan C and Cory D G 2007 *Phys. Rev. Lett.* **99** 250506
[28] Giorgi G L and Busch T 2013 *Phys. Rev. A* **88** 062309
[29] Bechmann-Pasquinucci H and Tittel W 2000 *Phys. Rev. A* **61** 062308
[30] Karimipour V, Bahraminasab A and Bagherinezhad S 2002 *Phys. Rev. A* **65** 052331
[31] Mafu M, Dudley A, Goyal S, Giovannini D, McLaren M, Padgett M J, Konrad T, Petruccione F, Lütkenhaus N and Forbes A 2013 *Phys. Rev. A.* **88** 032305
[32] Molina-Terriza G, Vaziri A, Ursin R and Zeilinger A 2005 *Phys. Rev. Lett.* **94** 040501
[33] Goyal S K, Boukama-Dzoussi P E, Ghosh S, Roux F S and Konrad T 2014 *Sci. Rep.* **4** 4543
[34] Wang C, Deng F G, Li Y S, Liu X S and Long G L 2005 *Phys. Rev. A* **71** 044305
[35] Jiang M, Huang X, Zhou L L, Zhou Y M and Zeng J 2012 *Chin. Sci. Bull.* **57** 2247
[36] Zou X F and Qiu D W 2014 *Sci. China-Phys. Mech. Astron.* **57** 1696
[37] Zheng C and Long G F 2014 *Sci. China-Phys. Mech. Astron.* **57** 1238
[38] Wang W Y, Wang C, Zhang G Y and Long G L 2009 *Chin. Sci. Bull.* **54** 158
[39] Deng F G, Zhou H Y and Long G L 2006 *J. Phys. A: Math Gen.* **39** 14089
[40] Lu Y, Feng G R, Li Y S and Long G L 2015 *Sci. Bull.* **60** 241
[41] Zhang C, Li C F and Guo G C 2015 *Sci. Bull.* **60** 249
[42] Gottesman D 1999 *Chaos, Solitons and Fractals* **10** 1749
[43] Cafaro C, Maiolini F and Mancini S 2012 *Phys. Rev. A* **86** 022308
[44] Nielsen M A, Bremner M J, Dodd J L, Childs A M and Dawson C M 2002 *Phys. Rev. A* **66** 022317
[45] Klimov A B, Guzmán R, Retamal J C and Saavedra C 2003 *Phys. Rev. A* **67** 062313
[46] Hugh D M and Twamley J 2005 *New J. Phys.* **7** 174
[47] Bullock S S, O, Leary D P and Brennen G K 2005 *Phys. Rev. Lett.* **94** 230502
[48] Bishop C A and Byrd M S 2008 *Phys. Rev. A* **77** 012314
[49] Paz-Silva G A, Rebić S, Twamley J and Duty T 2009 *Phys. Rev. Lett.* **102** 020503
[50] Strauch F W 2011 *Phys. Rev. A* **84** 052313
[51] Rousseaux B, Guérin S and Vitanov N V 2013 *Phys. Rev. A* **87** 032328
[52] Cao Y, Peng S G, Zheng C and Long G L 2011 *Commun. Theor. Phys.* **55** 790
[53] Luo M X and Wang X J 2014 *Sci. China-Phys. Mech. Astron.* **57** 1712
[54] Bayat A and Karimipour V 2007 *Phys. Rev. A* **75** 022321
[55] Bayat A 2014 *Phys. Rev. A* **89** 062302

[56] Romero-Isart O, Eckert K and Sanpera A 2007 *Phys. Rev. A* **75** 050303
[57] Qin W, Wang C and Long G L 2013 *Phys. Rev. A* **87** 012339
[58] Hayes J P and Mudge T 1989 *Proc. IEEE* **77** 1829
[59] Feder D L 2006 *Phys. Rev. Lett.* **97** 180502
[60] Chudzicki C and Strauch F W 2010 *Phys. Rev. Lett.* **105** 260501
[61] Qin W, Wang C, Cao Y and Long G L 2014 *Phys. Rev. A* **89** 062314
[62] Moore C and Russell A 2002 *Lecture Notes in Computer Science* (Berlin: Springer-Verlag)
[63] Košík J and Bužek V 2005 *Phys. Rev. A* **71** 012306
[64] Krovi H and Brun T A 2006 *Phys. Rev. A* **73** 032341
[65] Marquezino F L, Portugal R, Abal G and Donangelo R 2008 *Phys. Rev. A* **77** 042312
[66] Shenvi N, Kempe J and Whaley K B 2003 *Phys. Rev. A* **67** 052307
[67] Hein B and Tanner G 2009 *J. Phys. A: Math. Theor.* **42** 085303
[68] Patel A and Rahaman Md A 2010 *Phys. Rev. A* **82** 032330
[69] Patel A, Raghunathan K S and Rahaman Md A 2010 *Phys. Rev. A* **82** 032331
[70] Beineke L W and Wilson R J (eds.) 1978 *Selected Topics in Graph Theory* (London: Academic)
[71] Cucchietti F M, Paz J P and Zurek W H 2005 *Phys. Rev. A* **72** 052113
[72] Cai J M, Zhou Z W and Guo G C 2006 *Phys. Rev. A* **74** 022328
[73] Paganelli S, de Pasquale F and Giampaolo S M 2002 *Phys. Rev. A* **66** 052317
[74] Lucamarini M, Paganelli S and Mancini S 2004 *Phys. Rev. A* **69** 062308
[75] Yuan X Z, Goan H S and Zhu K D 2007 *New J. Phys.* **9** 219
[76] Yuan X Z, Goan H S and Zhu K D 2011 *New J. Phys.* **13** 023018
[77] Holstein T and Primakoff H 1940 *Phys. Rev.* **58** 1098
[78] Majlis N 2000 *The Quantum Theory of Magnetism* (Singapore: World Scientific)
[79] Schwinger J 1965 *Quantum Theory of Angular Momentum* (New York: Academic)
[80] Życzkowski K and Sommers H 2001 *J. Phys. A: Math. Gen.* **34** 7111
[81] Li H, Li Y S, Wang S H and Long G L 2014 *Commun. Theor. Phys.* **61** 273