Effect of the dispersion on multipartite continuous-variable entanglement in optical parametric amplifier*

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Based on the quantum fluctuations, we adopt the method of generalized V_1 criterion to investigate multipartite entanglement characteristics in an optical parametric amplification system with the consideration of dispersion. The nonlinear interaction becomes strong because of the existence of dispersion coefficient σ. Considering the influence of dispersion factor σ, with increasing the pump parameter μ, the value of minimum variance V_1 decreases and the squeezing curve nearly equals $1/(1+\mu)$. The larger particle number N results in a smaller variance and higher entanglement.

Keywords: dispersion, multipartite entanglement, optical parametric amplifier

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1. Introduction

The continuous variable (CV) entanglement light has been an interesting research subject theoretically and experimentally, as it could be used as a basic resource for this new type of information processing.[1] The experimental achievements include demonstrations of quantum teleportation,[2] quantum dense coding,[3] and quantum cryptographic communication[4] by using the squeezed light. The bipartite CV entanglement states of a light field were studied in Refs. [5] and [6] and demonstrated experimentally[7] for a non-degenerate optical parametric amplifier (NOPA). The production of tripartite CV entangled beams by mixing squeezed states with linear optical elements can be found in Refs. [8]–[13]. The genuine four-partite CV entanglement has been experimentally generated.[14] Pfister *et al*. proposed multipartite CV entanglement from concurrent nonlinearities through the generalization of bipartite interaction Hamiltonian $H_1 = \mathrm{i}\hbar\beta\chi(a_1^\dagger a_2^\dagger - a_1 a_2)$ to N-partite interaction Hamiltonian $H_N = \mathrm{i}\hbar\beta\chi\sum_{i=1}^{N}\sum_{j>i}^{N} a_i^\dagger a_j^\dagger + \mathrm{H.C.}$[15] Based on previous work,[16] we derive the generalized V_1 criterion for multipartite entanglement.[17]

At present, the squeezing and entanglement of pulse frequency combs have attracted a great deal of attention because of their potential applications in quantum communication. The dispersion is one of the critical factors. In order to avoid the dispersion effect, the interaction lengths may be restricted to being very short, which limits the degree of squeezing.[18–21] Takahashi *et al*. investigated the effects of dispersion on both quadrature squeezing and photon statistics by varying the interaction length when pumping a PPLN crystal with fs pulses.[22] We studied the phase mismatched phenomenon resulting from the dispersion effect, by periodically changing the polarity of crystal to obtain the optimal squeezing condition, and obtained the characteristic of EPR entanglements.[23] In this paper, we discuss the influence of the dispersion effect, find the solution of the time-dependent FPE with the dispersion, and investigate the entanglement characteristics for $N = 2$–5 partite. The investigation is especially important for the pulse OPO and the quantum characteristics of pulse frequency combs.

2. Theory model

The Hamiltonian describing this system for multipartite entanglement is

$$H = H_0 + V + W, \tag{1}$$

where

$$\begin{aligned} H_0 &= \hbar\omega_0 a_0^\dagger a_0 + \sum_{m=1}^{N}\hbar\omega_m a_m^\dagger a_m + \mathrm{i}\hbar(E\,\mathrm{e}^{-\mathrm{i}\omega_L t}a_0^\dagger - E^*\,\mathrm{e}^{\mathrm{i}\omega_L t}a_0) \\ &\quad + \sum_j\sum_{m=1}^{N}\hbar\omega_j B_{jm}^\dagger B_{jm}, \\ V &= \hbar\sum_j\sum_{m=1}^{N} k_j B_{jm} a_m^\dagger + \hbar\sum_j\sum_{m=1}^{N} k_j^* B_{jm}^\dagger a_m, \\ W &= \frac{\mathrm{i}\hbar}{2}\sum_{j\neq k}^{N}(2\varepsilon_{jk}a_j^\dagger a_k^\dagger - 2\varepsilon_{jk}^* a_j a_k). \end{aligned}$$

According to the non-degenerate mode operators $b_1,\ldots,b_N$ obtained by Ref. [17] and the standard technique,[24] we de-

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fine $\alpha_j = \langle b_j \rangle$, $j = 1 \sim N$, $\alpha_0 = \langle a_0 \rangle$. Assuming the dynamic equation to have the initial values $\alpha_{10} = \alpha_{20} = \cdots = \alpha_{N0} = 1$, $\alpha_{00} = 0$, we have $\alpha_1 = \alpha_2 = \cdots = \alpha_N$. Considering the interaction between the medium and the optical field, we have the nonlinear susceptibility $\chi = \chi' + \mathrm{i}\chi''$, where χ' is the revised refractive index, χ'' is the gain, the dispersion effect is neglected, and only the imaginary part χ'' is retained. The semi-classical equations describing the interactions between pump wave $\alpha_0(\tau)$ and parametric waves $\alpha_1(\tau)$, $\alpha_2(\tau)$ become[25]

$$\begin{aligned}
\frac{\mathrm{d}\alpha_1}{\mathrm{d}\tau} &= -\alpha_1 + m\frac{\chi''}{\gamma_1}\alpha_0\alpha_1^*, \\
\frac{\mathrm{d}\alpha_2}{\mathrm{d}\tau} &= -\alpha_2 - \frac{\chi''}{\gamma_1}\alpha_0\alpha_2^*, \\
\frac{\mathrm{d}\alpha_0}{\mathrm{d}\tau} &= -\gamma_\mathrm{r}\alpha_0 + \frac{\bar{\varepsilon}}{\gamma_1} - m\frac{\chi''}{2\gamma_1}(\alpha_1^2 - \alpha_2^2).
\end{aligned} \tag{2}$$

Here $\tau = \gamma_1 t$ is the dimensionless time, γ_1 and γ_2 are the damping rates, and decay ratio $\gamma_\mathrm{r} = \gamma_2/\gamma_1$, $\bar{\varepsilon}$ is the amplitude of external coherent field, and $m = N - 1$ is the number of particles.

Considering the influence of dispersion,

$$\varepsilon = \chi\alpha_0 = (\chi' + \mathrm{i}\chi'')\alpha_0$$

is the dielectric coefficient, the refractive index of the nonlinear susceptibility is

$$n = \sqrt{1+\varepsilon} = 1 + (\chi' + \mathrm{i}\chi'')\alpha_0/2 = n' + \mathrm{i}n'',$$

the real part n' denotes the revised refractive index when the light with different frequencies passes through a medium, whereas the imaginary part n'' denotes the gain.[23] When $\alpha_1 = \alpha_2$, in terms of Eq. (2), the semi-classical equation for the DOPA system becomes

$$\begin{aligned}
\frac{\mathrm{d}\alpha_1}{\mathrm{d}\tau} &= -\alpha_1 - \mathrm{i}m\frac{\chi}{\gamma_1}\alpha_0\alpha_1^*, \\
\frac{\mathrm{d}\alpha_0}{\mathrm{d}\tau} &= -\gamma_\mathrm{r}\alpha_0 + \frac{\bar{\varepsilon}}{\gamma_1} - m\frac{\chi''}{2\gamma_1}(\alpha_1^2 - \alpha_2^2).
\end{aligned} \tag{3}$$

Assuming α_0 to be real and writing the complex α_1 as $\alpha_1 = \beta_1 + \mathrm{i}\tilde{\beta}_1$ containing real and imaginary parts, equation (3) turns into

$$\begin{aligned}
\frac{\mathrm{d}\beta_1}{\mathrm{d}\tau} &= -\beta_1 + m\frac{\chi''}{\gamma_1}\alpha_0\beta_1 - m\frac{\chi'}{\gamma_1}\alpha_0\tilde{\beta}_1, \\
\frac{\mathrm{d}\tilde{\beta}_1}{\mathrm{d}\tau} &= -\tilde{\beta}_1 - m\frac{\chi''}{\gamma_1}\alpha_0\tilde{\beta}_1 - m\frac{\chi'}{\gamma_1}\alpha_0\beta_1, \\
\frac{\mathrm{d}\alpha_0}{\mathrm{d}\tau} &= -\gamma_\mathrm{r}\alpha_0 + \frac{\bar{\varepsilon}}{\gamma_1} - m\frac{\chi''}{2\gamma_1}[(\beta_1^2 + \tilde{\beta}_1^2) - (\beta_2^2 + \tilde{\beta}_2^2)].
\end{aligned} \tag{4}$$

We take the parameters $\chi''/\gamma_1 = \sqrt{2\gamma_\mathrm{r}\eta}$, $\bar{\varepsilon}/\gamma_1 = \mu\sqrt{\gamma_\mathrm{r}/2\eta}$, and assume the dispersion factor $\sigma = \chi'/\chi'' = 0.5$, the numerical solutions of α_0, $\bar{\alpha}_{0\mathrm{s}}$ and β_1 with $\gamma_\mathrm{r} = 0.5$, $\eta = 1/1000$, and $\mu = 2$ are depicted in Fig. 1.

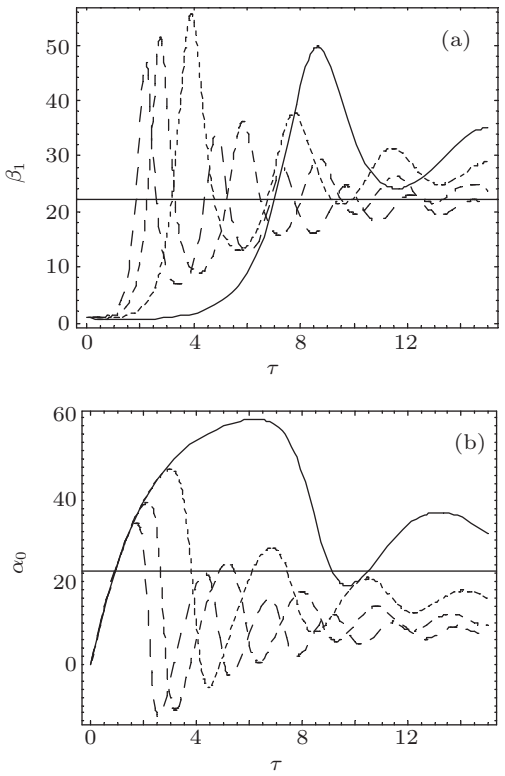


Fig. 1. Relations among the pump wave α_0 (solid line), steady state value $\bar{\alpha}_{0\mathrm{s}}$ of the pump wave (straight line), and the parameter wave β_1 (dashed line) for $\sigma = 0, 1$, $N = 2$–5 respectively.

Figures 1(a) and 1(b) describe the relations among pump wave α_0, steady state value $\bar{\alpha}_{0\mathrm{s}}$ of the pump wave, and parameter wave β_1 for $\sigma = 0, 1$, $N = 2$–5, respectively. Considering the effect of dispersion, the straight line corresponds to steady state value $\bar{\alpha}_{0\mathrm{s}}$, the solid line corresponds to pump wave α_0, and the dashed line corresponds to parameter wave β_1. Comparing Fig. 1(b) with Fig. 1(a), we have $\bar{\alpha}_{0\mathrm{s}} = \bar{\alpha}_0/\sqrt{1+\sigma^2}$, where $\bar{\alpha}_0 = \gamma_1/\chi''$ denotes the steady state value of the pump wave without dispersion. The squeezing curve contracts to the left, nearly equals $1/(1+\mu)$.

3. Multipartite entanglement for NOPA system with dispersion

Considering the effect of dispersion, according to Eq. (2), the complex $\alpha_1 = \beta_1 + \mathrm{i}\tilde{\beta}_1$ and $\alpha_2 = \beta_2 + \mathrm{i}\tilde{\beta}_2$, the semi-classical equations for the NOPA system turn into[26]

$$\begin{aligned}
\frac{\mathrm{d}\beta_1}{\mathrm{d}r} &= \frac{1}{\mu}\left(-\beta_1 + m\frac{\chi''}{\gamma_1}\alpha_0\beta_1 - m\frac{\chi'}{\gamma_1}\alpha_0\tilde{\beta}_1\right), \\
\frac{\mathrm{d}\tilde{\beta}_1}{\mathrm{d}r} &= \frac{1}{\mu}\left(-\tilde{\beta}_1 - m\frac{\chi''}{\gamma_1}\alpha_0\tilde{\beta}_1 - m\frac{\chi'}{\gamma_1}\alpha_0\beta_1\right), \\
\frac{\mathrm{d}\beta_2}{\mathrm{d}r} &= \frac{1}{\mu}\left(-\beta_2 - \frac{\chi''}{\gamma_1}\alpha_0\beta_2 + \frac{\chi'}{\gamma_1}\alpha_0\tilde{\beta}_2\right), \\
\frac{\mathrm{d}\tilde{\beta}_2}{\mathrm{d}r} &= \frac{1}{\mu}\left(-\tilde{\beta}_2 + \frac{\chi''}{\gamma_1}\alpha_0\tilde{\beta}_2 + \frac{\chi'}{\gamma_1}\alpha_0\beta_2\right), \\
\frac{\mathrm{d}\alpha_0}{\mathrm{d}r} &= \frac{1}{\mu}\left\{-\gamma_\mathrm{r}\alpha_0 + \frac{\bar{\varepsilon}}{\gamma_1} - m\frac{\chi''}{2\gamma_1}[(\beta_1^2 + \tilde{\beta}_1^2) - (\beta_2^2 + \tilde{\beta}_2^2)]\right\}.
\end{aligned} \tag{5}$$

Considering the initial condition $\beta_{10} = \tilde{\beta}_{10} = \beta_{20} = \tilde{\beta}_{20} = 1$, $\alpha_{00} = 0$, the numerical solutions of Eq. (5) yield the dependences of the parametric functions α_1, $\tilde{\alpha}_1$; α_2, $\tilde{\alpha}_2$ on the squeezing parameter r

$$\begin{aligned}
&\alpha_1(r) = \beta_1(r)/\beta_1(0),\\
&\tilde{\alpha}_1(r) = \exp\left[-\frac{2r}{\mu}\right](\beta_1(r)/\beta_1(0))^{-1},\\
&\alpha_2(r) = \exp\left[-\frac{(m+1)r}{m\mu}\right](\beta_1(r)/\beta_1(0))^{-1/m},\\
&\tilde{\alpha}_2(r) = \exp\left[-\frac{(m-1)r}{m\mu}\right](\beta_1(r)/\beta_1(0))^{1/m}.
\end{aligned} \tag{6}$$

The quantum fluctuation of the time-dependent linearly driven NOPA system is[27]

$$\begin{aligned}
\langle [X_1]^2\rangle = \langle [Y_j]^2\rangle = \frac{1}{2}\Bigg[&\frac{1}{2}(\alpha_1^2(r)+\tilde{\alpha}_1^2(r))\\
&+\frac{1}{\mu}\alpha_1^2(r)\int_0^r \alpha_1^{-2}(l)\,\mathrm{d}l + \frac{1}{\mu}\tilde{\alpha}_1^2(r)\int_0^r \tilde{\alpha}_1^{-2}(l)\,\mathrm{d}l\\
&+\frac{1}{2}(\alpha_2^2(r)+\tilde{\alpha}_2^2(r)) + \frac{1}{\mu}\alpha_2^2(r)\int_0^r \alpha_2^{-2}(l)\,\mathrm{d}l\\
&+\frac{1}{\mu}\tilde{\alpha}_2^2(r)\int_0^r \tilde{\alpha}_2^{-2}(l)\,\mathrm{d}l\Bigg].
\end{aligned} \tag{7}$$

The correlation between X_1 and Y_2 is

$$\begin{aligned}
\langle X_1 Y_j\rangle = \frac{1}{2}\Bigg[&\frac{1}{2}(\alpha_1^2(r)-\tilde{\alpha}_1^2(r)) + \frac{1}{\mu}\alpha_1^2(r)\int_0^r \alpha_1^{-2}(l)\,\mathrm{d}l\\
&-\frac{1}{\mu}\tilde{\alpha}_1^2(r)\int_0^r \tilde{\alpha}_1^{-2}(l)\,\mathrm{d}l\\
&-\frac{1}{2}(\alpha_2^2(r)-\tilde{\alpha}_2^2(r)) - \frac{1}{\mu}\alpha_2^2(r)\int_0^r \alpha_2^{-2}(l)\,\mathrm{d}l\\
&+\frac{1}{\mu}\tilde{\alpha}_2^2(r)\int_0^r \tilde{\alpha}_2^{-2}(l)\,\mathrm{d}l\Bigg].
\end{aligned} \tag{8}$$

The variance criterion V_1 is[23]

$$V_1(r) = \langle [X_1]^2\rangle - \sum_{j=2}^{N}\frac{\langle X_1 Y_j\rangle^2}{\langle [Y_j]^2\rangle}. \tag{9}$$

In the following, we investigate the influence of dispersion on multipartite entanglement. (i) $\mu = 0.5$ for below-threshold, (ii) $\mu = 1$ for the threshold, (iii) $\mu = 2$ for the above-threshold, and (iv) $\mu = 10$ for far-above-threshold. The results of numerical calculation of V_1 versus r with $\eta = 1/1000$ and $\gamma_r = 10$ for $N = 2$–5 are depicted in Figs. 2 and 3.

Figures 2 and 3 depict the minimum variance V_1 as a function of squeezing parameter $r = \mu\tau$ for $\sigma = 0, 1$, respectively. From bottom to top, the plots are for bipartite entanglement ($N = 2$, solid curve), tripartite entanglement ($N = 3$, narrow dashed curve), four-particle entanglement ($N = 4$, wide dashed curve), and five-particle entanglement ($N = 5$, dashed–dotted curve). Given dispersion factor σ, with the increase of pump parameter μ, the value of minimum variance V_1 decreases. Comparing Fig. 3 with Fig. 2, the scope of entanglement decreases from 10 to 2. The phase increases with the increase of pump parameter μ, namely $\varphi = \mu r = 10 \times 1.5 \approx 5\pi$, the polarity of gain medium changes many times, the net gain descends as shown in Figs. 2 and 3.

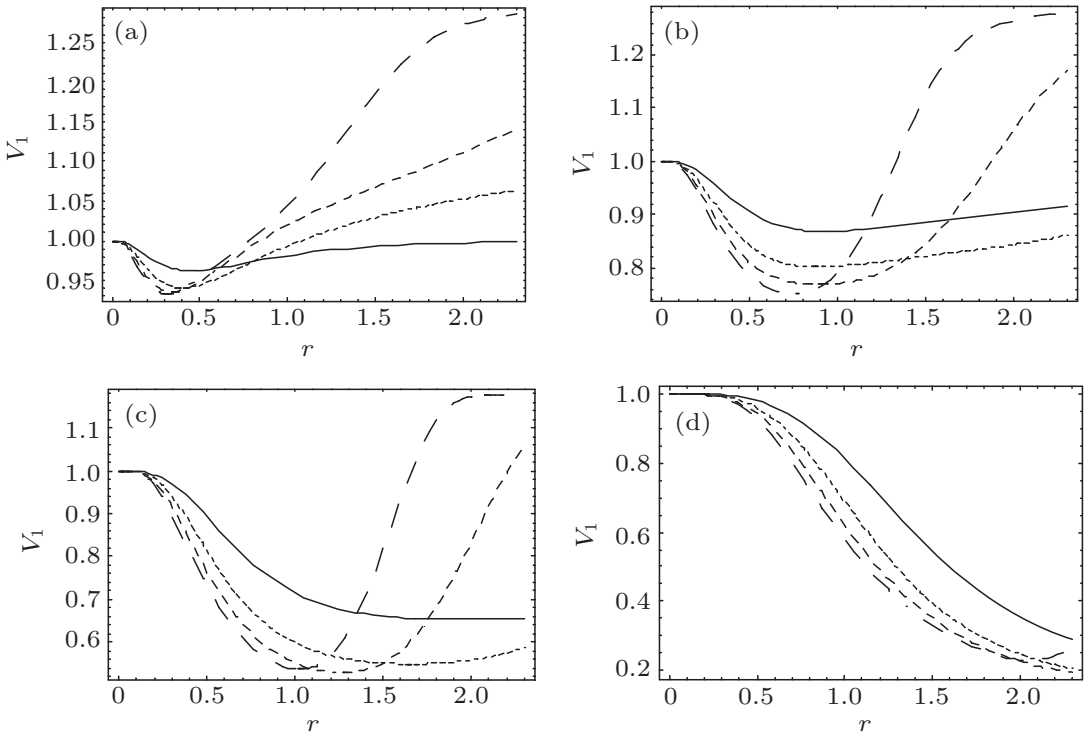


Fig. 2. Plots of variance V_1 versus τ with $\sigma = 0$, $\mu = 0.5$ (a), $\mu = 1$ (b), $\mu = 2$ (c), $\mu = 10$ (d).

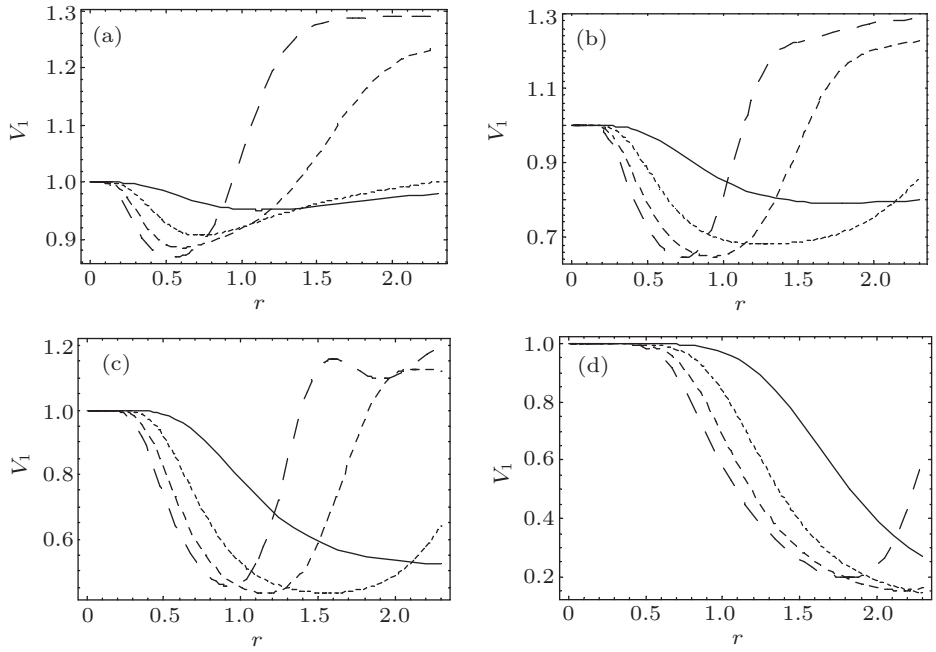


Fig. 3. Plots of variance V_1 versus τ with $\sigma = 1$, $\mu = 0.5$ (a), $\mu = 1$ (b), $\mu = 2$ (c), and $\mu = 10$ (d).

4. Conclusions

In this paper, we investigate the multipartite entanglement of FNOPA system with considering the effect of dispersion, the results are as shown in Figs. 1–3. The nonlinear interaction becomes strong due to dispersion coefficient σ. With the increase of pump parameter μ, the minimum variance V_1 decreases. The minimum variance V_1 decreases with the increase of the number N of particles.

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