Spatial snowdrift game in heterogeneous agent systems with co-evolutionary strategies and updating rules*

Xia Hai-Jiang(夏海江), Li Ping-Ping(李萍萍), Ke Jian-Hong(柯见洪)†, and Lin Zhen-Quan(林振权)

College of Physics and Electronic Information Engineering, Wenzhou University, Wenzhou 325035, *China*

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We propose an evolutionary snowdrift game model for heterogeneous systems with two types of agents, in which the inner-directed agents adopt the memory-based updating rule while the copycat-like ones take the unconditional imitation rule; moreover, each agent can change his type to adopt another updating rule once the number he sequentially loses the game at is beyond his upper limit of tolerance. The cooperative behaviors of such heterogeneous systems are then investigated by Monte Carlo simulations. The numerical results show the equilibrium cooperation frequency and composition as functions of the cost-to-benefit ratio r are both of plateau structures with discontinuous steplike jumps, and the number of plateaux varies non-monotonically with the upper limit of tolerance $\nu_{\rm T}$ as well as the initial composition of agents f_{a0}. Besides, the quantities of the cooperation frequency and composition are dependent crucially on the system parameters including $\nu_{\rm T}$, f_{a0}, and r. One intriguing observation is that when the upper limit of tolerance is small, the cooperation frequency will be abnormally enhanced with the increase of the cost-to-benefit ratio in the range of $0 < r < 1/4$. We then probe into the relative cooperation frequencies of either type of agents, which are also of plateau structures dependent on the system parameters. Our results may be helpful to understand the cooperative behaviors of heterogenous agent systems.

Keywords: cooperative behavior, evolutionary snowdrift game, heterogeneous agent system

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1. Introduction

The widespread phenomenon that cooperation emerges spontaneously in competitive groups consisting of selfish individuals has drawn much attention.[1–6] To understand such a phenomenon, scientists have taken advantage of game models to study the cooperative behaviors in competitive systems, and they have made great progress.[7–11] Especially, the prisoner's dilemma game (PD)[12] and the snowdrift game (SG)[13,14] have been employed and investigated widely in the literature. In these game models, two agents can choose to either cooperate (C-strategy) or defect (D-strategy) in their interactions. Each agent can get a payoff R if they both use the C-strategy and a payoff P if they both use D-strategy. If one chooses the C-strategy while another picks up D-strategy, the defector will receive the highest payoff T while the cooperator only gains the sucker's payoff S. The PD has the payoff ordering of $T > R > P > S$ and $2R > (T + S)$, while the SG differs from the PD only in the payoff ordering, namely, it is characterized with $T > R > S > P$.

Recently, considerable research works have been devoted to investigating the emerging and promoting of cooperation in the evolutionary PD or SG.[15–23] One key spirit of evolutionary games is that each agent need not persist in a sole strategy forever and can adapt it according to a certain updating rule. A variety of updating rules have been introduced in the literature,[15,24–33] which can be classified into two distinct categories. One is the class of imitation-type updating rules, such as the Fermi rule,[24] the replicator rule,[26,27] and the unconditional imitation rule.[15,29] For example, when an agent adopts the unconditional imitation rule, he always imitates the strategy of the agent with the highest payoff among his neighbors and himself. Imitation-type rules differ from one another only in the object and probability of imitation. Another is the class of self-directed rules, which depends on agents' own past experience. For instance, Wang *et al*. proposed a memory-based updating rule,[30] in which each agent records his own successful strategies in previous m rounds and then updates his strategy in the next round according to this pool of strategies. It is very sound to introduce the character of memory in many dynamical processes.[34–38]

It should also be pointed out that most of the research works only paid attention to the PD and SG in homogeneous agent systems, namely, all the agents obeying the same updating rule (see, e.g., Refs. [13], [30], [39]–[41] and references therein). However, for many real-world systems, the heterogeneity of agents is worthy of serious consideration. To the best of our knowledge, only a few papers are devoted to game dynamics in heterogenous systems, in which agents might adopt the same updating rule but with different parameters[42] or different imitation-type rules.[43–45] In our previous work,[46] we consider that different agents are of quite a distinct nature and thus divide the agents into two distinct types: copycat-like and inner-directed. Copycat-like agents are neighbor-dependent and follow one of the above

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†Corresponding author. E-mail: kejianhong@ymail.com

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imitation-type updating rules, while inner-directed agents are independent and take the memory-based updating rule. The results showed that the cooperative behaviors in the SG of such a heterogeneous system depend crucially on the composition of the two mixing agents.[46]

Since agents participating in the evolutionary games are intelligent, they might self-question their own updating style and change it after some failure experience. Thus, it might be more practical to consider that agents' updating rules are not unchangeable in the whole course of evolution. Our previous work was only devoted to the SG system with the updating rule of each agent keeping unchanged throughout the evolution.[46] In this work, we will further probe into the evolution dynamics of the SG in heterogeneous agent systems with co-evolutionary strategies and updating rules. The system consists of copycat-like and inner-directed agents. Copycat-like agents obey the unconditional imitation rule while inner-directed ones choose the memory-based updating rule proposed by Wang *et al*.[30] Moreover, in the course of evolution, a copycat-like agent could change into an inner-directed one after several successive failures due to adopting the imitation-type updating rule, and similarly, an inner-directed agent would become a copycat-like one if he suffers sequential failures by using the memory-based rule. By numerical simulations we investigate in detail the cooperative behaviors in the evolutionary SG of heterogenous agent systems.

The results show that the cooperation frequency in the heterogenous agent system also exhibits discontinuous plateau structures and the number of plateaux is dependent on the system parameters, as in the systems with a fixed composition of agents.[46] Besides, the equilibrium composition of agents is also of a discontinuous plateau structure. As for the system that the copycat-like agents adopt updating rules other than the unconditional imitation rule, we will defer it to a future work.

This paper is organized as follows. In Section 2, we briefly introduce the snowdrift game in heterogenous systems with co-evolutionary strategies and updating rules. In Section 3, we present the simulation results and then discuss the cooperative behaviors. A summary is given in Section 4.

2. The model of evolutionary snowdrift game in heterogenous agent systems

In this work, we aim to investigate the influence of the agent heterogeneity on the game dynamics. Consider a heterogenous system with two types of distinct agents: A-type and B-type. Agents are located on a two-dimensional (2D) square lattice with periodic boundary conditions, and each pair of neighboring agents plays the evolutionary snowdrift game (ESG). For simplicity, the payoffs in the SG can be represented by the following matrix:[12]

$$\begin{pmatrix} & C & D \\ C & 1 & 1-r \\ D & 1+r & 0 \end{pmatrix}.$$

Here, the parameter r denotes the cost-to-benefit ratio of mutual cooperation and $0 < r < 1$ in the SG. In a round, the agent uses the same strategy to play the pairwise SG with all of his neighbors and gets the total payoff by summing over all payoffs gained in the interactions with his neighbors. For instance, in one round, an agent adopts the C-strategy to play SGs with his four neighbors, among who there are x agents using the C-strategy and $4-x$ agents using D-strategy, then his total payoff is equal to $x\cdot 1+(4-x)\cdot(1-r)$; conversely, if he adopts the D-strategy to play SGs with the same neighbors, then his total payoff is equal to $x\cdot(1+r)+(4-x)\cdot 0$. Next, based on the value of the total payoff, this agent will change his type or update his strategy used in the next round according to a certain rule.

In our system, A-type agents are inner-directed and obey the memory-based rule (MB),[30] while B-type agents are copycat-like ones and take the unconditional imitation rule (UI).[14] The two updating rules are described briefly as follows:[46]

(i) Memory-based rule In each round, besides the actual total payoff obtained in practical interactions between neighbors, the agent also calculates a virtual total payoff by using the anti-strategy to play virtual games with his neighbors. The agent then makes a comparison between the actual and virtual payoffs and labels the strategy with higher payoff as the successful one in this round. In view of the finiteness of human memory, we assume that each agent has a limited memory chain of length m and can only note down the successful strategies in the recent m rounds. In the next round, the agent will count the number of C strategies recorded in his memory chain, say $m_{\rm c}$, and then adopts the C-strategy with a probability of $p_C = m_{\rm c}/m$ (or correspondingly, the D-strategy with a probability of $p_D = 1-p_C$).

(ii) Unconditional imitation rule In the end of each round, the agent analyzes the data of payoffs his neighbors obtained in the current round. If the agent himself has the maximum payoff, then he will continue to use his current strategy in the next round; otherwise, he will pick up the strategy of the neighboring agent(s) with the highest payoff as his next strategy. Moreover, if there is more than one neighbor having the same maximum payoff other than the agent himself, then one of them is selected randomly.

Since in real-world systems agents cannot update their strategies at the same time, we will adopt the asynchronous updating method in this work. In every Monte Carlo step (MCS), agents update their strategies in a random sequence and each

agent has a chance to update his strategy on average.[47] Meanwhile, in each round, besides the actual total payoff, each agent also computes out the virtual total payoff by using the anti-strategy to play virtual games. If the virtual total payoff is larger than the actual one, the agent then records this round as a failure (or losing the game). It is sound that agents can accept failure now and then but cannot endure too many sequential failures. On the other hand, agents might not change their updating rule easily due to inertia in mind and behavior. So, each agent has an upper limit of tolerance for the sequential failures, beyond which he will thoroughly dissatisfy his current updating rule and then change it into another one. The value of the upper limit of tolerance characterizes an agent's degree of inertia. In this work, for simplicity, all agents are assigned the same upper limit of tolerance v_{T} (here, v_{T} is a nonzero positive integer). In the course of evolution, each agent will count the number of his sequential failures; and once this number is beyond v_{T}, he will change his current updating rule into another one (correspondingly, his type is changed) and the number of his sequential failures resets. That is, a copycat-like agent will evolve into an inner-directed one, and *vice versa*, once the number of his sequential failures is larger than v_{T}.

3. Results and discussion

Consider a heterogenous agent system with N agents. The initial percentage concentration of A-type agents is set to be f_{a0} ($0 \leq f_{a0} \leq 1.0$), while that of B-type agents is $1 - f_{a0}$. Obviously, $f_{a0} = 1.0$ represents the homogenous system with pure A-type agents,[30] while $f_{a0} = 0$ indicates the pure system only with B-type agents.[41] The number of agents in all numerical simulations is the same, $N = L \times L = 10^4$. This work aims to investigate the dependence of evolution dynamics on the system parameters f_{a0}, r, and v_{T}. Besides, it is found that for the ESG in the system with pure A-type agents, the memory length is not the key parameter and has some effect only on the value of the cooperation frequency f_{c}.[30] Thus, the memory size of each A-type agent is set as $m = 7$. Initially, agents are assigned A-type with a probability f_{a0} or B-type with a probability $1 - f_{a0}$; and moreover, each agent chooses either the C or D strategy completely randomly. Each point in data figures is gained by measuring the mean over the last 1000 MCS after a transient MCS of 10^4 and then averaging over 20 realizations. Unless stated otherwise, the set of parameters remain unchanged for all numerical simulations.

We then perform a series of numerical simulations on a 2D $L \times L$ square lattice with periodical boundary conditions. Figure 1 illustrates a set of snapshots gained at MCS = 2×10^4 for different cost-to-benefit ratios, which can show both the strategy distribution and the type composition of agents. By careful statistic analysis of such snapshots one can understand the evolution of cooperation in the heterogenous agent system. In order to characterize the cooperative behavior quantitatively, we define three quantities: the global fraction of cooperators in the total agent of any type f_{c}, the relative fraction of cooperators among A-type agents g_{ca}, and the relative fraction of cooperators among B-type agents g_{cb}. Moreover, we define f_a as the fraction of A-type agents in the total agent of any type. Obviously, the above-introduced quantities satisfy the relationship $f_{\mathrm{c}} = f_a \cdot g_{ca} + (1 - f_a) \cdot g_{cb}$ at any time. At the beginning of the ESG, we have $g_{ca} = 0.5$ and $g_{cb} = 0.5$. In the following sections, we discuss in detail the cooperative behaviors and the equilibrium composition of agents in heterogeneous agent systems.

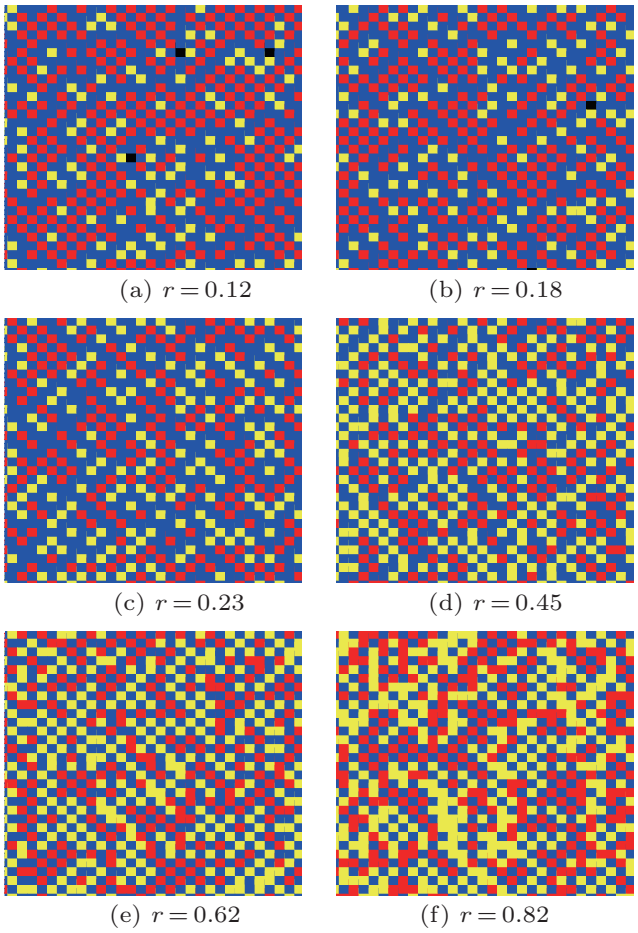


Fig. 1. (color online) Snapshots of spatial patterns of A-type cooperators (blue), B-type cooperators (black), A-type defectors (red), and B-type defectors (yellow) for different cost-to-benefit ratios. (a) $r = 0.12$; (b) $r = 0.18$; (c) $r = 0.23$; (d) $r = 0.45$; (e) $r = 0.62$; and (f) $r = 0.82$. Here, the other model parameters are the same, $v_{\mathrm{T}} = 3$ and $f_{a0} = 0.5$. Obviously, almost all the B-type agents will take the D-strategy eventually and the concentration of A-type agents decreases with the increase of r.

Since our model has three tunable parameters, we fix the value of the upper limit of tolerance to investigate the dependence of the cooperation frequency (i.e., the fraction of cooperators) and the equilibrium composition on the game parameters.

3.1. The $v_T = 3$ case

Due to a Chinese proverb, which says "Not to do anything more than three times", we first consider $v_T = 3$. Figure 2 shows a set of the global cooperation frequencies against the cost-to-benefit ratio r for different f_{a0}. Clearly, the global cooperation frequency f_c as a function of r is discontinuous and several steplike transitions occur at some particular values of r; moreover, f_c is independent of r in each piecewise cooperation level. Thus, f_c also has plateau structures in the heterogenous agent systems with co-evolutionary strategies and updating rules, as in the homogenous system with single-type agents[30,41] as well as the heterogenous system with the fixed composition of agents.[46]

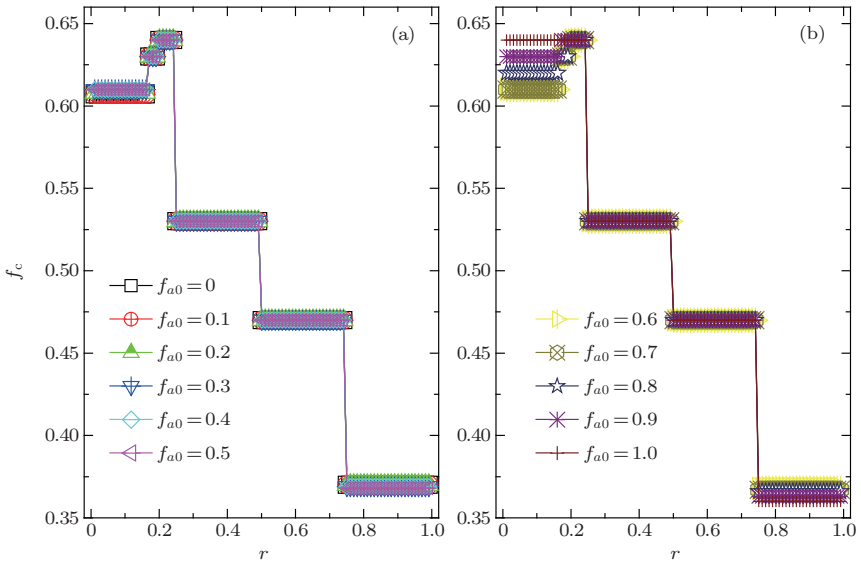


Fig. 2. (color online) The equilibrium cooperation frequency f_c as a function of the cost-to-benefit ratio r at $v_T = 3$. It is shown that f_c always exhibits a discontinuous steplike structure for an arbitrary composition f_{a0}, and the value of f_c is dependent on f_{a0} only in the range of $0 < r < 1/4$.

The phenomenon that the global cooperation frequencies are of such steplike structures can be interpreted by analyzing the local stability of configurations.[30,41] The X-agent surrounded by p C-agents and q D-agents in his neighborhood is denoted by $X(pCqD)$, where $X = C$ (D), p and q are two positive integers. In this work, we investigate the evolutionary snowdrift game on a 2D square lattice, in which each agent will play the pairwise SG only with his four neighbors. Thus, for the configuration $X(pCqD)$, we have the relation $p+q=4$, where $p \in \{0,1,2,3,4\}$. Obviously, only ten kinds of possible configurations exist in our system.

Table 1. The critical transition points in the system with pure A-type agents.

Case	The balance between the following configurations	The critical value r_c
1	$C(3C1D)$ and $D(3C1D)$	1/4
2	$C(2C2D)$ and $D(2C2D)$	1/2
3	$C(1C3D)$ and $D(1C3D)$	3/4

Moreover, a $C(pCqD)$-agent can obtain the total payoff $p+q(1-r)$, after he plays the SG with his p C-neighbors and q D-neighbors, and similarly, the total payoff that a $D(pCqD)$-agent gains is equal to $p(1+r)$. For the system with pure A-type agents, the $C(pCqD)$-agent will not change into a $D(pCqD)$-agent only when $p+q(1-r) > p(1+r)$, and inversely, the $D(pCqD)$-agent cannot change into a $C(pCqD)$-agent if $p(1+r) > p+q(1-r)$. So, the balance between the $C(pCqD)$-agent and $D(pCqD)$-agent results in the equation $p+q(1-r) = p(1+r)$, which yields a series of critical values $r_c = 1/4$, $1/2$, and $3/4$, as illustrated in Table 1. While, for the system with pure B-type agents, the $C(mCnD)$-agent (here, $m+n=4$) must have the same payoff as his neighboring $D(pCqD)$-agent so as to keep a local balance, which gives the equation $m+(4-m)(1-r) = p(1+r)$ with the possible solutions listed in Table 2.

Table 2. The critical transition points in the system with pure B-type agents.

Case	The balance between the following configurations	The critical value r_c
1	$C(0C4D)$ and $D(3C1D)$	1/7
2	$C(1C3D)$ and $D(3C1D)$	1/6
3	$C(2C2D)$ and $D(3C1D)$	1/5
4	$C(3C1D)$ and $D(3C1D)$	1/4
5	$C(0C4D)$ and $D(2C2D)$	1/3
6	$C(1C3D)$ and $D(2C2D)$	2/5
7	$C(2C2D)$ and $D(2C2D)$	1/2
8	$C(0C4D)$ and $D(1C3D)$	3/5
9	$C(3C1D)$ and $D(2C2D)$	2/3
10	$C(1C3D)$ and $D(1C3D)$	3/4

On the other hand, for each pair of configurations listed in Tables 1 and 2, one configuration can prevail over another in a certain range of r while another wins out in the remaining range of r. For instance, the $C(3C1D)$-agents dominate over the $D(3C1D)$-agents in the $r < 1/4$ case while it is just on the contrary in the $r > 1/4$ case (see Table 1). Thus, after the system evolves to an equilibrium state, only one or several of the ten configurations can survive finally. Since the value of f_c of the system is decided by the composition of configurations, the surviving configurations are somewhat different before and after each critical point, which leads to the discontinuity in f_c as a function of r and several apparent discontinuous jumps at those critical points. Thus, it can be safely concluded that the emergence of discontinuous jumps is caused by the competition between different configurations. Moreover, when r is in between two neighboring critical points, the dominating configurations are stable and their quantity is thus independent of the value of r, that is, the value of f_c will keep constant. So, the global cooperation frequency f_c as a function of r is of a discontinuous steplike structure, something like the energy level structure of atoms, due to the fact that our ESG system on the 2D regular lattice only has enumerable configurations and some distinct configurations survive eventually for different intervals of r. From a different perspective, each level of the global cooperation frequency corresponds to a set of configurations by which the system is constituted at the equilibrium state under some corresponding parameters.

In order to make a comparison between the homogenous and heterogenous systems, we recall the results of the pure B-agent system in the small-r case. One can find, by taking the instability of the configurations with D-core (such as $D(pCqD)$) in small-r cases into further consideration, that the discontinuous transitions in pure B-agent systems cannot emerge at $r_c < 1/4$.[41] For example, the transition point $r = 1/5$ is caused by the local configuration $D(3C1D)$ coexisting with $C(2C2D)$; but unfortunately, the configuration $D(3C1D)$ cannot survive in the pure B-agent system with $r < 1/4$ because such a system suppresses the survival of D-agents.[41] However, in the heterogenous agent system with both A-type and B-type agents, the rich-D configuration, such as $D(1C3D)$ and $D(2C2D)$, is possible to form due to the fact that A-type agents can occupy the core of the configurations and prefer the D-strategy in small-r cases.[46] Without any surprise, besides the above-mentioned discontinuous transitions at $r_c = 1/4$, $1/2$, and $3/4$, we also observe the secondary jumps at $r_c = 1/6$ and $1/5$ emerging in the system with co-evolutionary strategies and updating rules, as demonstrated in Fig. 2. As for the critical point $r_c = 1/7$, it is caused by the competition between the two configurations $C(0C4D)$ and $D(3C1D)$; however, the configuration $C(0C4D)$ cannot emerge in small-r cases in which C-agents dominate over D-ones, and thus, the secondary critical transition at $r_c = 1/7$ cannot be observed even in our heterogenous agent system.

Furthermore, as indicated in Fig. 2, the value of the equilibrium cooperation frequency f_c is independent of the initial composition of agents in the range of $3/4 > r > 1/5$; while, f_c slightly decreases with the increase of f_{a0} if $r > 3/4$ and it is just on the contrary if $r < 1/5$. More intriguingly, the piecewise level of f_c in the range of $1/6 < r < 1/5$ is somewhat higher than that in the range of $0 < r < 1/6$ for almost all f_{a0} except for $f_{a0} = 1.0$, while the value of the plateau in the range of $1/5 < r < 1/4$ is a little higher than that in the range of $1/6 < r < 1/5$ for most of f_{a0} except for $f_{a0} = 0.9$ and 1.0. This is an abnormal phenomenon. As we know, based on the scenario that the SG is constructed, agents will be more and more likely to adopt the D-strategy with the increase of r and, thus, the equilibrium cooperation level will decrease with increasing r.

Now we turn to investigate the composition of agents after a long-time evolution. The equilibrium fractions of A-type agents for different f_{a0} are demonstrated in Fig. 3. It is observed that f_a is also of a steplike plateau structure. The value of f_a depends crucially on the cost-to-benefit ratio r, and the discontinuous transition points are the same as those for the global cooperation frequency f_c. Moreover, the initial value of f_{a0} also takes an important role in the final composition of agents. For $f_{a0} \leq 0.8$, the equilibrium fraction of A-type agents f_a decreases consistently with the increase of r; while for $f_{a0} > 0.8$, f_a also decreases with increasing r in the range of $0 < r < 1/2$, but it is just on the contrary in the range of $1/2 < r < 1$. On the other hand, in the range of $1/4 < r < 1$, f_a always increases consistently with the increase of f_{a0}; while in the remaining range of r, f_a is independent of the value of f_{a0} for $f_{a0} \leq 0.7$ but increases along with f_{a0} for $f_{a0} > 0.7$. Making a comparison between the equilibrium fraction f_a and its initial f_{a0}, one can observe that the inequality $f_a \geq f_{a0}$ is always valid for almost all f_{a0} except for $f_{a0} = 1.0$ and the net difference consistently decreases with increasing f_{a0} and r. Especially, for $f_{a0} = 1.0$ only a little fraction of A-type agents will finally change into B-type ones. Hence, in the $v_T = 3$ case, A-type agents have fairly little probability of losing three successive games, while it is of high possibility for B-type agents to fail thrice in succession until the fraction of B-type agents decreases to a certain value that depends on the initial parameters f_{a0} and r.

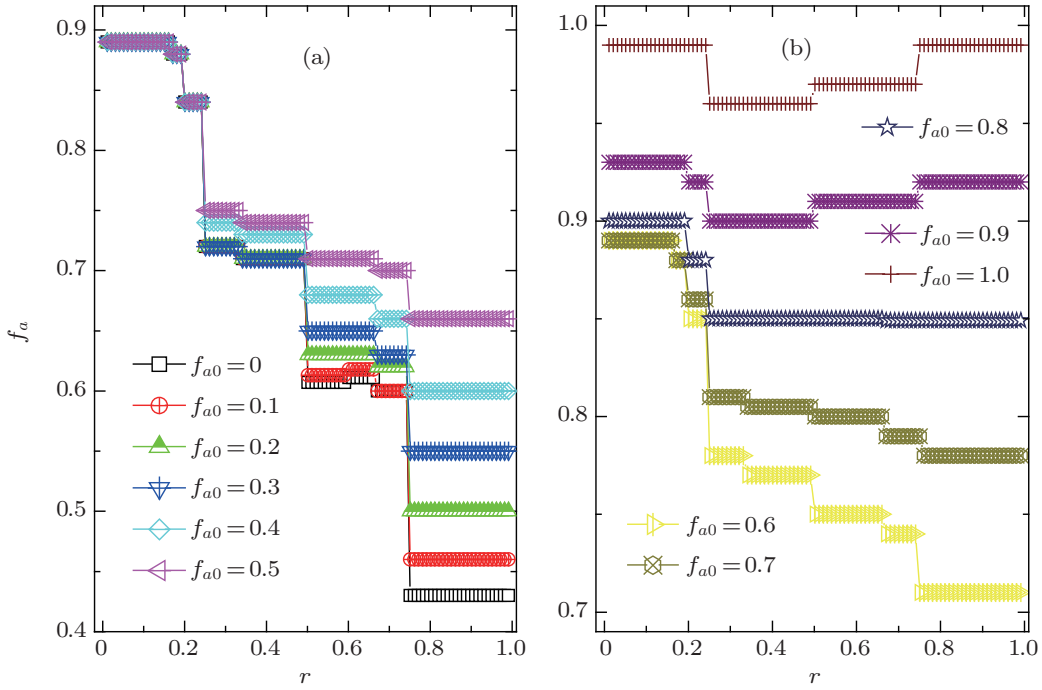


Fig. 3. (color online) The equilibrium composition f_a versus the cost-to-benefit ratio r at $v_T = 3$. It is shown that f_a always exhibits a discontinuous steplike structure for each initial composition f_{a0}, and the number of plateaux is related with the value of f_{a0}.

In order to understand the above-mentioned observations, we will probe deeply into the relative cooperative behaviors of both A- and B-type agents at $v_T = 3$. Figure 4 shows the data lines of the relative cooperation frequencies g_{ca} and g_{cb} against r for different f_{a0}. We again find that both g_{ca} and g_{cb} as the function of r are of plateau structures. For g_{cb}, there are only three plateaux; while for g_{ca}, the number of plateaux is eight when $f_{a0} \leq 0.7$, and with f_{a0} increasing from 0.7 to 1.0 the number of plateaux decreases to six due to the fact that the discontinuous transitions at $r_c = 0.4$ and $r_c = 2/3$ cannot happen. Moreover, g_{ca} is almost independent of the value of f_{a0} in the range of $0 < r < 1/4$ and $f_{a0} \leq 0.7$; while it decays with the increase of f_{a0} in the remaining range. It is worth noting that for small f_{a0} the relative cooperation frequency g_{ca} is fairly high (for instance, $g_{ca} > 0.65$ for $f_{a0} \leq 0.3$), i.e., most A-type agents adopt the C-strategy. More interestingly, the values of the three levels of g_{ca} in the range of $0 < r < 1/4$ anomalously enhance with the increase of r, which are in accord with those of the global cooperation frequency f_c. As for g_{cb}, it is identically equal to zero in the range of $r > 0.2$, i.e., all B-type agents adopt the D-strategy independent of the values of f_{a0} and r, while in the remaining range of $0 < r < 0.2$, very few of the B-type agents may take the C-strategy (here, the maximum value of g_{cb} is 0.15 at $f_{a0} = 1.0$) and the value of g_{cb} slightly increases with increasing f_{a0}.

By incorporating the three diagrams of g_{ca}, g_{cb}, and f_a against r, one can quantitatively understand the properties of the global cooperation frequency f_c as a function of r. Here, we only analyze qualitatively the evolution behaviors of A- and B-type agents. Making a comparison between Fig. 2 and the corresponding figure of f_c given by Wang *et al.*,[30] we find that (i) the global cooperation frequency f_c at $f_{a0} = 1.0$ in our model is almost the same as theirs, and (ii) f_c for other f_{a0} is also very similar to theirs, except for the range of $0 < r < 1/4$, in which it has three splitted levels in our system, in contrast to only one in the system with pure A-type agents. It follows from Ref. [30] that the typical equilibrium structure of the strategy distribution in the ESG with pure A-type agents is something like a chessboard with some embedded "C-lines" in the range of $0 < r < 1/4$ or a chessboard-like background with small sparsely-embedded "C-segments" in the range of $1/4 < r < 1/2$, while it is also like a chessboard but with a few "D-segments" in the range of $1/2 < r < 3/4$ or with "D-lines" in the range of $1/4 < r < 1$. Here, the four intervals of r, $(0, 1/4)$, $(1/4, 1/2)$, $(1/2, 3/4)$, and $(3/4, 1.0)$, can be called the main ones. Obviously, in a chessboard-like background, the agent adopting D-strategy will have higher payoff than the one using the C-strategy. Thus, B-type agents will take the D-strategy position in the chessboard-like pattern of strategies; otherwise, they will have a high probability of suffering successive failures in the evolutionary game and have to change into A-type ones. Of course, in the small-r case, B-type agents also have a little probability of occupying the inner sites of "C lines" embedded in the chessboard and survive finally. The above argument can be verified by the snapshots in the equi-

librium state, as illustrated in Fig. 5, in which we make a comparison between the original scattergram of the strategy and type distribution and the referenced one without regard to the agent type. Figures 4(c) and 4(d) indicate that all of the *B*-type agents will adopt the *D*-strategy if $r > 0.2$ and only a few of them may use the *C*-strategy if $0 < r < 0.2$. On the other hand, figure 2 shows that each of the first three main intervals will separate further into two or three secondary ones and, among the same main interval, the fraction of *B*-type agents for each secondary level increases with increasing r. Since one *B*-type agent occupying the *D*-strategy position of the chessboard pattern can compel four *A*-type agents in his neighborhood that they have to take over the *C*-strategy positions, the relative cooperation frequency g_{ca} for different secondary levels will be enhanced abnormally with the increase of *B*-type agents in the same main interval of r, as illustrated in Figs. 4(a) and 4(b). Especially, in the main interval of $0 < r < 1/4$, the slight decline in the fraction of *A*-type agents can cause a significant increase in g_{ca} but a little decrease in g_{cb}, which results in an abnormal improvement of the global cooperation frequency f_c for different secondary levels along with increasing r (see Fig. 2).

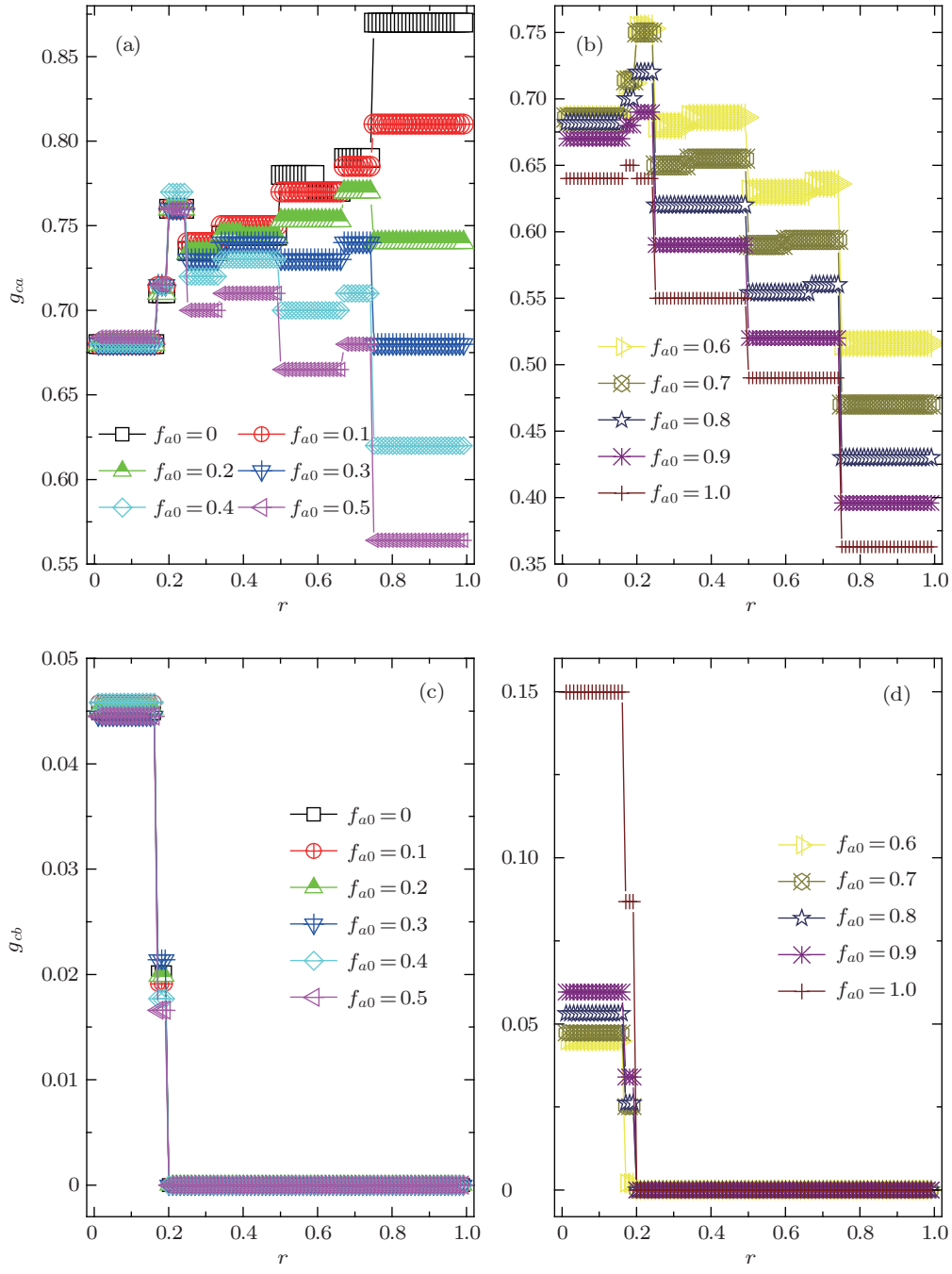


Fig. 4. (color online) The relative cooperation frequencies g_{ca} and g_{cb} versus the cost-to-benefit ratio r at $v_T = 3$. It is shown that both g_{ca} and g_{cb} have a discontinuous steplike structure for each initial composition f_{a0}. g_{ca} decreases with increasing f_{a0} while g_{cb} increases with f_{a0}. And g_{ca} always has a non-zero value for arbitrary r and f_{a0}, while g_{cb} consistently equals zero in the range of $1/5 < r < 1$.

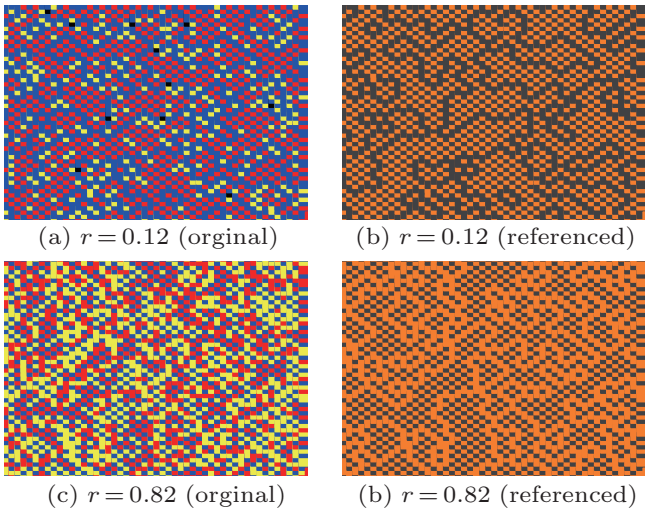


Fig. 5. (color online) Snapshots of spatial patterns of cooperators and defectors for different cost-to-benefit ratios: (a) and (b) $r = 0.12$ and (c) and (d) $r = 0.82$. Besides, panels (a) and (c) are original scattergrams of the strategy and type distribution, which consist of A-type cooperators (blue), B-type cooperators (black), A-type defectors (red), and B-type defectors (yellow); while, panels (b) and (d) are referenced scattergrams of the strategy distribution without regard to the agent type, which consist of cooperators (dark green) and defectors (orange). Here, the other model parameters are the same, $v_T = 3$ and $f_{a0} = 0.5$. Obviously, almost all of B-type agents will occupy the D-strategy sites in the chessboard-like pattern of strategies; while in the small-r case, a few of the B-type agents might also occupy the inner sites of "C lines" (dark green) embedded in the chessboard.

3.2. The $v_T = 36$ case

Next, we investigate the case with a moderate value of v_T. Here, we select $v_T = 36$ as an example. The global cooperation frequency f_c as a function of r for different f_{a0} is demonstrated in Fig. 6. We again observe that f_c is of a discontinuous plateau structure. As a contrast to the $v_T = 3$ case, the discontinuous jump in the global cooperation frequency f_c at $r = 1/6$ disappears in this case, and the phenomenon that the values of the secondary levels abnormally enhance with increasing r in the interval of $0 < r < 1/4$ does not occur any more. The disappearance of the secondary jump at $r_c = 1/6$ indicates that the configuration $C(1C3D)$ cannot form in the range of $0 < r < 1/6$ when $v_T = 36$. It is not surprising because the configuration $C(1C3D)$ is difficult to form in rich-C background and figure 6 shows $f_c > 0.64$ at $0 < r < 1/6$. While, new discontinuous transitions at $r_c = 1/3$ and $r_c = 2/5$ can occur in this case if $f_{a0} \leq 0.2$ and one more new jump will arise at $r = 2/3$ if $f_{a0} = 0$. According to Table 2, the competition between the configurations $C(0C4D)$ and $D(2C2D)$ leads to the jump at $r_c = 1/3$, while that between the configurations $C(1C3D)$ and $D(2C2D)$ causes the jump at $r_c = 2/5$. These configurations, such as $C(0C4D)$, $C(1C3D)$, and $D(2C2D)$, can easily form in rich-D background. Figure 6 shows that $f_c < 0.5$ in the range of $1/4 < r < 1/2$ only for $f_{a0} \leq 0.2$. So, the analyses on the basis of the local stability of configurations are in accordance with these observations. Moreover, when $1/2 < r < 1$, f_c in this case is the same as that in the $v_T = 3$ case, and when $1/4 < r < 1/2$, f_c is also similar to that in the $v_T = 3$ case, except for small f_{a0} (i.e., $f_{a0} \leq 0.2$); while for $0 < r < 1/4$, the value of f_c is distinctly larger than that in the $v_T = 3$ case.

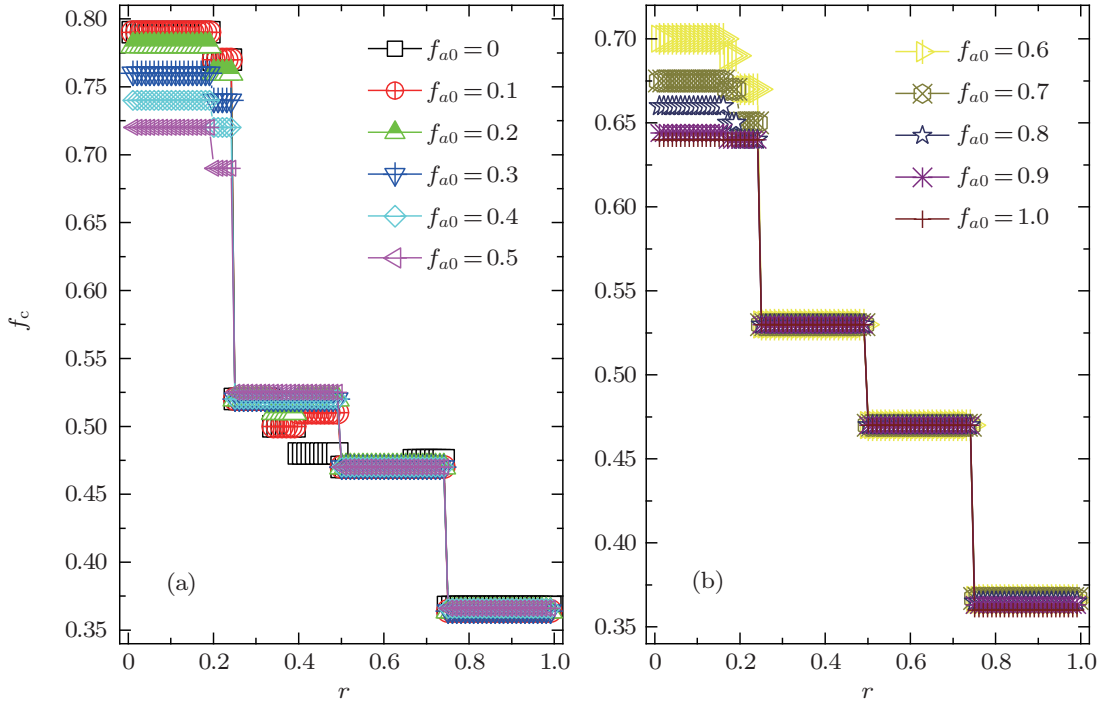


Fig. 6. (color online) The equilibrium cooperation frequency f_c as a function of the cost-to-benefit ratio r at $v_T = 36$. Each f_c exhibits a discontinuous steplike structure for an arbitrary composition f_{a0}. And the value of f_c is independent of f_{a0} in the range of $1/2 < r < 1$.

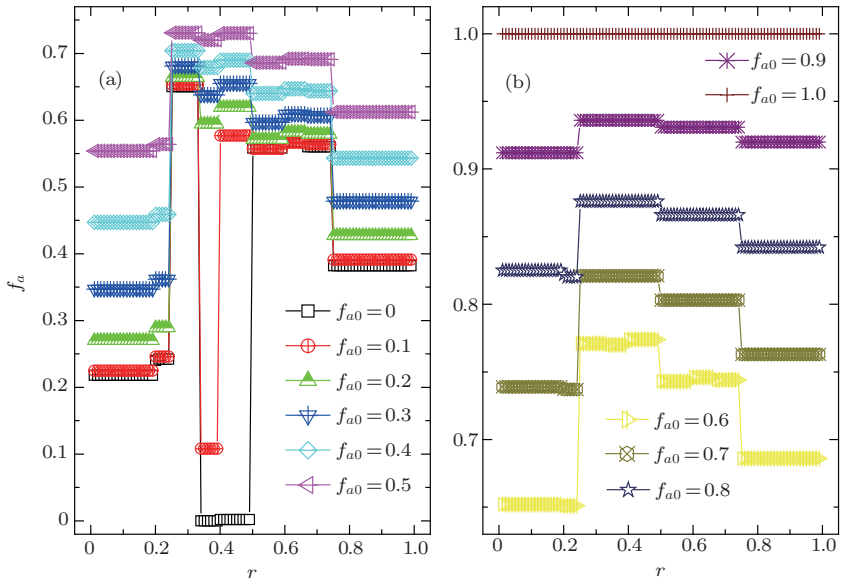


Fig. 7. (color online) The equilibrium composition f_a versus the cost-to-benefit ratio r at $v_T = 36$. Each f_a exhibits a discontinuous steplike structure for each initial composition f_{a0}, and the number of plateaux has relation with the value of f_{a0}. Especially, $f_a \simeq f_{a0}$ for $f_{a0} = 0$ in the range of $1/3 < r < 1/2$, and it is also the case for $f_{a0} = 0.1$ in the range of $1/3 < r < 2/5$.

In this case, the equilibrium fraction of A-type agents f_a also shows a discontinuous plateau structure, as demonstrated in Fig. 7. Besides the three main jumps occurring at $r = 1/4$, $1/2$, and $3/4$, some new discontinuous transitions arise within the first three main intervals, but which will disappear once f_{a0} is beyond a certain value. In the first main interval of $0 < r < 1/4$, there is a discontinuous jump occurring at $r = 1/5$ and this jump will disappear if $f_{a0} \geq 0.9$; in the second main interval of $1/4 < r < 1/2$, there are two discontinuous jumps arising respectively at $r = 1/3$ and $r = 2/5$, but which will disappear if $f_{a0} \geq 0.7$; and similarly, in the main interval of $1/2 < r < 3/4$, one can also observe two inapparent jumps emerging at $r = 3/5$ and $r = 2/3$ when $f_{a0} \leq 0.6$. Moreover, at a given r, the value of f_a always increases consistently with the increase of f_{a0}, and except for $f_{a0} = 0$, 0.1, and 1.0, f_a for other f_{a0} has the maximum value in the main interval of $1/4 < r < 1/2$ and the minimum value in the main interval of $0 < r < 1/4$. In more detail, when $f_{a0} \leq 0.5$, the level of f_a for the secondary range of $1/3 < r < 2/5$ has the minimum value within the main interval of $1/4 < r < 1/2$, while it has the maximum value for the secondary range of $3/5 < r < 2/3$ within the main interval of $1/2 < r < 3/4$. More intriguingly, when $f_{a0} = 0$ and 0.1, the value of $f_a - f_{a0}$ is equal to zero in the range of $1/3 < r < 2/5$, in other words, all B-type agents cannot change into A-type ones. While the value of f_a is very close to zero in the range of $2/5 < r < 1/2$ if $f_{a0} = 0$, namely, only a tiny proportion of B-type agents can become A-type ones under such a situation. It is also worth noting that when $f_{a0} = 1.0$, f_a is consistently equal to unity, independent of r. It is thus concluded that in the system initializing with pure A-type agents, all A-type agents cannot change into B-type ones. Similar to the above $v_T = 3$ case, the equilibrium fraction of A-type agents f_a is always larger than (or equal to) the corresponding initial quantity f_{a0} and the net difference between them consistently decreases with increasing f_{a0}.

Now we investigate the relative cooperation frequencies g_{ca} and g_{cb}. In this case, both g_{ca} and g_{cb} are also of plateau structures, as shown in Fig. 8. g_{ca} has eight plateaux when $f_{a0} \leq 0.5$; however, with the increase of f_{a0}, the secondary discontinuous transitions at $r = 1/5$, $1/3$, $2/5$, and $3/5$ will gradually disappear, and the number of plateaux finally decreases to 4 when $f_{a0} = 1.0$. Moreover, in the range of $r > 0.5$ and $1/4 < r < 1/3$, the equilibrium value of g_{ca} decreases with increasing f_{a0}; while in the remaining range, g_{ca} first increases and then decreases with the increase of f_{a0}, and the maximum value of g_{ca} is attained at $f_{a0} \simeq 0.6$ in the range of $0 < r < 1/4$, at $f_{a0} \simeq 0.3$ in the range of $1/3 < r < 2/5$, and at $f_{a0} \simeq 0.2$ in the range of $2/5 < r < 1/2$, respectively. It is also worth noting that when $f_{a0} \leq 0.2$, the value of g_{ca} for large r is larger than 0.8, namely, most A-type agents will adopt the C-strategy in such situations. Meanwhile, we observe that $g_{cb} = 0$ for all $f_{a0} \geq 0.1$ in the range of $r > 2/5$, namely, all B-type agents will adopt the D-strategy to survive under the large-r situation. In the main range of $0 < r < 1/4$, the value of g_{cb} decreases from about 0.8 to 0 with the increase of f_{a0}; while two secondary discontinuous jumps occur at $r = 1/6$ and $r = 1/5$, which will become more and more obvious with increasing f_{a0}, and the levels of g_{cb} for different secondary plateaux decrease along with r. Especially, when $f_{a0} = 0$, we observe $g_{cb} \simeq 0.5$ in the range of $1/3 < r < 1/2$, and when $f_{a0} = 0.1$, $g_{cb} \simeq 0.45$ in the range of $1/3 < r < 2/5$.

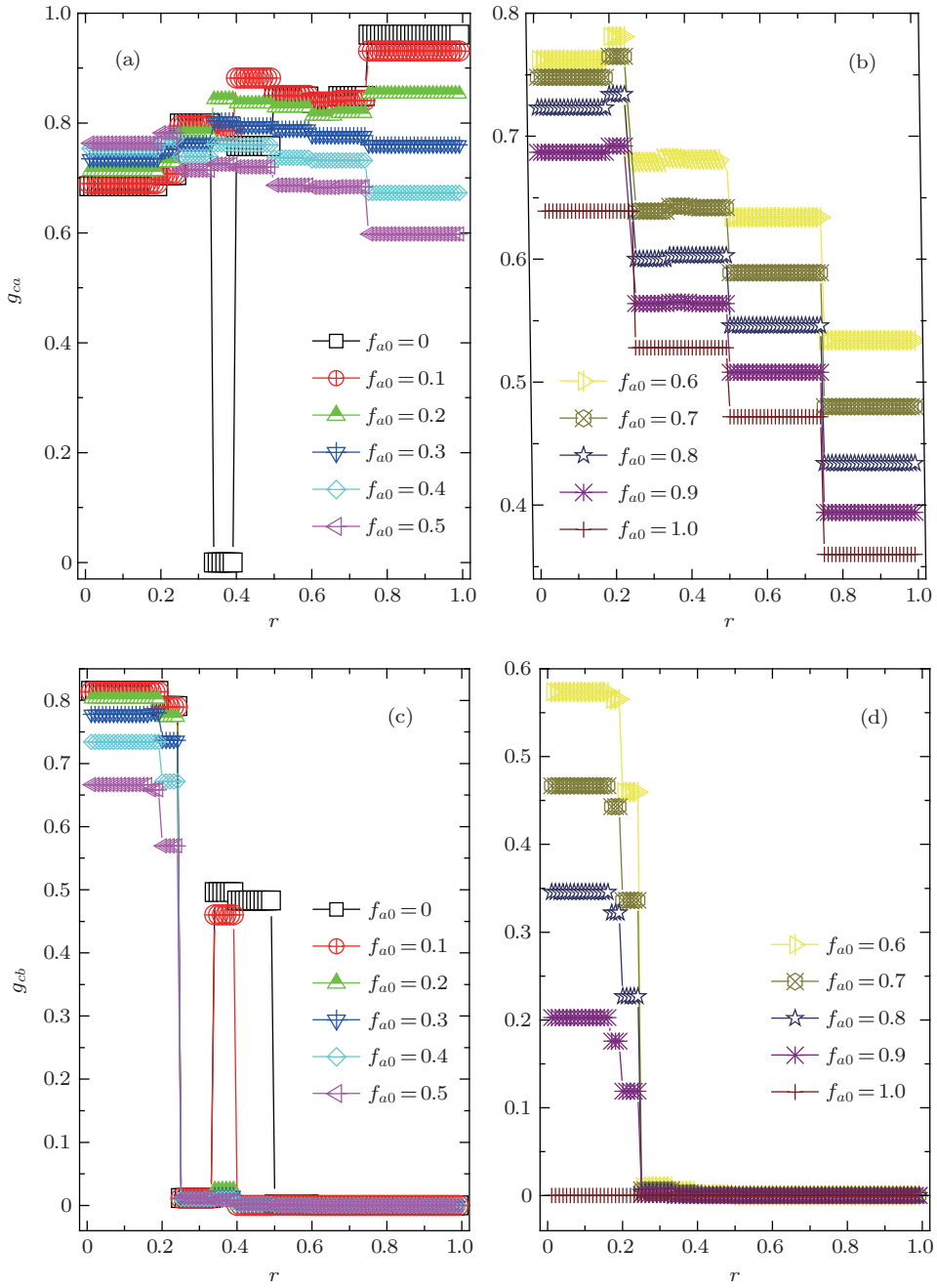


Fig. 8. (color online) The relative cooperation frequencies g_{ca} and g_{cb} versus the cost-to-benefit ratio r at $v_{\rm T}=36$. It is shown that both g_{ca} and g_{cb} have a discontinuous steplike structure for each initial composition f_{a0}. g_{cb} decreases with f_{a0} in the range of $0<r<1/2$, while it consistently equals zero in the range of $1/2<r<1$. g_{ca} always has a non-zero value for arbitrary r. Moreover, with increasing f_{a0} $f_{\rm c}$ first increases and then decreases in the range of $0<r<1/2$, and it consistently decreases with the increase of f_{a0} in the range of $1/2<r<1$.

Finally, by combining the relative cooperation frequencies g_{ca} and g_{cb} with the equilibrium composition f_a, we briefly interpret the above-mentioned behaviors of the global cooperation frequency $f_{\rm c}$ that differ from those in the $v_{\rm T}=3$ case. In the $v_{\rm T}=36$ case, with the increase of r, there are slight increases both in f_a and g_{ca} around the discontinuous point $r=1/5$, but a considerable decrease of g_{cb} also happens at the same time. It is the case for all $f_{a0}\leq 0.9$. Thus, before and after the transition point $r=1/5$, the global cooperation frequency $f_{\rm c}$ for different secondary levels decreases slightly along with r. Moreover, for $f_{a0}=0$, almost all B-type agents cannot change into A-type ones in the range of $1/3<r<1/2$ and, thus in such a situation our system is equivalent to the pure B-type agent system with the unconditional imitation rule, in which $f_{\rm c}\simeq 0.5$ and an inapparent jump occurs at $r=2/5$.[41] While for $f_{a0}=0.1$, all B-type agents cannot change into A-type ones in the range of $1/3<r<2/5$ and then our system is identical to the heterogenous system with a fixed composition of A- and B-type agents,[46] in which $f_{\rm c}\simeq 0.45$.

3.3. The general case

In this section, we discuss the overall dependence of the evolution behaviors of the heterogenous systems on the upper limit of tolerance v_T ($v_T \geq 2$). We select several values of f_{a0} as examples to show the equilibrium cooperation frequencies and the equilibrium fraction as functions of r and v_T. Figure 9 shows the equilibrium cooperation frequency f_c as a function of r and v_T for different values of f_{a0}. Obviously, as indicated by the contour lines in parallel with the v_T-axis, each f_c shows a plateau structure with three main discontinuous transitions at $r = 1/4$, $1/2$, and $3/4$, respectively. Besides, there are also several secondary discontinuous transitions occurring in f_c in the first two main intervals of $0 < r < 1/4$ and $1/4 < r < 1/2$, but which will gradually disappear with the increase of v_T as well as f_{a0}, and one can easily find the number of plateaux in f_c against r for an arbitrary value of v_T by making a horizontal cut at this v_T-value. On the other hand, figure 9 also indicates that the value of f_c increases consistently with increasing v_T in the interval of $0 < r < 1/4$, while it is independent of v_T in the interval of $3/5 < r < 1$.

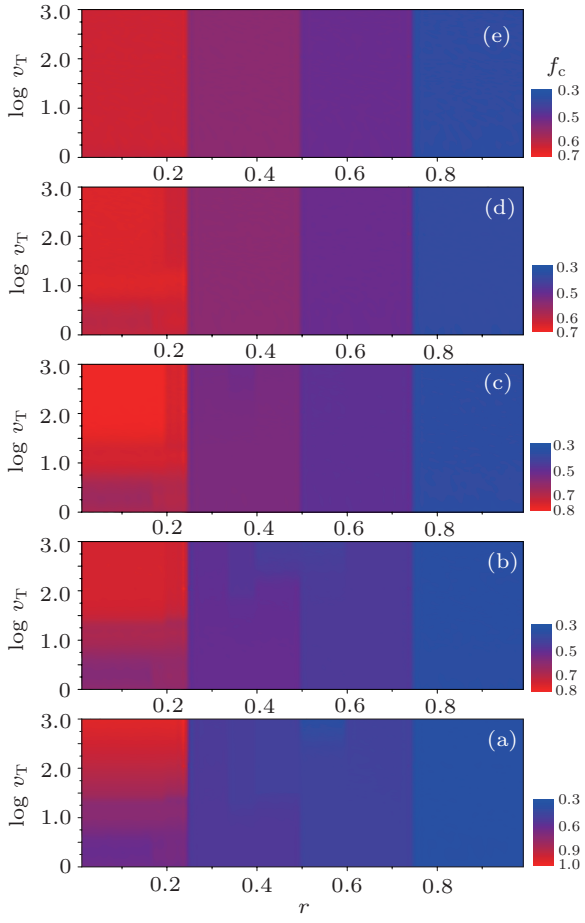


Fig. 9. (color online) The equilibrium cooperation frequencies f_c versus the cost-to-benefit ratio r versus the upper limit of tolerance v_T for different f_{a0} are plotted as a contour map. Here, (a) $f_{a0} = 0$; (b) $f_{a0} = 0.3$; (c) $f_{a0} = 0.5$; (d) $f_{a0} = 0.8$; and (e) $f_{a0} = 1.0$. f_c shows discontinuous steplike structures and neighboring plateaux are divided by several distinct contour boundaries at some particular values of r, i.e., the transition points.

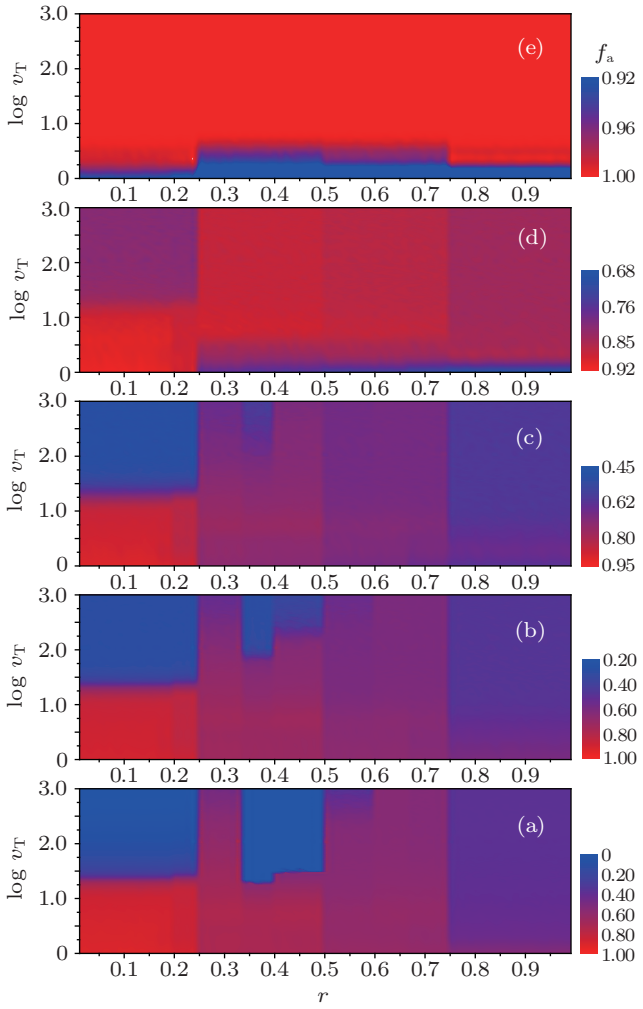


Fig. 10. (color online) The equilibrium composition f_a versus the cost-to-benefit ratio r versus the upper limit of tolerance v_T for different f_{a0} are plotted as a contour map. Here, (a) $f_{a0} = 0$; (b) $f_{a0} = 0.3$; (c) $f_{a0} = 0.5$; (d) $f_{a0} = 0.8$; and (e) $f_{a0} = 1.0$. f_a shows discontinuous steplike structures and neighboring plateaux are divided by several distinct contour boundaries at some particular values of r, i.e., the transition points.

Figure 10 illustrates the equilibrium composition f_a as a function of r and v_T for different values of f_{a0}, which is also of an obvious plateau structure. For small f_{a0}, i) f_a first keeps a constant independent of v_T until v_T is beyond a certain value, and it then rapidly decays to the initial value f_{a0} with increasing v_T in the ranges of $0 < r < 1/4$ and $1/3 < r < 1/2$; (ii) f_a slightly decreases to a steady value much larger than f_{a0} with increasing v_T in the range of $3/5 < r < 1$; and iii) f_a consistently decreases to a non-zero value finally in the remaining range of r. For very large f_{a0}, f_a first increases with increasing v_T and then rapidly reaches to a steady value. For mediate f_{a0}, f_a first keeps a constant independent of v_T and then rapidly decays to the steady value f_{a0} eventually in the range of $0 < r < 1/4$, while it first increases with the increase of v_T and then slightly decays to a non-zero value finally in the range of $1/4 < r < 1$. On the other hand, the quantity of f_a increases with the increase of f_{a0}. Moreover, when v_T is small (say, $v_T < 10$), f_a is close to unity in the range of

$0 < r < 1/4$, namely, most of the agents are A-type ones and adopt the memory-based updating rule; while in the remaining range of r, one can observe that $f_a \geq 0.7$ in the range of $1/4 < r < 1/2$, $f_a \geq 0.6$ in the range of $1/2 < r < 3/4$, and $f_a \geq 0.5$ in the range of $3/4 < r < 1$, and thus, at least half of the agents will take the A-type role even under the $f_{a0} = 0.0$ and $r > 3/4$ condition. When v_T is large (say, $v_T > 100$), we find that in the range of $0 < r < 1/4$, $f_a \simeq f_{a0}$, namely, it is almost impossible for an agent changing its type, and it is also the case for the $f_{a0} \leq 0.3$ system in the range of $1/3 < r < 1/2$; while in the remaining range of r, $f_a > f_{a0}$, that is, a certain percentage of B-type agents change into A-type ones finally. It should also be pointed out that a small percentage of A-type agents can change into B-type ones only when v_T is small (see, e.g., panel (e) in Fig. 10).

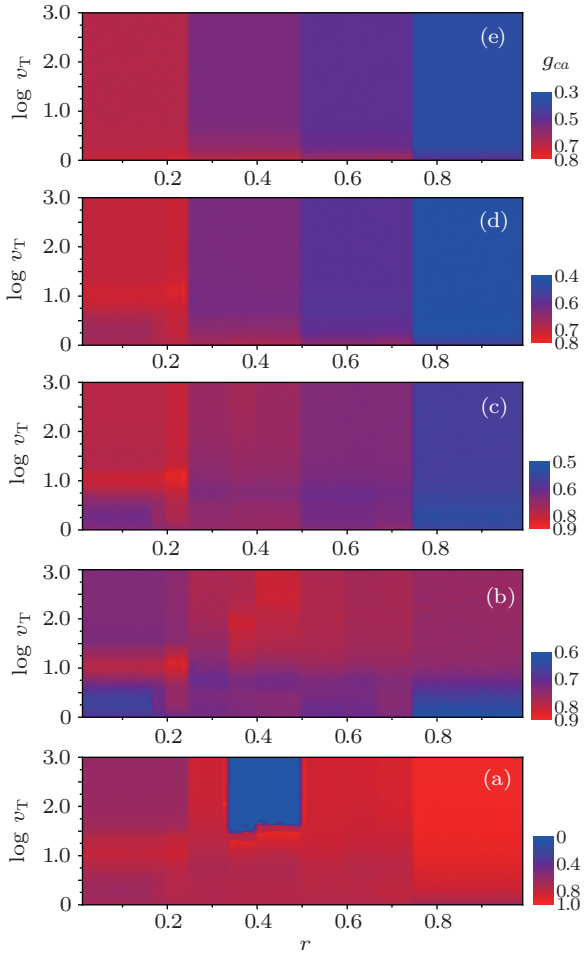


Fig. 11. (color online) The relative cooperation frequency g_{ca} versus the cost-to-benefit ratio r versus the upper limit of tolerance v_T for different f_{a0} is plotted as a contour map. Here, (a) $f_{a0} = 0$; (b) $f_{a0} = 0.3$; (c) $f_{a0} = 0.5$; (d) $f_{a0} = 0.8$; and (e) $f_{a0} = 1.0$. g_{ca} shows discontinuous steplike structures and neighboring plateaux are divided by several distinct contour boundaries at some particular values of r, i.e., the transition points.

Finally, we also analyze the relatively cooperation frequencies g_{ca} and g_{cb}. As illustrated in Fig. 11, for arbitrary v_T and f_{a0}, g_{ca} as a function of r is of an obvious plateau structure with three main discontinuity points at $r = 1/4$, $1/2$, and $3/4$, respectively. In each of the first three main intervals of r (namely, $0 < r < 1/4$, $1/4 < r < 1/2$, and $1/2 < r < 3/4$), the cooperation level will split further into several secondary plateaux. For large f_{a0}, say $f_{a0} = 1.0$, g_{ca} first decreases with the increase of v_T and soon decays to a steady value, which is independent of the value of r. While for small or moderate values of f_{a0}, the evolution of g_{ca} depends on the value of v_T as well as the value interval of r. Some remarkable features are listed as follows: a) the value of g_{ca} will reach to a steady value at large v_T; b) with v_T increasing from 2 to 1000, g_{ca} rapidly increases from the beginning in the range of $0 < r < 1/4$ and $3/4 < r < 1$, while it first decreases slightly and then rapidly increases to a steady value; c) g_{ca} has a maximum value at about $v_T = 15$ when $0 < r < 1/4$; and d) when $1/3 < r < 1/2$ and $f_{a0} = 0$, $g_{ca} = 0$ at large v_T. As a contrast to A-type agents, the cooperative behavior of B-type ones is quite simple, as illustrated in Fig. 12. For large r and f_{a0}, g_{cb} is almost equal to zero (namely, all B-type agents will adopt the D-strategy finally); while, only for small r and large v_T, g_{cb} has a non-zero value.

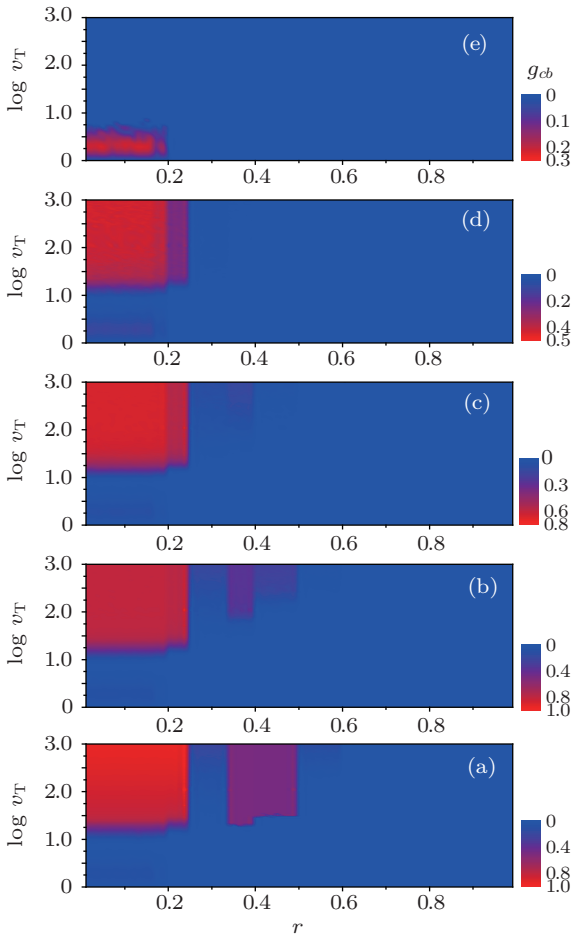


Fig. 12. (color online) The relative cooperation frequency g_{cb} versus the cost-to-benefit ratio r versus the upper limit of tolerance v_T for different f_{a0} is plotted as a contour map. Here, (a) $f_{a0} = 0$; (b) $f_{a0} = 0.3$; (c) $f_{a0} = 0.5$; (d) $f_{a0} = 0.8$; and (e) $f_{a0} = 1.0$. It is shown that g_{cb} equals zero for almost all experimental parameters, except for large v_T and small r.

4. Summary

In this work, we proposed an ESG in heterogenous agent systems with co-evolutionary strategies and updating rules. In the game systems, there are two types of agents: A-type and B-type. A-type agents adopt the memory-based rule to update their strategy while B-type ones use the unconditional imitation rule. Moreover, either type of agents can change into the other type if they suffer sequential failures beyond their upper limit of tolerance. We focused on the influences of the upper limit of tolerance and the initial composition of agents on the cooperative behavior of such heterogenous agent systems. Considering a simple ESG with the same upper limit of tolerance for all agents, we then studied its cooperative behavior by performing numerical simulations on a 2D square lattice with periodic boundary conditions.

It is found that the cooperation frequency f_c against r exhibits a steplike discontinuous plateau structure, and that the number of plateaux is independent of the upper limit of tolerance v_T as well as the initial composition f_{a0}. Generally, three main discontinuous transitions exist occurring at $r = 1/4$, $1/2$, and $3/4$, respectively, which divide the plateaux into four groups. In the first two groups, the plateaus may further split into several secondary discontinuous ones under some parameter conditions, and these secondary discontinuous jumps will gradually disappear with the increase of v_T as well as f_{a0}. More intriguingly, in the first main interval of $0 < r < 1/4$, the levels of secondary discontinuous plateaux abnormally increase with increasing r when v_T is small, say $v_T = 3$. Moreover, the value of f_c increases consistently to a steady quantity with increasing v_T in the interval of $0 < r < 1/4$ and it is just to the contrary in the interval of $1/4 < r < 3/4$, while it is independent of v_T in the interval of $3/5 < r < 1$.

We also observed that the equilibrium composition f_a as a function of r is of an obvious plateau structure. Besides, the quantity of f_a depends crucially on the initial composition f_{a0}, the upper limit of tolerance v_T, and the cost-to-benefit r. Firstly, f_a increases with the increase of f_{a0}. Secondly, when v_T is small, either type of agent can change their role into the other one, but the possibility of B-type agents changing into A-type ones is larger than that of the inverse process and, thus, for small or moderate f_{a0}, f_a is always larger than f_{a0}; when v_T is very large, an agent changing its type becomes impossible in the range of $0 < r < 1/4$ and it is similar for the $f_{a0} \leq 0.3$ system in the range of $1/3 < r < 1/2$, while in the remaining range of r, a certain percentage of B-type agents will eventually change into A-type ones. More interestingly, when v_T is small, almost all agents will be A-type ones in the range of $0 < r < 1/4$ and more than half of agents will take the A-type role even in the large-r case, independent of the initial composition f_{a0}.

In order to understand thoroughly our ESG system, we further analyzed the relative cooperative behavior of either type of agent. The relatively cooperation frequency g_{ca} as a function of r is of an obvious plateau structure with three main discontinuous jumps at $r = 1/4$, $1/2$, and $3/4$, respectively, and the cooperation level among the main interval of r will split further into several secondary plateaux. The quantity of g_{ca} depends on the value of v_T as well as the value of r. With increasing v_T, g_{ca} first increases rapidly and reaches to a steady value at large v_T. It is also worth noting that g_{ca} has a maximum value at about $v_T = 15$ when $0 < r < 1/4$. Besides, we observed $g_{ca} = 0$ at large v_T when $1/3 < r < 1/2$ and $f_{a0} = 0$. The relatively cooperative frequency g_{cb} of B-type agents has relatively simple behavior. When v_T is small, all B-type agents adopt the D-strategy in the range of $1/5 < r < 1$, i.e., $g_{cb} = 0$, while only very few B-type agents take the C-strategy in the remaining range of $0 < r < 1/5$. When v_T is large, we also observed $g_{cb} = 0$ in the range of $3/5 < r < 1$ but g_{cb} has a non-zero value in the range of $0 < r < 3/5$. Thus, for the practical case with small v_T, B-type agents almost contribute nothing to the cooperation level of the system.

Furthermore, numerical simulations on hexagonal and cubic lattices also exhibit the above-mentioned similar phenomena. So, the key feature that f_c versus r is of a plateau structure is typical of ESG with the mixed unconditional imitation and memory-based updating rules on regular lattices. This work is helpful for understanding in depth the ESG of heterogenous agent systems.

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