Drift effect on vacuum birefringence in a strong electric and magnetic field*

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(Received 27 August 2014; published online 30 January 2015)

As an important QED effect to detect the vacuum polarization, birefringence in the presence of a strong electric and magnetic field, $\boldsymbol{E}_0 \perp \boldsymbol{B}_0, E_0 \leq cB_0$, is considered. The directional dependence of birefringence is obtained. In two special cases: $E_0 = 0$ and $E_0 = cB_0$, our results are consistent with the previous ones. The refractive index of the probe wave propagating in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction decreases with E_0/cB_0, while that in the $-\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction increases with E_0/cB_0. The physics of the direction dependence of birefringence maybe the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ drift velocity of the virtual electrons and positrons.

Keywords: vacuum birefringence, critical Schwinger field, refractive index, $\boldsymbol{E} \times \boldsymbol{B}$ drift

PACS: 42.25.Bs, 42.50.–p, 03.70.+k, 42.50.Pq **DOI:** 10.1088/1674-1056/24/3/034201

1. Introduction

In the strong field, $\boldsymbol{B}_0 = B_0\widehat{\boldsymbol{y}}$, $B_0 \lesssim 4.3 \times 10^9$ T, which is close to the critical Schwinger field of electron–positron pair production,[1–4] the vacuum exhibits properties of nonlinear optical medium,[5–7] and leads to photon splitting,[8] dichroism,[9] birefringence,[10–14] and vacuum bragg scattering.[15] The results of PVLAS collaboration[13,14] are so close to the polarization rotation in a vacuum with a rotating transverse magnetic field, predicted by Adler,[10] Heyl and Hernquist.[11] Biswas and Melnikov[12] pointed out that the rotation of a magnetic field does not impact the birefringence. Adler also calculated the refractive indices of the probe wave in a rotation magnetic field and probed the dependence of the refractive indices on the frequency of the magnetic field. However, if there is a strong electric field, $\boldsymbol{E}_0$, perpendicular to the strong magnetic field, $\boldsymbol{B}_0$, the propagating characteristics of the wave in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction and in the $-\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction are quite different from each other. For $E_0 = cB_0$, several researchers[16–20] obtained the refractive index for the "head-on collision" probe wave (Eqs. (16) and (17) in Ref. [19] and Eq. (15) in Ref. [20]). Their results also showed that there is no interaction between the probe wave incident in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction, i.e., the refractive index is unit.

In general, in the field (E,B) of a tightly focused low-frequency laser, $|E| < c|B|$. If the laser frequency is smaller than 1×10^{20} rad/s, the scaling length of the field is much larger than the Compton wavelength of electron. In that case, the field can be equivalent to the homogenous field and the effective Lagrangian $\mathfrak{L}_{\rm eff}$[5] is available. Therefore, birefringence in a crossed field ($\boldsymbol{E}_0 \perp \boldsymbol{B}_0$ and $E_0 \leq cB_0$) should be reconsidered. In order to check birefringence in a more general regime, we consider the interaction between a high-frequency electromagnetic wave and a strong electromagnetic field: $\boldsymbol{E}_0 = E_0\widehat{\boldsymbol{x}}$, $\boldsymbol{B}_0 = B_0\widehat{\boldsymbol{y}}$, and $E_0 \leq cB_0$, where $\widehat{\boldsymbol{x}}$ and $\widehat{\boldsymbol{y}}$ are the unit vectors of the x and y directions, respectively.

2. Basic assumptions and theoretical model

In this paper, we show that the refractive index of a forward wave with the wave vector $\boldsymbol{k} \parallel \boldsymbol{E}_0 \times \boldsymbol{B}_0$ is different from that of a counter propagating wave in a vacuum with a strong homogeneous static electric and magnetic field, solving the Maxwell equations corrected by the effective Lagrangian[5]

$$\mathfrak{L}_{\rm eff} = \varepsilon_0 \frac{E^2 - c^2B^2}{2} + \kappa\varepsilon_0^2 \left[\left(E^2 - c^2B^2\right)^2 + 7c^2 \left(\boldsymbol{E}\cdot\boldsymbol{B}\right)^2 \right], \quad (1)$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ is the total electric and magnetic field respectively, using

$$\boldsymbol{D} = \varepsilon_0 \left\{ \boldsymbol{E} + \xi \frac{2\left(E^2 - c^2B^2\right)\boldsymbol{E} + 7c^2(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B}}{c^2B_0^2} \right\}, \quad (2)$$

and

$$\boldsymbol{H} = \mu_0^{-1} \left\{ \boldsymbol{B} + \xi \frac{2\left(E^2 - c^2B^2\right)\boldsymbol{B} - 7(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{E}}{c^2B_0^2} \right\}, \quad (3)$$

which are valid for $|E|, c|B| \ll E_{\rm crit} \approx 10^{18}$ V/m and the frequency of the field $\omega \ll \omega_e \approx 10^{20}$ rad/s. The quantum parameter, $\xi = 2\kappa\varepsilon_0 c^2 B_0^2$, where ε_0 is the permittivity of vacuum, $\kappa = 2\alpha^2\hbar^3/45m_{\rm e}^4c^5 \approx 3.3 \times 10^{-30}$ $({\rm J/m^3})^{-1}$, $m_{\rm e}$ is the electron static mass, and α is the fine-structure constant.

*Project supported by the National Basic Research Program of China (Grant No. 2011CB808104) and the National Natural Science Foundation of China (Grant No. 11105233).

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First, the external field is assumed to be $\boldsymbol{E}_0 = E_0\hat{\boldsymbol{x}}$ and $\boldsymbol{B}_0 = B_0\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are the unit vectors of the x and y directions respectively, $E_0 \lesssim cB_0 \lesssim 10^{18}$ V/m, which is the critical Schwinger field of electron–positron pair production in a vacuum.

Let $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_{\rm q}$ and $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_{\rm q}$ be the total electromagnetic field, and $\boldsymbol{E}_{\rm q}$ and $\boldsymbol{B}_{\rm q}$ are the nonclassical parts which are expressed as

$$\begin{aligned} \boldsymbol{E}_{\rm q} &= E_1\hat{\boldsymbol{y}} + E_2\hat{\boldsymbol{x}}, \\ \boldsymbol{B}_{\rm q} &= B_1\hat{\boldsymbol{y}} + B_2\hat{\boldsymbol{x}}. \end{aligned} \tag{4}$$

Let $|E_1|$, $|E_2|$, $|cB_1|$, $|cB_2| \ll |E_0|$, and $|cB_0|$, where c is the light velocity. The wave vector $\boldsymbol{k}//\hat{\boldsymbol{z}}$. Using the effective Lagrangian $\mathfrak{L}_{\rm eff}$,[5] the QED corrected electric displacement field $\boldsymbol{D}_{\rm q} = \boldsymbol{D} - \varepsilon_0\alpha_0\boldsymbol{E}_0$ and magnetic field strength $\boldsymbol{H}_{\rm q} = \boldsymbol{H} - \mu_0^{-1}\alpha_0\boldsymbol{B}_0$ are given by

$$\begin{aligned} \boldsymbol{D}_{\rm q} &= \varepsilon_0\{[(\alpha_0+7\xi)E_1 + 7\xi c_0 B_2]\hat{\boldsymbol{y}} \\ &\quad + [(\alpha_0 + 4\beta_0^2\xi)E_2 - 4\xi c_0 B_1]\hat{\boldsymbol{x}}\}, \\ \boldsymbol{H}_{\rm q} &= \mu_0^{-1}\left\{\left[4\frac{\beta_0}{c}\xi E_2 + (\alpha_0 - 4\xi)B_1\right]\hat{\boldsymbol{y}}\right. \\ &\quad \left. + \left[(\alpha_0 - 7\beta_0^2\xi)B_2 - 7\frac{\beta_0}{c}\xi E_1\right]\hat{\boldsymbol{x}}\right\}, \end{aligned} \tag{5}$$

where $\alpha_0 = 1 + 2\xi\hat{L}_0$, $c_0 = E_0/B_0$, $\beta_0 = c_0/c$, and $\hat{L}_0 = \beta_0^2 - 1$. In fact, c_0 is just the drift velocity of the virtual electrons and positrons in the field.

With the wave assumptions that $E_{1,2}, B_{1,2} \propto \exp[{\rm i}(kz - \omega t)]$, the linearization of the corrected source-free Maxwell equations

$$\begin{aligned} \partial_t \boldsymbol{B}_{\rm q} &= -\nabla\times\boldsymbol{E}_{\rm q}, \quad \nabla\cdot\boldsymbol{B}_{\rm q} = 0, \\ \partial_t \boldsymbol{D}_{\rm q} &= \nabla\times\boldsymbol{H}_{\rm q}, \quad \nabla\cdot\boldsymbol{D}_{\rm q} = 0, \end{aligned} \tag{6}$$

gives

$$\begin{aligned} &\omega B_1 = kE_2, \\ &\omega((\alpha_0 + 4\beta_0^2\xi)E_2 - 4c_0\xi B_1) \\ &= c^2k\left((\alpha_0 - 4\xi)B_1 + 4\frac{\beta_0}{c}\xi E_2\right), \end{aligned} \tag{7}$$

$$\begin{aligned} &\omega B_2 = -kE_1, \\ &-\omega((\alpha_0 + 7\xi)E_1 + 7c_0\xi B_2) \\ &= c^2k((\alpha_0 - 7\beta_0^2\xi)B_2 - 7\beta_0\xi E_1). \end{aligned} \tag{8}$$

3. Dispersion relationship and the refractive indices

With Eq. (8), the dispersion relationship for parallel polarization is obtained after some straightforward algebra

$$\omega^2(\alpha_0 + 7\xi) - 14\xi\omega c_0 k - (ck)^2(\alpha_0 - 7\beta_0^2\xi) = 0. \tag{9}$$

With Eq. (7), the dispersion relationship for perpendicular polarization is given by

$$\omega^2(\alpha_0 + 4\beta_0^2\xi) - 8\xi\omega c_0 k - (ck)^2(\alpha_0 - 4\xi) = 0. \tag{10}$$

Let $n = ck/\omega$ be the refractive index. With Eqs. (9) and (10), the refractive index of a probe wave propagating in the positive z direction, i.e., in the $\boldsymbol{E}_0\times\boldsymbol{B}_0$ direction for parallel polarization and perpendicular polarization, is respectively given by

$$\begin{aligned} n_{+,\rm par} &= \frac{-7\beta_0\xi + \sqrt{\alpha_0(\alpha_0 - 7\hat{L}_0\xi)}}{\alpha_0 - 7\beta_0^2\xi} \\ &\approx 1 - \frac{7}{2}\hat{L}_0\xi - 7\beta_0(1-\beta_0)\xi, \end{aligned} \tag{11}$$

and

$$\begin{aligned} n_{+,\rm per} &= \frac{-4\beta_0\xi + \sqrt{\alpha_0(\alpha_0 + 4\hat{L}_0\xi)}}{\alpha_0 - 4\xi} \\ &\approx 1 + 2\hat{L}_0\xi + 4(1-\beta_0)\xi. \end{aligned} \tag{12}$$

Similarly, the refractive index of a probe wave propagating in the negative z direction, i.e., in the $-\boldsymbol{E}_0\times\boldsymbol{B}_0$ direction for parallel polarization and perpendicular polarization, is respectively given by

$$\begin{aligned} n_{-,\rm par} &= \left|\frac{-7\beta_0\xi - \sqrt{\alpha_0(\alpha_0 - 7\hat{L}_0\xi)}}{\alpha_0 - 7\beta_0^2\xi}\right| \\ &\approx 1 - \frac{7}{2}\hat{L}_0\xi + 7\beta_0(1+\beta_0)\xi, \end{aligned} \tag{13}$$

and

$$\begin{aligned} n_{-,\rm per} &= \left|\frac{-4\beta_0\xi - \sqrt{\alpha_0(\alpha_0 + 4\hat{L}_0\xi)}}{\alpha_0 - 4\xi}\right| \\ &\approx 1 + 2\hat{L}_0\xi + 4(1+\beta_0)\xi. \end{aligned} \tag{14}$$

With Eqs. (11) and (14), the difference of the refractive indices for the parallel polarization and the perpendicular polarization in the $\boldsymbol{E}_0\times\boldsymbol{B}_0$ direction is obtained as

$$\Delta n_+ \approx -\frac{11}{2}\hat{L}_0\xi - (4 + 7\beta_0)(1-\beta_0)\xi. \tag{15}$$

In the $-\boldsymbol{E}_0\times\boldsymbol{B}_0$ direction, it is obtained as

$$\Delta n_- \approx -\frac{11}{2}\hat{L}_0\xi - (4 - 7\beta_0)(1+\beta_0)\xi. \tag{16}$$

Equation (15) shows that the difference of the refractive index for the parallel polarization wave and the perpendicular polarization wave is proportional to ξ. Here the wave propagates in the positive z direction, i.e., $\boldsymbol{E}_0\times\boldsymbol{B}_0$. Equation (16) shows the refractive index for the parallel polarization wave and the perpendicular wave propagating in the negative z direction, i.e., $-\boldsymbol{E}_0\times\boldsymbol{B}_0$. The results predict the difference of the refractive indices is proportional to ξ. Figure 1 shows the drift effect on vacuum birefringence in a homogenous strong crossed field. The birefringence is amplified by the drift effect for the counter-propagating probe wave and is reduced for the probe wave propagating in the direction of $\boldsymbol{E}_0\times\boldsymbol{B}_0$.

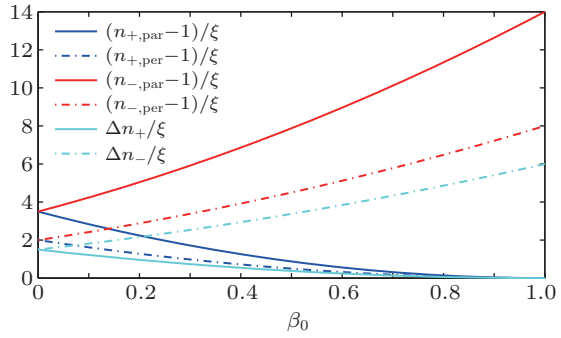


Fig. 1. (color online) Drift effect on vacuum birefringence in a homogenous strong crossed field.

4. Comparison and discussion

To compare our results with that of the literatures, two special cases are considered: (a) $E_0 = 0$, i.e., vacuum birefringence in the presence of a homogenous magnetic field; (b) $E_0 = cB_0$ in a crossed field.

For $E_0 = 0$, $c_0 = \beta_0 = 0$, $\hat{L}_0 = -1$, and the refractive indices are

$$n_{+,\mathrm{par}} = n_{-,\mathrm{par}} = 1 + \frac{7\xi}{2}, \tag{17}$$

for parallel polarization, and

$$n_{+,\mathrm{per}} = n_{-,\mathrm{per}} = 1 + 2\xi, \tag{18}$$

and the difference of the refractive indices is

$$\Delta n_+ = \Delta n_- = \frac{3\xi}{2}. \tag{19}$$

All the results are the same as those of Refs. [10]–[13]. Therefore, in a homogenous magnetic field, there is no drift velocity of virtual particles and there is no direction dependence of the refraction.

Another well-known case is $E_0 = cB_0$, $\beta_0 = 1$, and $c_0 = c$. Therefore, the refractive indices in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction are trivial and $n_{+,\mathrm{par}} = n_{+,\mathrm{per}} = 1$. There is no interaction between the probe wave and the crossed field. It is also consistent with Eqs. (16) and (17) in Ref. [19] and Eq. (15) in Ref. [20] in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction.

However, for the counter-propagating probe wave, the refractive index is given by

$$n_{-,\mathrm{par}} = 1 + 14\xi, \tag{20}$$

for parallel polarization, and

$$n_{-,\mathrm{per}} = 1 + 8\xi, \tag{21}$$

which is also consistent with Eqs. (16) and (17) in Ref. [19] and Eq. (15) in Ref. [20] in the $-\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction.

The refractive indices of the probe wave in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction and in the $-\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction are different from each other. The difference of the refractive indices for parallel polarization and perpendicular polarization are also different in the two directions. Define

$$\Delta n = \Delta n_- - \Delta n_+ = 6\beta_0\xi, \tag{22}$$

which tends to be zero as $E_0 \to 0$.

The physics of the direction dependence of the refractive index maybe the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ drift of the virtual electrons and positrons in the strong field. The virtual electrons and positrons have the same $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ drift velocity, $E_0/B_0\hat{\boldsymbol{z}} = c_0\hat{\boldsymbol{z}}$. Therefore, it induces the direction dependence of refraction. Equation (22) shows that the difference is proportional to the drift velocity.

In the experiment to measure the birefringence, the light is reflected and propagates through the field forward and backward repeatedly for N times. Set the longitudinal length of the field as L, the elipticity value[14] is given by

$$\psi = N\pi\frac{L}{\lambda}(\Delta n_+ + \Delta n_-) = N\pi\frac{L}{\lambda}3(1+\beta_0^2)\xi, \tag{23}$$

which is $1+\beta_0^2$ times that in a strong magnetic field.

5. Conclusion

In conclusion, vacuum birefringence is different in the $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction and in the $-\boldsymbol{E}_0 \times \boldsymbol{B}_0$ direction. The $\boldsymbol{E}_0 \times \boldsymbol{B}_0$ drift of the virtual electrons and positrons may be the key point. The influence of the drift effect on the ellipticity in the experiments has been given explicitly.

Acknowledgment

We are grateful to Felix Karbstein for the interesting and enlightening discussion.

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