# Measurement-induced nonlocality in the W and Greenberger–Horne–Zeilinger superposition states*

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Measurement-induced nonlocality (MIN) is a newly defined quantity to measure correlations in bipartite quantum states [Luo S and Fu S 2011 *Phys. Rev. Lett.* **106** 120401]. MIN in the $n$-qubit W and Greenberger–Horne–Zeilinger (GHZ) superposition states is considered. It is revealed that $n = 3$ and $n \geq 4$ states have very different characteristics, especially the monogamy relation about MIN, and the monogamy equality of MIN is held in all $n$-qubit W states ($n \geq 3$).

**Keywords:** measurement-induced nonlocality, monogamy, W state

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## 1. Introduction

Quantum nonlocality usually refers to correlations that cannot be explained by any local hidden variable theory and it has been widely studied through Bell's inequalities.[1–8] Traditionally, it is believed that nonlocality can only be proven by quantum entanglement, which plays a vital role in quantum information processing and the violation of Bell's inequalities.[9–11] However, it is demonstrated by many works that nonlocality can exist without entanglement. For example, there exist sets of locally indistinguishable orthogonal pure product states that show nonlocality without entanglement.[12] Measurement-induced nonlocality (MIN) was recently proposed by Luo and Fu, and is understood as some kind of correlation that is more general than nonlocality related to entanglement or violation of Bell's inequalities.[8]

The MIN quantifies the nonlocal correlation in bipartite states from a geometric perspective. It considers the distance between the original state and the local measurement-induced state, preserving the local density matrix.[8] Since the proposal of MIN, many researchers have studied this further. Sen *et al*. investigated the monogamy relation of MIN in some three and four-qubit states.[11] Xu and Tong analyzed the dynamics of MIN.[13] Hu and Fan proposed the entropic MIN.[14] Xi *et al*. quantify the measurement-induced nonlocality from the perspective of the relative entropy.[15] Moreover, some works have been devoted to comparing the MIN with other quantum correlations,[16–24] such as geometric measure of discord,[25,26] quantum discord,[27,28] and measurement-induced disturbance.[29] As a new correlation measure, a deep investigation about MIN is very desirable.

In this paper we study the MIN in $n$-qubit ($n \geq 3$) W and Greenberger–Horne–Zeilinger (GHZ) superposition states, which will be called W–GHZ states hereafter. We find that the three-qubit W–GHZ states and the other ones ($n \geq 4$) have different characteristics, especially the monogamy relation about MIN. Although all two-qubit reduced states of $n$-qubit W–GHZ states depend on the relative phases between the W and GHZ components, we find that the MIN depends on the relative phases only for the case of $n = 3$, i.e., the MIN for $n \geq 4$ does not depend on the relative phases. Furthermore, we find that the monogamy relation of MIN is satisfied in all of the three-qubit W–GHZ states but it is not satisfied in many states for $n \geq 4$. In addition, we also find that the monogamy equality of MIN holds in $n$-qubit W state for any $n \geq 3$. This paper is organized as follows. In Section 2, we review some basic properties of MIN and monogamy. In Section 3, we analyze the MIN in three-qubit W–GHZ states. Furthermore, we study the MIN in $n$-qubit ($n \geq 4$) W–GHZ states in Section 4. Finally, we give a brief conclusion.

## 2. MIN and correlation monogamy

The MIN of a bipartite quantum state $\rho$ shared by two particles $a$ and $b$ is defined as[8]

$$N(\rho_{a|b}) = \max_{\Pi^a} \| \rho - \Pi^a(\rho) \|^2, \tag{1}$$

where the maximum is taken over all the local von Neumann measurements $\Pi^a = \{\Pi_k^a\}$ which does not disturb $\rho^a$ locally, satisfying $\sum_k \Pi_k^a \rho^a \Pi_k^a = \rho^a$, and $\|X\|^2 = \mathrm{tr}(X^\dagger X)$ is taken to

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be Hilbert–Schmidt norm. In the following, when we consider the MIN in the state $\rho_{abcd\cdots}$ we regard $bcd\cdots$ as a group and use the notation $N(\rho_{a|bcd\cdots})$, and the local measurement is done on particle $a$. For bipartite pure states with the Schmidt decomposition $|\psi\rangle_{ab}=\sum_i\sqrt{s_i}|\alpha_i\rangle\otimes|\beta_i\rangle$, the MIN can be evaluated by[8]

$$N(|\psi\rangle\langle\psi|)=1-\sum_i s_i^2=1-\mathrm{tr}(\rho_a^2), \tag{2}$$

where $\rho_a=\mathrm{tr}_b(|\psi\rangle\langle\psi|)$.

In the generalized Bloch representation, a two-qubit mixed state can always be written as

$$\begin{aligned}\rho_{ab} &= \frac{1}{2}\frac{I_2}{\sqrt{2}}\otimes\frac{I_2}{\sqrt{2}}+\sum_{i=1}^{3}x_i\frac{\sigma_i}{\sqrt{2}}\otimes\frac{I_2}{\sqrt{2}}\\ &\quad+\frac{I_2}{\sqrt{2}}\otimes\sum_{j=1}^{3}y_j\frac{\sigma_j}{\sqrt{2}}+\sum_{i=1}^{3}\sum_{j=1}^{3}t_{ij}\frac{\sigma_i}{\sqrt{2}}\otimes\frac{\sigma_j}{\sqrt{2}},\end{aligned} \tag{3}$$

where $I_2$ is the identity operator for single qubits and $\{\sigma_i : i=1,2,3\}$ are three Pauli operators. It can be verified that

$$x_i=\mathrm{tr}\left(\rho_{ab}\frac{\sigma_i}{\sqrt{2}}\otimes\frac{I_2}{\sqrt{2}}\right),\quad t_{ij}=\mathrm{tr}\left(\rho_{ab}\frac{\sigma_i}{\sqrt{2}}\otimes\frac{\sigma_j}{\sqrt{2}}\right).$$

In this case, the MIN can be calculated as follows:[8]

$$N(\rho_{a|b})=\begin{cases}\mathrm{tr}(TT^{\mathrm{T}})-\dfrac{1}{\|X\|^2}X^{\mathrm{T}}TT^{\mathrm{T}}X, & \text{if } \|X\|^2\neq 0,\\ \mathrm{tr}(TT^{\mathrm{T}})-\lambda_3, & \text{if } \|X\|^2=0,\end{cases} \tag{4}$$

where $T^{\mathrm{T}}$ is the transpose of matrix $T=(t_{ij})$, $\lambda_3$ is the minimum eigenvalue of the matrix $TT^{\mathrm{T}}$, and $\|X\|^2=\sum_i x_i^2$ with $X=(x_1,x_2,x_3)^{\mathrm{T}}$.

In many-body quantum systems, monogamy is one of the most important properties, and several studies have been done on monogamy.[30–33] A correlation measure $M$ satisfies the monogamy relation in $n$-particle state $\rho$ if the relation

$$M_{(\rho_{a_i|a_1,a_2,\ldots,a_{i-1},a_{i+1},a_n)}}\geq\sum_{k=1,k\neq i}^{n}M(\rho_{a_i|a_k}) \tag{5}$$

holds for all $i=1,2,\ldots,n$. This relation means that the correlation between the particle $i$ and the rest is not less than the sum of the correlations between two particles $i$ and $k$.

## 3. MIN in three-qubit W–GHZ states

In this section, we analyze the MIN in a three-qubit W–GHZ state

$$|\psi_{abc}\rangle=\cos\theta|\mathrm{W}\rangle_3+\sin\theta\,\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{GHZ}\rangle_3,\quad \theta\in[0,\pi/2], \tag{6}$$

where $|\mathrm{W}\rangle_3=(|100\rangle+|010\rangle+|001\rangle)/\sqrt{3}$ and $|\mathrm{GHZ}\rangle_3=(|000\rangle+|111\rangle)/\sqrt{2}$.

We first express the two-qubit reduced state $\rho_{ab}=\mathrm{tr}_c(|\psi_{abc}\rangle\langle\psi_{abc}|)$ as the Bloch form shown in Eq. (3). The parameters $x_i$ can be calculated by the equation

$$x_i=\langle\psi_{abc}|\frac{\sigma_i}{\sqrt{2}}\otimes\frac{I_2}{\sqrt{2}}\otimes I_2|\psi_{abc}\rangle,$$

and there are

$$x_1 = \sin\theta\cos\theta\cos\varphi/\sqrt{6}, \tag{7}$$

$$x_2 = -\sin\theta\cos\theta\sin\varphi/\sqrt{6}, \tag{8}$$

$$x_3 = \cos^2\theta/6. \tag{9}$$

Since the state $\rho_{ab}$ is a symmetric state, there is $t_{ij}=t_{ji}$ and $T^{\mathrm{T}}=T$. The elements of correlation matrix $T$ are

$$t_{ij}=\langle\psi_{abc}|\frac{\sigma_i}{\sqrt{2}}\otimes\frac{\sigma_j}{\sqrt{2}}\otimes I_2|\psi_{abc}\rangle,$$

and we have

$$t_{11} = \frac{1}{3}\cos^2\theta+\frac{1}{\sqrt{6}}\sin\theta\cos\theta\cos\varphi, \tag{10}$$

$$t_{12} = \frac{1}{\sqrt{6}}\sin\theta\cos\theta\sin\varphi, \tag{11}$$

$$t_{13} = \frac{1}{\sqrt{6}}\sin\theta\cos\theta\cos\varphi, \tag{12}$$

$$t_{22} = \frac{1}{3}\cos^2\theta-\frac{1}{\sqrt{6}}\sin\theta\cos\theta\cos\varphi, \tag{13}$$

$$t_{23} = -\frac{1}{\sqrt{6}}\sin\theta\cos\theta\sin\varphi, \tag{14}$$

$$t_{33} = -\frac{1}{6}\cos^2\theta+\frac{1}{2}\sin^2\theta. \tag{15}$$

Now, we analyze the MIN $N(\rho_{a|b})$. Because $\mathrm{tr}(TT^{\mathrm{T}})=\Sigma_{ij}t_{ij}^2$, we find that for any parameter $\theta$ and $\varphi$ there is $\mathrm{tr}(TT^{\mathrm{T}})=1/4$. Note that

$$\|X\|^2=\sum_{i=1}^{3}x_i^2=\frac{1}{36}\cos^2\theta(6\sin^2\theta+\cos^2\theta), \tag{16}$$

we have the following discussion.

1) When $\cos\theta=0$, there is $\|X\|^2=0$ and we should use the second line of Eq. (4) to calculate $N(\rho_{a|b})$. Simple calculation shows $\lambda_3=0$, and there is $N(\rho_{a|b})=\mathrm{tr}(TT^{\mathrm{T}})-\lambda_3=1/4$.

2) When $\cos\theta\neq 0$, there is $\|X\|^2\neq 0$ and we should use the first line of Eq. (4) to calculate $N(\rho_{a|b})$. From the expression $X^{\mathrm{T}}TT^{\mathrm{T}}X=\|T^{\mathrm{T}}X\|^2$, we obtain

$$\begin{aligned}X^{\mathrm{T}}TT^{\mathrm{T}}X &= \frac{1}{36}\sin^2\theta\cos^6\theta+\frac{13}{144}\sin^4\theta\cos^4\theta\\ &\quad+\left(\frac{1}{36}\cos^4\theta\right)^2+\frac{1}{6\sqrt{6}}\sin^3\theta\cos^5\theta\cos(3\varphi),\end{aligned} \tag{17}$$

leading to

$$\begin{aligned}N(\rho_{a|b}) &= \frac{1}{4}-\frac{\sin^2\theta\cos^4\theta+\dfrac{13}{4}\sin^4\theta\cos^2\theta}{6\sin^2\theta+\cos^2\theta}\\ &\quad-\frac{\dfrac{1}{36}\cos^6\theta+\sqrt{6}\sin^3\theta\cos^3\theta\cos(3\varphi)}{6\sin^2\theta+\cos^2\theta}.\end{aligned} \tag{18}$$

We note that the above expression is also applicable when $\cos\theta = 0$.

It is obvious that $N(\rho_{a|b})$ depends on the relative phase $\varphi$, which may not be surprising since $\rho_{ab}$ depends on the parameter $\varphi$. However in the next section we will show that $N(\rho_{a|b})$ does not depend on $\varphi$ in the $n$-qubit ($n \geq 4$) W–GHZ state. Note that $\sin^3\theta\cos^3\theta \geq 0$, from Eq. (18) it can be observed that for any fixed $\theta$ not equal to 0 or $\pi/2$, $N(\rho_{a|b})$ will increase as $\cos(3\varphi)$ decreases. Figure 1 shows how $N(\rho_{a|b})$ varies as a function of $\cos^2\theta$ and $\cos(3\varphi)$. It can be found that $N(\rho_{a|b})$ gets its maximal value $1/4$ when $\cos\theta = 0$, and $N(\rho_{a|b})$ gets its minimal value 0 when $\cos(3\varphi) = 1$ and $\cos^2\theta = 3/5$.

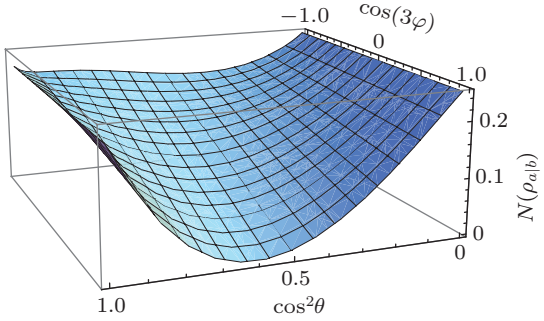



**Fig. 1.** (color online) $N(\rho_{a|b})$ of the two-qubit reduced state of the three-qubit W–GHZ state varies as a function of $\cos^2\theta$ and $\cos(3\varphi)$. For any fixed $\theta$ not equal to 0 or $\pi/2$, i.e., $\cos^2\theta \neq 0$ or 1, $N(\rho_{a|b})$ will increase as $\cos(3\varphi)$ decreases.

Next we analyze the monogamy property of MIN in the three-qubit W–GHZ state. Due to the symmetry, we only need to check whether $\Delta N = N(\rho_{a|bc}) - N(\rho_{a|b}) - N(\rho_{a|c}) \geq 0$ is satisfied. Since $|\psi_{abc}\rangle$ is a pure state, $N(\rho_{a|bc})$ can be obtained from the reduced density matrix $\rho_a$ according to Eq. (2). From the Bloch representation $\rho_a = I_2/2 + \sum_{i=1}^{3} x_i\sigma_i$, and we obtain

$$N(\rho_{a|bc}) = \frac{1}{2} - \frac{1}{18}\cos^2\theta(6\sin^2\theta + \cos^2\theta). \tag{19}$$

Furthermore, due to $N(\rho_{a|b}) = N(\rho_{a|c})$, there is

$$\Delta N = N(\rho_{a|bc}) - 2N(\rho_{ab}) = \frac{36\sin^2\theta\cos^2\theta}{6\sin^2\theta + \cos^2\theta} \times A, \tag{20}$$

where

$$A = \left(\frac{\cos\theta}{3\sqrt{3}}\right)^2 + \left(\frac{\sin\theta}{2\sqrt{2}}\right)^2 + 2\left(\frac{\cos\theta}{3\sqrt{3}}\right)\left(\frac{\sin\theta}{2\sqrt{2}}\right)\cos(3\varphi).$$

It is obvious that $A \geq 0$ for any $\theta$ and $\varphi$, and therefore the monogamy relation is satisfied in all three-qubit W–GHZ states. It can be observed that for any fixed $\theta$ not equal to 0 or $\pi/2$ , $\Delta N$ increases along the parameter $\cos(3\varphi)$. Figure 2 shows that how $\Delta N$ varies as a function of $\cos^2\theta$ and $\cos(3\varphi)$. It is found that $\Delta N$ gets its maximal value $2/5$ when $\cos(3\varphi) = 1$ and $\cos^2\theta = 3/5$, and $\Delta N$ gets its minimal value 0 when $\sin\theta\cos\theta = 0$ or in the state satisfying $\cos(3\varphi) = -1$ and $\cos\theta/(3\sqrt{3}) = \sin\theta/(2\sqrt{2})$.

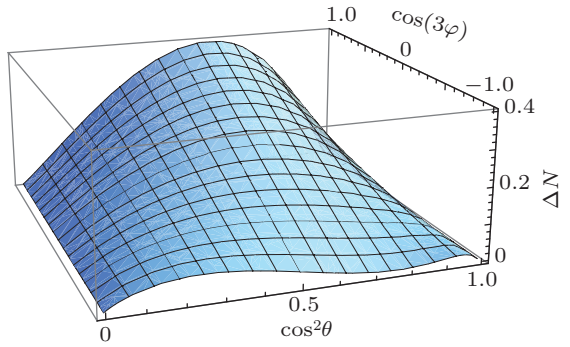



**Fig. 2.** (color online) $\Delta N = N(\rho_{a|bc}) - 2N(\rho_{ab})$ varies as a function of $\cos^2\theta$ and $\cos(3\varphi)$. For any fixed $\theta$ not equal to 0 or $\pi/2$, i.e., $\cos^2\theta \neq 0$ or 1, $\Delta N$ will increase as $\cos(3\varphi)$ increases. There is $\Delta N \geq 0$ for any $\theta$ and $\varphi$ that means the monogamy relation of MIN is always satisfied in the three-qubit W–GHZ state.

## 4. MIN in generalized *n*-qubit W–GHZ states

In this section, we investigate the MIN in the generalized $n$-qubit ($n \geq 4$) W–GHZ state

$$|\psi_{abcd\cdots}\rangle = \cos\theta|\mathrm{W}\rangle_n + \sin\theta\,\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{GHZ}\rangle_n, \tag{21}$$

where $\theta \in [0, \pi/2]$, the W and GHZ states are $|\mathrm{W}\rangle_n = (1/\sqrt{n})(|10\cdots0\rangle + |01\cdots0\rangle + \cdots + |00\cdots1\rangle)$ and $|\mathrm{GHZ}\rangle_n = (1/\sqrt{2})(|00\cdots0\rangle + |11\cdots1\rangle)$, respectively.

The discussion is parallel to three-qubit W–GHZ states. We first write the two-qubit reduced state $\rho_{ab}$ in the Bloch form shown in Eq. (3). To obtain $N(\rho_{a|b})$, we need to calculate $x_i$ and $t_{ij}$. After some derivation, we have

$$x_1 = \sin\theta\cos\theta\cos\varphi/\sqrt{2n}, \tag{22}$$

$$x_2 = -\sin\theta\cos\theta\sin\varphi/\sqrt{2n}, \tag{23}$$

$$x_3 = \frac{(n-2)\cos^2\theta}{2n}, \tag{24}$$

and the elements of the symmetric matrix $T$ are

$$t_{11} = \frac{1}{n}\cos^2\theta, \tag{25}$$

$$t_{12} = 0, \tag{26}$$

$$t_{13} = \frac{1}{\sqrt{2n}}\sin\theta\cos\theta\cos\varphi, \tag{27}$$

$$t_{22} = \frac{1}{n}\cos^2\theta, \tag{28}$$

$$t_{23} = -\frac{1}{\sqrt{2n}}\sin\theta\cos\theta\sin\varphi, \tag{29}$$

$$t_{33} = \frac{n-4}{2n}\cos^2\theta + \frac{1}{2}\sin^2\theta. \tag{30}$$

From the expression $\mathrm{tr}(TT^{\mathrm{T}}) = \sum_{ij} t_{ij}^2$, we obtain

$$\mathrm{tr}(TT^{\mathrm{T}}) = \left(\frac{6}{n^2} - \frac{1}{n}\right)\cos^4\theta - \frac{1}{n}\cos^2\theta + \frac{1}{4}. \tag{31}$$

Note that

$$\|X\|^2 = \sum_{i=1}^{3} x_i^2 = \frac{\cos^2\theta}{4n^2}((n^2 - 6n + 4)\cos^2\theta + 2n), \tag{32}$$

we have the following discussion.

1) When $\cos\theta = 0$, there is $\|X\|^2 = 0$ and we should use the second line of Eq. (4) to calculate $N(\rho_{a|b})$. Simple calculation shows $\lambda_3 = 0$, and there is $N(\rho_{a|b}) = \mathrm{tr}(TT^{\mathrm{T}}) - \lambda_3 = 1/4$.

2) When $\cos\theta \neq 0$, there is $\|X\|^2 \neq 0$ and we should use the first line of Eq. (4) to calculate $N(\rho_{a|b})$. From the expression $X^{\mathrm{T}}TT^{\mathrm{T}}X = \|T^{\mathrm{T}}X\|^2$, it is found that

$$X^{\mathrm{T}}TT^{\mathrm{T}}X = \left(\frac{(3n-4)^2}{4n^4} - \frac{1}{8n}\right)\cos^8\theta + \left(\frac{1}{8n} - \frac{3n-4}{4n^2}\right)\cos^6\theta + \frac{1}{16}\cos^4\theta. \quad (33)$$

The MIN of $\rho_{ab}$ is

$$N(\rho_{a|b}) = \frac{c_1\cos^6\theta + c_2\cos^4\theta - c_3\cos^2\theta + n^3}{2n^2[(n^2-6n+4)\cos^2\theta + 2n]}, \quad (34)$$

where $c_1 = -n^3 + 6n^2 - 32n + 16$, $c_2 = 3n^3 + 16n$, and $c_3 = 3n^3 + 2n^2$. The above expression is also applicable for the case $\cos\theta = 0$.

It is obvious that $N(\rho_{a|b})$ does not depend on the relative phase $\varphi$, which is a counterintuitive result since $\rho_{ab}$ does depend on $\varphi$. Figure 3 shows how $N(\rho_{a|b})$ varies as a function of $\cos^2\theta$ for the different value of $n$. It can be observed that for fixed $\theta$ not equal to $\pi/2$, $N(\rho_{a|b})$ will decrease along with $n$.

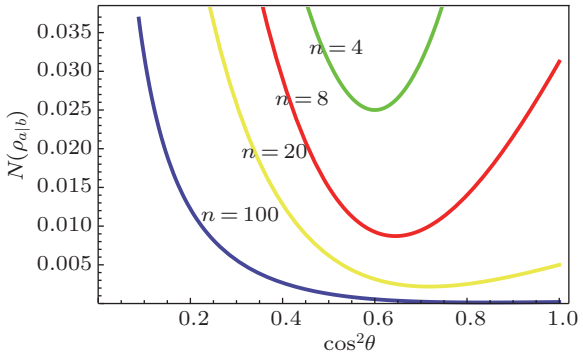



**Fig. 3.** (color online) $N(\rho_{a|b})$ of the two-qubit reduced state of the $n$-qubit W–GHZ state varies as a function of $\cos^2\theta$, which does not vary with the relative phase $\varphi$ although $\rho_{ab}$ depends on $\varphi$. For any fixed $\theta$ not equal to $\pi/2$, i.e., $\cos^2\theta \neq 0$, $N(\rho_{a|b})$ will decrease as $n$ increases.

Next we analyze the monogamy property of the MIN in this generalized $n$-qubit W–GHZ state. We need to check whether $\Delta N = N(\rho_{a|bcd\cdots}) - (n-1)N(\rho_{a|b}) \geq 0$ is satisfied. According to Eq. (2), the MIN $N(\rho_{a|bcd\cdots})$ has the form

$$N(\rho_{a|bcd\cdots}) = \frac{-n^2+6n-4}{2n^2}\cos^4\theta - \frac{\cos^2\theta}{n} + \frac{1}{2}. \quad (35)$$

Therefore, there is

$$\Delta N = \frac{d_1\cos^6\theta + d_2\cos^4\theta + d_3\cos^2\theta + 3n - n^2}{2[(n^2-6n+4)\cos^2\theta + 2n]}, \quad (36)$$

where $d_1 = 5n-6$, $d_2 = -3n^2 - n + 8$, and $d_3 = 4n^2 - 7n - 2$. When $\cos\theta = 0$, i.e., the state we consider is the $n$-qubit GHZ state, there is $\Delta N = (3-n)/4 < 0$ and the monogamy relation is not satisfied. When $\cos\theta = 1$, i.e., the state we consider is the $n$-qubit W state, there is $\Delta N = 0$ for any $n$, and the monogamy equality of MIN is satisfied. When $\cos\theta \neq 0$ and $n \to \infty$, we have

$$\Delta N(\infty) = \lim_{n\to\infty}\Delta N = 2 - \frac{1}{2}\left[3\cos^2\theta + \frac{1}{\cos^2\theta}\right]. \quad (37)$$

It can be derived that $\Delta N(\infty) \leq 2 - \sqrt{3}$ and the equality is achieved when $\cos^2\theta = 1/\sqrt{3}$. In addition, the quantity $\Delta N(\infty) \geq 0$ when $\cos^2\theta \in [1/3, 1]$. Figure 4 shows how $\Delta N$ varies as a function of $\cos^2\theta$ for different $n$. It can be observed that when $\cos^2\theta \in [1/3, 1]$ there is $\Delta N \geq 0$ for any $n$, which means the monogamy of MIN is satisfied in these states.

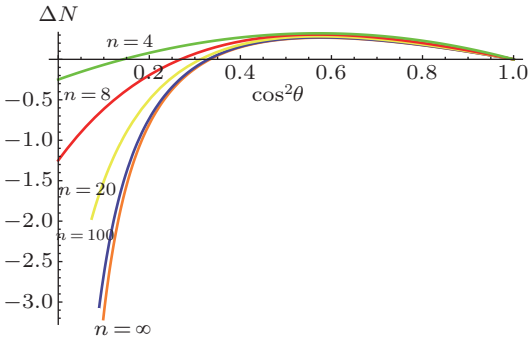



**Fig. 4.** (color online) $\Delta N = N(\rho_{a|bcd\cdots}) - (n-1)N(\rho_{a|b})$ varies as a function of $\cos^2\theta$. When $\cos^2\theta \in [1/3, 1]$ there is $\Delta N \geq 0$ for any $n$, which means the monogamy relation of MIN in these states is satisfied.

## 5. Conclusion

In this paper, we have explored the property of MIN in the superposition of $n$-qubit W and GHZ states. We provide three valuable results about this nonlocality measure: 1) The two-qubit reduced states of $n$-qubit W and GHZ superposition states depend on the relative phases for any $n \geq 3$, but the MIN of the reduced two-qubit states do not depend on the relative phases for $n \geq 4$; 2) The monogamy relation of MIN is satisfied in all three-qubit W and GHZ superposition states but it is not satisfied in all $n$-qubit ($n \geq 4$) W and GHZ superposition states; 3) The monogamy equality of MIN is held in a $n$-qubit W state for any $n \geq 3$.

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