Fast multi-copy entanglement purification with linear optics*

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We describe an entanglement purification protocol for a polarization Bell state. Different from the previous protocols, it does not require the controlled-not gate, and only uses linear optical elements to complete the task. This protocol requires multi-copy degraded mixed states, which can make this protocol obtain a high fidelity in one purification step. It can also be extended to purify the multi-photon Greenberger–Horne–Zeilinger (GHZ) state. This protocol may be useful in future long-distance communication.

Keywords: quantum computation, entanglement purification, entanglement

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1. Introduction

Quantum communication has its absolute security which has attracted a great deal of attention in the past ten years. Quantum teleportation,[1,2] quantum secret sharing (QSS),[3–5] quantum key distribution (QKD),[6,7] quantum secure direct communication (QSDC),[8,9] and other quantum communication and quantum information processing protocols have been widely developed.[10–39] In most quantum communication tasks, they usually require the maximally entangled state to set up the quantum channel. However, entanglement can only be prepared locally. In order to share the quantum channel, people should distribute the photons in the free space or in a fiber. Unfortunately, during the distribution, the photons will inevitably suffer from the noise. The channel noise will degrade the entanglement. The degraded entanglement will decrease the quality of quantum communication. Moreover, the quantum communication will become insecure. Therefore, before quantum communication, we should repair the degraded quantum entanglement channel.

Entanglement purification is a powerful tool to distill the maximally entangled states from the low quality mixed states,[40–72] which has been studied widely, since Bennett *et al.* proposed the concept of entanglement purification.[40] For an optical system, Pan *et al.* developed a feasible entanglement purification protocol (EPP) with linear optics.[41] Since then, a lot of works of entanglement purification were proposed. For example, in 2008, using a spontaneous parametric down-conversion (SPDC) source, Sheng *et al.* described a practical EPP with cross-Kerr nonlinearity.[45] Wang *et al.* described an efficient EPP for an electron-spin entangled state using quantum-dot in optical microcavities.[52] Ren *et al.* described an interesting EPP for hyperentanglement.[62] It has recently been shown that entanglement purification can be used to protect the secure blind quantum computation in a noisy environment.[64,65] Interestingly, except the deterministic EPPs with hyperentanglement, most of the EPPs all require two copies of low quality mixed states. There are several protocols using three copies of the low quality mixed state.[66–69] For example, in 2000, Feng *et al.* described an EPP with controlled-controlled-not (CCNOT) operations. Their EPP was improved by Metwally and Obada.[67] In 2012, Chi *et al.* described another three-to-one EPPs, which can improve the efficiency of the previous EPP.[68] In 2015, an efficient EPP with three Werner states were proposed.[69]

In this paper, we will describe a different EPP, which uses multi-copy low quality mixed states to realize the entanglement purification. Different from the previous multi-copy EPPs, what is most advantageous is that this protocol does not require the CCNOT gate or CNOT gate. We only exploit the polarization beam splitters (PBSs), which is feasible in the current experimental condition. After performing the purification, we can obtain a high fidelity mixed state. Moreover, we show that this EPP can be extended to purify the multi-photon Greenberger–Horne–Zeilinger (GHZ) state.

This paper is organized as follows. In Section 2, we describe the EPP for the polarization Bell state. In Section 3, we explain the EPP for the multi-photon GHZ state. In Section 4, we provide a discussion and conclusion.

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2. Multi-copy EPP for bit-flip error

Now we start to describe our protocol with a simple case, following the original protocol in Ref. [41]. Suppose that Alice and Bob want to share the maximally entangled state as

$$|\Phi^+\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b + |V\rangle_a|V\rangle_b). \tag{1}$$

However, because of the noise, it becomes $|\Psi^+\rangle_{ab}$ as

$$|\Psi^+\rangle_{ab} = \frac{1}{\sqrt{2}}(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b) \tag{2}$$

with the probability of $1-F$. It means that a bit-flip error occurs. Therefore, the whole mixed state can be written as

$$\rho_{ab} = F|\Phi^+\rangle_{ab}\langle\Phi^+| + (1-F)|\Psi^+\rangle_{ab}\langle\Psi^+|. \tag{3}$$

Here $|H\rangle$ denotes the horizontal-polarized photon and $|V\rangle$ denotes the vertical-polarized photon, respectively. As shown in Fig. 1, the entanglement sources S_1, S_2, and S_3 emit the mixed state ρ_{a1b1}, ρ_{a2b2}, and ρ_{a3b3} to Alice and Bob, respectively. State ρ_{a1b1} is in the spatial modes $a1b1$, state ρ_{a2b2} is in the spatial modes $a2b2$ and ρ_{a3b3} is in the spatial modes $a3b3$, respectively.

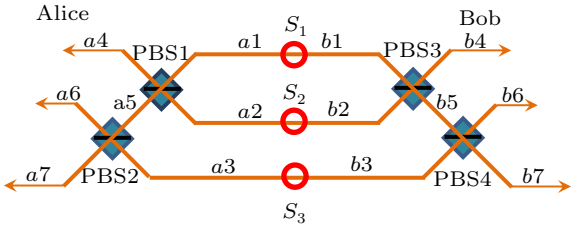


Fig. 1. (color online) The entanglement purification protocol (EPP) with three copies of degraded mixed states. PBSs are polarization beam splitters.

From Fig. 1, the polarization beam splitters can fully transmit the $|H\rangle$ polarization photon and reflect the $|V\rangle$ polarization photon, respectively. The whole system $\rho_{a1b1}\otimes\rho_{a2b2}\otimes\rho_{a3b3}$ can be described as follows.

(i) States are $|\Phi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}$ with a probability of F^3.

(ii) States are

$$|\Phi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3},\quad |\Psi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3},$$
$$|\Phi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}$$

with an equal probability of $F^2(1-F)$.

(iii) States are

$$|\Phi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3},\quad |\Psi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3},$$
$$|\Psi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}$$

with an equal probability of $F(1-F)^2$.

(iv) States are $|\Psi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3}$ with a probability of $(1-F)^3$.

Similar to Ref. [41], the key step is to select the case with only one photon in each output mode. We call it the "six-mode case". Only items $|\Phi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}$ and $|\Psi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3}$ can satisfy the condition of the six-mode case. For example, with the probability of F^3, states $|\Phi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}$ can be written as

$$\begin{aligned}
&|\Phi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}\\
=&\frac{1}{\sqrt{2}}(|H\rangle_{a1}|H\rangle_{b1}+|V\rangle_{a1}|V\rangle_{b1})\\
&\otimes\frac{1}{\sqrt{2}}(|H\rangle_{a2}|H\rangle_{b2}+|V\rangle_{a2}|V\rangle_{b2})\\
&\otimes\frac{1}{\sqrt{2}}(|H\rangle_{a3}|H\rangle_{b3}+|V\rangle_{a3}|V\rangle_{b3})\\
=&\frac{1}{2\sqrt{2}}(|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|H\rangle_{a3}|H\rangle_{b3}\\
&+|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|V\rangle_{a3}|V\rangle_{b3}\\
&+|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|H\rangle_{a3}|H\rangle_{b3}\\
&+|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|V\rangle_{a3}|V\rangle_{b3}\\
&+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|H\rangle_{a3}|H\rangle_{b3}\\
&+|V\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|V\rangle_{a3}|V\rangle_{b3}\\
&+|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|H\rangle_{a3}|H\rangle_{b3}\\
&+|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|V\rangle_{a3}|V\rangle_{b3}).
\end{aligned}\tag{4}$$

After passing through the PBS1, PBS2, PBS3, and PBS4, respectively, only items $|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|H\rangle_{a3}|H\rangle_{b3}$ and $|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|V\rangle_{a3}|V\rangle_{b3}$ will make each of the output modes $a4b4$, $a6b6$, and $a7b7$ exactly contain one photon. Certainly, with the probability of $(1-F)^3$, states $|\Psi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3}$ can be written as

$$\begin{aligned}
&|\Psi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3}\\
=&\frac{1}{\sqrt{2}}(|H\rangle_{a1}|V\rangle_{b1}+|V\rangle_{a1}|H\rangle_{b1})\\
&\otimes\frac{1}{\sqrt{2}}(|H\rangle_{a2}|V\rangle_{b2}+|V\rangle_{a2}|H\rangle_{b2})\\
&\otimes\frac{1}{\sqrt{2}}(|H\rangle_{a3}|V\rangle_{b3}+|V\rangle_{a3}|H\rangle_{b3})\\
=&\frac{1}{2\sqrt{2}}(|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}|H\rangle_{a3}|V\rangle_{b3}\\
&+|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}|V\rangle_{a3}|H\rangle_{b3}\\
&+|H\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}|H\rangle_{a3}|V\rangle_{b3}\\
&+|H\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}|V\rangle_{a3}|H\rangle_{b3}\\
&+|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}|H\rangle_{a3}|V\rangle_{b3}\\
&+|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}|V\rangle_{a3}|H\rangle_{b3}\\
&+|V\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}|H\rangle_{a3}|V\rangle_{b3}\\
&+|V\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}|V\rangle_{a3}|H\rangle_{b3}).
\end{aligned}\tag{5}$$

After passing through the PBSs, only items

$$|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{a2}|V\rangle_{b2}|H\rangle_{a3}|V\rangle_{b3}$$

and

$$|V\rangle_{a1}|H\rangle_{b1}|V\rangle_{a2}|H\rangle_{b2}|V\rangle_{a3}|H\rangle_{b3}$$

will make each of the output modes only contain one photon.

Interestingly, other states cannot satisfy the six-mode case. For example, states $|\Psi^+\rangle_{a1b1}|\Phi^+\rangle_{a2b2}|\Phi^+\rangle_{a3b3}$ will make no photon in the output mode b_7 and there are two photons in the output mode b_4. States $|\Phi^+\rangle_{a1b1}|\Psi^+\rangle_{a2b2}|\Psi^+\rangle_{a3b3}$, will make no photon in the output mode b_4 and there are two photons in the output mode b_7.

Finally, by selecting the six-mode case, they can obtain

$$\begin{aligned}|\phi_1\rangle = \frac{1}{\sqrt{2}}(&|H\rangle_{a1}|H\rangle_{b1}|H\rangle_{a2}|H\rangle_{b2}|H\rangle_{a3}|H\rangle_{b3}\\ &+|V\rangle_{a1}|V\rangle_{b1}|V\rangle_{a2}|V\rangle_{b2}|V\rangle_{a3}|V\rangle_{b3})\end{aligned} \tag{6}$$

with the probability of $F^3/4$, and

$$\begin{aligned}|\phi_2\rangle = \frac{1}{\sqrt{2}}(&|H\rangle_{a4}|V\rangle_{b4}|H\rangle_{a6}|V\rangle_{b6}|H\rangle_{a7}|V\rangle_{b7}\\ &+|V\rangle_{a4}|H\rangle_{b4}|V\rangle_{a6}|H\rangle_{b6}|V\rangle_{a7}|H\rangle_{b7})\end{aligned} \tag{7}$$

with the probability of $(1-F)^3/4$. In order to obtain the two-photon entangled state, they should measure the photon in spatial modes $a6b6$ and $a7b7$ in the basis $|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$. By classical communication, if the number of $|-\rangle$ is even, they can obtain a new mixed state

$$\rho'_{a4b4} = F_1|\Phi^+\rangle_{a4b4}\langle\Phi^+| + (1-F_1)|\Psi^+\rangle_{a4b4}\langle\Psi^+| \tag{8}$$

with

$$F_1 = \frac{F^3}{F^3+(1-F)^3}. \tag{9}$$

If $F > 1/2$, $F_1 > F$. Otherwise, if the number of $|-\rangle$ is odd, they can perform a phase-flip operation on one of the photons to convert the state to Eq. (8). In this way, the whole purification is successful.

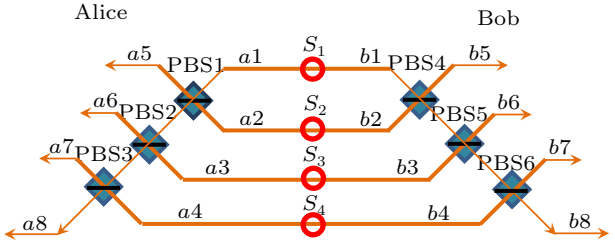


Fig. 2. (color online) The EPP with four copies of degraded mixed states.

Interestingly, this protocol can be easily extended to the multi-copy purification. As shown in Fig. 2, if they choose four copies of mixed states in Eq. (3), and select the cases that all the output modes $a5b5$, $a6b6$, $a7b7$, and $a8b8$ exactly contain one photon, they can finally obtain

$$\begin{aligned}|\phi_3\rangle = \frac{1}{\sqrt{2}}(&|H\rangle_{a5}|H\rangle_{b5}|H\rangle_{a6}|H\rangle_{b6}|H\rangle_{a7}|H\rangle_{b7}|H\rangle_{a8}|H\rangle_{b8}\\ &+|V\rangle_{a5}|V\rangle_{b5}|V\rangle_{a6}|V\rangle_{b6}|V\rangle_{a7}|V\rangle_{b7}|V\rangle_{a8}|V\rangle_{b8}),\end{aligned} \tag{10}$$

with the probability of $F^4/8$ and

$$\begin{aligned}|\phi_4\rangle = \frac{1}{\sqrt{2}}(&|H\rangle_{a5}|V\rangle_{b5}|H\rangle_{a6}|V\rangle_{b6}|H\rangle_{a7}|V\rangle_{b7}|H\rangle_{a8}|V\rangle_{b8}\\ &+|V\rangle_{a5}|H\rangle_{b5}|V\rangle_{a6}|H\rangle_{b6}|V\rangle_{a7}|H\rangle_{b7}|V\rangle_{a8}|H\rangle_{b8})\end{aligned} \tag{11}$$

with the probability of $(1-F)^4/8$. Finally, by measuring the photons in spatial modes $a6b6$, $a7b67$, and $a8b8$ in the basis $|\pm\rangle$, they can also obtain a new mixed state of the form

$$\rho''_{a5b5} = F_2|\Phi^+\rangle_{a5b5}\langle\Phi^+| + (1-F_2)|\Psi^+\rangle_{a5b5}\langle\Psi^+| \tag{12}$$

with

$$F_2 = \frac{F^4}{F^4+(1-F)^4}. \tag{13}$$

This protocol can also be extended to the cases of n copies purification. If they choose n copies of mixed states in Eq. (3) simultaneously and select the "$2n$-mode case", they can obtain a new mixed state with the fidelity of

$$F_n = \frac{F^n}{F^n+(1-F)^n}. \tag{14}$$

Certainly, if $F > 1/2$, $F_n > F$. For a Bell state, if a bit-flip error can be purified, the phase-flip can be also purified in the same way. They only require performing the Hadamard operation on two photons to change it to the bit-flip error. After transformation, $|\Phi^+\rangle$ does not change, while $|\Phi^-\rangle$ will become $|\Psi^+\rangle$. Therefore, using the same approach described above, they can also purify the phase-flip error.

3. Multi-copy of purification for GHZ state

It is straightforward to extend this protocol to the case of a GHZ state. Previous EPPs for multi-partite all exploit two copies of degraded states.[73–75] Recently, with the help of hyperentanglement, He *et al.* described an interesting and important EPP for four-photon GHZ states.[76] If the mixed state of the three-photon GHZ state is

$$\rho_{abc} = F|\Phi^+\rangle_{abc}\langle\Phi^+| + (1-F)|\Psi^+\rangle_{abc}\langle\Psi^+|. \tag{15}$$

Here

$$|\Phi^+\rangle_{abc} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b|H\rangle_c + |V\rangle_a|V\rangle_b|V\rangle_c), \tag{16}$$

and

$$|\Psi^+\rangle_{abc} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b|V\rangle_c + |V\rangle_a|V\rangle_b|H\rangle_c). \tag{17}$$

From Eq. (15), a bit-flip error occurs with the probability of $1-F$. As shown in Fig. 3, the three copies of mixed state

ρ_{a1b1c1}, ρ_{a2b2c2}, and ρ_{a3b3c3} are shared by Alice, Bob, and Charlie from S_1, S_2, and S_3, respectively.

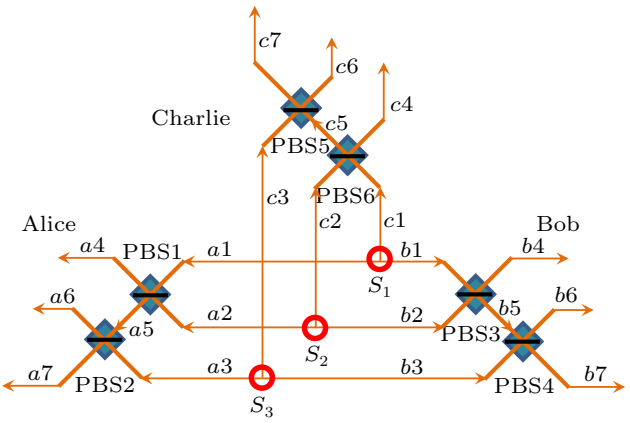


Fig. 3. (color online) The EPP for GHZ states with three copies.

The whole system $\rho_{a1b1c1}\otimes\rho_{a2b2c2}\otimes\rho_{a3b3c3}$ can also be regarded as the mixture of eight pure states. With the probability of F^3, states are $|\Phi^+\rangle_{a1b1c1}|\Phi^+\rangle_{a2b2c2}|\Phi^+\rangle_{a3b3c3}$. With the probability of $(1-F)^3$, it is in the states of $|\Psi^+\rangle_{a1b1c1}|\Psi^+\rangle_{a2b2c2}|\Psi^+\rangle_{a3b3c3}$, and so on. The purification principle is similar to the previous description. They select the "nine-mode case". The nine-mode case means that there is exactly one photon in each output mode. By selecting the nine-mode case, only the item $|\Phi^+\rangle_{a1b1c1}|\Phi^+\rangle_{a2b2c2}|\Phi^+\rangle_{a3b3c3}$ and $|\Psi^+\rangle_{a1b1c1}|\Psi^+\rangle_{a2b2c2}|\Psi^+\rangle_{a3b3c3}$ will satisfy this condition. In detail, with the probability of $F^3/4$, they can obtain

$$|\phi_5\rangle = \frac{1}{\sqrt{2}}(|H\rangle_{a4}|H\rangle_{b4}|H\rangle_{c4}|H\rangle_{a6}|H\rangle_{b6}|H\rangle_{c6}|H\rangle_{a7}|H\rangle_{b7}|H\rangle_{b7} + |V\rangle_{a4}|V\rangle_{b4}|V\rangle_{c4}|V\rangle_{a6}|V\rangle_{b6}|V\rangle_{c6}|V\rangle_{a7}|V\rangle_{b7}|V\rangle_{b7}), \quad (18)$$

and with a probability of $(1-F)^3/4$, they can obtain

$$|\phi_6\rangle = \frac{1}{\sqrt{2}}(|H\rangle_{a4}|H\rangle_{b4}|V\rangle_{c4}|H\rangle_{a6}|H\rangle_{b6}|V\rangle_{c6}|H\rangle_{a7}|H\rangle_{b7}|V\rangle_{b7} + |V\rangle_{a4}|V\rangle_{b4}|H\rangle_{c4}|V\rangle_{a6}|V\rangle_{b6}|H\rangle_{c6}|V\rangle_{a7}|V\rangle_{b7}|H\rangle_{b7}). \quad (19)$$

Finally, by measuring the photons in spatial modes $a6b6c6$, and $a7b7c7$, they will finally obtain a new mixed state as

$$\rho'_{abc} = F'|\Phi^+\rangle_{abc}\langle\Phi^+| + (1-F')|\Psi^+\rangle_{abc}\langle\Psi^+|. \quad (20)$$

with the fidelity F'

$$F' = \frac{F^3}{F^3+(1-F)^3}. \quad (21)$$

Certainly, they can also use n copies of the mixed state in Eq. (15) to realize the purification. In this way, they choose $3n$-mode cases. After performing the purification, they can also obtain a high fidelity F_n, which is the same as Eq. (14).

Finally, let us discuss the phase-flip error purification for the GHZ state. If a phase-flip error occurs, the initial mixed state can be written as

$$\rho^o_{abc} = F|\Phi^+\rangle_{abc}\langle\Phi^+| + (1-F)|\Phi^-\rangle_{abc}\langle\Phi^-|. \quad (22)$$

Here

$$|\Phi^-\rangle_{abc} = \frac{1}{\sqrt{2}}(|H\rangle_a|H\rangle_b|H\rangle_c - |V\rangle_a|V\rangle_b|V\rangle_c). \quad (23)$$

Before purification, they should add the Hadamard operation on each photon and transform the state ρ^o_{abc} to

$$\rho^{o'}_{abc} = F|\varphi^+\rangle_{abc}\langle\varphi^+| + (1-F)|\psi^+\rangle_{abc}\langle\psi^+|. \quad (24)$$

Here

$$|\varphi^+\rangle_{abc} = \frac{1}{2}(|H\rangle_a|H\rangle_b|H\rangle_c + |H\rangle_a|V\rangle_b|V\rangle_c + |V\rangle_a|H\rangle_b|V\rangle_c + |V\rangle_a|V\rangle_b|H\rangle_c), \quad (25)$$

and

$$|\psi^+\rangle_{abc} = \frac{1}{2}(|H\rangle_a|H\rangle_b|V\rangle_c + |H\rangle_a|V\rangle_b|H\rangle_c + |V\rangle_a|H\rangle_b|H\rangle_c + |V\rangle_a|V\rangle_b|V\rangle_c). \quad (26)$$

Subsequently, they choose three copies of the mixed states $\rho^{o'}_{a1b1c1}$, $\rho^{o'}_{a2b2c2}$, and $\rho^{o'}_{a3b3c3}$ and let them pass through the PBSs, respectively. By selecting the nine-mode case, only the items $|\varphi^+\rangle_{a1b1c1}|\varphi^+\rangle_{a2b2c2}|\varphi^+\rangle_{a3b3c3}$ and $|\psi^+\rangle_{a1b1c1}|\psi^+\rangle_{a2b2c2}|\psi^+\rangle_{a3b3c3}$ can satisfy the condition of the nine-mode case. For example, states $|\varphi^+\rangle_{a1b1c1}|\varphi^+\rangle_{a2b2c2}|\varphi^+\rangle_{a3b3c3}$ will become

$$|\phi_7\rangle = \frac{1}{2}(|H\rangle_{a4}|H\rangle_{b4}|H\rangle_{c4}|H\rangle_{a6}|H\rangle_{b6}|H\rangle_{c6}|H\rangle_{a7}|H\rangle_{b7}|H\rangle_{c7} + |H\rangle_{a4}|V\rangle_{b4}|V\rangle_{c4}|H\rangle_{a5}|V\rangle_{b6}|V\rangle_{c6}|H\rangle_{a7}|V\rangle_{b7}|V\rangle_{c7}) + |V\rangle_{a4}|H\rangle_{b4}|V\rangle_{c4}|V\rangle_{a5}|H\rangle_{b6}|V\rangle_{c6}|V\rangle_{a7}|H\rangle_{b7}|V\rangle_{c7} + |V\rangle_{a4}|V\rangle_{b4}|H\rangle_{c4}|V\rangle_{a5}|V\rangle_{b6}|H\rangle_{c6}|V\rangle_{a7}|V\rangle_{b7}|H\rangle_{c7}), \quad (27)$$

with the probability of $F^3/4$, and

$$|\phi_8\rangle = \frac{1}{2}(|H\rangle_{a4}|H\rangle_{b4}|V\rangle_{c4}|H\rangle_{a5}|H\rangle_{b6}|V\rangle_{c6}|H\rangle_{a7}|H\rangle_{b7}|V\rangle_{c7} + |H\rangle_{a4}|V\rangle_{b4}|H\rangle_{c4}|H\rangle_{a5}|V\rangle_{b6}|H\rangle_{c6}|H\rangle_{a7}|V\rangle_{b7}|H\rangle_{c7} + |V\rangle_{a4}|H\rangle_{b4}|H\rangle_{c4}|V\rangle_{a5}|H\rangle_{b6}|H\rangle_{c6}|V\rangle_{a7}|H\rangle_{b7}|H\rangle_{c7} + |V\rangle_{a4}|V\rangle_{b4}|V\rangle_{c4}|V\rangle_{a5}|V\rangle_{b6}|V\rangle_{c6}|V\rangle_{a7}|V\rangle_{b7}|V\rangle_{c7}), \quad (28)$$

with the probability of $(1-F)^3/4$. If the state is $|\phi_7\rangle$, by measuring the photons in spatial modes $a6b6c6$ and $a7b7c7$ in the basis $|\pm\rangle$, they will obtain the state $|\varphi^+\rangle_{a4b4c4}$, if the number of $|V\rangle$ is even. If the state is $|\phi_8\rangle$, by measuring the photons in spatial modes $a6b6c6$ and $a7b7c7$ in the basis $|\pm\rangle$, they will obtain the state $|\varphi^+\rangle_{a4b4c4}$, and the state $|\psi^+\rangle_{a4b4c4}$, if the number of $|V\rangle$ is even. Finally, they perform another Hadamard operation on each photon to convert $|\varphi^+\rangle_{a4b4c4}$ to $|\Phi^+\rangle_{abc}$, and $|\phi_7\rangle$ to $|\Psi^+\rangle$, respectively. In this way, they can obtain a new mixed state with the same fidelity F' as Eq. (21).

On the other hand, if the number of $|V\rangle$ is odd, $|\phi_7\rangle$ will become

$$|\varphi_1^+\rangle_{abc} = \frac{1}{2}(|H\rangle_a|H\rangle_b|H\rangle_c - |H\rangle_a|V\rangle_b|V\rangle_c - |V\rangle_a|H\rangle_b|V\rangle_c + |V\rangle_a|V\rangle_b|H\rangle_c), \quad (29)$$

and $|\phi_8\rangle$ will become

$$|\psi_1^+\rangle_{abc} = \frac{1}{2}(|H\rangle_a|H\rangle_b|V\rangle_c - |H\rangle_a|V\rangle_b|H\rangle_c - |V\rangle_a|H\rangle_b|H\rangle_c + |V\rangle_a|V\rangle_b|V\rangle_c). \quad (30)$$

After performing the Hadamard operation on each photon, they can obtain another new mixed state as

$$\rho_{abc}^{o''} = F|\Phi_1^+\rangle_{abc}\langle\Phi_1^+| + (1-F)|\Phi_1^-\rangle_{abc}\langle\Phi_1^-|. \quad (31)$$

Here

$$|\Phi_1^+\rangle_{abc} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|V\rangle + |V\rangle|V\rangle|H\rangle), \quad (32)$$

and

$$|\Phi_1^-\rangle_{abc} = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|V\rangle - |V\rangle|V\rangle|H\rangle). \quad (33)$$

One can convert state $\rho_{abc}^{o''}$ to ρ_{abc}^{o} by adding a bit-flip operation on the third photon. In this way, the phase-flip error can also be purified.

Certainly, if a phase-flip occurs, they can also choose n copies of mixed states in Eq. (15) to perform the purification, which they can obtain the higher fidelity F', which is the same as Eq. (14).

The EPP described above can also be extended to the arbitrary multi-photon GHZ state as

$$|\Phi^+\rangle_N = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle\cdots|H\rangle + |V\rangle|V\rangle\cdots|V\rangle). \quad (34)$$

If a bit-flip error occurs, it becomes

$$|\Phi_1^+\rangle_N = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle\cdots|V\rangle + |V\rangle|V\rangle\cdots|H\rangle). \quad (35)$$

The mixed state can be written as

$$\rho_N = F|\Phi^+\rangle_N\langle\Phi^+| + (1-F)|\Phi_1^+\rangle_N\langle\Phi_1^+|. \quad (36)$$

If they choose three copies of mixed states in Eq. (36), following the same principle, they can also obtain a high mixed state with the fidelity of F' as shown in Eq. (21).

4. Discussion and summary

So far, we have completely explained our protocol. We first described the EPP for two-photon Bell states. Subsequently, we showed that this approach can also be used to purify the multi-photon GHZ state. Both the bit-flip error and phase-flip error can be purified. It is interesting to compare this protocol with other protocols. In 2012, Chi *et al.* described an EPP with three copies of mixed states.[68] Their success probability is twice as large as that of the protocol by Feng *et al.* However, their protocol is still based on the CNOT gate. Moreover, they exploited the Bell-state measurement to complete the task. The recent EPP also exploited the CNOT gate.[69] It is known that the CNOT gate is hard to realize in the current experiment conditions. The deterministic Bell-state measurement is also a big challenge in linear optics, which greatly limits the application of these EPPs. In Fig. 4, we calculated the fidelity F_n altered with the initial fidelity F. As shown in Fig. 4, curve a represents the purification with two copies as described in Ref. [41]. It is the first EPP with linear optics. Curves b, c, and d represent the purification with three, four, and five copies. We let the initial fidelity be $F \in (0.5, 1)$ to ensure the initial mixed states are entangled. From Fig. 4, we can obtain a higher new fidelity F_n if n is larger. For example, if $F = 0.6$, we can obtain $F_n \approx 0.69$ with $n = 2$, while we can obtain $F_n \approx 0.89$ with $n = 5$.

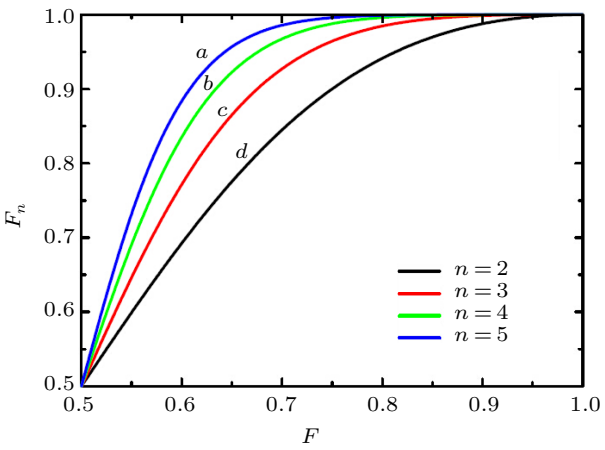


Fig. 4. (color online) The increased fidelity of F_n altered with the initial fidelity F. The curve with $n = 2$ is the original EPP in Ref. [41]. It is shown that they can obtain a high fidelity with multi-copy degraded mixed states.

Obtaining high quality entanglement in a rapid way is fundamental for many quantum communication protocols. In a traditional EPP, one can obtain a high fidelity photon pair by iterating the protocol many times. Unfortunately, this approach is unsuitable for an EPP with linear optics, for we should exploit the postselection principle to obtain the purified mixed state. In this way, the EPP can only be performed once. After purification, the photon pair will be detected by the single-photon detectors. Therefore, this approach is a great limitation to obtaining a high quality entangled state from the low quality entangled states. Certainly, we should point out that though our EPP can obtain the high quality rapidly, the total success probability is lower than the original protocol.

In conclusion, we have presented a fast multi-copy EPP with linear optics. We showed that this approach not only can

purify the Bell state, but also can purify the multi-photon GHZ state. Compared with the previous EPPs, this protocol does not require the CNOT gate and Bell-state measurement. Only feasible linear optical elements are required. Moreover, this EPP can obtain a high fidelity by performing this EPP once, which makes it useful in future quantum communication.

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