Quantum speed limits for Bell-diagonal states*

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The lower bounds of the evolution time between two distinguishable states of a system, defined as quantum speed limit time, can characterize the maximal speed of quantum computers and communication channels. We study the quantum speed limit time between the composite quantum states and their target states in the presence of nondissipative decoherence. For the initial states with maximally mixed marginals, we obtain the exact expressions of the quantum speed limit time which mainly depend on the parameters of the initial states and the decoherence channels. Furthermore, by calculating the quantum speed limit time for the time-dependent states started from a class of initial states, we discover that the quantum speed limit time gradually decreases in time, and the decay rate of the quantum speed limit time would show a sudden change at a certain critical time. Interestingly, at the same critical time, the composite system dynamics would exhibit a sudden transition from classical decoherence to quantum decoherence.

Keywords: quantum information, quantum decoherence, quantum speed limit time

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1. Introduction

Quantum mechanics establishes the fundamental bounds for the minimum evolution time between two states of a given system. Bounds of this evolution time, known as quantum speed limit time (QSLT), are intimately related to the evolution speed of quantum systems. The applications of these bounds are shown in different scenarios, including quantum communication,[1,2] quantum metrology,[3,4] quantum computation,[5] as well as quantum optimal control protocols.[6–9] Derivations of these basic bounds focused on the closed system with unitary evolution, are obtained by the Mandelstam–Tamm (MT) type bound and Margolus–Levitin (ML) type bound.[10–21] Moreover, the extensions of the QSLT for nonunitary evolution of open systems are also studied,[22–27] since the relevant influence of the environment on processing or information transferring systems cannot be ignored. It is shown that two unified bounds of the minimum evolution time for the pure or mixed initial states including both MT and ML types for the nonunitary dynamics process have been formulated, respectively.[24,25] These two unified bounds have already been used to illustrate the quantum evolution speed for a qubit state (whether it is a pure state or mixed state) under the decoherence process. We know that the correlated composite quantum states play a central role in quantum information and communication theory,[28–32] such as, in order to fulfil a long-distance high-fidelity quantum communication in a noisy channel, the entanglement of a composite system (two distant sites) must be used to implement quantum repeaters.[33,34] The QSLT on the evolution of an open composite quantum system would be helpful to analyze the robustness of quantum simulators and computers against decoherence.[35,36] The application of these unified bounds to assess the evolution speed of the arbitrary states in the composite quantum system with nonunitary dynamics, is not well studied.

In this work, we are interested in the evolution speed of two-qubit Bell-diagonal states under certain noise channels (i.e., phase flip, bit flip, and bit–phase flip). By using of the unified bound of QSLT for arbitrary mixed states,[25] we have calculated the exact expressions of QSLT for the initial two-qubit Bell-diagonal states, which mainly depend on the parameters of the initial states and the decoherence channels. In the following, we also focus on the QSLT for the time-dependent states started from the initial Bell-diagonal states in the whole dynamics process. We demonstrate that the QSLT will be reduced gradually with the starting point in time under the noise channels, that is to say, the two-qubit system executes a speeded-up dynamics evolution process. Moreover, for certain initial Bell-diagonal states, a remarkable result we find is the existence of a sudden change of the decay rate of the QSLT at a certain critical time τ_c. On the other hand, as we know that, at the same critical time τ_c, a sudden transition from classical decoherence to quantum decoherence for

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this class of initial Bell-diagonal states also appears.[37] Then we can specifically point out that the evolution speed of the whole dynamics process can be described qualitatively by the classical decoherence for $\tau < \tau_c$, and quantum decoherence for $\tau > \tau_c$. Finally, we also recognize a symmetry among the above three decoherence channels.

2. The quantum speed limit under decoherence channels

Firstly, we start with the definition of the QSLT for open quantum systems. A unified lower bound, including both MT and ML types, has been derived in Ref. [25]. With the help of the von Neumann trace inequality and the Cauchy–Schwarz inequality, the QSLT between an arbitrary initially mixed state ρ_0 and its target state ρ_{τ_D}, governed by the master equation $\dot{\rho}_t = L_t\rho_t$, with L_t the positive generator of the dynamical semigroup, is as follows:

$$\tau_D \geq \tau_{QSL} = \max\left\{\frac{1}{\overline{\sum_{i=1}^n \sigma_i\rho_i}}, \frac{1}{\overline{\sqrt{\sum_{i=1}^n \sigma_i^2}}}\right\} B(\rho_0, \rho_{\tau_D}), \quad (1)$$

with $\overline{X} = \tau_D^{-1}\int_0^{\tau_D} X\,\mathrm{d}t$, $B(\rho_0,\rho_{\tau_D}) = |\mathrm{tr}(\rho_0\rho_{\tau_D}) - \mathrm{tr}(\rho_0^2)|$ denotes a metric on the space of the initial state ρ_0 and the target state ρ_{τ_D} via the so-called relative purity,[22] and σ_i are the singular values of $\dot{\rho}_t$ and ρ_i those of the initial mixed state ρ_0. For a pure initial state $\rho_0 = |\phi_0\rangle\langle\phi_0|$, the singular value $\rho_i = \delta_{i,1}$, then the expression of τ_{QSL} thus recovers the unified bound given in Ref. [24]. So the QSLT formulated in Eq. (1) can effectually define the minimal evolution time for arbitrary initial states, and also be used to deal with the maximal speed of the evolution of open quantum systems.

By considering two qubits without mutual interaction, we mainly focus on the phase flip, bit flip, and bit-phase flip channels as the decoherence processes. The dynamics of each qubit is governed by a master equation that gives rise to a completely positive trace-preserving map $\varepsilon(\cdot)$. In the Born–Markov approximation, the operator-sum representation is given by

$$\varepsilon(\rho_{AB}) = \sum_{i,j} K_i^A K_j^B \rho_{AB} K_j^{B\dagger} K_i^{A\dagger},$$

with $K_i^{A(B)}$ are the Kraus operators that describe the noise channels A and B. In order to calculate the QSLT of a two-qubit initial mixed state to its target state in the above noisy channels, we take a two-qubit Bell-diagonal state as the initial state of the composite system,[37,38] described by

$$\rho_0 = \frac{1}{4}(\boldsymbol{I}_A\otimes\boldsymbol{I}_B + \sum_{i=1}^3 c_i\sigma_i^A\otimes\sigma_i^B), \quad (2)$$

where $c_i \in \Re$ such that $0 \leq |c_i| \leq 1$ for $i = 1,2,3$, and $\boldsymbol{I}_{A(B)}$ is the identity operator in subspace $A(B)$. The state in Eq. (2) includes the Werner states ($|c_1| = |c_2| = |c_3| = c$) and the Bell states ($|c_1| = |c_2| = |c_3| = 1$).

We first focus on the phase flip channel. For this channel, the Kraus operators are given by[28,37,38]

$$\begin{aligned}
K_0^A &= \mathrm{diag}(\sqrt{1-p_A/2}, \sqrt{1-p_A/2})\otimes\boldsymbol{I}_B,\\
K_1^A &= \mathrm{diag}(\sqrt{p_A/2}, \sqrt{p_A/2})\otimes\boldsymbol{I}_B,\\
K_0^B &= \boldsymbol{I}_A\otimes\mathrm{diag}(\sqrt{1-p_B/2}, \sqrt{1-p_B/2}),\\
K_1^B &= \boldsymbol{I}_A\otimes\mathrm{diag}(\sqrt{p_B/2}, \sqrt{p_B/2}),
\end{aligned}$$

here, $p_{A(B)}$ is used as the parametric time in channel $A(B)$, and by considering the symmetric situation in which the decoherence rate is equal in both channels, so $p_A = p_B = p$. For the initial state of Eq. (2), the time evolution of the total system is

$$\begin{aligned}
\rho(t) = {}& \lambda_\Psi^+(t)|\Psi^+\rangle\langle\Psi^+| + \lambda_\Phi^+(t)|\Phi^+\rangle\langle\Phi^+|,\\
&+\lambda_\Psi^-(t)|\Psi^-\rangle\langle\Psi^-| + \lambda_\Phi^-(t)|\Phi^-\rangle\langle\Phi^-|, \quad (3)
\end{aligned}$$

where

$$\begin{aligned}
\lambda_\Psi^\pm(t) &= (1\pm c_1(t)\mp c_2(t) + c_3(t))/4,\\
\lambda_\Phi^\pm(t) &= (1\pm c_1(t)\pm c_2(t) - c_3(t))/4, \quad (4)
\end{aligned}$$

and $|\Psi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Phi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are the four Bell states. The time dependent coefficients in Eq. (4) are $c_1(t) = (1-p)^2c_1$, $c_2(t) = (1-p)^2c_2$, and $c_3(t) = c_3$, with $p = 1-\exp(-\gamma t)$, and γ the phase damping rate.

Now, we calculate the QSLT between the initial Bell-diagonal state ρ_0 to ρ_{τ_D} by the driving time τ_D under the phase flip channel. According to the expression (1), we can clearly find that the distance between ρ_0 and ρ_{τ_D}, $B(\rho_0,\rho_{\tau_D}) = \frac{1}{4}|p_{\tau_D}^2 - 2p_{\tau_D}|(|c_1|^2+|c_2|^2)$, with $p_{\tau_D} = 1-\exp(-\gamma\tau_D)$. Thus our main task is to calculate the singular values of ρ_0 and $\dot{\rho}_t$, respectively. For $\dot{\rho}_t$, the singular values σ_i are

$$\begin{aligned}
\sigma_1 = \sigma_2 &= \frac{1}{2}|\dot{p}(1-p)(|c_1|+|c_2|)|,\\
\sigma_3 = \sigma_4 &= \frac{1}{2}|\dot{p}(1-p)(|c_1|-|c_2|)|. \quad (5)
\end{aligned}$$

While for the initial state ρ_0, the singular values ρ_i depend on the region of the coefficients c_i. Then, in the following, we divide the region of the coefficients c_i into four parts.

Case I If $|c_3| \geq |c_1| \geq |c_2|$ in Eq. (2), the singular values ρ_i of the initial states ρ_0 satisfy $\rho_1+\rho_2 = \frac{1}{2}(1+|c_3|), \rho_3+\rho_4 = \frac{1}{2}(1-|c_3|)$. In this case, $\sum_{i=1}^4\sigma_i\rho_i = \frac{1}{2}(1-p)|\dot{p}|(|c_1|+|c_2||c_3|)$ is always less than $\sqrt{\sum_{i=1}^4\sigma_i^2} = (1-p)|\dot{p}|\sqrt{|c_1|^2+|c_2|^2}$, so the QSLT for the class of initial states in this case can be obtained

$$\tau_{QSL}^{I} = \frac{\tau_D(|c_1|^2+|c_2|^2)}{|c_1|+|c_2||c_3|}.$$

Case II If $|c_3| \geq |c_2| \geq |c_1|$, the singular values ρ_i of the initial states ρ_0 are equal to those in case I. So we obtain $\sum_{i=1}^4\sigma_i\rho_i = \frac{1}{2}(1-p)|\dot{p}|(|c_2|+|c_1||c_3|)$, the ML type bound on

the QSLT is also tight for the initial states in the case II, then we acquire

$$\tau_{\rm QSL}^{\rm II}=\frac{\tau_{\rm D}(|c_1|^2+|c_2|^2)}{|c_2|+|c_1||c_3|}.$$

Case III When $|c_1|\geq|c_3|,|c_2|$, the singular values ρ_i are given by $\rho_1+\rho_2=\frac{1}{2}(1+|c_1|),\rho_3+\rho_4=\frac{1}{2}(1-|c_1|)$, and $\sum_{i=1}^4\sigma_i\rho_i=\frac{1}{2}(1-p)|\dot{p}||c_1|(1+|c_2|)$ is also less than $\sqrt{\sum_{i=1}^4\sigma_i^2}$. The QSLT of the initial states in case III can be written

$$\tau_{\rm QSL}^{\rm III}=\frac{\tau_{\rm D}(|c_1|^2+|c_2|^2)}{|c_1|(1+|c_2|)}.$$

The QSLT does not depend on the coefficient c_3.

Case IV Finally, if $|c_2|\geq|c_3|,|c_1|$, we acquire $\rho_1+\rho_2=\frac{1}{2}(1+|c_2|),\rho_3+\rho_4=\frac{1}{2}(1-|c_2|)$, and $\sum_{i=1}^4\sigma_i\rho_i=\frac{1}{2}(1-p)|\dot{p}||c_2|(1+|c_1|)$. With this, it is easy to show that the QSLT in this case yields

$$\tau_{\rm QSL}^{\rm IV}=\frac{\tau_{\rm D}(|c_1|^2+|c_2|^2)}{|c_2|(1+|c_1|)}$$

(also independent of the coefficient c_3).

Inspection of the expressions $\tau_{\rm QSL}^{\rm I}$, $\tau_{\rm QSL}^{\rm II}$, $\tau_{\rm QSL}^{\rm III}$, and $\tau_{\rm QSL}^{\rm IV}$ shows that in all the four cases the QSLT mainly depends on the coefficients c_i of the initial state ρ_0. Particularly, for all states in cases $|c_3|\geq|c_2|,|c_1|$ (cases I and II), the QSLT becomes inversely proportional to $|c_3|$. While for all states in which the coefficients c_i belong to cases III and IV, the QSLT is independent of c_3. In order to investigate the effects of different values of the coefficients c_i on the QSLT under the phase-flip channel, we plot the QSLT of the initial Bell-diagonal states ρ_0 as a function of $|c_1|$ and $|c_2|$ for a given choice $|c_3|=0.4$, in Fig. 1. Then, two remarkable features can be acquired from Fig. 1. (i) The QSLT is symmetrical with respect to the line $|c_1|=|c_2|$. That is to say, the states $\rho_0(|c_1|,|c_2|,|c_3|)$ and $\rho_0(|c_2|,|c_1|,|c_3|)$ have the same quantum speed of the evolution under phase flip channel. (ii) The QSLTs for the initial states in cases I and II are always smaller than those for the states in cases III and IV. So we note that, under the local phase-flip channel, the Bell-diagonal states in cases I and II have a smaller minimal evolution time than those in cases III and IV. That is to say, in the phase-flip channels, the evolution speed of the initial Bell-diagonal states in cases I and II would be faster than those in cases III and IV.

For the bit-flip channel, the Kraus operators are[15,17] $K_0^A=\mathrm{diag}(\sqrt{1-p/2},\sqrt{1-p/2})\otimes\boldsymbol{I}_B$, $K_1^A=\sqrt{p/2}\sigma_x^A\otimes\boldsymbol{I}_B$, $K_0^B=\boldsymbol{I}_A\otimes\mathrm{diag}(\sqrt{1-p/2},\sqrt{1-p/2})$, $K_1^B=\boldsymbol{I}_A\otimes\sqrt{p/2}\sigma_x^B$. The eigenvalue spectrum of $\rho_{AB}(t)$ is also given by Eq. (4), where the time dependent coefficients are $c_1(t)=c_1$, $c_2(t)=(1-p)^2c_2$, and $c_3(t)=(1-p)^2c_3$. The QSLT of the Bell-diagonal state ρ_0 under the bit-flip channel is symmetrical to that for the phase-flip channel. We just exchange the position of c_1 and c_3 in all expressions for the phase-flip channel, so the QSLT under the bit-flip channel can be obtained

$$\tau_{\rm QSL}^{B}=\begin{cases}\dfrac{\tau_{\rm D}(|c_2|^2+|c_3|^2)}{|c_3|+|c_1||c_2|},|c_1|\geq|c_3|\geq|c_2|,\\ \dfrac{\tau_{\rm D}(|c_2|^2+|c_3|^2)}{|c_2|+|c_1||c_3|},|c_1|\geq|c_2|\geq|c_3|,\\ \dfrac{\tau_{\rm D}(|c_2|^2+|c_3|^2)}{|c_3|(1+|c_2|)},|c_3|\geq|c_1|,|c_2|,\\ \dfrac{\tau_{\rm D}(|c_2|^2+|c_3|^2)}{|c_2|(1+|c_3|)},|c_2|\geq|c_1|,|c_3|.\end{cases}\tag{6}$$

For $|c_3|\geq|c_1|,|c_2|$ and $|c_2|\geq|c_1|,|c_3|$, under the bit-flip channel, the QSLT of the initial state does not depend on c_1. The QSLT is symmetrical with respect to the line $|c_2|=|c_3|$, i.e., $\tau_{\rm QSL}^B(\rho_0(|c_1|,|c_2|,|c_3|))=\tau_{\rm QSL}^B(\rho_0(|c_1|,|c_3|,|c_2|))$. Furthermore, the Bell-diagonal states in cases $|c_1|\geq|c_3|\geq|c_2|$ and $|c_1|\geq|c_2|\geq|c_3|$ have the smaller QSLTs than those in cases $|c_3|\geq|c_1|,|c_2|$ and $|c_2|\geq|c_1|,|c_3|$.

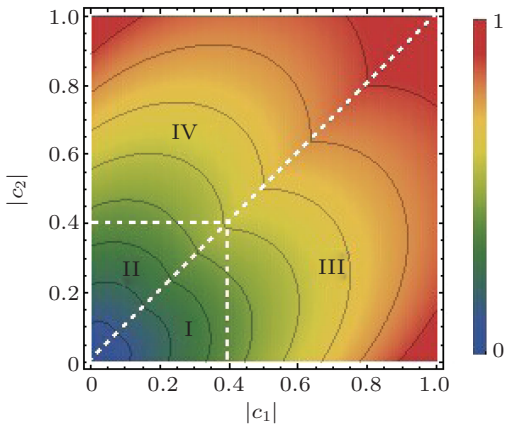


Fig. 1. (color online) The QSLT under the phase flip channel as a function of the coefficients $|c_1|$ and $|c_2|$ with $|c_3|=0.4$, here the driving time $\tau_{\rm D}=1$. The dashed white lines divide the entire range of $|c_1|$ and $|c_2|$ into four parts: case I, $|c_3|\geq|c_1|\geq|c_2|$; case II, $|c_3|\geq|c_2|\geq|c_1|$; case III, $|c_1|\geq|c_3|,|c_2|$; and case IV, $|c_2|\geq|c_3|,|c_1|$.

However, for the bit–phase-flip channel, the Kraus operators can be shown[28,38] $K_0^A=\mathrm{diag}(\sqrt{1-p/2},\sqrt{1-p/2})\otimes\boldsymbol{I}_B$, $K_1^A=\sqrt{p/2}\sigma_y^A\otimes\boldsymbol{I}_B$, $K_0^B=\boldsymbol{I}_A\otimes\mathrm{diag}(\sqrt{1-p/2},\sqrt{1-p/2})$, $K_1^B=\boldsymbol{I}_A\otimes\sqrt{p/2}\sigma_y^B$. In this noise channel, the time dependent coefficients are $c_1(t)=(1-p)^2c_1$, $c_2(t)=c_2$, and $c_3(t)=(1-p)^2c_3$. Once more, by swapping c_2 and c_3 in the phase-flip channel, we can obtain

$$\tau_{\rm QSL}^{\rm B-P}=\begin{cases}\dfrac{\tau_{\rm D}(|c_1|^2+|c_3|^2)}{|c_3|+|c_2||c_1|},|c_2|\geq|c_3|\geq|c_1|,\\ \dfrac{\tau_{\rm D}(|c_1|^2+|c_3|^2)}{|c_1|+|c_2||c_3|},|c_2|\geq|c_1|\geq|c_3|,\\ \dfrac{\tau_{\rm D}(|c_1|^2+|c_3|^2)}{|c_1|(1+|c_3|)},|c_1|\geq|c_2|,|c_3|,\\ \dfrac{\tau_{\rm D}(|c_1|^2+|c_3|^2)}{|c_3|(1+|c_1|)},|c_3|\geq|c_2|,|c_1|.\end{cases}\tag{7}$$

The fact that, for $|c_1|\geq|c_2|,|c_3|$ and $|c_3|\geq|c_2|,|c_1|$, the coefficient c_2 would not affect the QSLT under the bit–phase-flip

channel. It is also simple to see that, for the bit–phase-flip channel,

$$\tau_{\mathrm{QSL}}^{\mathrm{B-P}}(\rho_0(|c_1|,|c_2|,|c_3|)) = \tau_{\mathrm{QSL}}^{\mathrm{B-P}}(\rho_0(|c_3|,|c_2|,|c_1|)).$$

And the QSLTs for the initial states in cases $|c_2| \geq |c_3| \geq |c_1|$ and $|c_2| \geq |c_1| \geq |c_3|$ are always smaller than those for the states in cases $|c_1| \geq |c_2|, |c_3|$ and $|c_3| \geq |c_2|, |c_1|$.

It is worth mentioning that the QSLT of any Bell-diagonal states under phase flip, bit flip, and bit–phase-flip channels, can be accurately calculated; this is one of our main results in this work. Particularly, for the initial Werner state ($|c_1| = |c_2| = |c_3| = c$), the QSLTs under these three noise channels are all equal to $2\tau_{\mathrm{D}}/(1+1/c)$, which is proportional to c.

3. Quantum evolution speed for open dynamics process

In what follows, we shall illustrate the application of the QSLT to the quantum evolution speed of the whole dynamics process under different decoherence channels. The QSLT can demonstrate the speed of the dynamics evolution from a time-evolution state ρ_τ to another $\rho_{\tau+\tau_{\mathrm{D}}}$ by a driving time τ_{D}. It is easy to find that the time-evolution state ρ_τ at any point in time also maintains the form of a Bell-diagonal state. So the QSLT starting from an arbitrary time-evolution state ρ_τ to $\rho_{\tau+\tau_{\mathrm{D}}}$ can be calculated by the expressions of τ_{QSL} in Section 2. We would examine the evolution speed of the whole dynamics process where the system starts from the two-qubit Bell-diagonal state in Eq. (2), under phase-flip, bit-flip, and bit–phase-flip channels, respectively.

First of all, under the phase flip channel, $|c_3| \geq |c_1|, |c_2|$ is chosen for ρ_0. During the dynamics evolution process, c_3 remains unchanged, so the condition $|c_3| \geq |c_1(\tau)|, |c_2(\tau)|$ is fulfilled in $\tau \in [0,\infty)$. By considering an arbitrary mixed state ρ_τ evolves to another$\rho_{\tau+\tau_{\mathrm{D}}}$ under a driving time τ_{D}, the QSLT can be calculated

$$\tau_{\mathrm{QSL}}^{\mathrm{P}} = \begin{cases} \dfrac{\tau_{\mathrm{D}}(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_1|+|c_2||c_3|}, & |c_1| \leq |c_2|, \\ \dfrac{\tau_{\mathrm{D}}(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_2|+|c_1||c_3|}, & |c_1| \geq |c_2|. \end{cases} \tag{8}$$

From Eq. (8), one sees immediately that, the QSLT for the time-dependent state ρ_τ decays monotonically with the evolution time $\gamma\tau$, that is to say, in this case, the whole evolution of the two-qubit Bell-diagonal state can exhibit a speeded-up process under the local phase-flip noise channels. For the special case $c_3 = 1$, $c_1 = c_2 = 0$, which means the system would not evolve any further under the phase-flip channels (i.e., $\rho_\tau = \rho_{\tau+\tau_{\mathrm{D}}}$). Therefore, the minimum evolution time of ρ_τ to $\rho_{\tau+\tau_{\mathrm{D}}}$ is $\tau_{\mathrm{QSL}}^{\mathrm{P}} = 0$.

However, if ρ_0 satisfies $|c_1| \geq |c_2|, |c_3|$ or $|c_2| \geq |c_1|, |c_3|$, due to c_3 independence of the evolution time, it is easy to find that, for $\tau \leq \tau_{\mathrm{c}}$, with

$$\tau_{\mathrm{c}} = \frac{1}{2}\ln\Big(\frac{\max\{|c_1|,|c_2|\}}{|c_3|}\Big),$$

the conditions $|c_1(\tau)| \geq |c_2(\tau)|, |c_3|$ or $|c_2(\tau)| \geq |c_1(\tau)|, |c_3|$ are always true. By considering the dynamics process $\tau \in [0,\tau_{\mathrm{c}}]$, we can calculate the QSLT for the qubits to evolve from ρ_τ to $\rho_{\tau+\tau_{\mathrm{D}}}$,

$$\tau_{\mathrm{QSL}}^{\mathrm{P}} = \begin{cases} \dfrac{\tau_{\mathrm{D}}(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_1|[1+(1-p_\tau)^2|c_2|]}, & |c_1| \geq |c_2|, |c_3|, \\ \dfrac{\tau_{\mathrm{D}}(1-p_\tau)^2(|c_1|^2+|c_2|^2)}{|c_2|[1+(1-p_\tau)^2|c_1|]}, & |c_2| \geq |c_1|, |c_3|. \end{cases} \tag{9}$$

On the other hand, for $\tau > \tau_{\mathrm{c}}$, then $|c_3| \geq |c_1(\tau)| \geq |c_2(\tau)|$ or $|c_3| \geq |c_2(\tau)| \geq |c_1(\tau)|$, so when $\tau \in (\tau_{\mathrm{c}},\infty)$, the expressions of QSLT can be obtained by Eq. (8). Hence, in the conditions $|c_1| \geq |c_2|, |c_3|$ or $|c_2| \geq |c_1|, |c_3|$, we note that, the decay rate of the QSLT changes suddenly at $\tau = \tau_{\mathrm{c}}$. So we can conclude that, in the above case, the open system executes a speeded-up dynamics evolution process, but the increasing rate of the evolution speed would have a sudden change at $\tau = \tau_{\mathrm{c}}$. Let us, finally, study the QSLT, classical correlation and quantum correlation (quantum discord) on the class of initial states for which $c_1 = \pm 1$ and $c_2 = \mp c_3$, with $|c_3| < 1$, in Fig. 2. It is worth noting that, as noted in Ref. [37], for this initial condition, the behavior of sudden transition from classical correlation decoherence to quantum correlation decoherence can occur at $\tau = \tau_{\mathrm{c}}$, and at this time point, the decay rate of the QSLT can also change suddenly. Remarkably, the attenuations of the QSLT and classical correlation are synchronous in time until $\tau = \tau_{\mathrm{c}}$. However, for $\tau > \tau_{\mathrm{c}}$, the synchronous attenuation behavior occurs between the QSLT and quantum discord. This means that the speed of the dynamics evolution from a time-evolution state ρ_τ to another $\rho_{\tau+\tau_{\mathrm{D}}}$ can be clearly signatured by the classical correlation of the time-evolution state ρ_τ in the case $\tau < \tau_{\mathrm{c}}$. While for $\tau > \tau_{\mathrm{c}}$, we can use the quantum discord of ρ_τ to describe the speed of the dynamics process, qualitatively. This is a newly noticed phenomenon.

Finally, when we consider the bit-flip and the bit–phase-flip channels, the behaviors of the evolution speed of the whole dynamics process under the phase-flip channel described above can also occur under other conditions on the c_i in state (2). It is simple to see that, for the bit-flip and bit–phase-flip channels, the QSLT starting from an arbitrary time-evolution state ρ_τ to $\rho_{\tau+\tau_{\mathrm{D}}}$ can be obtained with c_1 and c_2 replacing c_3, respectively.

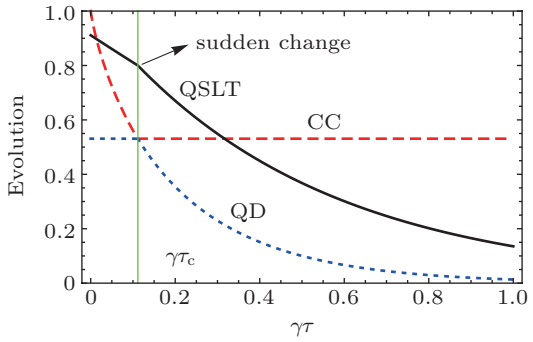


Fig. 2. (color online) The QSLT (solid black line), classical correlation (CC, dashed red line), and quantum correlation (QD, dotted blue line) under local phase-flip channels as a function of the initial time parameter $\gamma\tau$. We choose the class of initial Bell-diagonal states considered in Ref. [37], in which $c_1 = \pm 1$, $c_2 = \mp c_3$, here $c_1 = 1$, $c_2 = -0.8$, and $c_3 = 0.8$, and the driving time $\tau_D = 1$. On this initial condition, the behavior of sudden transition from classical correlation decoherence to quantum correlation decoherence occurs at $\tau = \tau_c$. The sudden change of the decay of QSLT has been marked in this figure. The classical correlation and quantum discord can be obtained by the method in Ref. [39].

4. Conclusion

Based on the definition of the QSLT for arbitrary mixed states, we have calculated the expressions of the QSLT to characterize the speed of evolution for an open composite quantum system. In particular, for the initial two-qubit Bell-diagonal state, we have identified four different expressions of the QSLT under decoherence, which depend on the coefficients of the initial composite state and on the noise channel. So these different expressions of the QSLT can be used to characterize the maximal evolution speed of the initial two-qubit Bell-diagonal states under different nondissipative decoherence channels. Moreover, by studying the bound for the arbitrary time-dependent states started from a class of two-qubit states cited in Ref. [37], a remarkable feature has been demonstrated that the decay rate of the QSLT can change suddenly at a critical time τ_c; at this moment, the behavior of sudden transition from classical decoherence to quantum decoherence also occurs. This evolution speed of the dynamics process would play an essential role in the understanding of the classical-to-quantum decoherence transition. Our results may be of interest in exploring the speed of quantum computation and quantum information processing under decoherence channels.

Appendix A

Firstly, we calculate the QSLT for the initial Bell-diagonal state ρ_0 in Eq. (2) under the bit-flip channel. For this channel, c_1 remains unchanged. Then the space of the initial state ρ_0 and the target state ρ_{τ_D} can be obtained $B(\rho_0, \rho_{\tau_D}) = \frac{1}{4}|p^2_{\tau_D} - 2p_{\tau_D}|(|c_2|^2 + |c_3|^2)$, and the singular values σ_i of $\dot{\rho}_t$ are $\sigma_1 = \sigma_2 = \frac{1}{2}|\dot{p}(1-p)(|c_2| + |c_3|)|$, $\sigma_3 = \sigma_4 = \frac{1}{2}|\dot{p}(1-p)(|c_2| - |c_3|)|$. In order to calculate the singular values of ρ_0, here we should divide the region of the coefficients c_i into four parts $\{|c_1| \geq |c_3| \geq |c_2|; |c_1| \geq |c_2| \geq |c_3|; |c_3| \geq |c_1|, |c_2|; |c_2| \geq |c_1|, |c_3|\}$. The singular values ρ_i of ρ_0 are given by

$$\begin{aligned}
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_1|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_1|), \\
&\qquad |c_1| \geq |c_3| \geq |c_2|, \\
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_1|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_1|), \\
&\qquad |c_1| \geq |c_2| \geq |c_3|, \\
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_3|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_3|), \\
&\qquad |c_3| \geq |c_1|, |c_2|, \\
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_2|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_2|), \\
&\qquad |c_2| \geq |c_1|, |c_3|.
\end{aligned} \tag{A1}$$

Similar to the phase-flip channel, we can obtain that $\sum_{i=1}^4 \sigma_i \rho_i$ is always less than $\sqrt{\sum_{i=1}^4 \sigma_i^2}$, and reach the result that the ML-type bound on the QSLT is tight for the open system. According to the definition of QSLT in Eq. (1), the different expressions of τ^B_{QSL} under the bit-flip channels in Eq. (6) would be clearly obtained.

With regard to the QSLT in the bit–phase-flip channels, where c_2 remains unchanged during the evolution process, we can also find $B(\rho_0, \rho_{\tau_D}) = \frac{1}{4}|p^2_{\tau_D} - 2p_{\tau_D}|(|c_1|^2 + |c_3|^2)$, and the singular values σ_i of $\dot{\rho}_t$ are $\sigma_1 = \sigma_2 = \frac{1}{2}|\dot{p}(1-p)(|c_1| + |c_3|)|$, $\sigma_3 = \sigma_4 = \frac{1}{2}|\dot{p}(1-p)(|c_1| - |c_3|)|$. We should divide the region of the coefficients c_i into four parts $\{|c_2| \geq |c_3| \geq |c_1|; |c_2| \geq |c_1| \geq |c_3|; |c_1| \geq |c_2|, |c_3|; |c_3| \geq |c_1|, |c_2|\}$. The singular values ρ_i of ρ_0 are given by

$$\begin{aligned}
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_2|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_2|), \\
&\qquad |c_2| \geq |c_3| \geq |c_1|, \\
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_2|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_2|), \\
&\qquad |c_2| \geq |c_1| \geq |c_3|, \\
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_1|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_1|), \\
&\qquad |c_1| \geq |c_2|, |c_3|, \\
&\rho_1 + \rho_2 = \frac{1}{2}(1 + |c_3|), \quad \rho_3 + \rho_4 = \frac{1}{2}(1 - |c_3|), \\
&\qquad |c_3| \geq |c_2|, |c_1|.
\end{aligned} \tag{A2}$$

Then we obtained $\tau^{\mathrm{B-P}}_{\mathrm{QSL}}$ for the bit–phase-flip channels in Eq. (7) as derived above.

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