A new kind of special function and its application*

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By virtue of the operator Hermite polynomial method and the technique of integration within the ordered product of operators we derive a new kind of special function, which is closely related to one- and two-variable Hermite polynomials. Its application in deriving the normalization for some quantum optical states is presented.

Keywords: new special function, new operator identities and integration formulas, Hermite-polynomial excitation state, integration within ordered product of operators (IWOP) method

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1. Introduction

The Hermite polynomial, as a special function, is very useful in quantum mechanics and mathematical physics. The one-variable Hermite polynomials are just eigenfunctions of the Harmonic oscillator, while the two-variable Hermite polynomials can represent transition amplitude from number state $|n\rangle$ to $|m\rangle$ in time evolution of a forced Harmonic oscillator.[1] The Hermite–Gaussian modes exist in optical propagation in quadratic gradient index (GRIN) lenses.[2] Mathematically, the one-variable Hermite polynomial $\mathrm{H}_n(x)$ can be introduced by its generating function

$$\mathrm{e}^{-t^2+2tx}=\sum_{n=0}^{\infty}\frac{t^n}{n!}\mathrm{H}_n(x) \tag{1}$$

or

$$\mathrm{H}_n(x)=\frac{\partial^n}{\partial t^n}\mathrm{e}^{-t^2+2tx}|_{t=0}=\sum_{l}^{[n/2]}\frac{n!(-1)^l}{l!(n-2l)!}(2x)^{n-2l}, \tag{2}$$

while the generating function of two-variable Hermite polynomial $\mathrm{H}_{n,m}(x,y)$ is[3]

$$\mathrm{e}^{-ts+tx+sy}=\sum_{n,m=0}^{\infty}\frac{s^nt^m}{n!m!}\mathrm{H}_{n,m}(x,y) \tag{3}$$

or

$$\begin{aligned}\mathrm{H}_{n,m}(x,y)&=\frac{\partial^{n+m}}{\partial s^n\partial t^m}\mathrm{e}^{-ts+tx+sy}|_{t=s=0}\\&=\sum_{l=0}^{\min(m,n)}\frac{n!m!(-1)^l}{l!(n-l)!(m-l)!}x^{n-l}y^{m-l}.\end{aligned} \tag{4}$$

An interesting question thus challenges us: if the power function $x^{n-l}y^{m-l}$ in Eq. (4) is replaced by suitable Hermite polynomials, like $\mathrm{H}_{n-l}(\mathrm{i}x)\mathrm{H}_{m-l}(\mathrm{i}y)$, then will a new meaningful special function appear? If yes, is it useful in some physics theories? The answer is affirmative, we shall employ the operator-Hermite polynomial method (OHPM)[4] to tackle this problem. The rest of the paper is organized as follows. In Section 2, we briefly review OHPM. In Section 3, we derive a new special function by using OHPM. In Section 4, we obtain some new operator identities and integration formulas. Their applications in deriving the normalization for some quantum optical states are presented in Section 5. Finally, some conclusions are presented in Section 6.

2. Brief review of OHPM

The essential point of OHPM is to replace $\mathrm{H}_n(x)$ by $\mathrm{H}_n(X)$, where X is the coordinate operator in quantum mechanics and expressed as

$$X=\frac{a+a^\dagger}{\sqrt{2}},\quad\left[a,a^\dagger\right]=1, \tag{5}$$

and then operator ordering (normal, anti-normal or Weyl ordering) and representation theory are used to obtain many operator identities. For instance, from Eq. (1) we have

$$\sum_{n=0}\frac{t^n}{n!}\mathrm{H}_n(X)=\mathrm{e}^{-t^2+2tX}=:\mathrm{e}^{2tX}:=\sum_{n=0}\frac{2^nt^n}{n!}:X^n:, \tag{6}$$

where : : denotes normal ordering, then we have the useful and concise operator identity

$$\mathrm{H}_n(X)=2^n:X^n:. \tag{7}$$

For a two-mode case, when we introduce the second-mode coordinate operator

$$Y=\frac{b+b^\dagger}{\sqrt{2}},\quad\left[b,b^\dagger\right]=1 \tag{8}$$

and noting $\left[a+b^\dagger,b+a^\dagger\right]=0$, from Eq. (3) we have

$$\sum_{n,m=0}\frac{s^nt^m}{n!m!}\mathrm{H}_{n,m}\left(a+b^\dagger,b+a^\dagger\right)$$

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$$
\begin{aligned}
&= \mathrm{e}^{t\left(a+b^{\dagger}\right)+s\left(b+a^{\dagger}\right)-st} \\
&= :\mathrm{e}^{t\left(a+b^{\dagger}\right)+s\left(b+a^{\dagger}\right)}: \\
&= \sum_{n,m=0} \frac{s^n t^m}{n!m!} :\left(a+b^{\dagger}\right)^n \left(b+a^{\dagger}\right)^m :, \qquad (9)
\end{aligned}
$$

which indicates

$$
\mathrm{H}_{n,m}\left(a+b^{\dagger}, b+a^{\dagger}\right) =: \left(a+b^{\dagger}\right)^n \left(b+a^{\dagger}\right)^m :, \qquad (10)
$$

which is another concise operator identity. Hence, the OHPM is powerful in deriving new operator identities.

3. Applying OHPM to Eq. (4) — appearance of a new special function

If we replace $x \to X$, $y \to Y$ in Eqs. (3) and (4), by noting $[X, Y] = 0$, we see

$$
\mathrm{e}^{-ts+tX+sY} = \sum_{n,m=0} \frac{s^n t^m}{n!m!} \mathrm{H}_{n,m}(X, Y) \qquad (11)
$$

and

$$
\begin{aligned}
&\mathrm{H}_{n,m}(X, Y) \\
&= \frac{\partial^{n+m}}{\partial s^n \partial t^m} \mathrm{e}^{-ts+tX+sY}|_{t=s=0} \\
&= \sum_{l=0}^{\min(m,n)} \frac{n!m!\,(-1)^l}{l!\,(n-l)!\,(m-l)!} X^{n-l} Y^{m-l}. \qquad (12)
\end{aligned}
$$

Then noticing

$$
X^n = (2\mathrm{i})^{-n} :\mathrm{H}_n(\mathrm{i}X): \qquad (13)
$$

which is a result of the following derivation

$$
\begin{aligned}
\sum_{n=0} \frac{(2t)^n}{n!} X^n &= \mathrm{e}^{2tX} \\
&= \mathrm{e}^{\sqrt{2}t\left(a+a^{\dagger}\right)} \\
&= :\mathrm{e}^{2(-\mathrm{i}t)\mathrm{i}X-(-\mathrm{i}t)^2}: \\
&= \sum_{n} \frac{(-\mathrm{i}t)^n}{n!} :\mathrm{H}_n(\mathrm{i}X): \qquad (14)
\end{aligned}
$$

and substituting Eq. (13) into Eq. (12), we have

$$
\begin{aligned}
&\frac{\partial^{n+m}}{\partial s^n \partial t^m} \mathrm{e}^{-ts+tX+sY}|_{t=s=0} \\
&= (2\mathrm{i})^{-n-m} \sum_{l=0}^{\min(m,n)} \frac{4^l n!m!}{l!\,(n-l)!\,(m-l)!} : \\
&\quad \times \mathrm{H}_{n-l}(\mathrm{i}X)\,\mathrm{H}_{m-l}(\mathrm{i}Y) :. \qquad (15)
\end{aligned}
$$

On the other hand

$$
\mathrm{e}^{-ts+tX+sY} =: \exp\left(sX + \frac{s^2}{4} + tY + \frac{t^2}{4} - st\right): , \qquad (16)
$$

thus

$$
\begin{aligned}
&\frac{\partial^{n+m}}{\partial s^n \partial t^m} :\exp\left(sX + \frac{s^2}{4} + tY + \frac{t^2}{4} - st\right): \Big|_{t=s=0} \\
&= (2\mathrm{i})^{-n-m} \sum_{l=0}^{\min(m,n)} \frac{4^l n!m!}{l!\,(n-l)!\,(m-l)!} : \\
&\quad \times \mathrm{H}_{n-l}(\mathrm{i}X)\,\mathrm{H}_{m-l}(\mathrm{i}Y) :, \qquad (17)
\end{aligned}
$$

where both sides are in normal ordering, which means

$$
\exp\left(sx + ty - st + \frac{s^2}{4} + \frac{t^2}{4}\right) = \sum_{n,m=0}^{\infty} \frac{s^n t^m}{n!m!} \mathfrak{F}_{n,m}(x, y), \qquad (18)
$$

where

$$
\begin{aligned}
&\mathfrak{F}_{n,m}(x, y) \\
&= (2\mathrm{i})^{-n-m} \sum_{l=0}^{\min(m,n)} \frac{4^l n!m!}{l!\,(n-l)!\,(m-l)!} \\
&\quad \times \mathrm{H}_{n-l}(\mathrm{i}x)\,\mathrm{H}_{m-l}(\mathrm{i}y) \qquad (19)
\end{aligned}
$$

is the new special function we are searching for, its generating function is $\exp\left(sx + ty - st + s^2/4 + t^2/4\right)$.

According to

$$
\mathrm{H}_n'(\mathrm{i}x) = 2\mathrm{i}n H_{n-1}(\mathrm{i}x), \qquad (20)
$$

from Eq. (19) we have

$$
\begin{aligned}
&\frac{\partial}{\partial x} \mathfrak{F}_{n,m}(x, y) \\
&= (2\mathrm{i})^{-n-m} \sum_{l=0}^{\min(m,n)} \frac{4^l n!m!}{l!\,(n-l)!\,(m-l)!} 2\mathrm{i}\,(n-l) \\
&\quad \times \mathrm{H}_{n-l-1}(\mathrm{i}x)\,\mathrm{H}_{m-l}(\mathrm{i}y) \\
&= n(2\mathrm{i})^{-n-m+1} \sum_{l=0}^{\min(m,n)} \frac{4^l (n-1)!m!}{l!\,(n-l-1)!\,(m-l)!} \\
&\quad \times \mathrm{H}_{n-l-1}(\mathrm{i}x)\,\mathrm{H}_{m-l}(\mathrm{i}y) \\
&= n\mathfrak{F}_{n-1,m}(x, y). \qquad (21)
\end{aligned}
$$

Similarly, we see

$$
\begin{aligned}
&\frac{\partial}{\partial y} \mathfrak{F}_{n,m}(x, y) \\
&= (2\mathrm{i})^{-n-m} \sum_{l=0}^{\min(m,n)} \frac{4^l n!m!}{l!\,(n-l)!\,(m-l)!} \mathrm{H}_{n-l}(\mathrm{i}x)\,\mathrm{H}'_{m-l}(\mathrm{i}y) \\
&= m\mathfrak{F}_{n,m-1}(x, y), \qquad (22)
\end{aligned}
$$

from which it then follows that

$$
\frac{\partial^2}{\partial x \partial y} \mathfrak{F}_{n,m}(x, y) = nm\mathfrak{F}_{n-1,m-1}(x, y).
$$

4. New operator identities and integration formulas

According to Eqs. (12), (15), and (19), we obtain a new operator identity

$$
\mathrm{H}_{n,m}(X, Y) =: \mathfrak{F}_{n,m}(X, Y): . \qquad (23)
$$

Further, from Eq. (18) we know

$$\begin{aligned}&\sum_{n,m}\frac{s^n t^m}{n!m!}\mathfrak{F}_{n,m}(\mathrm{i}X,\mathrm{i}Y)\\&=\exp\left(\mathrm{i}sX+\mathrm{i}tY-st+\frac{s^2}{4}+\frac{t^2}{4}\right)\\&=:\exp(\mathrm{i}sX+\mathrm{i}tY-st):\\&=\sum_{n,m}\frac{s^n t^m}{n!m!}:\mathrm{H}_{n,m}(\mathrm{i}X,\mathrm{i}Y):,\end{aligned}$$

which yields another operator identity

$$\mathfrak{F}_{n,m}(\mathrm{i}X,\mathrm{i}Y)=:\mathrm{H}_{n,m}(\mathrm{i}X,\mathrm{i}Y):. \tag{24}$$

Using the completeness relation of coordinate representation[5]

$$\int_{-\infty}^{\infty}\mathrm{d}x|x\rangle\langle x|=\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{\sqrt{\pi}}:\mathrm{e}^{-(x-X)^2}:=1, \tag{25}$$

we have

$$\begin{aligned}\mathrm{H}_{n,m}(X,Y)&=\iint_{-\infty}^{\infty}\frac{\mathrm{d}x\mathrm{d}y}{\pi}:\mathrm{e}^{-(x-X)^2-(y-Y)^2}:\mathrm{H}_{n,m}(x,y)\\&=:\mathfrak{F}_{n,m}(X,Y):,\end{aligned} \tag{26}$$

which implies a new integration formula

$$\begin{aligned}&\frac{1}{\pi}\iint_{-\infty}^{\infty}\mathrm{d}x'\mathrm{d}y'\,\mathrm{e}^{-(x'-x)^2-(y'-y)^2}\mathrm{H}_{n,m}(x',y')\\&=\mathfrak{F}_{n,m}(x,y).\end{aligned} \tag{27}$$

On the other hand, from Eq. (23) we have

$$\begin{aligned}&\mathfrak{F}_{n,m}(\mathrm{i}X,\mathrm{i}Y)\\&=\iint_{-\infty}^{\infty}\mathrm{d}x\mathrm{d}y\mathfrak{F}_{n,m}(\mathrm{i}x,\mathrm{i}y)|x,y\rangle\langle x,y|\\&=\frac{1}{\pi}\iint_{-\infty}^{\infty}\mathrm{d}x\mathrm{d}y:\mathrm{e}^{-(x-X)^2-(y-Y)^2}:\mathfrak{F}_{n,m}(\mathrm{i}x,\mathrm{i}y)\\&=\mathrm{H}_{n,m}(\mathrm{i}X,\mathrm{i}Y):,\end{aligned} \tag{28}$$

which implies another integration formula

$$\begin{aligned}&\frac{1}{\pi}\iint_{-\infty}^{\infty}\mathrm{d}x'\mathrm{d}y'\,\mathrm{e}^{-(x'-x)^2-(y'-y)^2}\mathfrak{F}_{n,m}(\mathrm{i}x,\mathrm{i}y)\\&=\mathrm{H}_{n,m}(\mathrm{i}x,\mathrm{i}y).\end{aligned} \tag{29}$$

5. Applications

Using Eqs. (19) and (26), we have the following two-variable Hermite excitation state

$$\begin{aligned}&\mathrm{H}_{n,m}(X,Y)|00\rangle\\&=\frac{1}{(2i)^{n+m}}\sum_{l=0}^{\min(m,n)}\frac{4^l n!m!}{l!(n-l)!(m-l)!}\\&\times\mathrm{H}_{n-l}\left(\frac{\mathrm{i}a^\dagger}{\sqrt{2}}\right)\mathrm{H}_{m-l}\left(\frac{\mathrm{i}b^\dagger}{\sqrt{2}}\right)|00\rangle,\end{aligned} \tag{30}$$

which is an entangled state. Further, from Eq. (18) and $\left[\left(a+b^\dagger\right),\left(b+a^\dagger\right)\right]=0$, we know

$$\begin{aligned}&\exp\left[s\left(a+b^\dagger\right)+t\left(b+a^\dagger\right)-st+\frac{s^2}{4}+\frac{t^2}{4}\right]\\&=\sum_{n,m}\frac{s^n t^m}{n!m!}\mathfrak{F}_{n,m}\left(a+b^\dagger,b+a^\dagger\right).\end{aligned} \tag{31}$$

Using the Baker–Hausdorff formula

$$\begin{aligned}&\exp\left[s\left(a+b^\dagger\right)+t\left(b+a^\dagger\right)-st+\frac{s^2}{4}+\frac{t^2}{4}\right]\\&=:\exp\left[s\left(a+b^\dagger\right)+t\left(b+a^\dagger\right)-\left(\frac{-\mathrm{i}s}{2}\right)^2-\left(\frac{-\mathrm{i}t}{2}\right)^2\right]:\\&=\sum_{n,m}\frac{(-\mathrm{i}s/2)^n(-\mathrm{i}t/2)^m}{n!m!}:\mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\\&\times\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right]:,\end{aligned} \tag{32}$$

and comparing Eq. (30) with Eq. (31), we obtain the following operator identity

$$\begin{aligned}&\mathfrak{F}_{n,m}\left(a+b^\dagger,b+a^\dagger\right)\\&=\left(\frac{-\mathrm{i}}{2}\right)^{m+n}:\mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right]:.\end{aligned} \tag{33}$$

Operating $\mathfrak{F}_{n,m}\left(a+b^\dagger,b+a^\dagger\right)$ on the two-mode vacuum state leads to

$$\begin{aligned}&\mathfrak{F}_{n,m}\left(a+b^\dagger,b+a^\dagger\right)|00\rangle\\&=\left(\frac{-\mathrm{i}}{2}\right)^{m+n}\mathrm{H}_n\left(\mathrm{i}b^\dagger\right)\mathrm{H}_m\left(\mathrm{i}a^\dagger\right)|00\rangle.\end{aligned} \tag{34}$$

On the other hand, from Eq. (18) we have

$$\begin{aligned}&:\exp\left[s\left(a+b^\dagger\right)+t\left(b+a^\dagger\right)-st+\frac{s^2}{4}+\frac{t^2}{4}\right]:\\&=\sum_{n,m}\frac{s^n t^m}{n!m!}:\mathfrak{F}_{n,m}\left(a+b^\dagger,b+a^\dagger\right):.\end{aligned} \tag{35}$$

Using the Baker–Hausdorff formula and Eq. (1), we see that the left hand side of Eq. (35) is equal to

$$\begin{aligned}&\exp\left[s\left(a+b^\dagger\right)+t\left(b+a^\dagger\right)+\frac{s^2}{4}+\frac{t^2}{4}\right]\\&=\sum_{n,m}\frac{(-\mathrm{i}s/2)^n(-\mathrm{i}t/2)^m}{n!m!}\mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\\&\times\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right],\end{aligned} \tag{36}$$

so comparing Eq. (35) with Eq. (36), we deduce another operator identity

$$\begin{aligned}&\mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right]\\&=(2\mathrm{i})^{m+n}:\mathfrak{F}_{n,m}\left(a+b^\dagger,b+a^\dagger\right):,\end{aligned} \tag{37}$$

from which it follows that

$$\begin{aligned}&\mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right]|00\rangle\\&=(2\mathrm{i})^{m+n}\mathfrak{F}_{n,m}\left(b^\dagger,a^\dagger\right)|00\rangle.\end{aligned} \tag{38}$$

Using the entangled state representation[6]

$$\int \frac{\mathrm{d}^2\xi}{\pi} |\xi\rangle\langle\xi| = \int \frac{\mathrm{d}^2\xi}{\pi} : \mathrm{e}^{-\left[\xi-\left(a+b^\dagger\right)\right]\left[\xi^*-\left(a^\dagger+b\right)\right]} : = 1, \tag{39}$$

where

$$|\xi\rangle = \exp\left(-\frac{|\xi|^2}{2} + a^\dagger\xi + b^\dagger\xi^* - a^\dagger b^\dagger\right)|00\rangle \tag{40}$$

obeys the eigenvector equations

$$\left(a+b^\dagger\right)|\xi\rangle = \xi|\xi\rangle, \quad \left(b+a^\dagger\right)|\xi\rangle = \xi^*|\xi\rangle. \tag{41}$$

Using Eqs. (38), (40), and (37), we have

$$\begin{aligned} &\mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right] \\ &= \mathrm{H}_n\left[\mathrm{i}\left(a+b^\dagger\right)\right]\mathrm{H}_m\left[\mathrm{i}\left(b+a^\dagger\right)\right]\int\frac{\mathrm{d}^2\xi}{\pi}|\xi\rangle\langle\xi| \\ &= \int\frac{\mathrm{d}^2\xi}{\pi}\mathrm{H}_n(\mathrm{i}\xi)\mathrm{H}_m(\mathrm{i}\xi^*) : \mathrm{e}^{-\left[\xi-\left(a+b^\dagger\right)\right]\left[\xi^*-\left(a^\dagger+b\right)\right]} : \\ &= (2\mathrm{i})^{m+n} : \mathfrak{F}_{n,m}\left(a+b^\dagger, b+a^\dagger\right) :, \end{aligned} \tag{42}$$

from which we deduce the integration formula

$$\begin{aligned} &\int\frac{\mathrm{d}^2\xi}{\pi}\mathrm{H}_n(\mathrm{i}\xi)\mathrm{H}_m(\mathrm{i}\xi^*)\,\mathrm{e}^{-[\xi-\lambda][\xi^*-\lambda^*]} \\ &= (2\mathrm{i})^{m+n}\,\mathfrak{F}_{n,m}(\lambda,\lambda^*). \end{aligned} \tag{43}$$

As its application, we consider how to normalize the Hermite-polynomial excitation state $\mathrm{H}_m\left(\mathrm{i}a^\dagger\right)|0\rangle$. Using the completeness relation of coherent state representation

$$\int\frac{\mathrm{d}^2z}{\pi}|z\rangle\langle z| = \int\frac{\mathrm{d}^2z}{\pi} : \mathrm{e}^{-\left[z^*-a^\dagger\right][z-a]} := 1,$$

$$|z\rangle = \mathrm{e}^{-|z|^2/2+za^\dagger}|0\rangle \tag{44}$$

and Eq. (42), we have

$$\begin{aligned} &\mathrm{H}_m(-\mathrm{i}a)\mathrm{H}_m\left(\mathrm{i}a^\dagger\right) \\ &= (-)^m\mathrm{H}_m(\mathrm{i}a)\mathrm{H}_m\left(\mathrm{i}a^\dagger\right) \\ &= (-)^m\int\frac{\mathrm{d}^2z}{\pi}\mathrm{H}_m(\mathrm{i}z)|z\rangle\langle z|\mathrm{H}_m(\mathrm{i}z^*) \\ &= (-)^m\int\frac{\mathrm{d}^2z}{\pi}\mathrm{H}_n(\mathrm{i}z)\mathrm{H}_m(\mathrm{i}z^*) : \mathrm{e}^{-\left[z^*-a^\dagger\right][z-a]} : \\ &= 4^m : \mathfrak{F}_{m,m}\left(a^\dagger, a\right) :, \end{aligned} \tag{45}$$

then the normalization for the Hermite-polynomial excitation state $\mathrm{H}_m\left(\mathrm{i}a^\dagger\right)|0\rangle$ is

$$\langle 0|\mathrm{H}_m(-\mathrm{i}a)\mathrm{H}_m\left(\mathrm{i}a^\dagger\right)|0\rangle = 4^m\mathfrak{F}_{m,m}(0,0). \tag{46}$$

6. Conclusion

In this paper, by virtue of the operator Hermite polynomial method and the technique of integration within ordered product of operators, we find a new kind of special function, which is closely related to one- and two-variable Hermite polynomials. Its application in deriving the normalization for some quantum optical states is presented.

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