Scintillation characterization for multiple incoherent uplink Gaussian beams*

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By means of numerical simulations, we analyze the scintillation characterization for multiple incoherent uplink Gaussian beams under weak fluctuations cases. Because truly independent beams are difficult to create, we present a more general but approximate model for the multiple of beams traveling through partially correlated paths. This model compares with wave-optics simulations and highlights the reduced correlation coefficient as the beam separation is increased. The scintillation index of three and six incoherent uplink Gaussian beams is also induced. The result shows that the scintillation index decreases with the increase of beams amount and beam separation. When the beams amount and strength of atmospheric turbulence are fixed, the reduction of scintillation index is affected by the ratio of beams separation and the Fried length. The corresponding physical mechanisms for the results are discussed.

Keywords: scintillation characterization, uplink beams, correlation coefficient, beam separation

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1. Introduction

When laser beams are transmitted from ground to the satellite, the variation in the index of refraction along the path causes the beam to randomly wander and scintillate, resulting in the fluctuation of power (called scintillation) at the receiver end of the propagation path.[1,2] Because the irradiance correlation length at satellite is many times large than probable size of the satellite, the receiver at the satellite always behaves like a point receiver.[3] For such a case, where a number of separate transmits were interfering with partially correlated paths, the signal arriving at the detector is sum of the signal from each uplink beam partially independent distributed statistics. Thus, the fluctuations on the uplink can be minimized by transmitting multiple independent lasers.

The experiment using multiple transmitters was carried out in order to examine the scintillation index (SI) for uplink Gaussian beams, especially the results of varying the number of beams and their diameters.[4,5] In theory, most of the reported research assumed that the multiple incoherent beams are completely independent.[6] The reduction in SI can be approximated by a N dependence, where N is the number of beams. But truly independent beams are practically impossible to create. There are some published research that cover the SI for multiple beams traveling through partially correlated horizontal and slant paths.[7–11] However, to the best of our knowledge, no corresponding work related with analysis of the scintillation characterization for partially independent uplink beams is carried out.

In this paper, we discuss the longitudinal (on-axis) component of scintillation index of three and six uplink collimated Gaussian beams under weak fluctuations cases. Based on the results of simulation, we propose an approximation to the scintillation index of uplink collimated beams as a function of number of the beams and beam separation.

2. Scintillation theory

The uplink longitudinal component of the scintillation index for an untracked beam in weak fluctuation regimes can be expressed as[12–14]

$$\sigma_{\mathrm{I}}^2 = 34.29\left(\frac{\Lambda L}{kr_0^2}\right)^{5/6}\left(\frac{\sigma_{\mathrm{pe}}}{W}\right)^2 + 8.7\mu_{\mathrm{u1}}k^{7/6}(H-h_0)^{5/6}\sec(\zeta), \tag{1}$$

$$\mu_{\mathrm{u1}} = \mathrm{Re}\int_{h_0}^{H} C_n^2(h)\{\xi^{5/6}[\Lambda\xi + \mathrm{i}(1-\overline{\Theta}\xi)]^{5/6} - \Lambda^{5/6}\xi^{5/3}\}\,\mathrm{d}h, \tag{2}$$

$$r_0 = \left[0.423k^2\sec(\zeta)\int_{h_0}^{H} C_n^2(h)\,\mathrm{d}h\right]^{-3/5}. \tag{3}$$

In these expressions, r_0 is the Fried length of plane wave, h denotes altitude (in units of m), H is the altitude of the receiver, h_0 is the altitude of the transmitter, $C_n^2(h)$ is the refractive index structural parameter as a function of altitude h, and ζ is the zenith angle. $\xi = 1-(h-h_0)/(H-h_0)$, Θ and Λ are the curvature parameter and the Fresnel ratio at the receive plane for vacuum propagation, which are given in terms of their respective values at the source plane are $\Theta = \Theta_0/(\Theta_0^2+\Lambda_0^2) = 1+L/F$, and $\Lambda = \Lambda_0/(\Theta_0^2+\Lambda_0^2) = 2L/W^2$. The curvature parameter and the Fresnel ratio at the source

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plane are $\Theta_0 = 1 - L/F_0$ and $\Lambda_0 = 2L/(kW_0^2)$ respectively, where L is the propagation distance, F_0 is the radius of curvature, W_0 denotes the radius at which the field amplitude falls to $1/e$ of that on the beam axis, and $k = 2\pi/\lambda$ is the wave number. The first term on the right-hand side of Eq. (1), which arises from the effective wander-induced pointing error, is the conventional weak-fluctuation radial term for the scintillation index evaluated at $r = \sigma_{\rm pe}$,[14] where $\sigma_{\rm pe}$ is the pointing error defined by

$$\sigma_{\rm pe}^2 = \langle r_{\rm c}^2\rangle \left[1 - \left(\frac{\pi^2 W_0^2/r_0^2}{1+\pi^2 W_0^2/r_0^2}\right)^{1/6}\right], \tag{4}$$

$$\langle r_{\rm c}^2\rangle = 7.25(H-h_0)^2 \sec^3(\zeta) W_0^{-1/3} \int_{h_0}^{H} C_n^2(h) \times [1-(h-h_0)/(H-h_0)]^2 \,{\rm d}h. \tag{5}$$

For multiple incoherent uplink Gaussian beams, the SI $\sigma_{{\rm I},M}^2$ with M beams would be[7,8]

$$\sigma_{{\rm I},M}^2 = F_{\rm B}\sigma_{\rm I}^2 = \frac{1}{M}(1-r)\sigma_{\rm I}^2 + r\sigma_{\rm I}^2, \tag{6}$$

where $F_{\rm B}$ is the beam averaging factor, r is the average correlation coefficient between all beams. When beams are completely correlated, that is, $r = 1$, the scintillation variance of beams is the same as that of single beam, that is, $F_{\rm B} = 1$. When beams are completely uncorrelated, that is, $r_{=}0$, the scintillation variance of beams is $1/M$ that of single beam, that is $F_{\rm B} = 1/M$.

The average correlation coefficient r between all beams can be expressed as

$$r = \sum_{p\in P} r_p \Big/ \left[\frac{M(M-1)}{2}\right], \quad M > 1, \tag{7}$$

where r_p is the correlation coefficient between the p-th pair of beams, and P represents the set of all pairs of beams

By substituting Eq. (7) into Eq. (6), we can obtain the SI of multiple incoherent uplink Gaussian beams namely

$$\sigma_{{\rm I},M}^2 = \frac{1}{M}\left(1 + \frac{2}{M}\sum_{p\in P} r_p\right)\sigma_{\rm I}^2. \tag{8}$$

The correlation coefficient r_p was determined by using wave optics simulations (WOS) of uplink Gaussian beam pairs of various beam separation d. The resulting relationship between beam separation and correlation coefficient was determined by[11]

$$r_p(d) = \frac{\langle I_{\rm A}(-d/2,0) - \mu_{\rm I}(-d/2,0))(I_{\rm A}(d/2,0) - \mu_{\rm I}(d/2,0))\rangle}{\sigma_{\rm I}(-d/2,0)\,\sigma_{\rm I}(d/2,0)}, \tag{9}$$

where $I_{\rm A}(x,y)$ is the received irradiance integrated over the receive aperture for a Gaussian beam centered on x, y. $\mu_{I_{\rm A}(x,y)}$ and $\sigma_{I_{\rm A}(x,y)}$ are the mean and variance of $I_{\rm A}(x,y)$, respectively. We define integrated irradiance for each beam as[11]

$$I_{\rm A}(x,y) = \iint_{\rm A} I(\xi - x, \eta - y)\,{\rm d}\eta\,{\rm d}\xi = \int_{-D/2}^{D/2}\int_{-\sqrt{(D/2)^2-(\xi-x)^2}}^{\sqrt{(D/2)^2-(\xi-x)^2}} \times I(\xi - x, \eta - y)\,{\rm d}\eta\,{\rm d}\xi, \tag{10}$$

where I represents the irradiance at the receiver plane, and D is the receive aperture diameter.

3. Wave optical simulations

Atmospheric turbulence is simulated with a split-step approach involving a series of random screens which assume a Kolmogorov spectrum.[15] We approached the simulation by considering each collimated beam separately and summing their resulting irradiance values at the receiver. The first beam is then propagated through 30 sets of equally spaced turbulence screens to obtain a record of 500 receiver responses due to this beam. Then a second beam with a different transmit plane position, is propagated through exactly the same sets of screens, and so on, until a desired number of beams is achieved.[16] The resulting irradiance values for the beams are summed for the 500 cases, which assumes that the beams are mutually incoherent.

Simulations were conducted for the link scenarios, where the beam separation d of the circle array was varied from 0 to around 30 cm. For a radius of zero, all the beams are completely overlapped at the center of the transmit plane. We select a grid size of $N = 256$ points on a square windows of 1 m on a side perpendicular to the propagation axis. This gives a grid interval of $\Delta x = \Delta y \approx 3.9$ mm at the transmitter. The grid interval should be varied according to the beam size at different turbulent phase screen to overcome the window effect and the loss-of-resolution problem. We use the coordinate transformation to achieve this purpose.[17] The inner scale and outer scale of turbulence are set to 0 mm and 100 m.

A refractive index structure parameter $C_n^2(h)$ profile model is needed to describe the varying strength of optical turbulence as a function of altitude h. One of the most widely used model is the Hufnagel–Valley (HV5/7) model, which is described by[15]

$$C_n^2(h) = 0.00594\times 10^{-50}(21/27)^2 h^{10}\,{\rm e}^{-h/1000} + 2.7\times 10^{-16}\,{\rm e}^{-h/1500} + A\,{\rm e}^{-h/100}, \tag{11}$$

where A is a nominal value of $C_n^2(0)$ at the ground ($A = 1.7\times 10^{-14}$ m$^{-2/3}$), h denotes altitude (in units of m). The zenith angle ζ is 0° and the propagation length L is chosen to be 100 km, so the Fried length of plane wave of r_0 is 11.2 cm in the engagement.

The results of 3- and 6-beams circular arrangements are showed in Fig. 1. The power and the separation between adjacent beams d_0 are distributed equally. The incoherent 1064-nm wavelength collimates uplink Gaussian beams with a beam waist radius of $W_0 = 5$ cm.

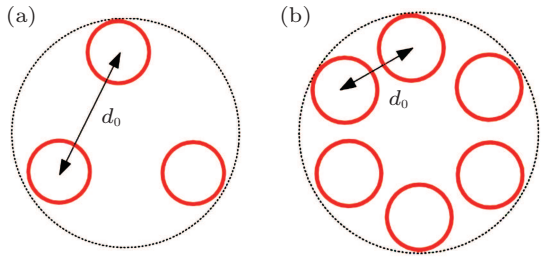


Fig. 1. (color online) Configuration used for (a) 3- and (b) 6-beams cases.

4. WOS and analytic results

In this section, we present examples of analytic and WOS scintillation index results. By using a least-squares fit to the simulation data, the relationship between beam separation and correlation coefficient is

$$r_p(d) \approx \exp(-0.65d/r_0). \tag{12}$$

This functional form provides the appropriate limiting conditions as d approaches zero and infinity when the strength of atmospheric turbulence is fixed. This behavior coincides with the general concept that when two beams are overlapped, they must be completely correlated, so r_p is equal to 1. However, when two beams are separated infinitely far apart, they must be completely uncorrelated, so r_p is equal to 0. It also shows that the correlation coefficient decreases monotonously with increasing ratio of beam separation and the Fried length.

For 3-beams case, the separation between beams d is the same, that is $d = d_0$. Hence, the correlation coefficient r_p is identical, and equation (8) can be expressed as

$$\sigma_{\mathrm{I},M}^2 = \frac{1}{M}(1-r_p)\sigma_{\mathrm{I}}^2 + r_p\sigma_{\mathrm{I}}^2. \tag{13}$$

For 6-beams case, the separation between adjacent beams is d_0 and the maximum beam separation (across diameter) is $2d_0$. When the separation between adjacent beams is 10 cm, we calculate the correlation coefficients between all pairs of beams using Eq. (12) are shown in Table 1.

Table 1. Correlation coefficient when the separation between adjacent beams is 10 cm.

d/cm	r_p	Number of pairs
10	0.575	6
17.32	0.3835	6
20	0.3306	3

Combining Eqs. (2), (8), and (11), we present the SI of 3- and 6-beams results as a function of separation between adjacent beams in Fig. 2. We can deduce from Fig. 2 that the WOS values generally follow the analytic results. The SI of single beam is about 0.55 when the separation between adjacent beams $d_0 = 0$. The SI of multiple beams decreases monotonously with increasing beam number and beam separation. When d_0 exceeds 60 cm, that is, $d_0/r_0 > 5.5$, the SI of 3- and 6-beams are about 0.2 and 0.1. This is because that the correlation coefficients between beams approach zero when the ratio of separation between adjacent beams and the Fried length exceeds 5.5. When separation between adjacent beams is the same, the SI of 6-beams is smaller than that of 3-beams.

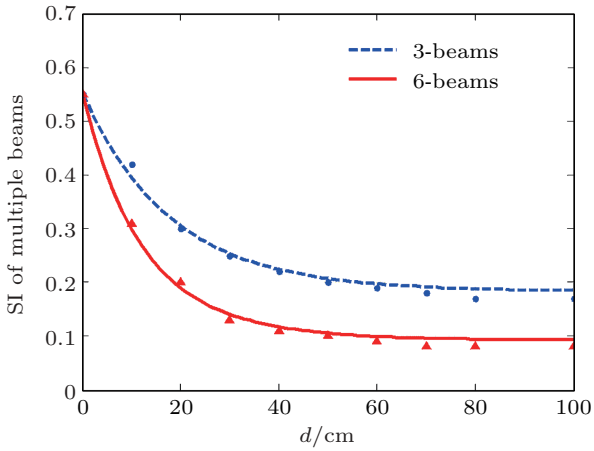


Fig. 2. (color online) SI of multiple incoherent uplink Gaussian beams as a function of d_0.

In Fig. 3 we plot the ratio of beam averaging factor F_{B} between 6-beams and 3-beams as a function of d_0/r_0. The ration of beam averaging factor increases quickly with the increase in d_0/r_0 from 1 to 3, but it increases slowly with the increase in d_0/r_0 from 3 to 6, and it will not change when $d_0/r_0 > 6$. Because beams are completely uncorrelated when the beam separation is big enough, the beam averaging factors of 6-beams and 3-beams are 1/6 and 1/3, so the ration of beam averaging factor is equal to 2 when $d_0/r_0 > 6$. This is because the average correlation coefficient r approaches zero when $d_0/r_0 > 6$, so N-beams approaches are completely correlated and the reduction in SI can be approximated by a N dependence. The results show that the number of beams and beam separation should be taken into consideration to obtain the minimum SI in practical utilization.

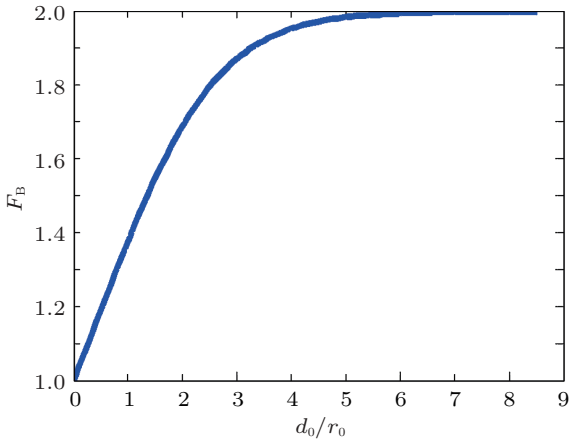


Fig. 3. (color online) Ratio of beam averaging factor F_B between 6-beams and 3-beams as a function of d_0/r_0.

5. Conclusions

In this paper, we analyze the correlation among multiple incoherent uplink Gaussian beams projected into a single receiver under weak fluctuations cases. We provide simple expression that shows the correlate coefficient approaches 1 and 0 as beam separation approaches zero and infinity. Spatial correlation between uplink beams decreases with increasing ration between the beam separation and the Fried length of turbulent atmosphere. The resulting analytic expressions for the scintillation index of 3- and 6-beams with wave optical simulation results are compared. The results show that the beam averaging factor of scintillation index increases with increasing number of beams, but the increase slows down when the ratio of beams separation and the Fried length exceed 3. From the application point of view, our formulation provides a theoretical way of evaluating the scintillation characterization of multiple uplink Gaussian beams under weak fluctuations cases.

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