Polarized radiative transfer considering thermal emission in semitransparent media*

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The characteristics of the polarization must be considered for a complete and correct description of radiation transfer in a scattering medium. Observing and identifying the polarizition characteristics of the thermal emission of a hot semitransparent medium have a major significance to analyze the optical responses of the medium for different temperatures. In this paper, a Monte Carlo method is developed for polarzied radiative transfer in a semitransparent medium. There are mainly two kinds of mechanisms leading to polarization of light: specular reflection on the Fresnel boundary and scattering by particles. The determination of scattering direction is the key to solve polarized radiative transfer problem using the Monte Carlo method. An optimized rejection method is used to calculate the scattering angles. In the model, the treatment of specular reflection is also considered, and in the process of tracing photons, the normalization must be applied to the Stokes vector when scattering, reflection, or transmission occurs. The vector radiative transfer matrix (VRTM) is defined and solved using Monte Carlo strategy, by which all four Stokes elements can be determined. Our results for Rayleigh scattering and Mie scattering are compared well with published data. The accuracy of the developed Monte Carlo method is shown to be good enough for the solution to vector radiative transfer. Polarization characteristics of thermal emission in a hot semitransparent medium is investigated, and results show that the U and V parameters of Stokes vector are equal to zero, an obvious peak always appear in the Q curve instead of the I curve, and refractive index has a completely different effect on I from Q.

Keywords: radiative transfer, thermal emission, polarization, Monte Carlo

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1. Introduction

In certain wavelengths, the radiatively participating medium or absorbing–emitting–scattering medium is commonly called as the semitransparent medium. For some applications, such as space target detection,[1,2] the thermal emission characteristics (infrared characteristics or ultraviolet characteristics) of a hot semitransparent medium have great significance. The research on thermal emission has developed for many years, but most of studies concentrate on the scalar radiation transfer which does not take the light polarized properties into account. In fact, polarization is one of the most important characteristics of light. The polarizition characteristics of the thermal emission of a hot semitransparent medium can be applied to analyzing the optical responses of the medium for different temperatures. The study of polarized radiative transfer has been developed for more than forty years in numerical simulation aspect, including Monte Carlo method,[3] F_{N} method,[4] adding-doubling method,[5] discrete-ordinate method,[6–8] multi-coupled single scattering method,[9] and other methods have been put to use. Compared with the classical scalar radiative transfer equation, the polarized radiative transfer equation is much more complex due to the introduction of Stokes vector and parameter matrices, which brings additional difficulties in the solution to the vector radiative transfer equation (VRTE). Most numerical methods based on the discretization of VRTE cannot avoid the tedious derivations and using a lot of mathematical skills, so they have certain limitations in solving VRTE. With obvious advantages such as high accuracy, good adaptability of complicated geometries and various optical boundary conditions, clear physical conception, simple programming and so on, the Monte Carlo method has become one of the most popular method in the investigation of polarized radiative transfer.

In 1968, Kattawar *et al.*[3] first attempted to calculate the status of polarization of the light after multiple scattering using a Monte Carlo method. Since then, many researchers published their works on Monte Carlo models of vector radiative transfer and their applications. In 1989, Kattawar *et al.*[10] wrote a large-scale Monte Carlo program to calculate the complete four-component Stokes vector at any region in a fully inhomogeneous atmosphere-ocean system with inclusion of a stochastic interface. Wang *et al.*[11] made an experimental and numerical investigation of polarized light transport through randomly scattering media, and the experimental data matched the theoretical results by the time-resolved Monte Carlo technique. Vaillona *et al.*[12] presented a vector Monte Carlo

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method to model the polarized radiative transfer in optically thick, multiple scattering and particle-laden semi-transparent medium and gave simulated results for a plane-parallel atmosphere layer; the model was also applied to the simulation of a nephelometric experiment to derive the effective Mueller matrix of the complete system. Ramella-Roman *et al.*[13] summarized two Monte Carlo methods including Meridian Planes Monte Carlo and Euler Monte Carlo, and then based on quaternion algebra, they originated a new way to build a quaternion Monte Carlo model for light propagation with polarization into scattering media. Some authors studied 3D Monte Carlo implementations of radiative transfer with polarization. Davis *et al.*[14] developed a 3D polarized radiative transfer model to assess the influence of cirrus clouds on radiances measured by the Earth Observing System Microwave Limb Sounder instrument. Côté *et al.*[15] presented a validated 3D polarization-sensitive Monte Carlo model to study the concentration determination of optically active species in moderately turbid suspensions. They pointed that to avoid systematic errors in concentration determination of optically-active molecular species in turbid samples, the scattered light in the forward direction should be used. Gay *et al.*[16] developed an integral-vector Monte Carlo method to analyze the transfer of polarized radiation in 3D multiple scattering particle-laden media.

From the published papers available, it is seen that the cold media are usually considered, and the work on polarization behavior of thermal emission in hot semitransparent media has not been reported up to now. In this paper we develop a Monte Carlo model of polarized radiative transfer in semitransparent media with consideration of Fresnel or Lambertian boundaries. Section 2 discusses the basic theory of Monte Carlo simulation of vector radiative transfer, including concept of Stokes vector used to describe the polarization state of a light, stochastic process description for emitting, transferring, absorbing, scattering, reflection and transmission of photons, and normalization of Stokes vector. We focus on how to determine scattering directions using the modified reject method and the definition of VRTM. In addition, the normalization of Stokes vector is also important. In Section 3, the validation of Monte Carlo model built in this paper is performed through comparison with published results, and the Rayleigh scattering and Mie scattering are considered. Finally, we focus on the problem of polarized radiative transfer considering thermal emission in a hot semitransparent medium.

2. Theory

2.1. Propagating

The radiative transfer problem considering polarized effects is described by the vector radiative transfer equation (VRTE). The steady state 3D VRTE for polarized monochromatic radiation in media with randomly oriented particles can be written as

$$\begin{aligned}&\boldsymbol{\Omega}\cdot\nabla\boldsymbol{S}(\boldsymbol{r},\boldsymbol{\Omega})+\boldsymbol{\kappa}_{\mathrm{e}}\boldsymbol{S}(\boldsymbol{r},\boldsymbol{\Omega})\\ &=\boldsymbol{\kappa}_{\mathrm{a}}\boldsymbol{I}_{\mathrm{b}}(\boldsymbol{r})+\int_{4\pi}\boldsymbol{Z}(\boldsymbol{r},\boldsymbol{\Omega}'\to\boldsymbol{\Omega})\boldsymbol{S}(\boldsymbol{r},\boldsymbol{\Omega}')\,\mathrm{d}\boldsymbol{\Omega}',\end{aligned}\tag{1}$$

where $\boldsymbol{S}$ is the Stokes vector; $\boldsymbol{\Omega}$ is the discrete direction vector; $\boldsymbol{r}$ is the spatial coordinate vector; $\boldsymbol{\kappa}_{\mathrm{e}}$ is the extinction matrix; $\boldsymbol{\kappa}_{\mathrm{a}}$ is the absorption vector; $\boldsymbol{I}_{\boldsymbol{b}}$ is the black body intensity; and $\boldsymbol{Z}$ is the scattering phase matrix. In the following sections, we will introduce how to use Monte Carlo technique to solve VRTE in detail.

In this paper we use Stokes vector to characterize the intensity and state of polarization of the radiation. According to Chandrasekhar,[17] the Stokes vector is defined as

$$\boldsymbol{S}=(I,Q,U,V)^{\mathrm{T}}.\tag{2}$$

From Eq. (2), there are four parameters (I, Q, U, V)$^{\mathrm{T}}$ constructing the Stokes vector $\boldsymbol{S}$, where I is the intensity, Q is the linear polarization aligned parallel or perpendicular to the z axis, U is the linear polarization aligned $\pm45^{\circ}$ to the z axis, and V is the circular polarization.

The photons are tracked in the semitransparent medium which experience free propagation, scattering, reflection and transmission and other events until the photons escape from the boundaries or are absorbed. The deterministic behavior of light is determined by the random behavior of a finite number of photons in the medium, and each photon randomly propagates along a possible path. The distance a photon travelling without being absorbed, scattered, reflected or refracted is determined by

$$L=-\frac{\ln(1-R_{\mathrm{L}})}{\kappa},\tag{3a}$$

where L is the free distance, κ is the extinction coefficient, and R_{L} is a pseudo random number with an uniform distribution. After travelling a free distance, the new coordinate position (x', y', z') of the photon is obtained by

$$\begin{cases}x'=x+L\sin\theta\cos\varphi,\\ y'=y+L\sin\theta\sin\varphi,\\ z'=z+L\cos\theta.\end{cases}\tag{3b}$$

2.2. Scattering

The following Monte Carlo analysis of multiple scattering events of light in semitransparent media is based on the assumption that the scattering event of light is independent and has no coherence effects. Also it is based on statistics. The deterministic behavior of incoming light is determined by the random behavior of a finite number of photons in the media, and each photon randomly simulates the possible path the light propagates. By analyzing the distribution of all photon bundles from any observation direction, we can obtain the physical and realistic radiation fields for the outcome of the simulation experiment.

In the propagation of the photons, the photons will probably interact with the particles in the semitransparent medium. The photons may be absorbed by the medium or scattered to other directions by the particles. We compare a random number to the scattering albedo ω of the medium to judge whether the photon is absorbed or scattered. If the random number $R_\omega > \omega$, the photon will be absorbed by the medium, otherwise the photon will be scattered by the particle.

The scattering problem for polarized radiative transfer is much more complicated to deal with than that for scalar one. Figure 1 shows the geometry of the problem. The photon directions, before and after scattering, are represented as points A and B respectively on the unit sphere. The unit vector of incident Stokes vector $\boldsymbol{S}_\mathrm{i}$ and the unit vector of scattering Stokes vector $\boldsymbol{S}_\mathrm{s}$ determine a plane AOB, defined as the scattering plane. The vector of incident Stokes vector $\boldsymbol{S}_\mathrm{i}$ and the z axis determine a plane, and the AOC plane is the meridian plane. When a scattering event occurs a new meridian plane is created by the new direction of propagation $\boldsymbol{S}_\mathrm{s}$ (scattering Stokes vector) and the z axis, the Stokes vector must be transformed so that the polarization is properly represented with respect to the new meridian plane BOC. Two rotations of the Stokes vector, in and out of the scattering plane, are required by the Monte Carlo method. Based on the above analysis, the resulting Stokes vector after scattering is

$$\boldsymbol{S}_\mathrm{s} = \boldsymbol{L}(\pi - i_2)\boldsymbol{M}(\Theta)\boldsymbol{L}(-i_1)\boldsymbol{S}_\mathrm{i}, \tag{4}$$

where the matrix $\boldsymbol{M}$ is the single scattering Mueller matrix defined in van de Hulst,[18] and $\boldsymbol{L}$ is a rotational matrix. The phase matrix $\boldsymbol{Z}$ in Eq. (1) is equal to $\boldsymbol{L}(\pi - i_2)\boldsymbol{M}(\Theta)\boldsymbol{L}(-i_1)$.

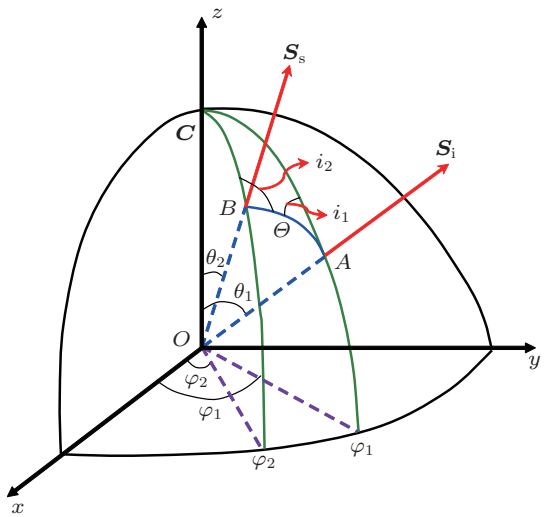


Fig. 1. (color online) Geometry of scattering plane and Meridian planes. The photon's direction of propagation before and after scattering is (θ_1, φ_1) and (θ_2, φ_2), respectively.

The matrix $\boldsymbol{M}$ is a matrix with 4×4 elements used for describing the transformation of the Stokes vector of the incident wave into that of the scattered wave. For spherical particle scattering, the matrix $\boldsymbol{M}$ can be simplified as one that has eight non-zero elements,[19] of which only four are independent. The form is as follows:

$$M(\Theta) = \begin{pmatrix} M_1 & M_2 & 0 & 0 \\ M_2 & M_1 & 0 & 0 \\ 0 & 0 & M_3 & M_4 \\ 0 & 0 & -M_4 & M_3 \end{pmatrix}, \tag{5}$$

where Θ is the scattering angle between the incident direction and the scattering direction. The simplified Muller matrix applies to both Mie scattering and Rayleigh scattering.

The matrix $\boldsymbol{L}$ represents a rotation in the clockwise direction with respect to an observer looking into the direction of the photon propagation.[20] The form is as follows:

$$\boldsymbol{L}(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi & 0 \\ 0 & -\sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

Then we can see the physical meaning of Eq. (3). We rotate the incident Stokes vector $\boldsymbol{S}_\mathrm{i}$ counterclockwise i_1 angle from the meridian plane AOC to the scattering plane AOB. After that the incident Stokes vector $\boldsymbol{S}_\mathrm{i}$ interacts with the particle and scatters into a new direction. Then, we rotate the scattering Stokes vector clockwise π–i_2 angle from the scattering plane AOB to the meridian plane BOC. Finally, the scattering Stokes vector $\boldsymbol{S}_\mathrm{s}$ is obtained.

For a given incident direction (θ_1, φ_1) as shown in Fig. 1, if we know i_1 and Θ, then the angles i_2, θ_2 and φ_1–φ_2 can be calculated from the spherical laws of sine and cosine.[21] In order to obtain these angles, some papers use the cumulative distribution method.[22] Whitney[23] present a rejection method to obtain i_1 and Θ by using a brute-force numerical method over all angles. In this paper we developed an optimized rejection method to obtain i_1 and Θ, which obviously saves the computing time. The specific steps are given as follows.

Firstly we should do some preparatory works. Before we rotate the scattering Stokes vector from the scattering plane to the meridian plane BOC, equation (4) can be rewritten as

$$\boldsymbol{S}'_\mathrm{s} = \boldsymbol{M}(\Theta)\boldsymbol{L}(-i_1)\boldsymbol{S}_\mathrm{i}. \tag{7a}$$

Then the I Stokes parameter can be computed according to Eq. (7a) as

$$I'_\mathrm{s} = I_\mathrm{i}M_1(\Theta) + Q_\mathrm{i}M_2(\Theta)\cos 2i_1 - U_\mathrm{i}M_2(\Theta)\sin 2i_1. \tag{7b}$$

By using trigonometric function formula, equation (7b) can be rewritten as

$$I'_\mathrm{s} = I_\mathrm{i}M_1(\Theta) + \sqrt{Q_\mathrm{i}^2 + U_\mathrm{i}^2}M_2(\Theta)\cos(2i_1 + \phi), \tag{7c}$$

where ϕ is some angle we are not interested in. Regarding i_1 as independent variable and considering the maximum absolute value of cosine function, we can obtain the maximum of

$I'_{s,\Theta}$:

$$I'_{s,\Theta,\max} = I_i M_1(\Theta) + \sqrt{Q_i^2 + U_i^2}\,|M_2(\Theta)|\,. \tag{7d}$$

Then we regard Θ as independent variable and compute the maximum of I'_s with numerical method over Θ. Ultimately, the resulting expression of the maximum of I'_s is written as

$$I'_{s,\max} = \max\left\{I_i M_1(\Theta) + \sqrt{Q_i^2 + U_i^2}\,|M_2(\Theta)|\right\}, \quad \Theta \in [0,\pi]. \tag{7e}$$

Now we can use the rejection method to obtain these angles.

(i) Random sample i_1 and Θ from a uniform distribution

$$\Theta \in [0,\pi], \quad i_1 \in [0,2\pi). \tag{8}$$

As shown in Fig. 1, i_1 is the angle between the meridian plane AOC and the scattering plane AOB, and Θ is the scattering angle. As $i_1 \in [0,\pi)$, $\varphi_2 < \varphi_1$; as $i_1 \in (\pi,2\pi)$, $\varphi_2 > \varphi_1$; as $i_1 = \pi$, $\varphi_2 = \varphi$.

(ii) Insert i_1 and Θ into Eq. (7b) and obtain $I'_s(i_1,\Theta)$. Then compare the value of $I'_s(i_1,\Theta)$ with that of $\xi I'_{s,\max}$ where ξ is a uniform random number between 0 and 1. If $I'_s(i_1,\Theta) \geq \xi I'_{s,\max}$, we accept i_1 and Θ as the new scattering angles. Otherwise, if $I'_s(i_1,\Theta) < \xi I'_{s,\max}$, we reject (i_1,Θ) as the new scattering angles and go back to step (i) to resample them again.

(iii) Once getting the new scattering angles i_1 and Θ, we are able to determine the new propagation direction and Stokes vectors after scattering in the observer's frame (the x–y–z frame in Fig. 1). The angles i_2, θ_2, θ_2, φ_1–φ_2, and φ_2 can be derived from the spherical laws of sine and cosine.[24]

From the definition of Stokes vector, the first component stores the energy carried by photon, and the other three components store the states of polarization in the Stokes vector. Therefore, in the Monte Carlo simulation, in order to ensure the conservation of energy, the normalization must be applied to the Stokes vector in the propagation of the photon. That is to say, the I-Stokes parameter of the photon is equal to 1 as it propagates through. The expression of the normalized Stokes vector is written as

$$\boldsymbol{S}' = \frac{\boldsymbol{S}}{I} = \left(1, \frac{Q}{I}, \frac{U}{I}, \frac{V}{I}\right)^{\mathrm{T}}, \tag{9}$$

where $\boldsymbol{S}'$ is just used to describe the polarized state of the photon and has nothing to do with the intensity of photons. The normalization procedure should be carried out for every state-change event such as emitting, scattering, reflection, and transmission.

2.3. Reflection and transmission

When the photon hits the boundary of semitransparent medium, the reflection or transmission will occur on the boundary. According to the reflection characteristics, the medium surfaces include Fresnel surface and Lambertian surface. The reflection Stokes vector and refraction Stokes vector are written as

$$\boldsymbol{S}_{\mathrm{ref}} = \boldsymbol{R}\boldsymbol{S}_{\mathrm{i}}, \tag{10a}$$

$$\boldsymbol{S}_{\mathrm{tra}} = \boldsymbol{T}\boldsymbol{S}_{\mathrm{i}}, \tag{10b}$$

where $\boldsymbol{R}$ and $\boldsymbol{T}$ are reflection matrix and transmission matrix, respectively. Note that $\boldsymbol{S}_{\mathrm{ref}}$ and $\boldsymbol{S}_{\mathrm{tra}}$ also need to be normalized.

For the Lambertian surface, the reflection matrix is given as follows:

$$\boldsymbol{R} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{11a}$$

where ρ is the reflection coefficient of the boundary which is independent of the incident beam and just related to the property of the surface. The reflected direction is determined by the following equations:

$$\theta_{\mathrm{r}} = \arccos\left(\sqrt{1-R_\theta}\right), \varphi_{\mathrm{r}} = 2\pi R_\varphi, \tag{11b}$$

where R_θ and R_φ are uniform random numbers. Both the incident angle and reflection angle are measured with respect to the normal direction of the reflection plane. We compare a uniform random number R_{ref} with ρ to judge whether the reflection takes place or not on the surface. If $R_{\mathrm{ref}} < \rho$ the photon is reflected, otherwise absorbed by the surface.

The Fresnel reflection and transmission matrices for a flat dielectric interface have been presented in a few papers.[25,26] The reflection matrix is written as

$$\boldsymbol{R}(\theta_i) = \frac{1}{2}\begin{pmatrix} r_\parallel^2 + r_\perp^2 & r_\parallel^2 - r_\perp^2 & 0 & 0 \\ r_\parallel^2 - r_\perp^2 & r_\parallel^2 + r_\perp^2 & 0 & 0 \\ 0 & 0 & 2\mathrm{Re}\{r_\perp r_\parallel^*\} & 2\mathrm{Im}\{r_\parallel r_\perp^*\} \\ 0 & 0 & 2\mathrm{Im}\{r_\perp r_\parallel^*\} & 2\mathrm{Re}\{r_\perp r_\parallel^*\} \end{pmatrix}, \tag{12a}$$

in which

$$r_\perp = \frac{n_i\cos(\theta_i) - \sqrt{n_t^2 - n_i^2\sin^2(\theta_i)}}{n_i\cos(\theta_i) + \sqrt{n_t^2 - n_i^2\sin^2(\theta_i)}},$$

$$r_\parallel = \frac{n_t^2\cos(\theta_i) - n_i\sqrt{n_t^2 - n_i^2\sin^2(\theta_i)}}{n_t^2\cos(\theta_i) + n_i\sqrt{n_t^2 - n_i^2\sin^2(\theta_i)}}, \tag{12b}$$

where $\perp$ and $\parallel$ denote the components perpendicular and parallel to the plane of incidence, respectively; Re{ } and Im{ } denote the real and imaginary parts, respectively; n_i and n_t are the refractive indexes of the media for the incident side and

the transmitting side, respectively. The transmission matrix is expressed as

$$\boldsymbol{T}(\theta_{\rm i}) = \frac{1}{2}\frac{n_t\cos(\theta_t)}{n_{\rm i}\cos(\theta_{\rm i})} \times \begin{pmatrix} t_\parallel^2+t_\perp^2 & t_\parallel^2-t_\perp^2 & 0 & 0 \\ t_\parallel^2-t_\perp^2 & t_\parallel^2+t_\perp^2 & 0 & 0 \\ 0 & 0 & 2{\rm Re}\{t_\perp t_\parallel^*\} & 2{\rm Im}\{t_{\parallel/}t_\perp^*\} \\ 0 & 0 & 2{\rm Im}\{t_\perp t_\parallel^*\} & 2{\rm Re}\{t_\perp t_\parallel^*\} \end{pmatrix}, \tag{13a}$$

where

$$t_\perp = \frac{2n_{\rm i}\cos(\theta_{\rm i})}{n_{\rm i}\cos(\theta_{\rm i})+\sqrt{n_t^2-n_{\rm i}^2\sin^2(\theta_{\rm i})}},$$
$$t_\parallel = \frac{2n_{\rm i}n_t\cos(\theta_{\rm i})}{n_t^2\cos(\theta_{\rm i})+n_{\rm i}\sqrt{n_t^2-n_{\rm i}^2\sin^2(\theta_{\rm i})}}. \tag{13b}$$

The reflection and transmission events abide by the laws of retraction and reflection. The angles after reflection and transmission are decided by the following equations:

$$\begin{aligned} &\theta_{\rm r} = \theta_{\rm i}, \\ &n_{\rm t}\sin\theta_{\rm t} = n_{\rm i}\sin\theta_{\rm i}, \\ &\varphi_{\rm r} = \varphi_{\rm t} = \varphi_{\rm i}. \end{aligned} \tag{14}$$

We also need to compare the reflection coefficient ρ with a uniform random number $R_{\rm ref}$ to judge whether the reflection or transmission take place when the photon hits the Fresnel surface. For the Fresnel surface, the reflection coefficient is written as

$$\rho = \frac{r_\parallel^2+r_\perp^2}{2}. \tag{15}$$

2.4. Photon's statistics and post-processing

For one-dimensional problem, in order to obtain the angular distribution of Stokes vector at a cross section of the medium, we need to record numbers of photons passing through the cross section in each direction. In this paper, we present the concept of VRTM used to solve the Stokes vector. For one-dimensional problem, the VRTM denoted by $\boldsymbol{D}_{{\rm i},\theta,\varphi}$ is defined as

$$\boldsymbol{D}_{{\rm i},\theta,\varphi} = \frac{\sum_{q=1}^{N_{\rm t}}\boldsymbol{S}_{\rm q}}{N}, \tag{16}$$

where N indicates the number of photons emitted from the light source; $N_{\rm t}$ denotes the total number of the photons passing through the cross-section i along the $(\theta,\ \varphi)$ direction in the solid angle $\Delta\Omega$; and $\boldsymbol{S}_q$ is the normalized Stokes vector carried by photon q. The physical meaning of VRTM is clear and it represents the transfer of intensity and polarized states of photons from the emission position to receiving position. Finally, we obtain the Stokes vector in terms of VRTM

$$\boldsymbol{S}_{{\rm i},\theta,\varphi} = \frac{I_0\,|\cos\theta_0|}{|\cos\theta|\,\Delta\Omega}\boldsymbol{D}_{{\rm i},\theta,\varphi}, \tag{17}$$

where I_0 is the intensity of incident wave, and $\cos\theta_0$ is the incident-direction cosine.

3. Results and discussion

3.1. Validation of the Monte Carlo model

Case 1 Polarization by Rayleigh scattering The Monte Carlo model built in the present work is validated against the results by Monte Carlo method developed in Ref. [12] for a pure Rayleigh scattering atmosphere layer. As shown in Fig. 2, a homogeneous plane-parallel slab subjected to collimated irradiation is considered. The number of the photons for the simulation is taken as 1.0×10^8. The angular discretization is uniform and taken as $N_\theta\times N_\varphi = 40\times24$. Figures 3(a) and 3(b) plot values of I and Q Stokes parameters for the radiation exiting the area element at $z=0$ and $z_{\max}$ in the case of an atmosphere with $\tau_{\max}=\kappa z_{\max}=0.5$ and a collimated radiation Stokes vector $I_0=(\pi,0,0,0)^{\rm T}$ incident at $z=0$ with $\mu_0=\cos\theta_0=0.4$ and $\varphi_0=0^\circ$. The scattering Mueller matrix for Rayleigh scattering is given by

$$\boldsymbol{M}(\Theta) = \frac{3}{4}\begin{pmatrix} \cos^2\Theta+1 & \cos^2\Theta-1 & 0 & 0 \\ \cos^2\Theta-1 & \cos^2\Theta+1 & 0 & 0 \\ 0 & 0 & 2\cos\Theta & 0 \\ 0 & 0 & 0 & 2\cos\Theta \end{pmatrix}. \tag{18}$$

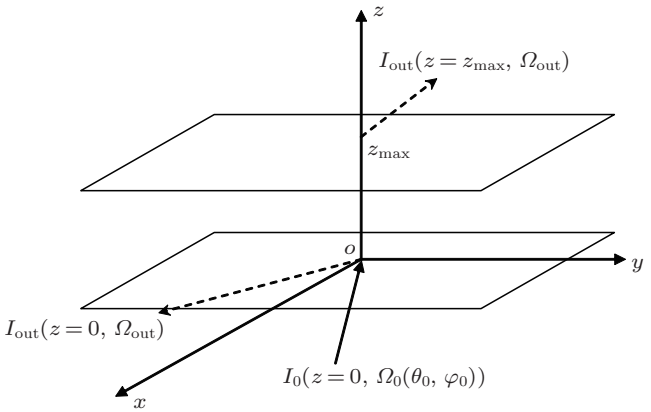


Fig. 2. Schematic diagram of the plane–parallel slab configuration.

The simulation results are in perfect agreement with the reference values, indicating the good accuracy of the Monte Carlo algorithm developed in this paper. Reference [12] did not give the U and V Stokes parameters, and in this paper we present the U Stokes parameter of the simulation example in Figs. 3(c) and 3(d). The V Stokes parameter has been verified to be equal to zero constant.

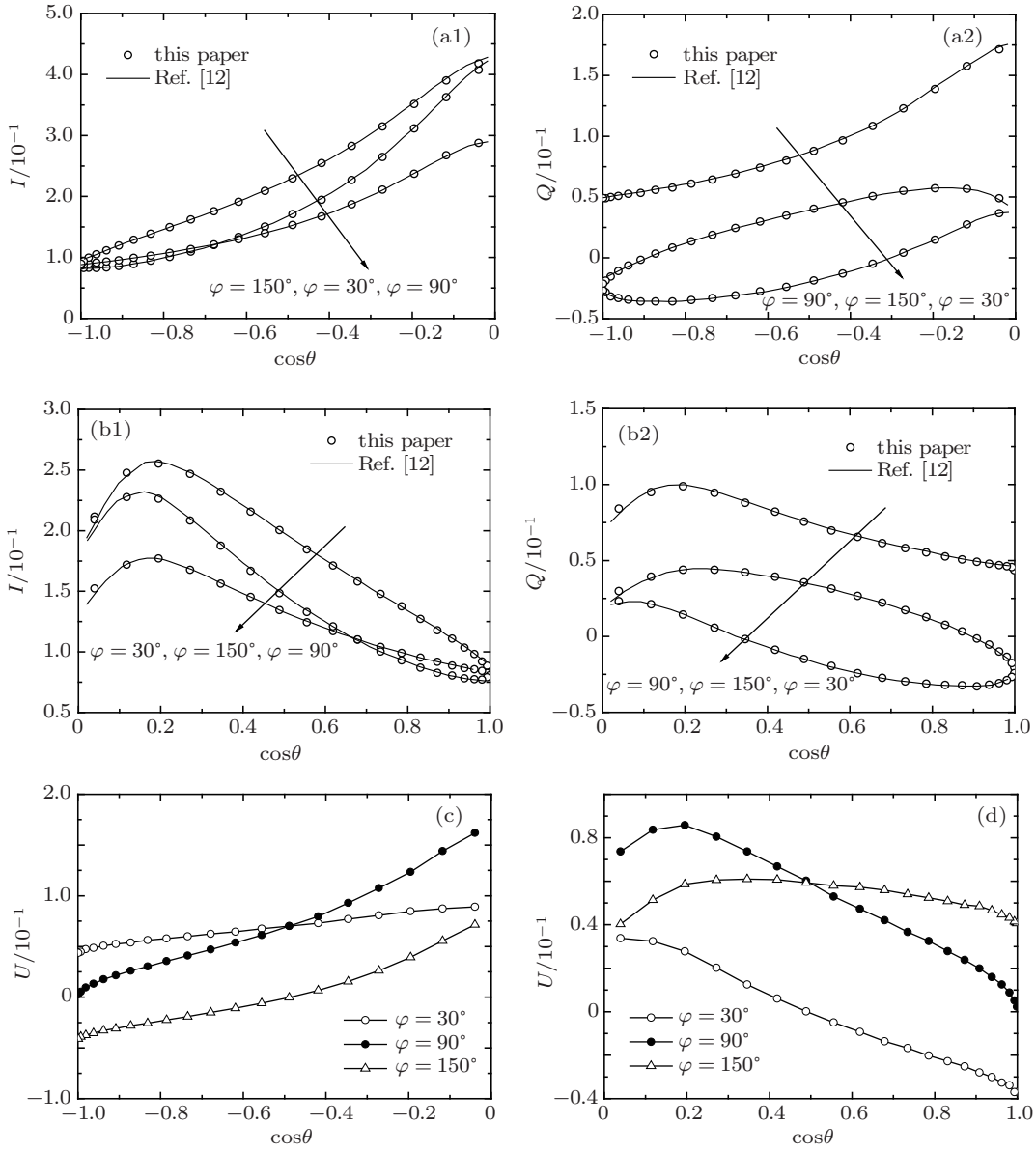


Fig. 3. Comparisons of I and Q Stokes parameters against the data from Ref. [12]: (a) $z = 0$ and (b) $z = z_{\max}$; U Stokes parameter of the simulated example: (c) $z = 0$ and (d) $z = z_{\max}$.

Case 2 Polarization by Mie scattering In this case, the '$L = 13$' problem was investigated by Garcia and Siewert.[4] As seen in Fig. 4, an oblique solar beam (oblique angle is $\mu_0 \cos\theta_0 = 0.2$, $\varphi_0 = 0$) penetrates into a scattering atmosphere layer from above. The irradiation flux intensity of the beam is π. The optical thickness τ based on the thickness of the atmosphere H is 1, and the single scattering albedo is 0.99. The phase matrix is for Mie scattering at a wavelength of 0.951 μm from a gamma distribution of particles with effective radius of 0.2 μm, effective variance of 0.07, and refractive index $n = 1.44$. The Legendre coefficients for the four unique elements of the phase matrix have been obtained by Evens and Stephens.[5] The lower boundary of the atmosphere layer is of diffuse reflection with $\rho = 0.1$. The emission of the atmosphere and the boundary are omitted. The Monte Carlo method is applied to obtaining the angular distribution of Stokes parameters along $\mu = \cos\theta$ at $\varphi - \varphi_0 = \pi/2$. The number of the photons for the simulation is taken as 3.0×10^9. The angular discretization is given by $N_\theta \times N_\varphi = 20 \times 40$.

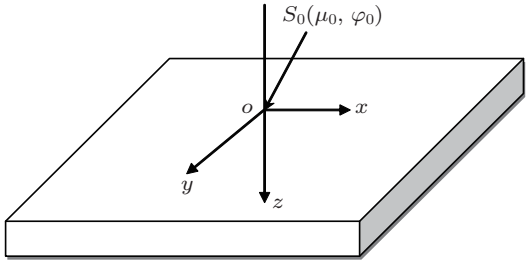


Fig. 4. Schematic of the oblique incident solar beam irradiating into a layer of atmosphere.

At $z/H = 0.5$, all components of Stokes vector are non-zero for both upward and downward directions resulting from multiple scattering. As displayed in Fig. 5, the Stokes parameters at different positions obtained by the Monte Carlo method in this paper agree very well with the results of Garcia and Siewert[4] by using the F_N method. The selected angular discretization is demonstrated to be enough to obtain convergent results with good accuracy.

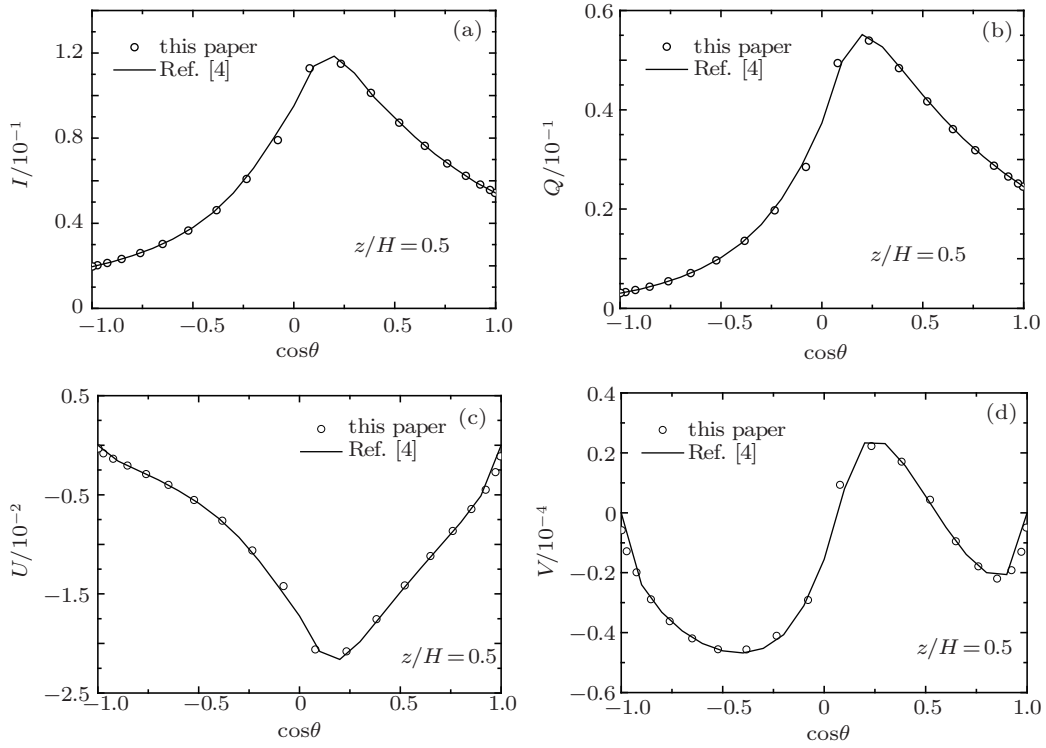


Fig. 5. Comparisons of Stokes parameters against the data by F_N method.[4] (a) I vs. $\cos\theta$; (b) Q vs. $\cos\theta$; (c) U vs. $\cos\theta$; (d) V vs. $\cos\theta$.

3.2. Polarization characteristics of thermal emission in hot media

The work of above sections only considered cold medium, and the term of $\kappa_a I_b$ was ignored in Eq. (1). In this section, we investigate the polarization behavior of thermal emission in a plane-parallel hot semitransparent medium. According to the physical meaning, using the VRTM, the upwelling Stokes vector from the top surface of the one-dimensional medium as shown in Fig. 6, can be obtained by

$$\boldsymbol{S}(\theta,\varphi) = \left[\sum_{j=1}^{N} 4\pi\kappa_a \mathrm{d}V_j n_j^2 I_b(T_j)\boldsymbol{D}_{js_2}(\theta,\varphi) + \varepsilon\pi n^2 I_b(T_{s_1})\boldsymbol{D}_{s_1 s_2}(\theta,\varphi)\right](\cos\theta\Delta\Omega)^{-1}, \quad (19)$$

where $I_b(T) = \sigma T^4/\pi$, $\sigma = 5.6703\times10^{-8}$, N is the number of control volumes, $\boldsymbol{D}$ is the VRTM, and ε is the emissivity of the bottom surface of the medium.

Through the calculation, we find that the Stokes parameters of U and V are equal to zero, so figure 7 just plots the curves for the results of the upwelling I and Q parameters of Stokes vector from the medium top surface. The top surface of the medium is Fresnel semitransparent. Figures 7(a) and 7(b) show the results for the medium with a Lambertian opaque bottom surface with an emissivity of ε, while figures 7(c) and 7(d) show those for the medium with a Fresnel semitransparent bottom surface. The Mie scattering is considered and the

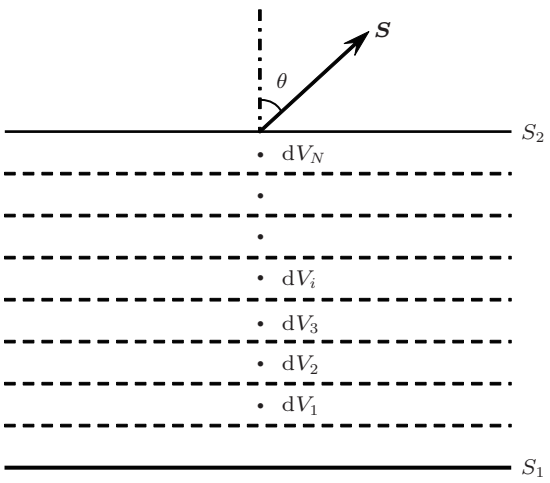


Fig. 6. Sketch for polarization transfer of thermal emission in a hot plane-parallel participating medium.

scattering phase matrix of particles is the same as that taken in Case 2 of Section 3.1. The refractive index of background environment surrounding the medium is equal to 1.0, and the scattering albedo of the background environment is zero. The number of the photons we used is 1.0×10^9, the angular discretization is taken as $N_\theta \times N_\varphi = 60 \times 20$ and the medium is equally divided into $N = 20$ control volumes for the simulation.

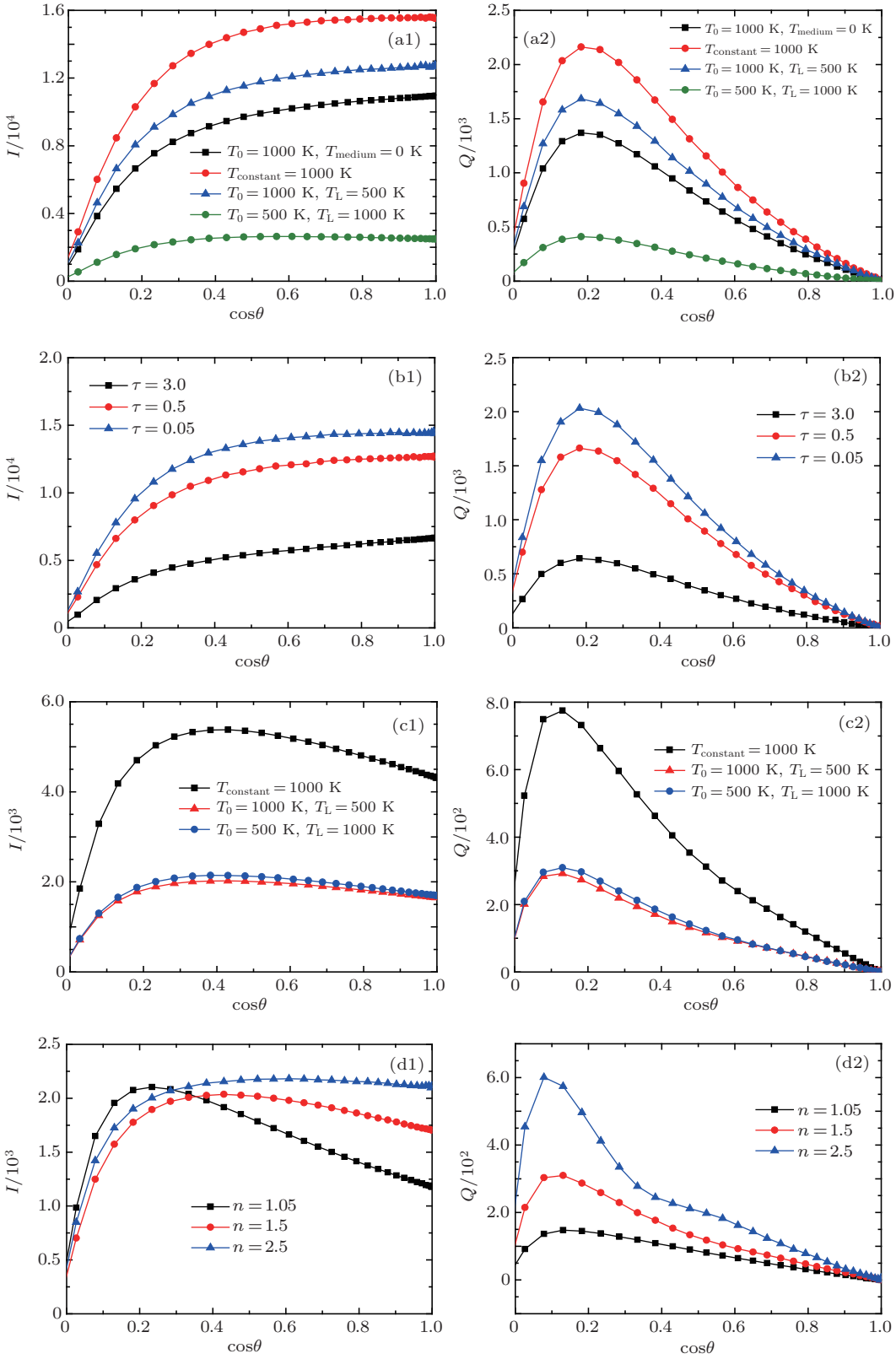


Fig. 7. (color online) Upwelling Stokes vector from the top surface of the hot medium with homogeneous refractive index, (a) $\varepsilon = 0.7$, $\omega = 0.7$, $\tau = 0.5$, $n = 1.44$; (b) $\varepsilon = 0.7$, $\omega = 0.7$, $n = 1.44$, linear distribution of temperature for $T_0 = 1000$ K and $T_L = 500$ K; (c) $\omega = 0.7$, $\tau = 0.5$, $n = 1.44$; and (d) $\omega = 0.7$, $\tau = 0.5$, linear distribution of temperature for $T_0 = 1000$ K and $T_L = 500$ K.

Figure 7(a) shows the curves of Stokes vector changing with different temperature distributions. For the medium with a Lambertian opaque bottom surface, with the increase in $\cos\theta$, parameter I monotonically increases, and parameter Q first increases rapidly to some maximum value at about $\cos\theta = 0.2$ and then decreases slowly to some small value. The effects of temperature distributions on the results are great. For the distribution of temperature decreasing linearly from the bottom surface to the top surface, the curves of I and Q are far higher than those for the distribution of temperature increasing linearly from the bottom surface to the top surface, respectively. Figure 7(b) shows the curves of Stokes vector changing with different optical thickness τ for a distribution of the temperature decreasing linearly from the bottom to the top. The medium with a Lambertian opaque bottom surface is also considered in the case. It is seen that the value of the Stokes vector increases with decreasing optical thickness.

Figure 7(c) shows the results with the same parameters as Fig. 7(a) except for the optical boundary condition. The semitransparent Fresnel bottom surface is considered in Figs. 7(c) and 7(d). The results plotted in Fig. 7(c) is obviously different from Fig. 7(a). The parameter I of Stokes vector first climbs up to some peak value and then declines with increasing $\cos\theta$. Two curves of Stokes vector for two linear distributions of temperature are slightly different, and the curves for the distribution of temperature increasing from the bottom to the top are slightly higher than those for the distribution of temperature decreasing from the bottom to the top, which is just opposite to the case shown in Fig. 7(a). Figure 7(d) presents the results of the Stokes vector for different refractive indexes. We can find that there is a peak in the curve of I parameter for the small refractive index. The smaller the refractive index is, the steeper the I curve looks, and the more obvious the peak becomes. Thus, the intersection of the I curves can be observed for various refractive indexes. While for Q parameter, the peak appears in the curve regardless of the refractive index, and intersection phenomenon does not exist in the curves. From Fig. 7(d), it can be seen that the smaller the refractive index is, the more gentle the slope of curves for Q is, and the curve for a big refractive index is always above that for small refractive index, which is completely different from the tendency of I curve changing with the refractive index.

4. Conclusion

In this work, we developed a Monte Carlo model for vector radiative transfer including Fresnel boundaries. One of the difficulties in solving polarized radiative transfer by using Monte Carlo is the determination of scattering direction. We put forward the concept of VRTM used to describe the polarized state transfer. Using the VRTM solved by Monte Carlo method, we obtained four components of Stokes vector.

We compared the simulated results against those available in published papers. The cases of Rayleigh scattering and Mie scattering were tested. The results of Stokes vector obtained by the present solution scheme are in good agreement with those reported by other works. Finally, we investigated the polarization behavior of thermal emission in a hot semitransparent medium and analyzed the effects of temperature distributions, optical boundary condition of the bottom surface, optical thickness and refractive index on the upwelling Stokes vector from the top surface. Results show that the U and V parameters are equal to zero, compared with the I parameter, the significant peaks of the Q curves are always observed, and refractive index has a completely different effect on I and Q. Polarization characteristics of thermal emission of hot media are expected to be applied in the field of infrared detection, target recognition, and remote sensing.

References

[1] Yuan Y, Sun C M, Huang F Z, Zhao H J and Wang Q 2011 *Acta Phys. Sin.* **60** 089501 (in Chinese)
[2] Sun C M, Yuan Y and Zhang X B 2010 *Acta Phys. Sin.* **59** 7523 (in Chinese)
[3] Kattawar G W and Plass G N 1968 *Appl. Opt.* **7** 1519
[4] Garcia R D M and Siewert C E 1989 *J. Quant. Spectrosc. Radiar. Transfer* **41** 117
[5] Evans K F and Stephens G L 1991 *J. Quant. Spectrosc. Radiar. Transfer* **46** 413
[6] Weng F 1992 *J. Quant. Spectrosc. Radiar. Transfer* **47** 19
[7] Haferman J L, Smith T F and Krajewski W F 1997 *J. Quant. Spectrosc. Radiar. Transfer* **58** 379
[8] Siewert C E 2000 *J. Quant. Spectrosc. Radiar. Transfer* **64** 227
[9] Sun B, Wang H, Sun X B, S, Hong J and Zhang Y J 2012 *Chin. Phys. B* **21** 129501
[10] Kattawar G W and Adams C N 1989 *Limnol. Oceanogr* **34** 1453
[11] Wang X D, Wang L V, Sun C W and Yang C C 2003 *J. Biomed. Opt.* **8** 608
[12] Vaillona R, Wongb B T and Mengüç M P 2004 *J. Quant. Spectrosc. Radiar. Transfer* **84** 383
[13] Ramella-Roman J C, Prahl S A and Jacques S L 2005 *Opt. Express* **13** 4420
[14] Davis C, Emde C and Harwood R 2005 *IEEE Trans. Geosci. Rem. Sens.* **43** 1096
[15] Côté D and Vitkin I A 2005 *Opt. Express* **13** 148
[16] Gay B, Vaillon R and Mengüç M P 2010 *J. Quant. Spectrosc. Radiar. Transfer* **111** 287
[17] Chandrasekhar S 1950 *Radiative Transfer* (Oxford: Oxford University Press)
[18] Van de Hulst H C 1981 *Light Scattering by Small Particles* (New York: Dover)
[19] Mishchenko M I, Travis L D and Lacis A A 2002 *Scattering, Absorption, and Emisson of Light by Small Particles* (New York: Cambridge University)
[20] Tynes H H, Kattawar G W, Zege E P, Katsev I L, Prikhach A S and Chaikovskaya L I 2001 *Appl. Opt.* **40** 400
[21] Green R 1985 *Spherical Astronomy* (Cambridge: Cambridge University Press)
[22] Cornet C, Labonnote L C and Szczap F 2010 *J. Quant. Spectrosc. Radiar. Transfer* **111** 174
[23] Whitney B A 2011 arXiv:1104.4990
[24] Masuda K and Takashima 1986 *Pap. Met. Geophys.* **37** 1
[25] Kattawar G W and Adams C N 1989 *Limnol. Oceanogr.* **34** 1453
[26] Zhai P W, Hu Y X, Cdhary J, Trepte C R, Lucker P L and Josset D B 2010 *J. Quant. Spectrosc. Radiar. Transfer* **111** 1025