A novel model and behavior analysis for a swarm of multi-agent systems with finite velocity*

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Inspired by the fact that in most existing swarm models of multi-agent systems the velocity of an agent can be infinite, which is not in accordance with the real applications, we propose a novel swarm model of multi-agent systems where the velocity of an agent is finite. The Lyapunov function method and LaSalle's invariance principle are employed to show that by using the proposed model all of the agents eventually enter into a bounded region around the swarm center and finally tend to a stationary state. Numerical simulations are provided to demonstrate the effectiveness of the theoretical results.

Keywords: multi-agent systems, swarm, finite velocity, attraction and repulsion

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1. Introduction

During the past decade, the distributed coordinated control of multi-agent systems has attracted a considerable amount of attention in various fields, such as physics, biology, management science, computer science, and control theory.[1–10] This has mainly been due to its ability to reveal the emergence properties of many natural complex systems, and has wide applications in distributed and parallel computing, wireless sensor networks, unmanned aerial vehicles, and so on. As one of the important issues in the distributed coordinated control of multi-agent systems, the swarm has been studied by many researchers.[11–25] One reason for this is that many kinds of natural life-forms tend to perform a swarm behavior, such as flocks of birds, schools of fish, herds of animals, and colonies of bacteria. Another reason is that a good understanding of the mechanisms and principles of swarm behavior can help us to design some practical engineering systems with a perfect performance, such as swarm home robots and multiple unmanned aerial vehicles.

In order to design high-performance automated systems in engineering by using swarm theory, we need to understand the mechanisms and principles of swarm behavior by establishing the swarm models of multi-agent systems and then carry out stability analyses to prove the effectiveness of the proposed models. Until now, a great deal of work has been done on the models and the stability analysis of a swarm of multi-agent systems. Vicsek *et al.* established a swarm model by using three rules, including separation, alignment, and cohesion, where the velocity direction of an agent is the average of the motion directions of its nearby agents. They have validated the proposed model by numerical simulations.[26] On the basis of Ref. [26], Gazi and Passion proposed a simple swarm model that consists of a number of agents with the attraction/repulsion rule and an identical interaction strength among the agents. They carried out their stability analysis of a swarm by using the Lyapunov function method.[27] It was shown that their model can capture the basic features of aggregation, cohesion, and stability of a swarm. In Ref. [28], Gazi and Passion further analyzed the stability of a swarm with an environment profile, where it was assumed that each agent can sense the goal position. Obviously, this assumption is too idealistic to be in accordance with the practical situations. In order to avoid collision among the agents, Liu *et al.* proposed an attraction/repulsion model where the repulsion is infinite as the distance between two agents goes to zero.[29] In Ref. [30], Liu *et al.* further studied the aggregation of a more general swarm by considering the different weights of each agent.

A common feature of the swarm models proposed in Refs. [27]– [30] is that the velocity of each agent might be infinite; however, this is impossible in real applications. For example, in multi-robot systems, the velocity of each robot is always bounded due to the constraints of the robot hardware. Therefore, it is hard to design automation systems by using the swarm models proposed in Refs. [27]–[30]. This compels us to establish a novel swarm model of multi-agent systems with finite velocity that is in accordance with real applications. Based on the above considerations, the aim of this paper is to propose a novel swarm model of multi-agent systems with finite velocity.

The remainder of this paper is organized as follows. In

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Section 2, the problem is formulated and a novel swarm model is proposed. The behavior analysis of the proposed swarm model is given in Section 3. Numerical simulations are provided in Section 4. Finally, some concluding remarks are stated in Section 5.

2. Problem statement and swarm modeling

Consider a swarm system composed of N agents in an n-dimensional Euclidean space, where both N and n are positive integers. In most of the existing swarm models, the dynamics of the agent i are given by

$$\dot{\boldsymbol{x}}^i = \sum_{j=1,j\neq i}^{N} g(\boldsymbol{x}^i - \boldsymbol{x}^j), \quad i \in S \triangleq \{1,2,\cdots,N\}, \tag{1}$$

where $\boldsymbol{x}^i \in \mathbf{R}^n$ is the position of agent i, and the function $g(\boldsymbol{x}^i - \boldsymbol{x}^j)$ represents the attraction/repulsion force between agent i and agent j. Obviously, the velocity $\dot{\boldsymbol{x}}^i$ of agent i is determined by the sum of the attraction/repulsion forces $g(\boldsymbol{x}^i - \boldsymbol{x}^j)$ from all of the other agents. This relationship between $\dot{\boldsymbol{x}}^i$ and $g(\boldsymbol{x}^i - \boldsymbol{x}^j)$ might lead to the unboundedness of the velocity $\dot{\boldsymbol{x}}^i$ of agent i. The reason for this is that the attraction force is inclined to infinity as the distance between agent i and agent j increases.[27,29] Another reason is that the repulsion force is inclined to infinity as the distance between agent i and agent j goes to zero.[29] However, both the velocity of a life-form in the nature and the velocity of a swarm unit in the engineering field are finite. Thus, the unboundedness of the velocity $\dot{\boldsymbol{x}}^i$ (i.e., the unboundedness of the attraction/repulsion force $g(\boldsymbol{x}^i - \boldsymbol{x}^j)$) is not in accordance with the practical situations. Therefore, we have to remodel the multi-agent system (1) by designing a bounded attraction/repulsion force function $g(\boldsymbol{x}^i - \boldsymbol{x}^j)$.

To this end, in this paper we choose $g(\cdot)$ in model (1) as

$$g(\boldsymbol{y}) = -\boldsymbol{y}\left(\frac{a}{\|\boldsymbol{y}\|} - b\exp\left(-\frac{\|\boldsymbol{y}\|^2}{c}\right)\right), \tag{2}$$

where a, b, and c are positive constants, and $||\boldsymbol{y}|| = \sqrt{\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y}}$, $\boldsymbol{y} \in \mathbf{R}^n$. The first term $-a\boldsymbol{y}/||\boldsymbol{y}||$ is the attraction force and the second term $b\boldsymbol{y}\exp(-||\boldsymbol{y}||^2/c)$ represents the repulsion force.

Notice that $\|b\boldsymbol{y}\exp(-\|\boldsymbol{y}\|^2/c)\| = b\|\boldsymbol{y}\|\exp(-\|\boldsymbol{y}\|^2/c)$, and $b\|\boldsymbol{y}\|\exp(-\|\boldsymbol{y}\|^2/c)$ is a bounded function whose maximum is $b\sqrt{c/2}\exp(-1/2)$ obtained at $\|\boldsymbol{y}\| = \sqrt{c/2}$, thus $\|b\boldsymbol{y}\exp(-\|\boldsymbol{y}\|^2/c)\| \leq b\sqrt{c/2}\exp(-1/2)$. In addition, $||-a\boldsymbol{y}/||\boldsymbol{y}|||| = a$. Therefore, we have

$$\|g(\boldsymbol{y})\| \leq a + b\sqrt{c/2}\exp(-1/2),$$

which further results in

$$\|\dot{\boldsymbol{x}}^i\| \leq (N-1)(a + b\sqrt{c/2}\exp(-1/2)), \quad i \in S. \tag{3}$$

Therefore, the velocity $\dot{\boldsymbol{x}}^i$ of agent i, $i \in S$ is bounded. Generally speaking, the maximum speed $\dot{\boldsymbol{x}}_{\max}$ of a swarm unit is a known quantity, which can help us to approximately choose the parameters a, b, and c on the basis of $\|\dot{\boldsymbol{x}}_{\max}\|/(N-1)$.

In the following, we will investigate the dynamic behavior of the swarm system (1) with the attraction/repulsion force function (2), including aggregation and stability.

3. Swarm behavior analysis

First, notice that

$$\begin{aligned}
\dot{\bar{\boldsymbol{x}}} &= \frac{1}{N}\sum_{i=1}^{N}\dot{\boldsymbol{x}}^i \\
&= -\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\boldsymbol{x}^i - \boldsymbol{x}^j) \\
&\quad \times \left(\frac{a}{\|\boldsymbol{x}^i - \boldsymbol{x}^j\|} - b\exp\left(-\frac{\|\boldsymbol{x}^i - \boldsymbol{x}^j\|^2}{c}\right)\right) \\
&= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\boldsymbol{x}^j - \boldsymbol{x}^i) \\
&\quad \times \left(\frac{a}{\|\boldsymbol{x}^j - \boldsymbol{x}^i\|} - b\exp\left(-\frac{\|\boldsymbol{x}^j - \boldsymbol{x}^i\|^2}{c}\right)\right) \\
&= \frac{1}{N}\sum_{j=1}^{N}\sum_{i=1,i\neq j}^{N}(\boldsymbol{x}^j - \boldsymbol{x}^i) \\
&\quad \times \left(\frac{a}{\|\boldsymbol{x}^j - \boldsymbol{x}^i\|} - b\exp\left(-\frac{\|\boldsymbol{x}^j - \boldsymbol{x}^i\|^2}{c}\right)\right) \\
&= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\boldsymbol{x}^i - \boldsymbol{x}^j) \\
&\quad \times \left(\frac{a}{\|\boldsymbol{x}^i - \boldsymbol{x}^j\|} - b\exp\left(-\frac{\|\boldsymbol{x}^i - \boldsymbol{x}^j\|^2}{c}\right)\right) \\
&= \mathbf{0}_n,
\end{aligned} \tag{4}$$

where $\mathbf{0}_n$ denotes the n-dimensional column vector with all zeros, thus the center $\bar{\boldsymbol{x}} = \sum_{i=1}^{N}\boldsymbol{x}^i/N$ of the swarm system (1) with the attraction/repulsion force function (2) is stationary for all t.

However, we still do not know anything about the relative motion of the agents. Now we consider the relative motion of the agents in the swarm system (1). The main results are given in the following Theorem 1.

Theorem 1 Consider the swarm system (1) with the attraction/repulsion force function (2). All of the agents will eventually enter into the region

$$B_\alpha(\bar{\boldsymbol{x}}) = \left\{\boldsymbol{x} : \sum_{i=1}^{N}\left\|\boldsymbol{x}^i - \bar{\boldsymbol{x}}\right\| \leq \alpha\right\},$$

where $\alpha \triangleq= (N-1)bc\exp(-1)/a$.

Proof Let $V = (1/2)\sum_{i=1}^{N}\boldsymbol{e}^{i\mathrm{T}}\boldsymbol{e}^i$ be the candidate Lyapunov function, where $\boldsymbol{e}^i = \boldsymbol{x}^i - \bar{\boldsymbol{x}}$. It follows from Eq. (4)

that $\dot{\boldsymbol{e}}^i = \dot{\boldsymbol{x}}^i$. Taking the time derivative of V, we have

$$\dot{V} = -\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}} \times\left(\frac{a}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}-b\exp\left(-\frac{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2}{c}\right)\right)\boldsymbol{e}^i. \quad (5)$$

Considering

$$\boldsymbol{e}^i-\boldsymbol{e}^j=\left(\boldsymbol{x}^i-\bar{\boldsymbol{x}}\right)-\left(\boldsymbol{x}^j-\bar{\boldsymbol{x}}\right)=\boldsymbol{x}^i-\boldsymbol{x}^j$$

and

$$\begin{aligned}\|\boldsymbol{e}^i\| &= \|\boldsymbol{x}^i-\bar{\boldsymbol{x}}\|=\left\|\boldsymbol{x}^i-\frac{1}{N}\sum_{j=1}^{N}\boldsymbol{x}^j\right\| \\ &= \frac{1}{N}\left\|\sum_{j=1}^{N}(\boldsymbol{x}^i-\boldsymbol{x}^j)\right\| \\ &\leq \frac{1}{N}\sum_{j=1}^{N}\|\boldsymbol{x}^i-\boldsymbol{x}^j\|,\end{aligned}$$

we have

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{a(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}\boldsymbol{e}^i}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|} \\ &=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{a(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}\boldsymbol{e}^i}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}+\frac{a(\boldsymbol{x}^j-\boldsymbol{x}^i)^{\mathrm{T}}\boldsymbol{e}^j}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}\right) \\ &=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{a(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}\left(\boldsymbol{e}^i-\boldsymbol{e}^j\right)}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}\right) \\ &=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{a(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}\left(\boldsymbol{x}^i-\boldsymbol{x}^j\right)}{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|}\right) \\ &=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(a\|\boldsymbol{x}^i-\boldsymbol{x}^j\|\right) \\ &=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\left(a\|\boldsymbol{x}^i-\boldsymbol{x}^j\|\right) \\ &\geq \frac{aN}{2}\sum_{i=1}^{N}\|\boldsymbol{e}^i\|. \end{aligned} \quad (6)$$

Notice that $\exp(-\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2/c)\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2$ is a bounded function whose maximum is $c\exp(-1)$ obtained at $\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2=c$, thus $\exp(-\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2/c)\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2\leq c\exp(-1)$. We have

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}b\exp\left(-\frac{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2}{c}\right)(\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}\boldsymbol{e}^i \\ &=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}b\exp\left(-\frac{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2}{c}\right) \\ &\quad\times\left((\boldsymbol{x}^i-\boldsymbol{x}^j)^{\mathrm{T}}\boldsymbol{e}^i+(\boldsymbol{x}^j-\boldsymbol{x}^i)^{\mathrm{T}}\boldsymbol{e}^j\right) \\ &=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}b\exp\left(-\frac{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2}{c}\right)\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2 \\ &=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}b\exp\left(-\frac{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2}{c}\right)\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2 \\ &\leq \frac{1}{2}N(N-1)bc\exp(-1). \end{aligned} \quad (7)$$

Substituting Eqs. (6) and (7) into Eq. (5) yields

$$\begin{aligned}\dot{V} &\leq -\frac{aN}{2}\sum_{i=1}^{N}\|\boldsymbol{e}^i\|+\frac{1}{2}N(N-1)bc\exp(-1) \\ &= -\frac{1}{2}aN\left(\sum_{i=1}^{N}\|\boldsymbol{e}^i\|-\frac{bc(N-1)}{a}\exp(-1)\right).\end{aligned}$$

Therefore, $\dot{V}<0$ as long as

$$\textstyle\sum_{i=1}^{N}\|\boldsymbol{e}^i\|>(N-1)\exp(-1)bc/a=\alpha. \quad (8)$$

The proof of Theorem 1 is completed.

From Theorem 1, we gain the aggregation motion of the swarm system (1) with the attraction/repulsion force function (2). In addition, from inequality (8), we can find that the aggregation region decreases as the parameter a increases (attraction increases) and increases as the parameters b and c increase (repulsion increases). The expected aggregation region $B_\alpha(\bar{\boldsymbol{x}})$ is the goal of our swarm engineering systems. We can approximately obtain the parameters a, b and c by using Eq. (3) and α when we design a practical swarm engineering system.

From Theorem 1, we know that all of the agents will enter into the region $B_\alpha(\bar{\boldsymbol{x}})$ around the swarm center $\bar{\boldsymbol{x}}$. However, we still do not know the motion of agents in this region: motionless or relative motion. To this end, we need to carry out a stability analysis. The main results are given in the following Theorem 2.

Theorem 2 Consider the swarm system (1) with the attraction/repulsion force function (2). As $t\to\infty$, $x\to\Omega_\beta\triangleq\{\boldsymbol{x}:\dot{\boldsymbol{x}}=0\}$, where $\boldsymbol{x}=[\boldsymbol{x}^{1\mathrm{T}},\cdots,\boldsymbol{x}^{M\mathrm{T}}]^{\mathrm{T}}$.

Proof We choose the candidate Lyapunov function

$$J(\boldsymbol{x})=\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}G(\boldsymbol{x}^i-\boldsymbol{x}^j),$$

where

$$G(\boldsymbol{x}^i-\boldsymbol{x}^j)=a\|\boldsymbol{x}^i-\boldsymbol{x}^j\|+\frac{1}{2}bc\exp\left(-\frac{\|\boldsymbol{x}^i-\boldsymbol{x}^j\|^2}{c}\right).$$

Then $\nabla_{\boldsymbol{x}^i}J(\boldsymbol{x}^i)=-\dot{\boldsymbol{x}}^i$. This further yields that for all t,

$$\begin{aligned}\dot{J}(\boldsymbol{x}) &= [\nabla_x\boldsymbol{J}(\boldsymbol{x})]^{\mathrm{T}}\dot{\boldsymbol{x}}=\sum_{i=1}^{N}[\nabla_{\boldsymbol{x}^i}J(\boldsymbol{x}^i)]^{\mathrm{T}}\dot{\boldsymbol{x}}^i \\ &= \sum_{i=1}^{N}\left[-\dot{\boldsymbol{x}}^i\right]^{\mathrm{T}}\dot{\boldsymbol{x}}^i=-\sum_{i=1}^{N}\|\dot{\boldsymbol{x}}^i\|^2\leq 0.\end{aligned}$$

From LaSalle's invariance principle, we obtain

$$\Omega=\{\boldsymbol{x}:\dot{J}(\boldsymbol{x})=0\}=\{\boldsymbol{x}:\dot{\boldsymbol{x}}=0\}=\Omega_\beta.$$

The proof of Theorem 2 is completed.

The aforementioned discussion shows that all of the agents move along the negative direction of the gradient and the swarm system (1) converges to an equilibrium point of the system. In other words, all of the agents are eventually stationary.

4. Simulations

In this section, we provide numerical simulations to demonstrate the effectiveness of the above theoretical results. Without loss of generality, we choose $n = 2$. In the region of 1×1, we randomly choose 20 points as the initial positions of the agents in a swarm system, as shown in Fig. 1, where the blue circles represent the initial positions of the agents and the blue star represents the position of the swarm center $\bar{x}$. We choose $a = 1$, $b = 5$, and $c = 0.01$. The aggregation trajectories of the agents are shown in Fig. 2, where the red circles represent the final positions of the agents in the swarm system. We show the final positions of the agents in Fig. 3 for legibility again. From these simulations, we can find that all agents move toward the center of the swarm to form a cohesive swarm cluster around the swarm center. This coincides with the above theoretical analysis.

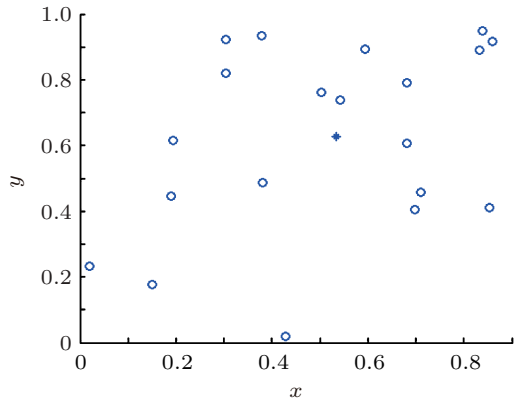


Fig. 1. (color online) The initial positions of the swarm agents.

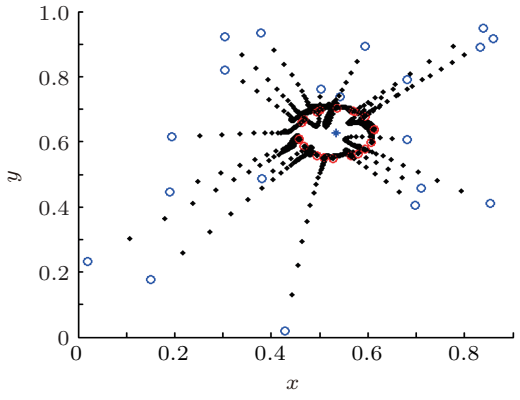


Fig. 2. (color online) The trajectories of the swarm agents.

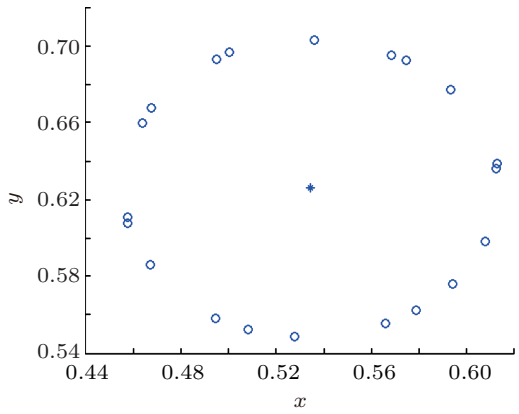


Fig. 3. (color online) The final positions of the swarm agents.

5. Conclusion

In this paper we have proposed a novel swarm model of multi-agent systems with finite velocity. Using the Lyapunov function method and LaSalle's invariance principle, we have proved that all of the agents using the proposed swarm model eventually enter into a bounded region around the swarm center and finally tend to a stationary state. Our future work will establish more practical swarm models with finite sense ability, time delay, and so on.

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