Effect of disorder on hyperbolic metamaterials*

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Based on a modified retrieving method, we demonstrate that hyperbolic metamaterials (HMs) have considerable robustness against disorders, even when the disorder strength is quite large. Our retrieval method is more precise when retrieving the parameters for anisotropic metamaterials. We also show that the light's negative refraction of an HM is nearly unaffected when relatively large disorders exist. These results help us to understand the HMs and they have a direct significance for experiments.

Keywords: hyperbolic metamaterials, disorder, retrieving method

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1. Introduction

Metamaterials are engineered artificial media containing sub-wavelength inclusions with designed shapes, sizes, and compositions, leading to many new phenomena, such as a negative index of refraction,[1–6] limitless focusing,[1] and cloaking.[7–11] In particular, a class of metamaterials with hyperbolic dispersion,[12,13] in which the permittivities in orthogonal directions have opposite signs, enable sub-wavelength imaging[14–18] and the control of spontaneous emission.[19–21] When manufacturing hyperbolic metamaterials (HMs), disorder[22] is difficult to avoid, even for the two simplest structures; that is, one-dimensional (1D) HMs composed of periodic metallic and dielectric multilayers[14] and two-dimensional (2D) HMs consisted of rod arrays.[15] The influence of disorder on the optical property of HMs has not been revealed and it is not very clear whether or not the hyperbolic dispersion relation would be destroyed if the strength of the disorder is large. The effect of the disorder on HMs is a fundamental subject and has a direct significance for experiments.

In this work, we present a modified retrieval method, which is based on the method introduced in Ref. [23], to quantitatively study the effect of disorder on HMs. This method is more precise when retrieving anisotropic metamaterials. By using this method, we are able to show the robustness of HMs against disorder.

2. Model and methods

First, let us start with an ordered HM, the system considered here is comprised of a periodic array of dielectric and metal slabs possessing dielectric permittivity $\varepsilon_{\rm d}$ and $\varepsilon_{\rm m}$, respectively, as shown in Fig. 1(a). The widths of the dielectric and metal slabs are $d_{\rm d}$ and $d_{\rm m}$, respectively, and $d_{\rm cell} = d_{\rm d} + d_{\rm m}$ is the width of one cell, $d_{\rm tot} = N d_{\rm cell}$ is the total width of the HM, where N is the cell number. Under the assumption of a long wavelength limit, the anisotropic effective permittivities along the directions parallel and perpendicular to the interface of the two media are

$$\varepsilon_x^{\rm A} = \frac{\varepsilon_{\rm d} d_{\rm d} + \varepsilon_{\rm m} d_{\rm m}}{d_{\rm d} + d_{\rm m}}, \tag{1a}$$

$$\varepsilon_z^{\rm A} = \frac{d_{\rm d} + d_{\rm m}}{d_{\rm d}/\varepsilon_{\rm d} + d_{\rm m}/\varepsilon_{\rm m}}. \tag{1b}$$

To study the effect of the disorder on the HMs, two types of disorders are added to the ordered HMs, as shown in Fig. 1(b). The width disorder means that the width of each layer is disordered, which becomes $d_i(1+\Delta)$, $i = {\rm d}, {\rm m}$, where

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Δ is a random number in the range $[-W, W]$ for every single layer, and W is the disorder strength. For $W = 0.2$, the width of each layer will distribute in $0.8d_i \sim 1.2d_i$, $i = \mathrm{d}, \mathrm{m}$. This type of disorder represents the unavoidable random fluctuation of the width when growing materials. The scattering disorder (i.e., the small metal squares added at the interfaces of the layers) represents the roughness of the interfaces caused by the nonuniform growth of materials. The side length of the square is fixed to $0.5d_\mathrm{d}$, making the square a rather larger scatterer in the HMs. These two types of disorders are very common when manufacturing real HMs, and we will show their effect on the effective permittivities of the HMs.

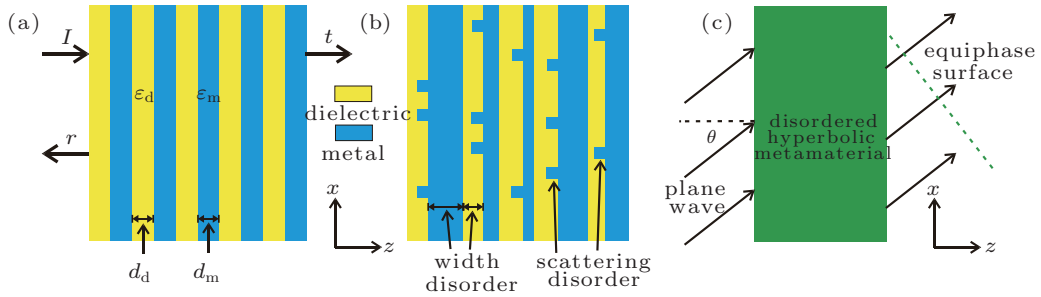


Fig. 1. (color online) Schemas of (a) 1D HM constituted by alternate dielectric and metal slabs without disorder, and (b) 1D HM with width disorder and scattering disorder. (c) When the scattering disorder is added, the transmission coefficient t should be the average value at different positions on an equi-phase surface.

Although equation (1) is an easy way to calculate the effective permittivity of an ideal HM, it cannot be used directly for real HMs. Because the width of one cell in an HM is usually not smaller enough than the wavelength, and disorders always exist, so the real effective permittivity is different from the analytical value in Eq. (1) and a retrieval method is often used to calculate the real effective parameters.

The usual method that is used to retrieve the effective permittivity ε and permeability μ of an isotropic metamaterial is to combine the transmission coefficient t and the reflection coefficient r to solve the refractive index n and the impedance z

$$\cos(nkd) = \frac{1 - r^2 + t^2}{2t}, \tag{2a}$$

$$z = \sqrt{\frac{(1-r)^2 - t^2}{(1+r)^2 - t^2}}, \tag{2b}$$

where k is the wave vector in air and d is the width of the metamaterial. Equation (2a) is a multi-value function of n, and can be solved as shown in Ref. [23]. We then have $\varepsilon^\mathrm{R} = n/z$ and $\mu^\mathrm{R} = nz$, where superscript R represents the retrieved value. Here, the transmission coefficient t and reflection coefficient r of the normal incidence are enough to calculate the permittivity and the permeability because they are both isotropic.

For anisotropic metamaterials, the above method can also solve ε_x^R, ε_z^R, and μ^R using t and r of two different incident angles θ; for example, $\theta_1 = 0$ and $\theta_2 = 45°$. For an arbitrary incident angle θ,

$$t(\theta) = \frac{1}{\cos n(\theta)k_0 d - \frac{\mathrm{i}}{2}\left(z(\theta) + \frac{1}{z(\theta)}\right)\sin n(\theta)k_0 d}, \tag{3a}$$

$$r(\theta) = \frac{-\frac{\mathrm{i}}{2}\left(z(\theta) - \frac{1}{z(\theta)}\right)\sin n(\theta)k_0 d}{\cos n(\theta)k_0 d - \frac{\mathrm{i}}{2}\left(z(\theta) + \frac{1}{z(\theta)}\right)\sin n(\theta)k_0 d}. \tag{3b}$$

Notice that t, r, n, and z are all functions of incident angle θ now, and $n(\theta) = k_z/k_{z0}$, $z(\theta) = k_z/(\varepsilon_x^\mathrm{R} k_{z0})$, where $k_z = \sqrt{\varepsilon_x^\mathrm{R}(\mu^\mathrm{R} - \sin^2\theta/\varepsilon_z^\mathrm{R})}k_0$ and $k_{z0} = k_0\cos\theta$ are the wave vectors along the z direction in the HM and air, respectively.

We can first solve ε_x^R and μ^R when $\theta = 0$

$$\varepsilon_x^\mathrm{R} = \frac{n(0)}{z(0)}, \tag{4a}$$

$$\mu^\mathrm{R} = n(0)z(0), \tag{4b}$$

and then calculate ε_z^R for an arbitrary incident angle $\theta \neq 0$

$$\varepsilon_z^\mathrm{R} = \frac{\sin^2\theta}{\mu^\mathrm{R} - \varepsilon_x^\mathrm{R}\cos^2\theta z^2(\theta)}. \tag{4c}$$

Because it is quite difficult to measure both the transmission coefficient t and reflection coefficient r exactly in time-domain simulations or experiments, the errors of the effective parameters calculated with Eq. (4) are unavoidable. It is clear from Eq. (4c) that a small error of μ^R will lead to a large inaccuracy of ε_z^R, although the real permeability is $\mu \equiv 1$ for our HMs. Here, we show our modified retrieving method to calculate the effective parameters precisely, which only uses the transmission coefficient t.

For the HMs comprised of dielectric and metal, the effective permeability μ must be 1 and we do not need to retrieve it. The transmission coefficient t of the HM can be calculated by using the transmission matrix method or it can be measured in experiments. For the HM with width d,

$$t(\theta) = \frac{1}{\cos k_z d - \frac{\mathrm{i}}{2}\left(\frac{k_z}{\varepsilon_x^\mathrm{R} k_{z0}} + \frac{\varepsilon_x^\mathrm{R} k_{z0}}{k_z}\right)\sin k_z d}, \tag{5}$$

where $k_{z0} = k_0 \cos\theta$ and $k_z = \sqrt{\varepsilon_x^{\mathrm{R}}(1-\sin^2\theta/\varepsilon_z^{\mathrm{R}})}k_0$. For the normal incidence, when the transmission coefficient $t(0)$ is obtained by a calculation or measurement, the above equation becomes a transcendental equation for $\varepsilon_x^{\mathrm{R}}$. By setting $\sqrt{\varepsilon_x^{\mathrm{R}}}$ as x, equation (5) becomes

$$f(x) = \cos(xk_0d) - \frac{\mathrm{i}}{2}\left(x + \frac{1}{x}\right)\sin(xk_0d) - \frac{1}{t(0)} = 0. \quad (6)$$

By using the Newton iteration method, the effective permittivity $\varepsilon_x^{\mathrm{R}}$ can be obtained as follows. Set an initial value $x = x_0$ near to the solution of $f(x) = 0$, and calculate $x_1 = x_0 - f(x_0)/f'(x_0)$, $x_2 = x_1 - f(x_1)/f'(x_1), \ldots, x_n = x_{n-1} - f(x_{n-1})/f'(x_{n-1})$ in order. When n increases, x_n gradually approaches to the solution of $f(x) = 0$. Generally, $n > 10$ is necessary to get a precise value of $\varepsilon_x^{\mathrm{R}}$. In a similar way, for an oblique incidence, we can also find the effective permittivity $\varepsilon_z^{\mathrm{R}}$ after we know the transmission coefficient $t(\theta)$ and $\varepsilon_x^{\mathrm{R}}$. By using the above method, the anisotropic permittivities can be retrieved from transmission coefficients t of normal and oblique incidences only, leading to a considerable increase in the precision of $\varepsilon_{x(z)}^{\mathrm{R}}$.

When the width disorder is added, the transmission coefficient t can also be calculated by using the transmission matrix method. However, when the scattering disorder is added, the electromagnetic wave will be scattered by these small metal squares, the transmission matrix method is no longer applicable for calculating the transmission coefficient t, and in this case we use the finite difference time domain (FDTD) method. When an incidence plane wave with incidence angle θ propagates in the HM with scattering disorder, the transmission wave is an approximate plane wave with the same angle, although the transmission coefficients at different positions are different. We can then consider the average of many transmission coefficients at an equi-phase surface as the effective transmission coefficient t of the HM, as shown in Fig. 1(c).

3. Results and discussion

First we study the influence of the width disorder on the effective permittivities $\varepsilon_x^{\mathrm{R}}$ and $\varepsilon_z^{\mathrm{R}}$. The parameters chosen are $\varepsilon_{\mathrm{d}} = 16.5$, $\varepsilon_m = -1.5$, $d_{\mathrm{d}} = d_{\mathrm{m}} = d_{\mathrm{cell}}/2 = d_{\mathrm{tot}}/(2N)$, where N is the cell number. With these parameters, we can obtain the effective permittivities $\varepsilon_x^{\mathrm{A}} = 7.5$, $\varepsilon_z^{\mathrm{A}} = -3.3$ by using Eq. (1). Figures 2(a) and 2(b) show the average $E[\varepsilon_{x(z)}^{\mathrm{R}}]$ and the standard deviation $\delta[\varepsilon_{x(z)}^{\mathrm{R}}]$ of $\varepsilon_x^{\mathrm{R}}$ and $\varepsilon_z^{\mathrm{R}}$ versus W for different N, respectively. The total width of the HM is $d_{\mathrm{tot}} = \lambda$ before the width disorder is added, here λ is the wavelength in air. The cell number $N = 100$, 50, 25 means that the width of one cell is $d_{\mathrm{cell}} = \lambda/100$, $\lambda/50$, and $\lambda/25$, respectively. The sample number is 10000 for each W and N. It is easy to understand that the distributions of $\varepsilon_x^{\mathrm{R}}$ and $\varepsilon_z^{\mathrm{R}}$ are both near to the Gaussian distribution when the width disorder is introduced, as shown in Figs. 2(c) and 2(d). It is clear that the averages are almost unchanged and that the standard deviations increase linearly when W increases for both $\varepsilon_x^{\mathrm{R}}$ and $\varepsilon_z^{\mathrm{R}}$, here the standard deviation shows the range in which most $\varepsilon_x^{\mathrm{R}}$ or $\varepsilon_z^{\mathrm{R}}$ distribute in for different W and N. From Fig. 2, we can learn that the smaller the width of one cell is, the closer the average of $\varepsilon_{x(z)}^{\mathrm{R}}$ and the analytical value $\varepsilon_{x(z)}^{\mathrm{A}}$ calculated by Eq. (1) will be. This is easy to understand if we think about the effective medium theory, the smaller d_{cell} means that the approximation of long wavelength limit is better satisfied, so the $\varepsilon_{x(z)}^{\mathrm{R}}$ calculated by the transmission matrix method and the retrieving method is closer to the theoretical value and the standard deviations of smaller d in Figs. 2(a) and 2(b) are also small. Although the standard deviations $\delta[\varepsilon_{x(z)}^{\mathrm{R}}]$ increase linearly with increasing W, the dispersion relation of the metamaterial remains hyperbolic for all N, even when $W = 0.5$, which shows the startling robustness of the HMs against width disorder.

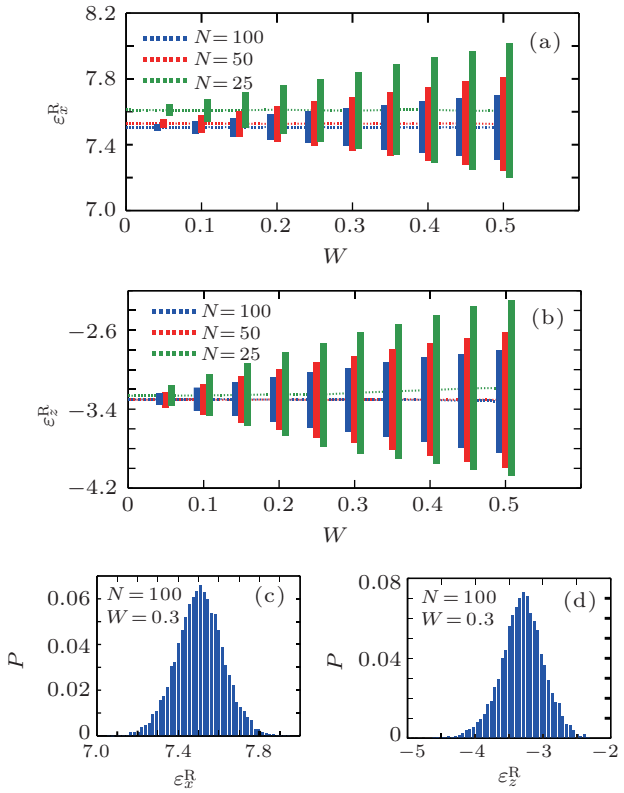


Fig. 2. (color online) Averages and standard deviations of (a) $\varepsilon_x^{\mathrm{R}}$ and (b) $\varepsilon_z^{\mathrm{R}}$ versus the strength of the width disorder W, where W increases from 0 to 0.5 in a step of 0.05, and the dot lines connect the average values of $\varepsilon_{x(z)}^{\mathrm{R}}$ of different W, with blue, red, and green lines corresponding to $N = 100$, 50, 25, respectively. The bound of the bar, $E[\varepsilon_{x(z)}^{\mathrm{R}}] \pm \delta[\varepsilon_{x(z)}^{\mathrm{R}}]$, shows the range in which most of $\varepsilon_{x(z)}^{\mathrm{R}}$ distribute in when the width disorder is introduced. Gaussian distributions of (a) $\varepsilon_x^{\mathrm{R}}$ and (b) $\varepsilon_z^{\mathrm{R}}$ at $N = 100$, $W = 0.3$, the sample numbers are both 10000.

Now, we mix the two types of disorders together. We calculate the effective permittivities $\varepsilon_{x(z)}^{\mathrm{R}}$ of several configurations using our modified method, these effective permittivities are

almost all distributed in the ranges shown in Fig. 2, confirming that the HMs have considerable robustness against these disorders.

To show the hyperbolic dispersion relation of the HMs with these disorders, we present the negative refraction of the HM for an incident Gaussian beam with incident angle $\theta = 45^\circ$ in Fig. 3, the cell number of the HM is $N = 25$. To get a large excursion of the negative refraction, the width of one cell is increased to $d_{\rm cell} = 0.08\lambda$, and the total width of the HM is $d_{\rm tot} = 2\lambda$. The strength of the width disorder is $W = 0.2$ in both Figs. 3(a) and 3(c), and lots of small metal squares with side length $a = 0.5d_{\rm d}$ are added at the interfaces of different layers in Fig. 3(c). To increase the transmission, the permittivities of the dielectric and metal slabs are changed to meet the impedance match condition $\varepsilon_x^{\rm A} = 1$. The negative refraction shown in Fig. 3 matches well with the hyperbolicity of the metamaterial, which also confirms the robustness of the HMs against these disorders.

To better understand the above results, we present a theoretic derivation of the effective permittivities $\varepsilon_{x(z)}$ of the HM with width disorder. We select an arbitrary cell in the HM, and consider other cells as an anisotropic background medium with dielectric permittivities $\varepsilon_x^{\rm R}$ and $\varepsilon_z^{\rm R}$, as shown in Fig. 4. The effective medium parameters $\varepsilon_{x(z)}^{\rm R}$ are determined

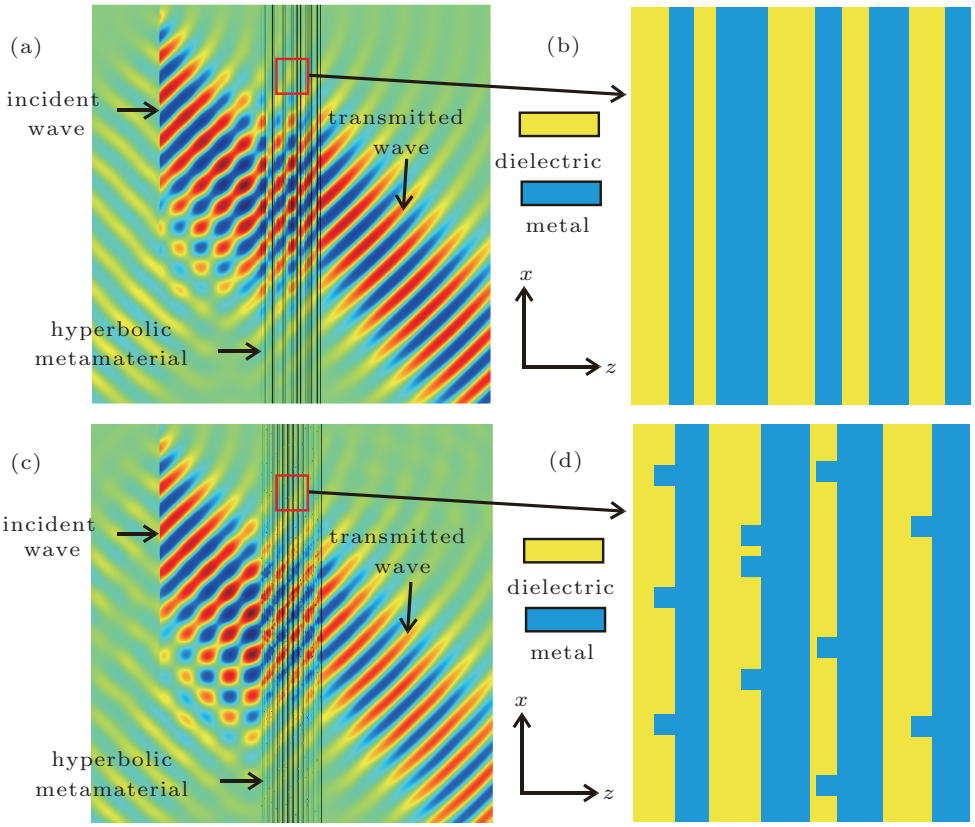


Fig. 3. (color online) (a) The negative refraction of the HM with width disorder. (b) Part of the configuration of the HM with width disorder. (c) The negative refraction of the HM with two types of disorders. (d) Part of the configuration of the HM with two types of disorders.

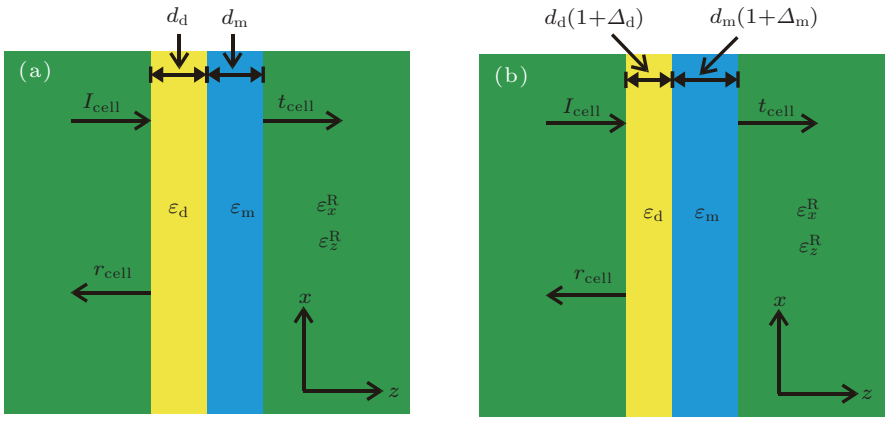


Fig. 4. (color online) (a) Schema of the effective medium theorem that consider the HM as an anisotropic background medium except one cell in it. (b) Using the effective medium theorem to calculate the effect of the width disorder of the chosen cell on the effective permittivities.

by the condition that the scattering of a unit cell vanishes[24] in the limit of $k_z d \ll 1$ for all incidence angles, which means $r_{\rm cell} = 0$. Assume that the widths of the dielectric layer and the metal layer in the cell we considered are changed to $d_{\rm d}(1+\Delta_{\rm d})$ and $d_{\rm m}(1+\Delta_{\rm m})$, respectively, where $|\Delta_i| \leq W$, $i = {\rm d}, {\rm m}$, and $W \ll 1$. By using the transmission matrix method, we can obtain the equation

$$\begin{pmatrix} I_{\rm cell} \\ r_{\rm cell} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} t_{\rm cell} \\ 0 \end{pmatrix}, \tag{7}$$

where $I_{\rm cell}$, $r_{\rm cell}$, and $t_{\rm cell}$ are the incident, the reflected, and the transmitted components of the wave, respectively, and T_{ij} are the elements of the transmission matrix. The $r_{\rm cell} = 0$ requires the matrix element $T_{21} = 0$. Under the long wavelength limit assumption, we have $k_{z{\rm d}} d_{\rm d} \ll 1$, $k_{z{\rm m}} d_{\rm m} \ll 1$, and $T_{21} = 0$ can be simplified to the form

$$\begin{aligned} &\frac{k_z^2}{(\varepsilon_x^{\rm R})^2}[\varepsilon_{\rm d} d_{\rm d}(1+\Delta_{\rm d}) + \varepsilon_{\rm m} d_{\rm m}(1+\Delta_{\rm m})] \\ &= \frac{k_{z{\rm d}}^2}{\varepsilon_{\rm d}} d_{\rm d}(1+\Delta_{\rm d}) + \frac{k_{z{\rm m}}^2}{\varepsilon_{\rm m}} d_{\rm m}(1+\Delta_{\rm m}), \end{aligned} \tag{8}$$

where $k_z = \sqrt{\varepsilon_x(k_0^2 - k_x^2/\varepsilon_z)}$, $k_{z{\rm d}} = \sqrt{\varepsilon_{\rm d} k_0^2 - k_x^2}$, $k_{z{\rm m}} = \sqrt{\varepsilon_{\rm m} k_0^2 - k_x^2}$. For the normal incidence, $k_x = 0$, equation (8) becomes

$$\begin{aligned} \varepsilon_x^{\rm R} &= \frac{\varepsilon_{\rm d} d_{\rm d}(1+\Delta_{\rm d}) + \varepsilon_{\rm m} d_{\rm m}(1+\Delta_{\rm m})}{d_{\rm d}(1+\Delta_{\rm d}) + d_{\rm m}(1+\Delta_{\rm m})} \\ &\approx \varepsilon_x^{\rm A}\left[1 + \frac{\varepsilon_{\rm d} - \varepsilon_{\rm m}}{\varepsilon_{\rm d}(1+d_{\rm d}/d_{\rm m}) + \varepsilon_{\rm m}(1+d_{\rm m}/d_{\rm d})}(\Delta_{\rm d} - \Delta_{\rm m})\right]. \end{aligned} \tag{9a}$$

For the oblique incidence with incident angle θ, $k_x = k_0 \sin\theta$, equation (8) becomes

$$\begin{aligned} \varepsilon_z^{\rm R} &= \frac{d_{\rm d}(1+\Delta_{\rm d}) + d_{\rm m}(1+\Delta_{\rm m})}{d_{\rm d}(1+\Delta_{\rm d})/\varepsilon_{\rm d} + d_{\rm m}(1+\Delta_{\rm m})/\varepsilon_{\rm m}} \\ &\approx \varepsilon_z^{\rm A}\left[1 + \frac{\varepsilon_{\rm d} - \varepsilon_{\rm m}}{\varepsilon_{\rm d}(1+d_{\rm m}/d_{\rm d}) + \varepsilon_{\rm m}(1+d_{\rm d}/d_{\rm m})}(\Delta_{\rm d} - \Delta_{\rm m})\right]. \end{aligned} \tag{9b}$$

From Eq. (9), we can see that when W is small, which means $|\Delta_{\rm d} - \Delta_{\rm m}| \ll 1$, the difference between numerical $\varepsilon_{x(z)}^{\rm R}$ and analytical $\varepsilon_{x(z)}^{\rm A}$ is also small, and the fluctuation of $\varepsilon_{x(z)}^{\rm R}$ is in direct proportion to the strength of the width disorder, W, which is consistent with Figs. 2(a) and 2(b).

4. Conclusion

We present a modified retrieval method that can retrieve the parameters of anisotropic metamaterials more precisely. By using this method, we find that the HMs still have a hyperbolic dispersion relation, even when the disorder strength of the width disorder is $W = 0.5$. The influence of the scattering disorder on the effective dielectric permittivities are also small. Based on these results, we can conclude that the HMs have considerable robustness against the disorders, even when the disorder strength is quite large.

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