Scattering of a general partially coherent beam from a diffuse target in atmospheric turbulence*

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The second-order and fourth-order statistical moments of the speckle field from a diffuse target in atmospheric turbulence are studied which have great influence on the performance of lidar systems. By expanding a general rotationally symmetric beam as a sum of Gaussian–Schell model (GSM) beams, the mean intensity of the general beam propagating over a distance in an atmospheric turbulence is formulated. Expressions for the degree of coherence (DOC) and the normalized intensity variance of the scattered field of a general beam from a rough surface in turbulence are derived based on the extended Huygens–Fresnel principle. The general expressions reduce to the well-known forms for a GSM beam. Another example for the general beam used in this paper is the collimated flat-topped beam. The results of both kinds of beams show that the intensity profile on the target plane is a key factor affecting the statistical characteristics of the speckle field. A larger beam spot on the target plane induces a smaller coherence length and a smaller normalized intensity variance of the received field. As turbulence gets stronger, the coherence length becomes smaller, and the normalized intensity variance firstly increases and declines to unity finally.

Keywords: atmospheric optics, extended Huygens–Fresnel principle, partially coherent beam, degree of coherence

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1. Introduction

The "speckle effect" has been used in many applications, such as target recognition and non-contact measurement of displacement and speed.[1] Some applications of the speckle effect in atmosphere (e.g., detecting targets in space) inevitably involve the challenge of wave propagation through atmospheric turbulence, which has a large influence on the statistical characteristics of speckle field.

Several theories have been developed to study the problem of speckle field in turbulence. Holmes *et al.* have used the extended Huygens–Fresnel principle to investigate the second-order and fourth-order statistical moments of the speckle field in turbulence, and observed the speckle pattern changing with the type of laser beam, the strength of the turbulence, and the length of the propagation path.[2] Wei *et al.* have extended the analysis to the situation of light propagation along a slant path.[3] Gudimetla *et al.* found another way to analyze the intensity fluctuation of a speckle field in turbulence[4] by proposing a physical model giving the K distribution of the intensity and calculating the second-order moment of the intensity on the basis of the probability model. Aksenov and Banakh have used the location Green function to calculate the fluctuations of the laser radiation field retro-reflected from a target in a turbulence.[5] Andrews *et al.* used the effective beam parameters to solve the problem of double-pass propagation of a Gaussian beam in turbulence.[6]

Partially coherent beams have been widely used and investigated since the 1970s because it is less affected by turbulence compared with the fully coherent beams. Leader firstly studied the MCF and intensity variance of a partially coherent beam propagating in turbulence.[7,8] Ricklin derived the expressions for average intensity, beam size, phase front radius of curvature, and wave-front coherence length of a partially coherent beam from the cross-spectral density function.[9] Later, Wu and Boardman[10] and Korotkova *et al.*[11] have studied the scintillation effect of partially coherent beams in an atmospheric turbulence. Recently, Wu and Li have derived the second-order and fourth-order moments of the scattering field of a GSM beam in turbulence.[12]

In recent years, propagations of various new types of beams in turbulence have been reported, such as Hermite-cosh-Gaussian,[13] flat-topped,[14] and Bessel–Gaussian beams.[15] Two expanding forms of general beams were proposed by Baykal and the scintillation index of a general beam in turbulence was deduced.[16,17] However, attention has been mostly paid on the second-order and fourth-order moments of the beams after single-pass propagation through turbulence. The study on the scattering field of a general source from a

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rough target has not been found.

In Section 2, a model of a partially coherent beam with an arbitrary rotationally symmetric intensity profile is introduced, and the mean intensity on the target in turbulence is formulated. The single GSM beam and the collimated flat-topped beam are used as special cases for the general beam. In Section 3, the expressions of the MCF and DOC of reflected wave from the target are derived. The coherence lengths for the two special kinds of beams are calculated. In Section 4, the expression of the intensity variance of the reflected wave is derived and calculated.

In this work, the intensity profile of a general beam after propagating through turbulence is formulated by expanding the general beam as a sum of GSM beams. The coherence property and the intensity fluctuation of the scattered field from a rough surface are studied. The single GSM beam and the collimated flat-topped beam are used as special cases for the general beam. All calculations are made in the case of horizontal path, whereas the formulations can also be used for the case of slant path.

2. Mean intensity of a general beam in turbulence

A general beam can be obtained from a Gaussian beam by passing through a suitable filter plate, and the coherence properties of the beam can be controlled by passing through a random phase screen. Thus, the expression of a general rotationally symmetric source in the plane $z = 0$ can be written as

$$U_0(\boldsymbol{r},0) = U_\mathrm{d}(r,0)\exp[\mathrm{i}\varphi(\boldsymbol{r})], \tag{1}$$

where $U_\mathrm{d}(r,0)$ is a coherent beam field, and $\varphi(\boldsymbol{r})$ is the random phase with a Gaussian correlation function

$$\langle\exp[\varphi(\boldsymbol{\rho},\boldsymbol{r}_1)+\varphi(\boldsymbol{\rho},\boldsymbol{r}_2)]\rangle \simeq \exp(-r_-^2/l_\mathrm{c}^2), \tag{2}$$

where $r_- = |\boldsymbol{r}_-| = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$.

It can be known from Ref. [16] that the coherent field can be expanded as a sum of complex Gaussian fields

$$U_\mathrm{d}(r,0) = \sum_{n=1}^{N} A_n \exp(-\frac{1}{2}k\alpha_n r^2), \tag{3}$$

where N is the order of the expansion, and

$$\alpha_n = 1/(kW_{\mathrm{sn}}^2) + \mathrm{i}/F_n, \tag{4}$$

where A_n, W_n, and F_n are the complex amplitude, the beam radius, and the focal length of the nth Gaussian field, respectively.

Let $N = 1$ in Eq. (3), then a GSM beam can be obtained as

$$U_0(\boldsymbol{r},0) = \exp\left(-\frac{1}{2}k\alpha r^2\right)\exp[\mathrm{i}\varphi(\boldsymbol{r})],$$

$$\alpha = 2/(kW_0^2) + \mathrm{i}/F_0. \tag{5}$$

A general beam can be seen as a sum of GSM beams. Another special case used in this paper is the collimated flat-topped beam, the coherent part of which is written as

$$U_\mathrm{d}(r,0) = \sum_{n=1}^{N}(-1)^{n-1}\frac{N!}{n!(N-n)!}\exp\left(-\frac{r^2}{W_n^2}\right), \tag{6}$$

where $W_n = W_0/\sqrt{nP_N}$, W_0 is the radius of the source, and P_M is the modified scaling factor

$$P_\mathrm{M} = 2\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{n+m}\frac{(N!)^2}{n!m!(N-n)!(N-m)!}. \tag{7}$$

Figure 1 shows the amplitude of collimated flat-topped sources with different orders. The source with a higher order has a flatter top, which shows that when $N \to \infty$, the beam becomes perfectly flat-topped. The mean square error of the beam with a finite N is defined as

$$\begin{aligned}\sigma_\mathrm{err}^2 &= 2W_0^{-2}\int_0^{W_0}\left[1-\sum_{n=1}^{N}(-1)^{n-1}\frac{N!}{n!(N-n)!}\exp\left(-\frac{r^2}{W_n^2}\right)\right]^2 r\mathrm{d}r \\ &+2\int_{W_0}^{\infty}\left[\sum_{n=1}^{N}(-1)^{n-1}\frac{N!}{n!(N-n)!}\exp\left(-\frac{r^2}{W_n^2}\right)\right]^2 r\mathrm{d}r.\end{aligned} \tag{8}$$

Then the mean-square errors of the beams with $N = 1$, 5, 10, 40 are 21.2%, 9.3%, 4.5%, and 3.7%, respectively. Commonly, 5% is small enough for the mean-square error, so a flat-topped beam with $N = 10$ is used for the numerical calculation in the paper.

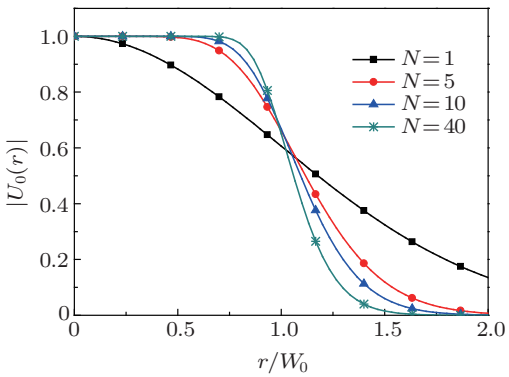


Fig. 1. (color online) Amplitude of collimated flat-topped sources with different orders.

Using the extended Huygens–Fresnel principle, the field of the beam in the plane $z = L$ after propagating through turbulence is written as

$$\begin{aligned}U(\boldsymbol{\rho},L) &= [k/(2\pi\mathrm{i}L)]\exp(\mathrm{i}kL)\int \mathrm{d}\boldsymbol{r}U_0(\boldsymbol{r},0) \\ &\times\exp[\mathrm{i}k|\boldsymbol{r}-\boldsymbol{\rho}|^2/(2L)+\psi(\boldsymbol{r},\boldsymbol{\rho})],\end{aligned} \tag{9}$$

where $\psi(\boldsymbol{r},\boldsymbol{\rho})$ is the field fluctuation of a spherical wave propagating from point $(\boldsymbol{r},0)$ at the target to the receiving point $(\boldsymbol{\rho},L)$ in turbulence.

Based on Eq. (8), the mean intensity at $z=L$ from a general source can be formulated as

$$\begin{aligned}\langle I(\boldsymbol{\rho},L)\rangle &= \langle U(\boldsymbol{\rho},L)U^*(\boldsymbol{\rho},L)\rangle \\ &= [k/(2\pi L)]^2 \iint \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2[U_\mathrm{d}(r_1,0)U_\mathrm{d}^*(r_2,0)] \\ &\quad\times\langle\exp[\mathrm{i}\varphi(\boldsymbol{r}_1)-\mathrm{i}\varphi(\boldsymbol{r}_2)]\rangle \\ &\quad\times\exp[\mathrm{i}k(|\boldsymbol{\rho}-\boldsymbol{r}_1|^2-|\boldsymbol{\rho}-\boldsymbol{r}_2|^2)/(2L)] \\ &\quad\times\langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho})+\psi^*(\boldsymbol{r}_2,\boldsymbol{\rho})]\rangle, \end{aligned} \tag{10}$$

where the superscript $*$ denotes the conjugate value, and

$$\begin{aligned}&U_\mathrm{d}(\boldsymbol{r}_1,0)U_\mathrm{d}^*(\boldsymbol{r}_2,0) \\ &= \sum_{n=1}^{N}\sum_{m=1}^{N} A_nA_m^*\exp\left[-\frac{1}{2}k(\alpha_n+\alpha_m^*)r^2\right], \end{aligned} \tag{11}$$

$$\begin{aligned}&\langle\exp[\psi_i(\boldsymbol{\rho},\boldsymbol{r}_1)+\psi_i^*(\boldsymbol{\rho},\boldsymbol{r}_2)]\rangle \\ &= \exp[-D(0,\boldsymbol{r}_-)/2]\simeq\exp[-r_-^2/\rho_\mathrm{T}^2], \end{aligned} \tag{12}$$

where $D(0,\boldsymbol{r}_-)$ is the wave structure function of a spherical wave in turbulence, $p_\mathrm{T}=[1.46k^2L\int_0^1 C_n^2(\xi\cos(\theta)L)(1-\xi)^{5/3}\mathrm{d}\xi]^{-3/5}$ is the coherence length of the spherical wave,[12] $C_n^2[tL\cos(\theta)]$ is the model for the fractive-index structure constant in the atmosphere, and θ is the zenith angle of the target.

Substituting Eqs. (2), (11), and (12) into Eq. (10), the mean intensity can be obtained by

$$\begin{aligned}\langle I(\boldsymbol{\rho},L)\rangle &= [k/(2\pi L)]^2\sum_{n}^{N}\sum_{m}^{N}A_nA_m^*\iint\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2 \\ &\quad\times\exp\left[-\frac{1}{2}k(\alpha_n r_1^2+\alpha_m^* r_2^2)\right] \\ &\quad\times\exp[-r_-(l_\mathrm{c}^{-2}+2\rho_\mathrm{T}^{-2})] \\ &\quad\times\exp[\mathrm{i}k(2\boldsymbol{r}_+\cdot\boldsymbol{r}_- - 2\boldsymbol{r}_-\cdot\boldsymbol{\rho})/(2L)] \\ &= \sum_{n}^{N}\sum_{m}^{N}\frac{A_nA_m^*}{B_{nm}E_{nm}}\exp\left[-\frac{\rho^2}{E_{nm}}\right], \end{aligned} \tag{13}$$

where $\boldsymbol{r}_+=(\boldsymbol{r}_1+\boldsymbol{r}_2)/2$, $B_{nm}=k(\alpha_n+\alpha_m^*)/2$, $C_{nm}=k(\alpha_n-\alpha_m^*)/2$, and $E_{nm}=(1+\mathrm{i}C_{nm}L/k)^2/B_{nm}-(B_{nm}+l_\mathrm{c}^{-2}+\rho_\mathrm{T}^{-2})L^2/k^2$.

For a GSM beam, $B=2/W_0^2$ and $C=\mathrm{i}/F_0$, then equation (13) reduces to the well-known form[12]

$$\langle I(\boldsymbol{\rho}_1,L)\rangle = \frac{W_0^2}{W_\mathrm{G}^2}\exp[-\frac{2\rho^2}{W_\mathrm{G}^2}], \tag{14}$$

where

$$W_\mathrm{G}^2 = W_n^2[(1-L/F_n)^2+2L(1+W_n^2/l_\mathrm{c}^2+W_n^2/\rho_\mathrm{T}^2)/(kW_n^2)].$$

The mean intensity from other sources after propagating over a distance in turbulence can be calculated by directly using Eq. (11). The results for collimated flat-topped beams with $N=10$ are presented in Fig. 2, and those from Dan's theory[14] for the mean intensity of flat-topped beams propagating in turbulence are also given for comparison. It shows that the curves of the two theories agree well.

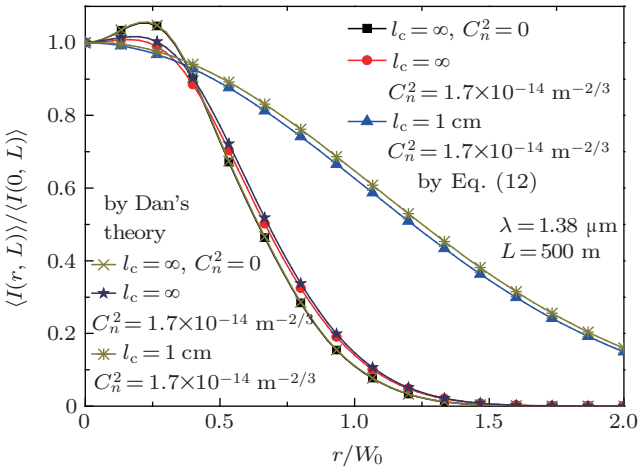


Fig. 2. (color online) Normalized intensity of collimated flat-topped beams after propagating over a distance of L.

3. The second moment of the received field from a rough target

Assume the field of a general beam is scattered by a target in turbulence and received in the plane $z=0$, but not exactly the same place with the source. The target surface is perpendicular to the axis of the beam and much larger than the beam spot. The surface is fully rough that the second and fourth moments of the scattered field can be expressed as

$$\begin{aligned}&\langle U_\mathrm{s}(\boldsymbol{\rho}_1,L)U_\mathrm{s}^*(\boldsymbol{\rho}_2,L)\rangle \\ &= (4\pi R_\mathrm{d}/k^2)\langle I(\boldsymbol{\rho}_1,L)\rangle\delta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2), \end{aligned} \tag{15}$$

$$\begin{aligned}&\langle U_\mathrm{s}(\boldsymbol{\rho}_1,L)U_\mathrm{s}^*(\boldsymbol{\rho}_2,L)U_\mathrm{s}(\boldsymbol{\rho}_3,L)U_\mathrm{s}^*(\boldsymbol{\rho}_4,L)\rangle \\ &= (4\pi R_\mathrm{d}/k^2)^2\langle I(\boldsymbol{\rho}_1,L)\rangle\langle I(\boldsymbol{\rho}_3,L)\rangle \\ &\quad\times[\delta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)\delta(\boldsymbol{\rho}_3-\boldsymbol{\rho}_4)+\delta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_4)\delta(\boldsymbol{\rho}_3-\boldsymbol{\rho}_2)], \end{aligned} \tag{16}$$

where R_d is the equivalent average reflection index of the rough surface.

Again by using the extended Huygens–Fresnel principle, the MCF of the field at the receiver is written as

$$\begin{aligned}\Gamma_2(\boldsymbol{p}_1,\boldsymbol{p}_2,2L) &= \langle U(\boldsymbol{p}_1,2L)U^*(\boldsymbol{p}_2,2L)\rangle \\ &= [k/(2\pi L)]^2\int\int\mathrm{d}\boldsymbol{\rho}_1\,\mathrm{d}\boldsymbol{\rho}_2\langle U_s(\boldsymbol{\rho}_1,L)U_s^*(\boldsymbol{\rho}_2,L)\rangle \\ &\quad\times\exp[\mathrm{i}k(|\boldsymbol{p}_1-\boldsymbol{\rho}_1|^2-|\boldsymbol{p}_1-\boldsymbol{\rho}_2|^2)/(2L)] \\ &\quad\times[\exp[\psi(\boldsymbol{\rho}_1,\boldsymbol{p}_1)+\psi^*(\boldsymbol{\rho}_2,\boldsymbol{p}_2)]\rangle. \end{aligned} \tag{17}$$

Using Eq. (15), equation (17) can be transformed into

$$\begin{aligned}&\Gamma(\boldsymbol{p}_1,\boldsymbol{p}_2,2L) \\ &= \frac{R_\mathrm{d}}{\pi L^2}\int\mathrm{d}\boldsymbol{\rho}\langle I(\boldsymbol{\rho},L)\rangle \\ &\quad\times\exp\{\mathrm{i}k[p_1^2-p_2^2-2\boldsymbol{p}_-\cdot\boldsymbol{\rho}]/(2L)\}\exp[-D(0,\boldsymbol{p}_-)/2] \\ &= \exp[-(p_-/\rho_\mathrm{T})^{5/3}]\exp[\mathrm{i}k(p_1^2-p_2^2)/(2L)] \end{aligned}$$

$$\times \frac{2R_{\rm d}}{L^2}\int_0^{\infty} {\rm d}\rho\rho\langle I(\rho,L)\rangle J_0(kp_-\rho/L), \tag{18}$$

where $\boldsymbol{p}_- = \boldsymbol{p}_1 - \boldsymbol{p}_2$.

Letting $\boldsymbol{p}_1 = \boldsymbol{p}_2$ in Eq. (18), the mean intensity at the receiver is obtained as

$$\langle I(\boldsymbol{p},2L)\rangle = \Gamma(\boldsymbol{p}_1,\boldsymbol{p}_2,2L) = \frac{2R_{\rm d}}{L^2}\int {\rm d}\rho\rho\langle I(\rho,L)\rangle. \tag{19}$$

Substituting Eq. (13) into Eqs. (18) and (19), the MCF and the mean intensity of the received field from a general source can be written as

$$\begin{aligned}\Gamma(p_-) &= \frac{2R_{\rm d}}{L^2}\exp[-D(p_-)/2 + {\rm i}k(p_1^2 - p_2^2)/(2L)] \\ &\quad\times \sum_{n=1}^{N}\sum_{m=1}^{N}\frac{2A_nA_m^*}{B_{nm}}\exp\left[-\frac{k^2E_{nm}p_-^2}{4L^2}\right],\end{aligned} \tag{20}$$

$$I(\boldsymbol{\rho},L) = \frac{2R_{\rm d}}{L^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{2A_nA_m^*}{B_{nm}}. \tag{21}$$

The coherent properties of the speckle field can be characterized by using the degree of coherence (DOC),

$$\begin{aligned}&\gamma(\boldsymbol{p}_1,\boldsymbol{p}_2,2L) \\ &= |\Gamma(\boldsymbol{p}_1,\boldsymbol{p}_2,2L)|/[\langle I(\boldsymbol{p}_1,2L)\rangle\langle I(\boldsymbol{p}_2,2L)\rangle]^{1/2}.\end{aligned} \tag{22}$$

Substituting Eqs. (20) and (21) into Eq. (22) yields

$$\begin{aligned}\gamma(p_-,2L) &= \exp[-(p_-/p_{\rm T})^{5/3}]\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{2A_nA_m^*}{B_{nm}} \\ &\quad\times\exp\left[-\frac{k^2E_{nm}p_-^2}{4L^2}\right]\Big/\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{2A_nA_m^*}{B_{nm}}.\end{aligned} \tag{23}$$

For a GSM beam, equation (23) reduces to

$$\gamma(p_-,2L) = \exp[-(p_-/p_{\rm T})^{5/3}]\exp\left[-\frac{k^2W_G^2p_-^2}{8L^2}\right]. \tag{24}$$

Figure 3 shows the curves of DOC of the speckle field from the GSM sources and collimated flat-topped sources. It can be found from Fig. 3(a) that the curves of the partially coherent and divergent sources decrease more rapidly than those of the coherent and convergent ones, respectively. In other words, the larger the beam spot on the target, the steeper the curve of DOC of the received field. This is also applicable for other kinds of beams. Equation (18) shows that the MFC of the received field can be divided into two parts: one part is the coherence function of a spherical wave in turbulence, which is independent of the source; while the other part is the MCF of the speckle field without considering turbulence effect in the backward path, which is the Fourier transform of the intensity profile in the plane of target. The property of the Fourier transform requires that the wider the intensity profile on the target, the sharper the coherent function for all kinds of sources. Thus, the curves in Fig. 3(b) for collimated flat-topped beams are sharper, but with longer tails than Gaussian functions.

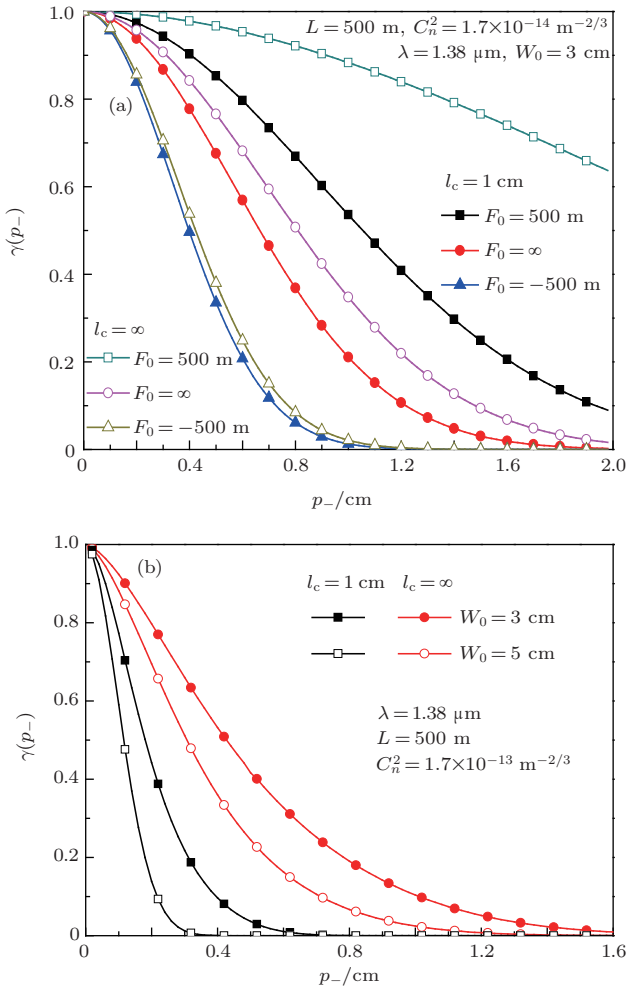


Fig. 3. (color online) DOC of the received field scattered by a rough surface in turbulence. (a) GSM beam; (b) collimated flat-topped beam.

The falling speed of the DOC of the received field can be described by coherence length ρ_0 which is defined by $\gamma(p_0) = \exp(-1)$. The ρ_0 is also the diameter of the speckle of the received field. By using the approximation $(p_-/\rho_{\rm T})^{5/3} \approx (p_-/\rho_{\rm T})^2$, the coherent length of the reflected wave of a GSM beam can be expression as

$$\rho_0 = 2\sqrt{2}L\rho_{\rm T}/\sqrt{k^2W_{\rm G}^2\rho_{\rm T}^2 + 8L^2}. \tag{25}$$

While the coherence length of the speckle field from the collimated flat-topped beam can be numerically calculated with the definition.

Figure 4 presents the curves of coherence length of the received field from two kinds of sources with turbulence strength which is measured by $\sigma_{\rm T}^2 = 0.5623k^{7/6}L^{11/6}\int_0^1 C_n^2[tL\cos(\theta)]t^{5/6}(1-t)^{5/6}\,{\rm d}t$. In weak turbulence ($\sigma_{\rm T}^2 \le 0.3$), the coherence length of the speckle field from a partially coherent source is larger than that from a coherent one, which is consistent with Fig. 3. As turbulence gets stronger, the divergence between the curves becomes smaller, while in a very strong turbulence the curves may coincide with each other as shown in Fig. 4(a).

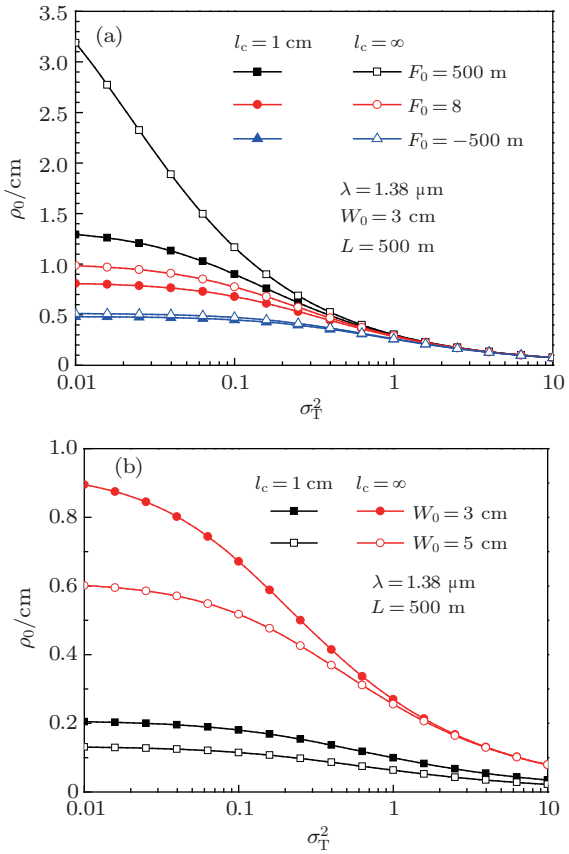


Fig. 4. (color online) Coherence length of the received field scattered by a rough target versus turbulence strength. (a) GSM beam; (b) collimated flat-topped beam.

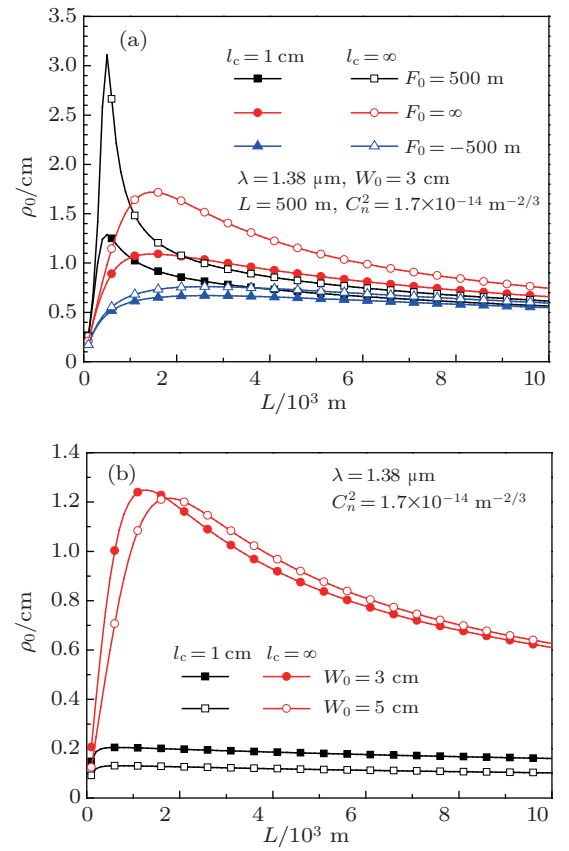


Fig. 5. (color online) Coherence length of the received field scattered by a rough target versus propagating distance. (a) GSM beam; (b) collimated flat-topped beam.

Figure 5 presents the curves of coherence length of the received field from two kinds of sources with the propagating distance. As the propagating distance increases, the coherence length increases at first, and then tends to be a constant value. However, the curves for some beams, such as the convergent GSM beams and coherent collimated flat-topped beams, may reach a peak value at some distance and then decrease to a constant value. It is also found that a flat-topped source may generate smaller speckles than a GSM source with the same beam radius and coherence length, which means that higher-order beams generate wider spots in the same environment.

4. Normalized variance of the received intensity

Similar to Eq. (17) for the MCF, the correlation function of the received intensity is expressed as[2]

$$\begin{aligned} & B_{\mathrm{I}}(\boldsymbol{p}_1,\boldsymbol{p}_2,2L) \\ &= \langle U(\boldsymbol{p}_1,2L)U^*(\boldsymbol{p}_1,2L)U(\boldsymbol{p}_2,2L)U^*(\boldsymbol{p}_2,2L)\rangle \\ &= [k/(2\pi L)]^4 \int\int\int\int \mathrm{d}\boldsymbol{\rho}_1\,\mathrm{d}\boldsymbol{\rho}_2\,\mathrm{d}\boldsymbol{\rho}_3\,\mathrm{d}\boldsymbol{\rho}_4 \\ &\quad \times \langle U_{\mathrm{s}}(\boldsymbol{\rho}_1,L)U_{\mathrm{s}}^*(\boldsymbol{\rho}_2,L)U_{\mathrm{s}}(\boldsymbol{\rho}_3,L)U_{\mathrm{s}}^*(\boldsymbol{\rho}_4,L)\rangle \\ &\quad \times \exp[\mathrm{i}k(|\boldsymbol{p}_1-\boldsymbol{\rho}_1|^2-|\boldsymbol{p}_1-\boldsymbol{\rho}_2|^2+|\boldsymbol{p}_2-\boldsymbol{\rho}_3|^2 \\ &\quad -|\boldsymbol{p}_2-\boldsymbol{\rho}_4|^2)/(2L)+H(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\boldsymbol{\rho}_3,\boldsymbol{\rho}_4;\boldsymbol{p}_1,\boldsymbol{p}_2)], \end{aligned} \tag{26}$$

where $H(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\boldsymbol{\rho}_3,\boldsymbol{\rho}_4;\boldsymbol{p}_1,\boldsymbol{p}_2) = \langle \psi(\boldsymbol{p}_1,\boldsymbol{\rho}_1)+\psi^*(\boldsymbol{p}_1,\boldsymbol{\rho}_2)+\psi(\boldsymbol{p}_2,\boldsymbol{\rho}_3)+\psi^*(\boldsymbol{p}_2,\boldsymbol{\rho}_4)\rangle$.

Substituting Eq. (16) into Eq. (26), the correlation function becomes

$$\begin{aligned} & B_{\mathrm{I}}(\boldsymbol{p}_1,\boldsymbol{p}_2,2L) \\ &= \frac{R_{\mathrm{d}}^2}{\pi^2L^4}\int\int \mathrm{d}\boldsymbol{\rho}_1\,\mathrm{d}\boldsymbol{\rho}_3\langle I(\boldsymbol{\rho}_1,L)\rangle\langle I(\boldsymbol{\rho}_3,L)\rangle H_1(\boldsymbol{\rho}_1,\boldsymbol{\rho}_3;\boldsymbol{p}_1,\boldsymbol{p}_2) \\ &\quad + \frac{R_{\mathrm{d}}^2}{\pi^2L^4}\int\int \mathrm{d}\boldsymbol{\rho}_1\,\mathrm{d}\boldsymbol{\rho}_3\langle I(\boldsymbol{\rho}_1,L)\rangle\langle I(\boldsymbol{\rho}_3,L)\rangle \\ &\quad \times \exp(\mathrm{i}k\boldsymbol{p}_-\cdot\boldsymbol{\rho}_-/L)H_2(\boldsymbol{\rho}_1,\boldsymbol{\rho}_3;\boldsymbol{p}_1,\boldsymbol{p}_2), \end{aligned} \tag{27}$$

where

$$H_1 = \exp[4C_\chi(\boldsymbol{\rho}_-,\boldsymbol{p}_-)], \tag{28}$$

$$\begin{aligned} H_2 = \exp\{&4C_\chi(\boldsymbol{\rho}_-,\boldsymbol{p}_-)-[2D(\boldsymbol{\rho}_-,0)-D(\boldsymbol{\rho}_-,\boldsymbol{p}_-) \\ &+2D(\boldsymbol{p}_-,0)-D(\boldsymbol{\rho}_-,-\boldsymbol{p}_-)]/2\}, \end{aligned} \tag{29}$$

where $C_\chi(\boldsymbol{\rho}_-,\boldsymbol{p}_-)$ is the log-amplitude covariance of spherical wave in turbulence.

The normalized variance of the received intensity is defined as

$$\sigma_{\mathrm{I}}^2 = B_{\mathrm{I}}(\boldsymbol{p},\boldsymbol{p},2L)/\langle I(\boldsymbol{p},2L)\rangle^2 - 1. \tag{30}$$

Substituting Eqs. (19) and (27) into Eq. (30), the normalized variance becomes

$$\sigma_{\mathrm{I}}^2 = -1+\left\{\frac{1}{\pi}\int\int \mathrm{d}\boldsymbol{\rho}_1\,\mathrm{d}\boldsymbol{\rho}_3\langle I(\boldsymbol{\rho}_1,L)\rangle\langle I(\boldsymbol{\rho}_3,L)\rangle\right.$$

$$\times \exp[4C_\chi(\rho_-,0)]\Big\}\Big/\left(\int_0^\infty \rho\langle I(\rho)\rangle \mathrm{d}\rho\right)^2$$
$$= -1+\sum_n^N\sum_m^N\sum_{n'}^N\sum_{m'}^N \frac{2A_nA_m^*A_{n'}A_{m'}^*}{B_{nm}B_{n'm'}(E_{nm}+E_{n'm'})}$$
$$\times \int_0^\infty \mathrm{d}\rho_-\rho_- \exp[-\rho_-^2/(E_{nm}+E_{n'm'})]$$
$$\times \exp[4C_\chi(\rho_-)]\Big/\left(\sum_{n=1}^N\sum_{m=1}^N \frac{2A_nA_m^*}{B_{nm}}\right)^2. \quad (31)$$

For a further simplification of Eq. (31), we use the form of the log-amplitude covariance proposed by Baykal[16]

$$C_\chi(\rho_-) = \sigma_\chi^2 \exp(-\rho_-^2/\rho_0^2), \quad (32)$$

where σ_χ^2 is the log-amplitude variance of the spherical wave in turbulence with the expression for any turbulence strength[18]

$$\sigma_\chi^2 = \begin{cases} \sigma_\mathrm{T}^2, & \sigma_\mathrm{T}^2 \le 0.3, \\ 2.95\int_0^1 \sigma_\mathrm{T}^2[u(1-u)]^{5/6}\mathrm{d}u\int_0^\infty \mathrm{d}y\frac{\sin^2 y}{y^{11/6}}\exp\{-\sigma_\mathrm{T}^2[u(1-u)]f(y)\}, & \sigma_\mathrm{T}^2 > 0.3, \end{cases} \quad (33)$$

where

$$f(y) = 7.02y^{5/6}\int_{0.7y}^\infty \mathrm{d}x x^{-8/3}[1-J_0(x)]. \quad (34)$$

Additionally, the approximation is utilized as

$$\exp[4C_\chi(\rho_-)] = \exp[4\sigma_\chi^2\exp(-\rho_-^2/\rho_0^2)]$$
$$\simeq 1+[\exp(4\sigma_\chi^2)-1]\exp(-\rho_-^2/\rho_0^2). \quad (35)$$

In weak turbulence, equation (35) reduces to $\exp[4C_\chi(\rho_-)] \simeq 1+4\sigma_\chi^2\exp(-\rho_-^2/\rho_0^2)$. While in strong turbulence, equation (32) ensures that $\exp[4C_\chi(0)] = \exp(4\sigma_\chi^2)$ and $\exp[4C_\chi(\infty)] = 1$.

Substituting Eq. (35) into Eq. (30), the normalized variance becomes

$$\sigma_\mathrm{I}^2 = -1+\sum_n^N\sum_m^N\sum_{n'}^N\sum_{m'}^N \frac{A_nA_m^*A_{n'}A_{m'}^*}{B_{nm}B_{n'm'}}$$
$$\times\{1+[\exp(4\sigma_\chi^2)-1]\rho_0^2/(E_{nm}+E_{n'm'}+\rho_0^2)\}. \quad (36)$$

Figure 6 shows the variation of the normalized intensity variance of the speckle field versus the log-amplitude variance. Compared with Fig. 4, it can be seen that in a fixed turbulence strength, the larger the coherence length, the larger the normalized variance. Combining with the above analysis, it can be known that a larger beam spot on the target induces a smaller normalized variance of the received intensity, which is the so-called "target aperture averaging effect". As the fields scattered from each point on the rough target add incoherently at the receiving point, which is equivalent to the process that intensity from a point source in turbulence adds in a receiving aperture, and the illuminating area on the target acts as the aperture and has an averaging effect on the intensity fluctuation caused by turbulence.

In weak turbulence, the intensity fluctuation caused by turbulence is small, which induces a small variance of the received intensity. As turbulence increases, intensity fluctuation caused by turbulence gets saturated and the averaged normalized variance also reaches a maximum value. As turbulence further increases, the beam spot continues to increase and the averaging effect is also enhanced, which leads the intensity variance caused by turbulence to approach zero. It is notable that the normalized variance caused by a rough target is not affected by the beam spot which is always unity.

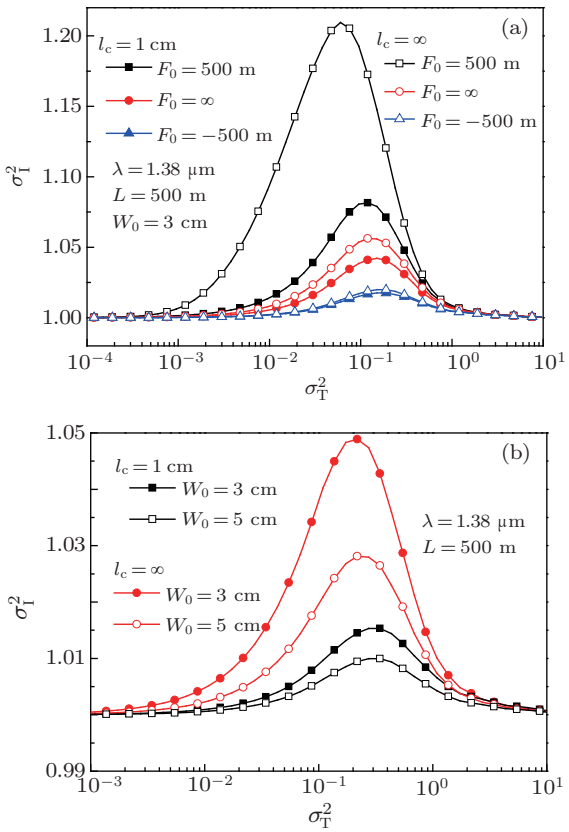


Fig. 6. (color online) Normalized variance of the received intensity versus turbulence strength. (a) GSM beam; (b) collimated flat-topped beam.

5. Conclusion

In the paper, an applicable method is described to analyze the second-order and fourth-order moments of the speckle field in atmospheric turbulence from an arbitrary rotationally

symmetric source. The analytical expressions for the coherence property and the scintillation property of the speckle field are derived by decomposing the general beam into a sum of partially coherent Gaussian beams. Some general conclusion can be deduced from the results of two special beams.

The statistical moments of speckle field from a rough target in turbulence are mainly affected by the intensity profile on the target as well as the complex fluctuations of a spherical wave propagating from the target to the receiver. A larger spot on the target produces smaller speckles and a smaller normalized variance of the received intensity.

Turbulence effects influence the intensity fluctuations of the received wave in two ways. (i) In the forward propagation, turbulence causes beam spreading which makes the DOC decrease more rapidly and the "aperture averaging effect" increase. (ii) In the backward propagation, turbulence causes additional fluctuations on the field, which decrease the coherence of the received field and increase the intensity fluctuation. The intensity fluctuation caused by turbulence becomes saturated in strong turbulence, but the beam spot size will continuously increase. Therefore, in a very strong turbulence, the turbulence effects only affect the coherence length of the speckle field and do not induce intensity fluctuation.

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