Solving the atmospheric scattering optical transfer function using the multi-coupled single scattering method*

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The atmospheric scattering optical transfer function (OTF) is solved by applying the multi-coupled single scattering (MCSS) method to the three-dimensional radiative transfer equation (RTE) under the periodic ground condition. This approach is a direct hit to the atmospheric scattering OTF using the same original context of modulation transfer function (MTF) measurement, i.e., images of sinusoidal grating at different spatial frequencies. Both the amplitude and phase shift of the OTF at various zenith and azimuth angles can be obtained at an arbitrary spatial frequency.

Keywords: atmospheric scattering, optical transfer function, modulation transfer function, multi-coupled single scattering method

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1. Introduction

In high-resolution images from satellites, blurring caused by atmospheric scattering and turbulence severely impairs the image quality.[1–3] Atmospheric turbulence is usually corrected by self-adaptive or active optics,[5–8] while atmospheric scattering can only be partially corrected by using software approaches, such as the image enhancement based on the atmospheric optical transfer function (OTF),[4] which is directly connected to the atmospheric conditions, e.g., aerosol and cloud optical depths and their micro-physical properties.

The first step in correcting image blurring caused by atmospheric scattering is accurately modeling the atmospheric scattering OTF. Solving the atmospheric scattering OTF is a three-dimensional (3D) radiative transfer problem in nature, and has been studied since the late 1950s.[9,10] There are two different approaches: most works focus on the solution of the point-spread function (PSF),[11,12] while Lyapustin [13] developed an excellent approach based on the Fourier transformation of the radiative transfer equation (RTE), which indicates a periodic ground condition (not point δ).

In the PSF, it is hard to describe the multi-reflection between the atmosphere and the ground, while Lyapustin's approach is a straightforward simulation of the ground–atmosphere radiative transfer over a spatially non-uniform surface. So in this paper, we use the spatially periodic condition of both the ground reflecting ratio and the radiative field in the atmosphere, reduce two derivative terms in the 3D RTE , and apply the multi-coupled single scattering (MCSS)[14] method to the RTE for the final solution.

2. Model and method

2.1. 3D radiative transfer equation

The most general form of the RTE is a description of an infinitesimal variation of radiative intensity $I(x,y,z;\vartheta,\varphi)$ along a line segment $\mathrm{d}\boldsymbol{s} = (\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$

$$-\frac{1}{\alpha(x,y,z)}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)I(x,y,z;\vartheta,\varphi) = I(x,y,z;\vartheta,\varphi)+\Im(x,y,z;\vartheta,\varphi), \quad (1)$$

where $l=\cos\varphi\sin\vartheta$, $m=\sin\varphi\sin\vartheta$, $n=\cos\vartheta$, and $\alpha(x,y,z)$ is the extinction ratio at point (x,y,z). We have three derivative terms and one integral term in the RTE, making it impossible to have an analytic solution and hard to apply numerical methods.

2.2. DC and AC parts of RTE

The 3D RTE can be reduced significantly under the following plausible conditions:

1) Lambert ground, albedo varies only along the x axis:[13] $I^{\uparrow}(x,z=0;\vartheta,\varphi)=R(x)E^{\downarrow}(x)/\pi$;

2) ground's reflection albedo is periodic: $R(x)=R_0+R_f\,\mathrm{e}^{\mathrm{i}fx}$;

3) the extinction ratio varies only in the z direction: $\alpha(x,y,z)=\alpha(z)$.

The term $m\partial/\partial y$ in RTE (1) disappears under condition 1. Furthermore, we can separate the radiative intensity and the ground albedo into DC and AC parts as

$$I(x,z;\vartheta,\varphi) = \bar{I}(z;\vartheta,\varphi)+\widetilde{I}(x,z;\vartheta,\varphi), \quad (2)$$

$$\Im(x,z;\vartheta,\varphi) = \bar{\Im}(z;\vartheta,\varphi)+\widetilde{\Im}(x,z;\vartheta,\varphi), \quad (3)$$

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$$R(x) = \bar{R} + \widetilde{R}(x). \tag{4}$$

Then RTE (1) becomes

$$\begin{aligned} & -\frac{1}{\alpha(z)}\, l\, \frac{\partial}{\partial z}\, \bar{I}(z;\vartheta,\varphi) \\ &= \bar{I}(z;\vartheta,\varphi) + \bar{\mathfrak{I}}(z;\vartheta,\varphi)\,, \end{aligned} \tag{5}$$

$$\begin{aligned} & -\frac{1}{\alpha(z)}\left(l\frac{\partial}{\partial x} + n\frac{\partial}{\partial z}\right)\widetilde{I}(x,z;\vartheta,\varphi) \\ &= \widetilde{I}(x,z;\vartheta,\varphi) + \widetilde{\mathfrak{I}}(x,z;\vartheta,\varphi), \end{aligned} \tag{6}$$

with the boundary condition

$$\begin{aligned} & \bar{I}^{\uparrow}(z=0;\vartheta,\varphi) \\ &= \frac{\bar{R}}{\pi}\left[\,|\mu_0|\,\pi F_0 + \int_{2\pi} |\mu'|\,\bar{I}^{\downarrow}(z=0;\vartheta',\varphi')\,\mathrm{d}\Omega'\right], \end{aligned} \tag{7}$$

$$\begin{aligned} & \widetilde{I}^{\uparrow}(x,z=0;\vartheta,\varphi) \\ &= \frac{\widetilde{R}(x)}{\pi}\left[\,|\mu_0|\,\pi F_0 + \int_{2\pi} |\mu'|\,\bar{I}^{\downarrow}(z=0;\vartheta',\varphi')\,\mathrm{d}\Omega'\right] \\ &\quad + \frac{[\bar{R}+\widetilde{R}(x)]}{\pi}\int_{2\pi} |\mu'|\,\widetilde{I}^{\downarrow}(x,z=0;\vartheta',\varphi')\,\mathrm{d}\Omega'. \end{aligned} \tag{8}$$

The DC component is not influenced by the AC part of intensity and ground albedo, yet the DC part of radiative intensity and ground albedo is coupled to the AC intensity through boundary condition (8) at the ground. The AC component of frequency f may stimulate harmonics (e.g., $2f, 3f, \ldots$) through multiple ground–atmosphere reflections specified by boundary condition (8), whereas the realistic aerosol atmosphere is forward-scattering predominant, leading the harmonic components to be negligible.[13] So one simplification of Eq. (8) can be made for the aerosol or cloudy atmosphere as follows:

$$\begin{aligned} & \widetilde{I}^{\uparrow}(x,z=0;\vartheta,\varphi) \\ &= \frac{\widetilde{R}(x)}{\pi}\left[\,|\mu_0|\,\pi F_0 + \int_{2\pi} |\mu'|\,\bar{I}^{\downarrow}(z=0;\vartheta',\varphi')\,\mathrm{d}\Omega'\right] \\ &\quad + \frac{\bar{R}}{\pi}\int_{2\pi} |\mu'|\,\widetilde{I}^{\downarrow}(x,z=0;\vartheta',\varphi')\,\mathrm{d}\Omega'. \end{aligned} \tag{9}$$

So the complete solution needs two steps: (i) using the ordinary one-dimensional (1D) radiative transfer code to obtain the DC part of the light intensity, (ii) solving the AC part using the DC downward light as the energy source as described in Eq. (9).

2.3. Simplification with periodic ground condition and forward scattering predominant atmosphere

If we have the DC part of the light intensity in hand, and the ground albedo is periodic $R(x) = R_f\,\mathrm{e}^{\mathrm{i}fx}$, then boundary condition (8) guarantees that the ground reflected light stimulated by the downward DC intensity has spatial frequency f, while the harmonic components of higher frequencies are neglected due to condition (9).

So we approximate the intensity field as

$$\widetilde{I}(x,z;\vartheta,\varphi) = \widetilde{I}(z;\vartheta,\varphi)\,\mathrm{e}^{\mathrm{i}f(x\,-\,z\tan\vartheta\cos\varphi)}. \tag{10}$$

Note that $\widetilde{I}(z;\vartheta,\varphi)$ may be a complex number which permits a spatial phase shift in the process of radiative transfer. The phase element $-fz\tan\vartheta\cos\varphi$ is intrinsic for the propagation of the reflected light at the ground, which is the main portion of the radiative energy inside the atmosphere.

This assumption of the light intensity in the atmosphere is precise in order to describe the upward radiative transfer of light stimulated by the DC intensity at periodic ground albedo, while only approximately tenable because the harmonic component of higher frequency is neglected. Substituting Eq. (10) into the AC part of Eq. (6), we obtain

$$\begin{aligned} & -\frac{1}{\alpha(z)/\mu}\frac{\partial}{\partial z}\widetilde{I}(z;\vartheta,\varphi) \\ &= \widetilde{I}(z;\vartheta,\varphi) + \frac{1}{4\pi}\int_{4\pi}\widetilde{I}(z;\vartheta',\varphi')\,\mathrm{e}^{-\mathrm{i}fz(\tan\vartheta'\cos\varphi'-\tan\vartheta\cos\varphi)} \\ &\quad \times p(\vartheta,\varphi;\vartheta',\varphi')\,\mathrm{d}\Omega'. \end{aligned} \tag{11}$$

Equation (11) has only one derivitive term which makes it look like an ordinary 1D RTE with the following formal solution:

$$\begin{aligned} & \widetilde{I}(z;\vartheta,\varphi) \\ &= \widetilde{I}(0;\vartheta,\varphi)\,\mathrm{e}^{-\frac{\tau(0,z)}{|\mu|}} + \frac{1}{4\pi}\int_0^z \mathrm{e}^{-\frac{\tau(z',z)}{|\mu|}}\,\frac{\alpha(z')\,\mathrm{d}z'}{|\mu|} \\ &\quad \times \int_{4\pi}\widetilde{I}(z';\vartheta',\varphi')\,\mathrm{e}^{-\mathrm{i}fz'(\tan\vartheta'\cos\varphi'-\tan\vartheta\cos\varphi)} \\ &\quad \times p(\vartheta,\varphi;\vartheta',\varphi')\,\mathrm{d}\Omega'. \end{aligned} \tag{12}$$

Equation (12) has a very comprehensible physical meaning that the interference effect caused by light of different direction $\widetilde{I}(z';\vartheta',\varphi')\,\mathrm{e}^{-\mathrm{i}fz'(\tan\vartheta'\cos\varphi'-\tan\vartheta\cos\varphi)}$ makes the scattering source function and the modulation transfer function (MTF) degraded.

Now the basic algorithm of MCSS[13] is still applicable with the formal solution modified to the complex version (12):

1) approximate $\widetilde{I}(z';\vartheta',\varphi')$ by $\widetilde{I}(0;\vartheta',\varphi')\,\mathrm{e}^{-\frac{\tau(0,z')}{|\mu'|}}$;

2) single scattering approximation: no further extinction of scattering source function;

3) the extinction coefficient is uniform inside the optically thin layer: $\alpha(z) = \alpha$.

Finally, we obtain the single scattering kernel and use the incident light at the boarder of a single scattering layer to calculate the reflected and the transmitted lights

$$\widetilde{I}(z;\vartheta,\varphi) = \widetilde{I}(0;\vartheta,\varphi)\,\mathrm{e}^{-\frac{\tau(0,z)}{|\mu|}} + \frac{1}{4\pi}\int_{4\pi}\widetilde{I}'(\vartheta',\varphi')\;p(\vartheta,\varphi;\vartheta',\varphi')\,\mathrm{d}\Omega',$$

$$\widetilde{I}'(\vartheta',\varphi') = \int_0^z \widetilde{I}(z';\vartheta',\varphi')\,\mathrm{e}^{-\mathrm{i}fz'(\tan\vartheta'\cos\varphi'-\tan\vartheta\cos\varphi)}\,\frac{\alpha(z')\,\mathrm{d}z'}{|\mu|}$$

$$= \frac{|\mu'|}{|\mu|}\,\frac{\alpha\cos\vartheta\cos\vartheta'[1-\mathrm{e}^{-\frac{\alpha z}{|\mu'|}}\,\mathrm{e}^{-\mathrm{i}fz(\tan\vartheta'\cos\varphi'-\tan\vartheta\cos\varphi)}]}{\alpha\cos\vartheta\cos\vartheta'+\mathrm{i}\;f\;|\mu'|(\cos\vartheta\cos\varphi'\sin\vartheta'-\sin\vartheta\cos\varphi\cos\vartheta')}\,\widetilde{I}(0,\vartheta',\varphi'). \tag{13}$$

3. Validation and error analysis

3.1. Result at zero frequency

When the spatial frequency is 0, the theory in Section 2 degenerates to the normal 1D radiative transfer of plane-parallel atmosphere. This serves as a special case of the OTF calculation, validates both the algorithm design and the software implementation. Let us assume the DC component of the ground albedo to be $\bar{R} = 0.25$, and the AC one to be $\widetilde{R}(x) = 0.55\,\mathrm{e}^{\mathrm{i}10x}$. Then the problem is equivalent to the 1D radiative transfer with ground albedo $R = 0.8$. Here we show the results calculated for the Rayleigh scattering atmosphere with optical thickness $\tau = 1$, the values of $\bar{R}$, $\widetilde{R}$, and τ are chosen to compare with the results calculated by Natraj.[15] When dealing with the ground reflection, we choose Eq. (8) instead of Eq. (9) because zero frequency will not stimulate harmonics and the Rayleigh scattering atmosphere is not predominantly forward scattering.

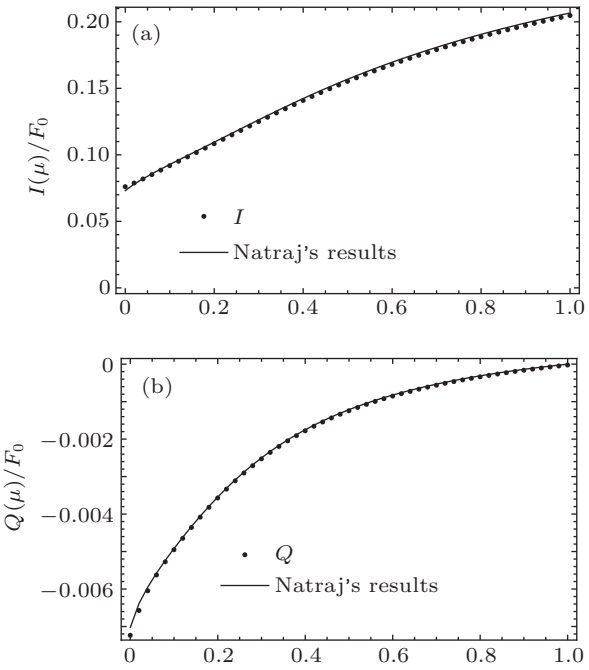


Fig. 1. Comparison of OTF calculation at $\bar{R} = 0.25$, $\widetilde{R} = 0.55$, $f = 0$ to the substraction of Natraj's result at albedo 0.8 and 0.25: (a) up-scattering $I(\mu)$, (b) up-scattering $Q(\mu)$. Solar zenith is 126.9° ($\mu_0 = -0.6$), scattering light's azimuth is 0.

The result (Fig. 1) shows that the difference of up-scattering light intensity between ground albedo 0.25 and 0.8 is indeed the same as that in the OTF calculation with $\bar{R} = 0.25$ and $\widetilde{R} = 0.55$. This verifies the MCSS method in atmosphere OTF calculation and its software realization. Polarization is also incorporated in the OTF calculation without introducing too much complexity by virtue of the simplicity of the MCSS algorithm.

3.2. Convergence of solutions and accuracy

The error in the MCSS method is generated from the approximation from integral to summation and single scattering.[14] Since similar results are rarely found, we analyze the accuracy of the solution by its property of convergency as the number of zenith/azimuth's quadrature points N_p and the number of single-scattering layers N_l approach $+\infty$.

Limited by the ability of computers, we currently use the most accurate case with $N_\mathrm{p} = 360$ for a quadrature region of $[0,\ \pi]$ and $N_\mathrm{l} = 1000$ for an aerosol optical thickness of 0.42 as the standard solution, the error increase due to the decreases of N_p and N_l is presented in Tables 1 and 2.

Table 1. Relative error at different N_p, same $N_\mathrm{l} = 1000$.

N_p	90	120	180	360
Amplitude error/%	–0.010	–0.017	–0.0012	0
Phase error/%	–0.044	–0.0053	0.0086	0

Table 2. Relative error at different N_l, same $N_\mathrm{p} = 180$.

N_l	200	500	800	1000
Amplitude error/%	–8.12	–4.75	–1.80	–0.012
Phase error/%	–72.71	–37.20	–11.71	0.0086

The result shows that increasing N_p and N_l will bring better results, especially the increase of N_l. The amplitude is more steady than the phase with the variations of N_p and N_l. This complex version of MCSS needs finer angle and layer division than the 1D one because the interference effect caused by the phase shift in formal solution (12) is very sensitive. According to Table 2, the amplitude accuracy is higher than 95%.

4. Example

4.1. Definition of atmosphere scattering OTF and MTF

Before diving into the realistic aerosol atmosphere, we first define the atmosphere scattering MTF, which is the module of the OTF. The OTF is a measure of the capability of an

optical system preserving the original contrast ratio of an object at different spatial frequencies. As the result at $f = 0$ is $I_0^{\uparrow}$, and that at arbitrary spatial frequency f is $I_f^{\uparrow}$, the OTF can be suitably defined as

$$H(f) = I_f^{\uparrow}/I_0^{\uparrow}, \tag{14}$$

and the MTF is given by

$$M(f) = |H(f)|. \tag{15}$$

The $M(f)$ should be less than or equal ($f = 0$) to 1. Although the DC part of the up-scattering light (e.g., light scattered purely by atmosphere) will definitely reduce the contrast ratio of the image, we do not embrace it in the OTF here, because it is easy to be taken out using image processing methods, which is a standard step in the on-board MTF testing.[16]

Suppose we have an optical system (with MTF $= M_{\text{inst}}$) loaded on board a satellite, and include atmosphere as part of the imaging system, then the MTF of the whole system should be the multiplication of the atmosphere's and instrument's MTFs

$$M(f) = M_{\text{atm}}(f) \times M_{\text{inst}}(f). \tag{16}$$

4.2. MTF of aerosol atmosphere

The example (Fig. 2) used here has ground condition $\bar{R} = 0.3$ and $\widetilde{R} = 0.1$ at two different aerosol optical depths (0.42 and 0.63 at 555 nm). The aerosol atmosphere has the ordinary two-peak ($r_1 = 0.18$ μm, $r_1 = 4.2$ μm, $\sigma_1 = 0.73$ μm, $\sigma_2 = 0.99$ μm) log-normal size spectrum of volume,[17,18] the particle's refractive index is $n_{\text{Re}} = 1.61$, $n_{\text{Im}} = 0.033$ for $r < 1$ μm, and $n_{\text{Re}} = 1.50$, $n_{\text{Im}} = 0.051$ for $r > 1$ μm, and the volume ratio of the two peaks is 0.65.

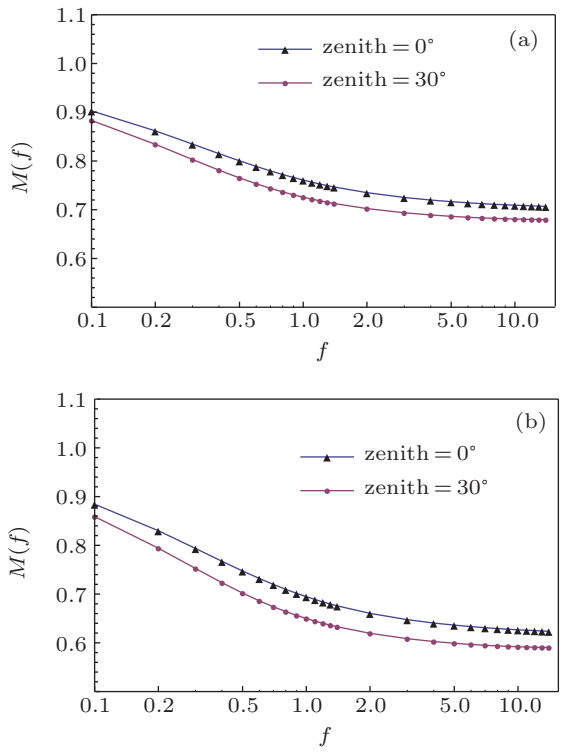


Fig. 2. (color online) MTFs for aerosol optical depths of (a) 0.42 and (b) 0.63.

In this example, at spatial frequency 1 m^{-1} (Fig. 3), the MTF degrade caused by atmosphere scattering is 0.76 and 0.69 for aerosol optical thickness 0.42 and 0.63, respectively. The polarization components (Q and U) are not shown here because they are relatively small compared to the intensity (I), this is understandable because the blurring of a sinusoidal stripe is caused mainly by forward scattering, which is not strongly polarized.

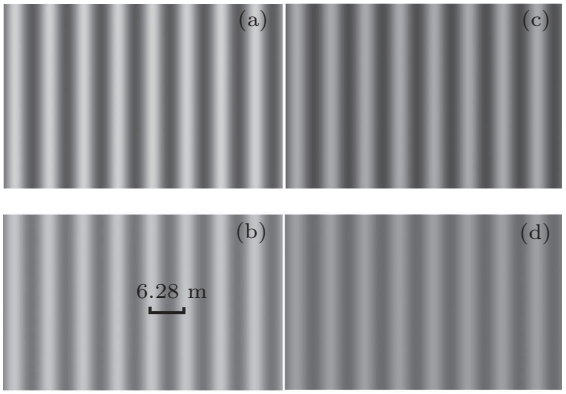


Fig. 3. Visualization of sinusoidal stripes with $f = 1$ at aerosol optical depth: (a), (b) $\tau_a = 0.42$ and (c), (d) $\tau_a = 0.63$. Panels (a) and (c) are the boundary reflecting pictures at ground, and panels (b) and (d) are the degraded ones at the top of the atmosphere.

5. Conclusion

We have developed a rigorous analytical method for calculating a simplified 3D radiative transfer problem with the periodic ground condition, which is physically equivalent to solving the atmosphere scattering OTF. We follow the basic deduction of Lyapustin,[13] and apply the reliable MCSS[13] 1D RT algorithm at the source code level, which insures the reliability of the OTF calculation.

This algorithm could be the basic procedure of image enhancement copying the pixel adjacent effect, e.g., determining the parameter of MTF compensation in the spatial frequency domain. According to Lyapustin,[3] the OTF solution serves as a fundamental core in the radiative transfer with a spatially heterogeneous anisotropic surface. So, the results in this paper can provide a theoretical support to the design of correction equipments to the high-resolution space camera which suffers from atmosphere blurring. The measurement of MTF degeneracy and a numerical example of the pixel adjacent effect based on this algorithm will be described in succeeding works.

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