Synchronous implementation of optoelectronic NOR and XNOR logic gates using parallel synchronization of three chaotic lasers*

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The parallel synchronization of three chaotic lasers is used to emulate optoelectronic logic NOR and XNOR gates via modulating the light and the current. We deduce a logical computational equation that governs the chaotic synchronization, logical input, and logical output. We construct fundamental gates based on the three chaotic lasers and define the computational principle depending on the parallel synchronization. The logic gate can be implemented by appropriately synchronizing two chaotic lasers. The system shows practicability and flexibility because it can emulate synchronously an XNOR gate, two NOR gates, and so on. The synchronization can still be deteceted when mismatches exist with a certain range.

Keywords: chaos, synchronization, optical logic, semiconductor laser

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1. Introduction

Chaotic synchronization has been extensively investigated in the last two decades[1–3] due to the unpredictability, randomness, sensitivity to initial conditions, and easy realization of synchronization of chaotic signals. Chaotic synchronization has been developed as a promising technique for improving security in communications.[4–8] Recently, a new research area involving the chaotic laser dynamics based computation received much attention.[9–12] A chaotic system can be used to perform simple mathematical calculations or to simulate logic operations. If the systems have the capability of emulating logic gates, they can be used to perform universal computing with appropriate combinations, all possible functions can be programmed. Over the past ten years, Ditto *et al.* made great efforts to design or improve many logic gates.[9,10] Recently, Chlouverakis reported an optoelectronic logic gate.[11] In this paper, the realization of the fundamental XNOR and NOR gates is implemented via controlling the parallel synchronization of three lasers. The logic operations are achieved. The system is used to emulate synchronously an XNOR gate, two NOR gates, and so on. It can program more logic functions than the dual-laser-gate system.[12–15] So it is more practicable and flexible. This method is a new application involving the chaotic optics computation.

2. Parallel synchronization

Dynamic behaviors of injection lasers have been studied theoretically and experimentally, investigating a route to chaos from bifurcation with adding injection, and finding self-pulse, quasi-period, undamped oscillation and chaos.[1] A lot of chaotic lasers and their synchronization systems have been presented.[16–18] In our work, an injection laser as a drive system drives three injection lasers with feedback paths as the response systems; the three response lasers obtain synchronization themselves. The synchronization in the model uses the closed-loop scheme. This parallel synchronization scheme can be used in logic implementation, as shown in Fig. 1. The equations used to describe the parallel synchronization and the logical operation are[1,10–18]

$$\begin{aligned}
\frac{\mathrm{d}E_{\mathrm{I}}}{\mathrm{d}t} &= \frac{1}{2}(G_{\mathrm{I}}-\gamma)E_{\mathrm{I}}+\frac{k}{\tau_L}E_{\mathrm{m}}\cos(\varphi_{\mathrm{I}}),\\
\frac{\mathrm{d}\varphi_{\mathrm{I}}}{\mathrm{d}t} &= \frac{1}{2}\beta_{\mathrm{c}}(G_{\mathrm{I}}-\gamma)+\frac{k}{\tau_L}\frac{E_{\mathrm{m}}}{E_{\mathrm{I}}}\sin(-\varphi_{\mathrm{I}})-\Delta\omega_{\mathrm{I}},\\
\frac{\mathrm{d}N_{\mathrm{BI}}}{\mathrm{d}t} &= \eta_{\mathrm{i}}\frac{I}{q}-\gamma_{\mathrm{BQ}}N_{\mathrm{BI}}+\gamma_{\mathrm{QB}}N_{\mathrm{I}},\\
\frac{\mathrm{d}N_{\mathrm{I}}}{\mathrm{d}t} &= \gamma_{\mathrm{BQ}}N_{\mathrm{BI}}-(\gamma_{\mathrm{e}r}+\gamma_{\mathrm{QB}})N_{\mathrm{I}}-G_{t,r}V_{\mathrm{p}}E_{\mathrm{I}}^2,\\
\frac{\mathrm{d}E_r}{\mathrm{d}t} &= \frac{1}{2}(G_r-\gamma)E_r+\frac{k}{\tau_L}E_{\mathrm{m}}\cos(\varphi_r)+(f/\tau_L)\\
&\quad\times[(E_{\mathrm{I}}+\varepsilon_r\times S_r)\cos(\varphi_{\mathrm{I}}-\varphi_r)-E_r],\\
\frac{\mathrm{d}\varphi_r}{\mathrm{d}t} &= \frac{1}{2}\beta_{\mathrm{c}}(G_r-\gamma)+\frac{k}{\tau_L}\frac{E_{\mathrm{m}}}{E_r}\sin(-\varphi_r)\\
&\quad+(f/\tau_L)[(E_{\mathrm{I}}+S_r)/E_r]\sin(\varphi_{\mathrm{I}}-\varphi_r)-\Delta\omega_r,\\
\frac{\mathrm{d}N_{\mathrm{B}r}}{\mathrm{d}t} &= \eta_{\mathrm{i}}\frac{I\times(1+\rho_r\times S_r)}{q}-\gamma_{\mathrm{BQ}}N_{\mathrm{B}r}+\gamma_{\mathrm{QB}}N_r,\\
\frac{\mathrm{d}N_r}{\mathrm{d}t} &= \gamma_{\mathrm{BQ}}N_{\mathrm{B}r}-(\gamma_{\mathrm{e}r}+\gamma_{\mathrm{QB}})N_r-G_{1,2}V_{\mathrm{p}}E_r^2,
\end{aligned}\tag{1,2}$$

where subscripts I and r ($r=1,2,3$) represent the drive system and the response systems, respectively, $E(t)$ and φ(t) are the amplitude and the phase, respectively, N is the number of

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carriers in the active region, N_B is the number of carriers in the barrier regions, E_m is the injection light amplitude, k is the injection factor, $\gamma_p = v_p(\alpha_m + \alpha_{int})$ represents the total photon loss with α_m and α_{int} being the cavity and the internal losses, $\tau_L = 2Ln_g/c$ represents the round-trip time in the cavity of length L, c is the light velocity in a vacuum, $n_g = c/v_g$ is the group refractive index, η_i represents the internal quantum efficiency, β_c represents the linewidth enhancement, I represents the current, q represents the unit charge, γ_{BQ} represents the loss of carriers going away from the SCH layer, γ_{QB} represents the loss of carriers escaping from the active region, $\Delta\omega$ is the detuning between the frequencies of the master laser (M-L) and the salver laser (S-L), f is the drive-feedback factor, S is the logic input signal, which is used to modulate the current or the output light, and ρ_r and ε_r are the modulation depths of the current intensity and the light amplitude, respectively. The nonlinear gain is described by

$$G = [(\Gamma g_0 v_g)/(1 + E^2/E_s^2)]\lg[(N + N_s)/(N_{th} + N_s)],$$

where $\Gamma = V/V_p$ represents the mode confinement coefficient, V and V_p are the cavity and the mode volumes, respectively, g_0 represents the gain constant, v_g represents the group velocity of the photons, $E_s = (P_s/V_p)^{1/2}$ represents the amplitude at saturation with saturation photon number P_s, $N_s = n_s V$ represents the logarithmic gain with the density n_s, and $N_{th} = n_{th}V$ represents the carrier number at transparency with n_{th} being the density. The total carrier loss in the active layer is described by

$$\gamma_e = A_n + B_n(N/V) + C_A(N/V)^2,$$

where A_n, B_n, and C_A represent the non-radiative recombination rate, the radiative recombination coefficient, and the Auger recombination coefficient, respectively.

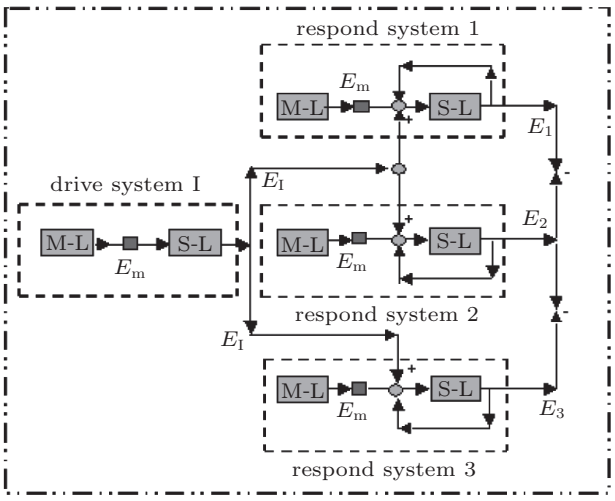


Fig. 1. Synchronization used for a fundamental logic gate.

The chaos shift key (CSK) is widely used in chaotic cryptography.[1–4] This technique improves on the system for a logic gate. The system can be used to emulate logic gates via modulation of the current and the drive light. Expressions (1) and (2) are regarded as logical computational equations that govern chaotic synchronization, logical input, and logical output. By controlling the logic input signal, a logical computational principle is deduced as: when the two response systems obtain synchronization themselves, the logical implementation results in output 1; when the two response systems desynchronize, the logical implementation results in output 0.

3. Three NOR implementations

The system is used to emulate three NOR gates via modulating the current and the drive light based on the CSK. The parameters are listed in Table 1.[12–15] The normalized parameters are $k = 0.19$, $E_m = 0.26$, and $f = 0.8$. The detunings are $\Delta\omega_I = 2.02$ GHz, $\Delta\omega_1 = 2$ GHz, $\Delta\omega_2 = 1.98$ GHz, and $\Delta\omega_3 = 1.9$ GHz, considering the laser frequency alteration and parameter mismatches in all logical calculations. To obviously distinguish synchronization from desynchronization, the modulation depths are set to be $\rho_1 = 2\%$, $\rho_2 = 0$, $\rho_3 = 0$, $\varepsilon_1 = 0$, $\varepsilon_2 = 1.5\%$, and $\varepsilon_3 = 3\%$, where the system 1 current is modulated by the current modulator and the drive light is modulated by the optical amplitude modulators before systems 2 and 3. The logic input signals S_1, S_2, and S_3 alter between 0 and 1 while their modulation rates are 30 Mbit/s, 60 Mbit/s, and 90 Mbit/s, respectively, where the square wave signals in Matlab are used to simulate the logic input signals.

Table 1. Laser parameters.

Symbol	Value	Unit	Symbol	Value	Unit
L	1200	μm	B_r	1.0×10^{-10}	cm^3/s
V	50.4	μm^3	C_A	5.0×10^{-29}	cm^6/s
Γ	0.045		P_s	2.2×10^7	
n_g	3.6		g_0	2700	cm^{-1}
n_s	0.1×10^{18}		I	50	mA
α_m	11.5	cm^{-1}	n_i	0.8	
α_{int}	20	cm^{-1}	γ_{BQ}	2.5×10^{10}	s^{-1}
n_{th}	2.1×10^{18}	cm^{-3}	γ_{QB}	5×10^9	s^{-1}
A_{nr}	2.5×10^8	s^{-1}			

When response system 2 cooperates with response system 1, a logical implementation is operated. The logical computational principle is expressed as: when $(S_2, S_1) = (0,0)$, then the lasers operate at the same currents, the two chaotic lights are equal, and the synchronization is realized. If $(S_2, S_1) = (1,1)$, or $(S_2, S_1) = (1,0)$, or $(S_2, S_1) = (0,1)$, then a synchronization error or a desynchronization arises. Hence, one can easily observe the logic performances of the NOR gate by detecting the difference, $(E_2 - E_1)$, between the two chaotic signals. The result is illustrated by Fig. 2. Figure 2(a) shows the subtraction of $E_2 - E_1$. When the synchronization is achieved between the systems, namely, $E_2 = E_1$, $S_2 = 0$, and $S_1 = 0$, the logical operation is accomplished while the logical

output is 1. When $(S_2,\ S_1) \neq (0,0)$, a synchronization error arises, then the logical output is 0. Thus, simply detecting the desynchronization and the synchronization regions, one observes the performances of the logical NOR gate. Figure 2(b) shows the logical output sequences.

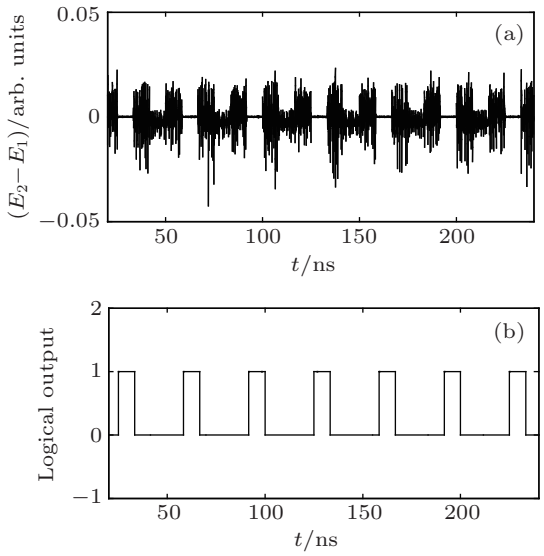


Fig. 2. Realization of NOR implementation: (a) the difference between the two chaotic signals, (b) the output of the NOR gate.

When response system 2 cooperates with response system 3, the second logical implementation is realized. If $(S_2,\ S_3) = (0,0)$, then the lasers operate at the same currents, the two chaotic lights are equal, and the synchronization is achieved. If $(S_2,\ S_3) = (1,1)$, or $(S_2,\ S_3) = (1,0)$, or $(S_2,\ S_3) = (0,1)$, i.e., $(S_2,\ S_3) \neq (0,0)$, then a synchronization error or a desynchronization occurs. The operation of the NOR gate is based on detecting the difference, $(E_2 - E_3)$, between the two chaotic signals. The result is illustrated in Fig. 3. The NOR gate characterizes all optics.

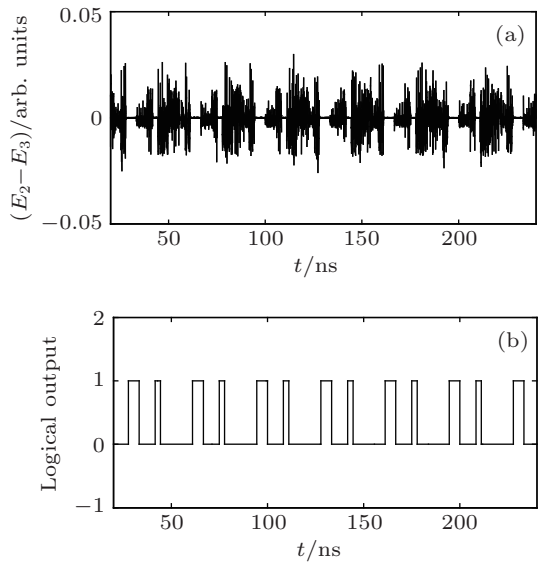


Fig. 3. Realization of the second NOR implementation: (a) the difference between the two chaotic signals, and (b) the output of the NOR gate.

When response system 1 cooperates with response system 3, one achieves the third logical implementation. If $(S_1,\ S_3) = (0,0)$, then the lasers operate at the same currents and the two chaotic lights are equal, which results in synchronization. If $(S_1,\ S_3) = (1,0)$, or $(S_1,\ S_3) = (0,1)$, or $(S_1,\ S_3) = (1,1)$, i.e., $(S_1,\ S_3) \neq (0,0)$, then a desynchronization or a synchronization error ocurrs. The operations of the logic NOR gate are based on detecting the difference $(E_1 - E_3)$. The result is shown in Fig. 4.

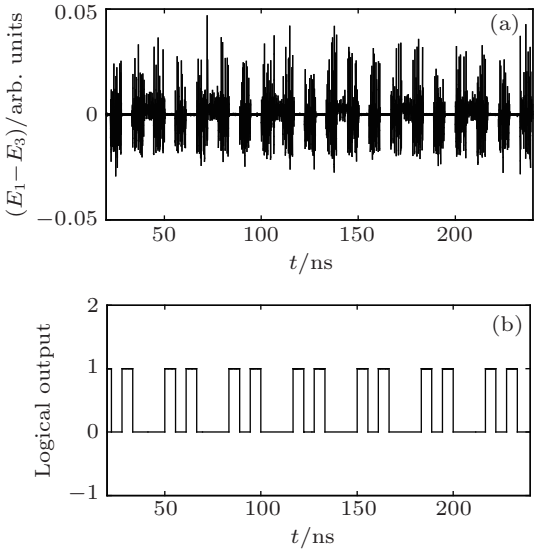


Fig. 4. Realization of the third NOR implementation: (a) the difference between the two chaotic signals, and (b) the output of the NOR gate.

4. Synchronous XNOR and NOR implementations

The system is used to emulate synchronously an XNOR gate and two NOR gates. It shows practicability and flexibility. The modulation depths are set to be $\rho_1 = \rho_2 = 2\%$, $\rho_3 = 0$, $\varepsilon_1 = \varepsilon_2 = 0$, and $\varepsilon_3 = 1\%$. The logic input signals S_1, S_2, and S_3 alter between 0 and 1, and their bit rates are 20 Mbit/s, 40 Mbit/s, and 80 Mbit/s, respectively.

4.1. XNOR implementation

When we let response system 1 coordinate with response system 2, a logical XNOR operation is implemented. The system is used as an XNOR gate to simulate logic operations. The logical calculation principle is defined as: if $(S_1,\ S_2) = (1,1)$, or $(S_1,\ S_2) = (0,0)$, i.e., $S_1 = S_2$, the inputs are identical, then the synchronization is realized, and the logical output is 1. If $(S_3,\ S_2) = (0,1)$, or $(S_1,\ S_2) = (1,0)$, i.e., $S_2 \neq S_1$, then a synchronization error or a desynchronization arises, and the logical output is 0. Detecting the difference, $(E_2 - E_1)$, between the two chaotic signals, one can observe the operations of the logic XNOR gate. The operation is implemented and shown in Fig. 5. The difference $E_2 - E_1$ is illustrated in Fig. 5(a). When $S_2 = S_1$, the synchronization is achieved between response systems 2 and 1, and the logical operation gives output

1. When $S_1 \neq S_2$, a desynchronization occurs, the logical operation gives output 0. Figure 5(b) shows the logical output sequences.

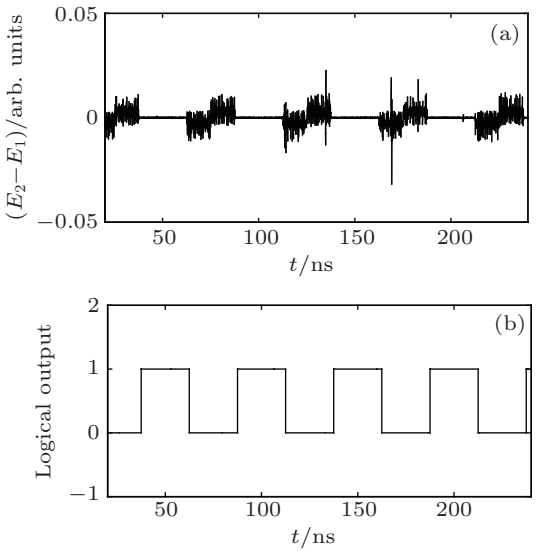


Fig. 5. Realization of XNOR implementation: (a) the difference between the two chaotic signals, and (b) the output of the XNOR gate.

4.2. Two NOR implementations

One NOR logical implementation can be realized. If $(S_2,\ S_3) = (0,0)$, then the lasers operate at the same currents, the two chaotic lights are equal, and the synchronization is achieved. If $(S_2,\ S_3) = (1,1)$, or $(S_2,\ S_3) = (1,0)$, or $(S_2,\ S_3) = (0,1)$, i.e., $(S_2,\ S_3) \neq (0,0)$, then a desynchronization or a synchronization error arises. The operations of the logic NOR gate can be observed by detecting the difference $(E_2 - E_3)$. The results are shown in Fig. 6.

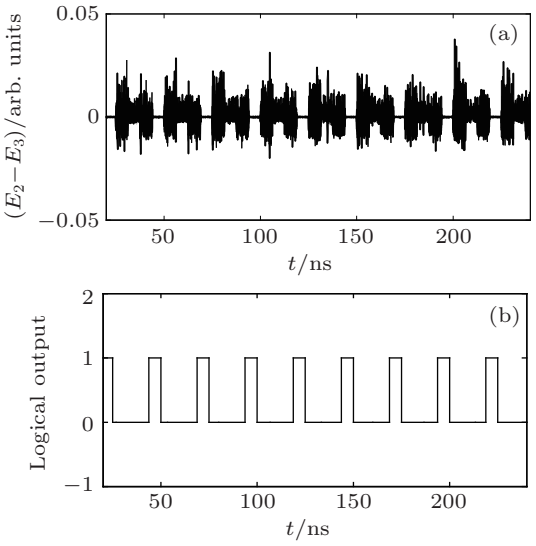


Fig. 6. NOR implementation: (a) the difference between the two chaotic signals, and (b) the output of the NOR gate.

When response system 1 cooperates with response system 3, another NOR logical implementation is realized. When $(S_1,\ S_3) = (0,0)$, the synchronization is achieved. When $(S_1,\ S_3) = (1,1)$, or $(S_1,\ S_3) = (1,0)$, or $(S_1,\ S_3) = (0,1)$, i.e., $(S_1,\ S_3) \neq (0,0)$, then a desynchronization or a synchronization error arises. The results are shown in Fig. 7.

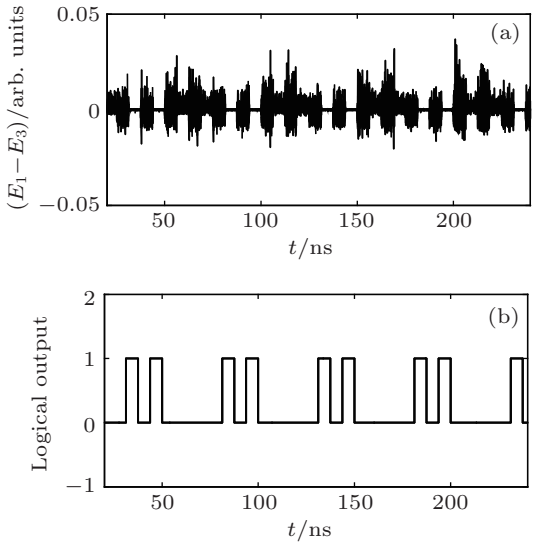


Fig. 7. Another NOR implementation: (a) the difference between the two chaotic signals, and (b) the output of the gate.

5. Parameter mismatch

We simply discuss a parameter mismatch case. Unsynchronization differences are $\langle|E_1 - E_3|\rangle = 0.0733$, $\langle|E_2 - E_3|\rangle = 0.0732$, and $\langle|E_1 - E_3|\rangle = 0.0722$. When we have injection E_m for laser I, $1.01E_\mathrm{m}$ for laser 1, $0.99E_\mathrm{m}$ for laser 2, and $1.015E_\mathrm{m}$ for laser 3, the mismatch results in synchronization errors $\langle|E_1 - E_3|\rangle = 1.56 \times 10^{-4}$, $\langle|E_2 - E_3|\rangle = 7.97 \times 10^{-4}$, and $\langle|E_1 - E_3|\rangle = 6.38 \times 10^{-4}$. Due to $0.0733/1.6 \times 10^{-4} \approx 456$, $0.0732/7.97 \times 10^{-4} \approx 94$, and $0.0722/6.38 \times 10^{-4} = 113$, the synchronization or unsynchronization regions can be distinguished by detecting the three ratios. When the detunings are $\Delta\omega_\mathrm{I} = 2.02$ GHz, $\Delta\omega_1 = 2$ GHz, $\Delta\omega_2 = 1.98$ GHz, and $\Delta\omega_3 = 1.9$ GHz, the synchronization errors are $\langle|E_1 - E_3|\rangle = 1.29 \times 10^{-4}$, $\langle|E_2 - E_3|\rangle = 4.3 \times 10^{-5}$, and $\langle|E_1 - E_3|\rangle = 8.59 \times 10^{-5}$. Owing to $0.0733/1.29 \times 10^{-4} = 568$, $0.0732/4.3 \times 10^{-5} = 1.7 \times 10^3$, and $0.0722/8.59 \times 10^{-5} = 840$, one can easily distinguish the synchronization regions from the unsynchronization ones when the ratio of unsynchronization difference to synchronization error is greater than 50. When the currents are taken as $I_1 = 0.999I_\mathrm{I}$, $I_2 = 0.998I_\mathrm{I}$, and $I_3 = 0.997I_\mathrm{I}$, the mismatch results in ratios of unsynchronization difference to synchronization error being 260, 520, and 514, the synchronization or unsynchronization regions are distinguished. When taking $\beta_{c1} = 1.001\beta_{cI}$, $\beta_{c2} = 0.998\beta_{cI}$, and $\beta_{c3} = 1.003\beta_{cI}$, the mismatch results in ratios of unsynchronization difference to synchronization error being 123, 48.8, and 83.9. We can still detect the synchronization regions. So the system allows a certain range of mismatch, i.e., the injection mismatch should be lower than $1\%E_\mathrm{m}$, the detuning mismatch should be lower than 0.02 GHz, the current mismatch

should be lower than $0.1\%I_1$, and the linewidth enhancement mismatch should be lower than $0.2\%\beta_{cI}$.

6. Conclusion

The parallel synchronization scheme has been used to implement logic gates. The logic gates are successfully implemented because the system can emulate synchronously an XNOR gate, two NOR gates, and so on. It has more logic functions than the dual-laser-gate system.[3–5] The detection of synchronization can still be realized when the mismatch is within a certain range. If the gates are appropriately programmed, all possible functions can be realized. The system shows practicability and flexibility and has good prosperity. The results show an important chaotic application in the field of optical logic computing. The proposed system can be further simplified. The M-Ls in the response systems can be removed and the output from the M-L in the drive system can be used as the injection light of the S-Ls in response systems.

References

[1] Xiao Y, Deng T, Wu Z M, Wu J G, Lin X D, Tang X, Zeng L B and Xia G Q 2012 *Opt. Commun.* **285** 1442
[2] Yan S L 2005 *Chin. Phys. Lett.* **22** 2504
[3] Annovazzi-Lodi V, Donati S and Scirè A 1996 *J. IEEE Quantum Electronics* **32** 953
[4] Zhu S Q, Lü X, Chen X F, Thornbrug K S Jr, VanWiggeren G D and Roy R 2000 *Chin. Phys.* **9** 337
[5] Fan L, Xia G Q and Wu Z M 2009 *Acta Phys. Sin.* **58** 0989 (in Chinese)
[6] Yan S L 2012 *Acta Phys. Sin.* **61** 160505 (in Chinese)
[7] Liu M, Zhang M J, Wang A B, Wang L S, Ji Y N and Ma Z 2013 *Acta Phys. Sin.* **62** 064209 (in Chinese)
[8] Li X F, Pan W, Ma D, Luo B, Zhang W L and Xiong Y 2006 *Acta Phys. Sin.* **55** 5094 (in Chinese)
[9] Sinha S and Ditto W L 1998 *Phys. Rev. Lett.* **81** 2156
[10] Murali K, Sina S and Ditto W L 2003 *Int. J. Bifur. Chaos Appl. Sci. Eng.* **13** 2669
[11] Chlouverakis K E and Adams M J 2005 *Electron. Lett.* **41** 359
[12] Yan S L 2010 *Chin. Opt. Lett.* **8** 1147
[13] Yan S L 2012 *Optics & Laser Technology* **44** 83
[14] Yan S L 2012 *Acta Phys. Sin.* **61** 160505 (in Chinese)
[15] Yan S L 2011 *Chin. Sci. Bull.* **56** 1264
[16] Zhang M J, Liu T G, Li P, Wang A B, Zhang J Z and Wang Y C 2011 *IEEE Photon. Technol. Lett.* **23** 1872
[17] Wu J, Wu Z, Xia G Q, Deng T, Lin X, Tang X and Feng G 2011 *IEEE Photon. Technol. Lett.* **23** 1854
[18] Xiang S Y, Pan W, Wen A J, Li N Q, Zhang L Y, Shang L and Zhang H X 2013 *IEEE Photon. Technol. Lett.* **25** 587