Generation of countless embedded trumpet-shaped chaotic attractors in two opposite directions from a new three-dimensional system with no equilibrium point*

Sun Chang-Chun(孙常春)†

School of Science, Shenyang Jianzhu University, Shenyang 110168, *China*

(Received 18 December 2013; revised manuscript received 27 February 2014; published online 9 July 2014)

A new three-dimensional (3D) continuous autonomous system with one parameter and three quadratic terms is presented firstly in this paper. Countless embedded trumpet-shaped chaotic attractors in two opposite directions are generated from the system as time goes on. The basic dynamical behaviors of the strange chaotic system are investigated. Another more complex 3D system with the same capability of generating countless embedded trumpet-shaped chaotic attractors is also put forward.

Keywords: three-dimensional system, trumpet-shaped chaotic attractor, equilibrium point

PACS: 05.45.Gg, 05.45.–a, 05.45.Pq **DOI:** 10.1088/1674-1056/23/9/090502

1. Introduction

With the development of natural science and technology, some new research results on data-driven[1,2] and chaos synchronization[3–6] have been obtained. Design approaches of chaotic circuits[7,8] have been presented and improved gradually. New theories and research fruits on chaos can be applied to the engineering fields such as communications, signal processing, and complex networks. Therefore, much attention has been paid to generating and utilizing chaos instead of understanding and restraining chaos in the past.

So far, many strange chaotic attractors have been found in three-dimensional (3D) or higher-dimensional continuous autonomous systems, such as the Lorenz attractor,[9] the Chen attractor,[10] the Lü attractor,[11] and the Chua attractor.[12] There are various basic shapes for the chaotic attractors. Two classes of representative shapes are wing shapes and scroll shapes. Wing-shaped attractors include one-wing,[13,14] double-wing,[9–11] three-wing,[13,14] four-wing,[15–18] n-wing,[19–22] and $n \times m$-wing[19] attractors. Scroll-shaped attractors include double-scroll,[12,23] three-scroll,[24] and n-scroll[25–28] attractors. Most multi-wing or multi-scroll chaotic attractors are generated by constructing a piecewise function and employing a switching approach. Recently, a new switching control approach of generating a compound multi-scroll multi-wing chaotic attractor was put forward and a circuit simulation was designed.[29] Other shapes of known chaotic attractors include the basin shape,[30,31] torus,[32] funnel shape,[33] rectangular,[34] umbrella shape, and ball shape.[35] Hitherto, the quantity of wing-shaped or scroll-shaped attractors generated from the discoverable or designed chaotic systems is countable as time goes on.

The traditional approach to generating multi-wing or multi-scroll chaotic attractors in a 3D continuous system relies on constructing a piecewise function based on the discoverable 3D chaotic systems. The quantity of wing-shaped or scroll-shaped attractors is countable. Moreover, the designed chaotic systems have many equilibrium points.

In contrast to the aforementioned results, a new and important result is derived in this paper. A new 3D quadratic polynomial continuous autonomous chaotic system with no piecewise function is presented. The new system has a strong capability of generating multiple embedded trumpet-shaped chaotic attractors. The quantity of trumpet-shaped attractors is countless as time goes on. Moreover, the new system does not have any equilibrium point. To the best of my knowledge, the novel attractors generated from the presented system are the most complex in topological structure among all the discoverable attractors generated from 3D quadratic polynomial continuous autonomous chaotic systems with no piecewise function up to this time.

In 1994, Sprott[36] presented 19 simple chaotic systems. The first one is different from the rest. That is, the system does not have any equilibrium point. The fact shows that a 3D continuous autonomous system with no equilibrium point can also generate chaos under the condition of particular parameters. To the best of my knowledge, other chaotic systems with no equilibrium point have been little reported so far.

In this paper, the new 3D continuous autonomous chaotic system with one parameter and three quadratic terms is constructed. The chaotic system does not have any equilibrium point. As time goes on, countless embedded trumpet-shaped chaotic attractors in two opposite directions are generated

*Project supported by the Science Research Foundation of Liaoning Provincial Education Department, China (Grant No. L2013229).

†Corresponding author. E-mail: changchunsun@sina.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

from the system. Basic dynamical behaviors including symmetry and dissipation, system equilibria, impact of time, response of the system, impact of the initial value, and variation of the parameter are investigated respectively. Trumpet-shaped chaotic attractors can be generated from the system within a wide range of parameters. Finally, another more complex 3D system with the same capability of generating countless embedded trumpet-shaped chaotic attractors is also presented.

2. A new 3D system

Consider a new 3D continuous autonomous system with one parameter and three quadratic terms

$$\begin{cases} \dot{x} = -2x + 3y, \\ \dot{y} = -18x + 10yz, \\ \dot{z} = a - 2y^2 + 3xy, \end{cases} \tag{1}$$

where a is a positive real parameter.

When $a = 80$ and the initial values are $(x_0, y_0, z_0) = (1, 1, 1)$, system (1) can generate multiple embedded trumpet-shaped chaotic attractors in two opposite directions within $t = 500$ s (see Figs. 1–4).

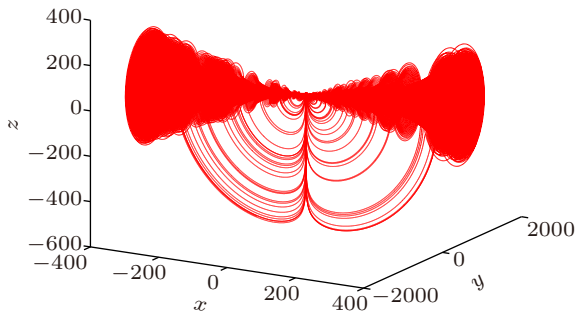


Fig. 1. (color online) Multiple embedded trumpet-shaped chaotic attractors.

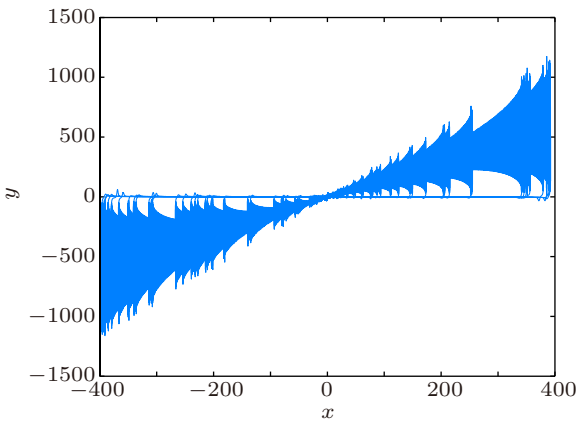


Fig. 2. (color online) Projection of the chaotic attractors in the x–y plane.

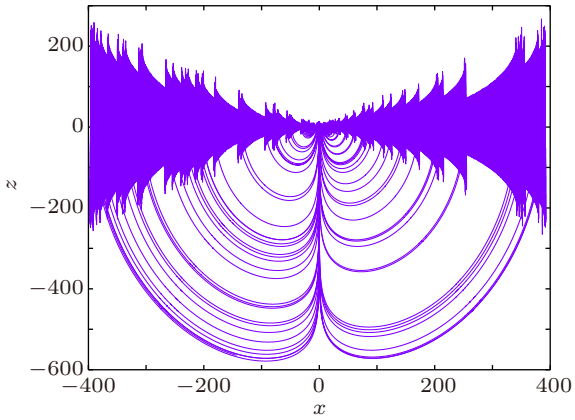


Fig. 3. (color online) Projection of the chaotic attractors in the x–z plane.

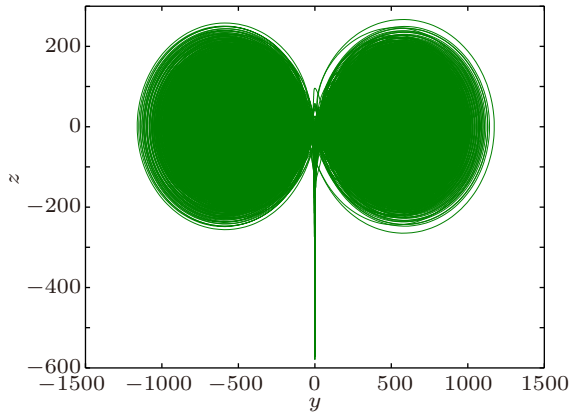


Fig. 4. (color online) Projection of the chaotic attractors in the y–z plane.

3. Analysis of the basic dynamical behaviors

The basic dynamical behaviors of system (1) are investigated in detail by theoretical analysis and numerical simulations in this section.

3.1. Symmetry and dissipation

System (1) exists in symmetry under the coordinate transformation $(x, y, z) \to (-x, -y, z)$. The divergence of system (1) is defined as

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -2 + 10z. \tag{2}$$

When $z < 0.2$, $\nabla V < 0$, system (1) is dissipative.

3.2. System equilibria

By solving the equation $\dot{x} = \dot{y} = \dot{z} = 0$, i.e.,

$$\begin{cases} -2x + 3y = 0, \\ -18x + 10yz = 0, \\ a - 2y^2 + 3xy = 0, \end{cases} \tag{3}$$

an equation can be obtained as follows:

$$a + 2.5y^2 = 0. \tag{4}$$

Because of $a > 0$, equation (4) does not have real solutions. Therefore, system (1) does not have any equilibrium point.

3.3. Impact of time

Take the parameter $a = 80$ and the initial values $(x_0, y_0, z_0) = (1, 1, 1)$, as time goes on, the quantity of embedded trumpet-shaped chaotic attractors increases (see Fig. 5). Finally, countless embedded trumpet-shaped chaotic attractors in two opposite directions will be generated from system (1).

3.4. Response of the system

Take the parameter $a = 80$ and the initial values $(x_0, y_0, z_0) = (1, 1, 1)$, the state response of system (1) within $t = 500$ s is shown in Fig. 6. It shows that the amplitude of every state variable starting from a smaller value becomes relatively larger.

3.5. Impact of the initial value

Let the initial value vary when the parameter is fixed at $a = 80$, the variation of phase diagram of system (1) within $t = 30$ s is shown in Fig. 7. When the initial values are taken as $(x_0, y_0, z_0) = (0, 0, z_0)$, the solution of system (1) is

$$\begin{cases} x = x_0 = 0, \\ y = y_0 = 0, \\ z = at + z_0. \end{cases} \tag{5}$$

Equation (5) shows that the orbit of system (1) will run vertically from the starting point at speed a. Therefore, system (1) cannot generate any chaotic attractor under the condition that the starting point is $(0, 0, z_0)$.

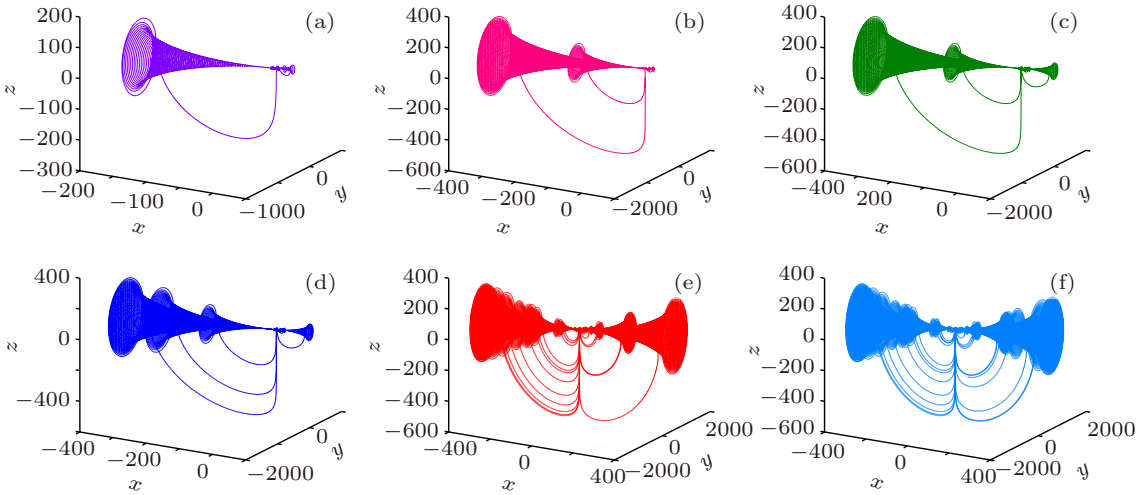


Fig. 5. (color online) The variation of chaotic attractors in quantity: (a) $t = 10$ s, (b) $t = 20$ s, (c) $t = 28$ s, (d) $t = 38$ s, (e) $t = 150$ s, (f) $t = 200$ s.

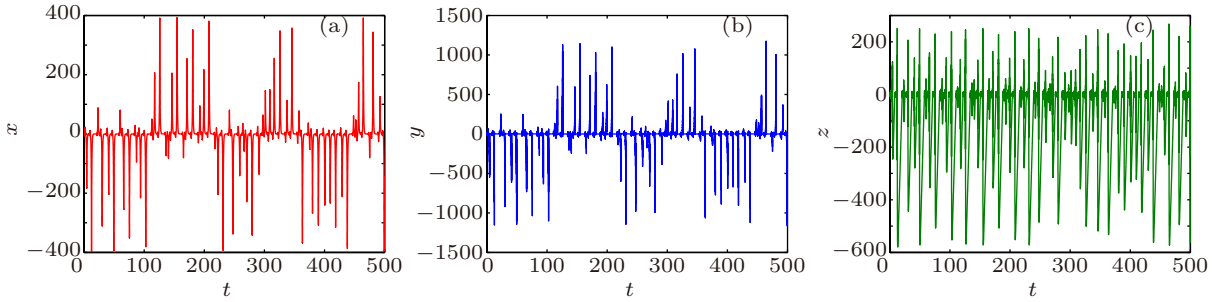


Fig. 6. (color online) State response of system (1): (a) x, (b) y, (c) z.

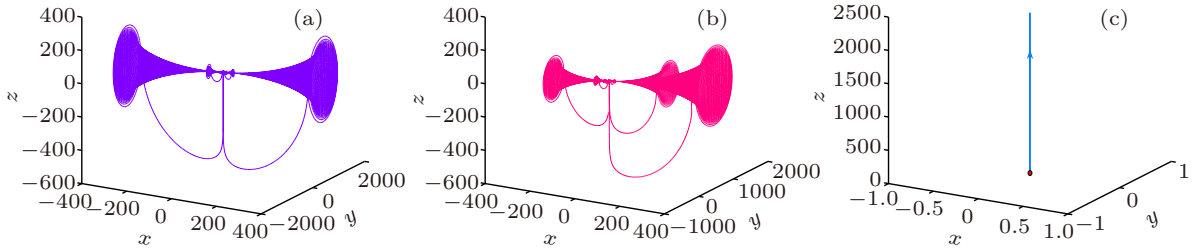


Fig. 7. (color online) The variation of phase diagram with different initial values: (a) $(x_0, y_0, z_0) = (1, 0, 0)$, (b) $(x_0, y_0, z_0) = (0, 1, 0)$, (c) $(x_0, y_0, z_0) = (0, 0, 1)$.

3.6. Variation of the parameter

Let the parameter a vary when the initial values are fixed at $(x_0, y_0, z_0) = (1, 1, 1)$. When $50 \leq a \leq 100$, multiple embedded trumpet-shaped chaotic attractors can be generated from system (1) within $t = 50$ s, as shown in Fig. 8.

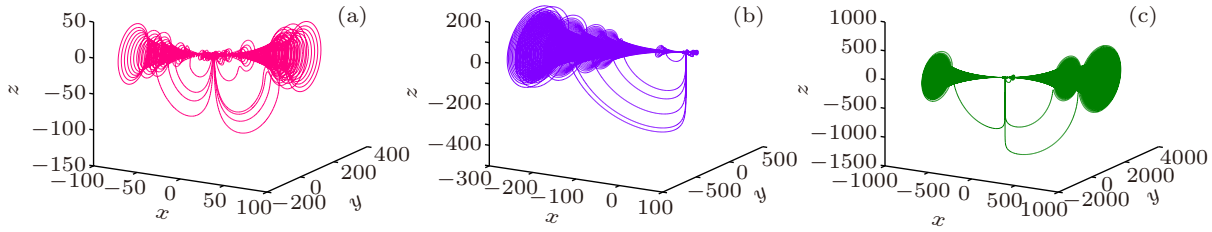


Fig. 8. (color online) The variation with different parameter a: (a) $a = 50$, (b) $a = 75$, (c) $a = 100$.

4. Another more complex 3D system

In this section, the existence of similar embedded trumpet-shaped chaotic attractors in other 3D systems is investigated. Compared to system (1), another more complex 3D system with the same capability of generating countless embedded trumpet-shaped chaotic attractors is put forward as follows:

$$\begin{cases} \dot{x} = -2x + 3y + b, \\ \dot{y} = -18x + 10yz + cy + dz + e, \\ \dot{z} = a - 2y^2 + 3xy. \end{cases} \tag{6}$$

When $a = 90$, $b = 2$, $c = 6$, $d = 3$, $e = -1$, and the initial values are $(x_0, y_0, z_0) = (1, 1, 1)$, system (6) with no equilibrium point can also generate multiple embedded trumpet-shaped chaotic attractors in two opposite directions within $t = 100$ s (see Figs. 9–12).

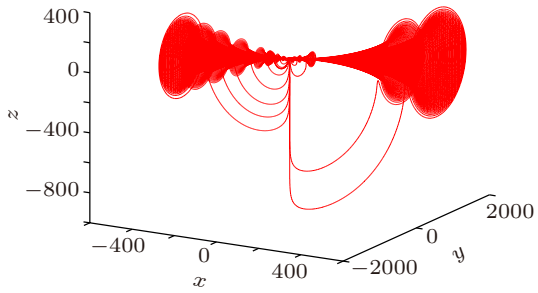


Fig. 9. (color online) Multiple embedded trumpet-shaped chaotic attractors.

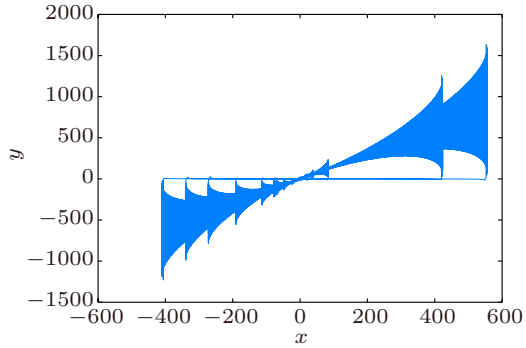


Fig. 10. (color online) Projection of the chaotic attractors in the x–y plane.

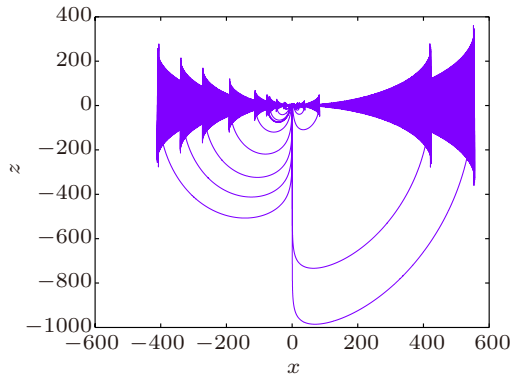


Fig. 11. (color online) Projection of the chaotic attractors in the x–z plane.

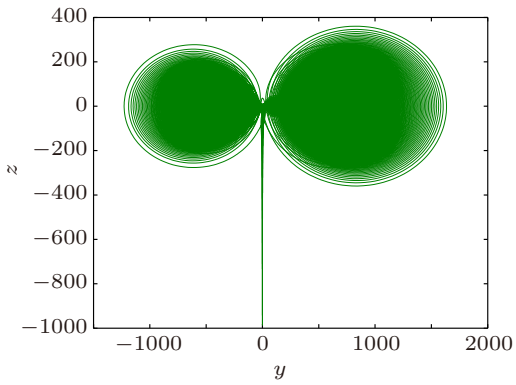


Fig. 12. (color online) Projection of the chaotic attractors in the y–z plane.

5. Conclusions

In this paper, a new 3D chaotic system with one parameter and three quadratic terms is constructed and countless embedded trumpet-shaped chaotic attractors in two opposite directions are generated from the system as time goes on. The system does not have any equilibrium point and has a strong capability of generating embedded trumpet-shaped chaotic attractors within a wide range of parameters. The basic dynamical behaviors of the system are investigated. Finally, another more complex 3D system with the same capability of generating countless embedded trumpet-shaped chaotic attractors in two opposite directions is also presented. Under the condition that the amplitude is large and the attractors are countless as time goes on, an important and challenging problem about the

estimation of the boundary for the strange chaotic attractors is worth investigation.

References

[1] Yin S, Ding S X, Haghani A, Hao H and Zhang P 2012 *J. Process Control* **22** 1567
[2] Yin S, Luo H and Ding S X 2014 *IEEE Trans. Ind. Electron.* **61** 2402
[3] Zhu D R, Liu C X and Yan B N 2012 *Chin. Phys. B* **21** 090509
[4] Zhang R X and Yang S P 2012 *Chin. Phys. B* **21** 080505
[5] Wang Z and Sun W 2013 *Acta Phys. Sin.* **62** 020511 (in Chinese)
[6] Lü L, Yu M, Wei L L, Zhang M and Li Y S 2012 *Chin. Phys. B* **21** 100507
[7] Zhou X Y 2012 *Acta Phys. Sin.* **61** 030504 (in Chinese)
[8] Li G L and Chen X Y 2011 *Acta Phys. Sin.* **60** 090508 (in Chinese)
[9] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130
[10] Chen G R and Ueta T 1999 *Int. J. Bifurc. Chaos* **9** 1465
[11] Lü J H and Chen G R 2002 *Int. J. Bifurc. Chaos* **12** 659
[12] Chua L O, Komuro M and Matsumoto T 1986 *IEEE Trans. Circ. Sys.* **33** 1072
[13] Dadras S, Momeni H R and Qi G 2010 *Nonlinear Dyn.* **62** 391
[14] Yu F, Wang C H, Yin J W and Xu H 2011 *Chin. Phys. B* **20** 110505
[15] Dong E Z, Chen Z P, Chen Z Q and Yuan Z Z 2009 *Chin. Phys. B* **18** 2680
[16] Cang S, Qi G and Chen Z 2010 *Nonlinear Dyn.* **59** 515
[17] Liu X Y 2009 *Chin. Phys. Lett.* **26** 090504
[18] Yu F, Wang C H, Yin J W and Xu H 2012 *Acta Phys. Sin.* **61** 020506 (in Chinese)
[19] Yu S, Tang W K S, Lü J and Chen G 2008 *IEEE Trans. Circ. Sys. II* **55** 1168
[20] He S B, Sun K H and Zhu C X 2013 *Chin. Phys. B* **22** 050506
[21] Luo M W, Luo X H and Li H Q 2013 *Acta Phys. Sin.* **62** 020512 (in Chinese)
[22] Zhou X, Wang C H and Guo X R 2012 *Acta Phys. Sin.* **61** 200506 (in Chinese)
[23] Sprott J C 2000 *Phys. Lett. A* **266** 19
[24] Li D 2008 *Phys. Lett. A* **372** 387
[25] Yang X S and Li Q 2003 *Chaos Soliton. Fract.* **18** 25
[26] Lü J, Yu S, Leung H and Chen G 2006 *IEEE Trans. Circ. Sys. I* **53** 149
[27] Yu S, Lü J and Chen G 2007 *Phys. Lett. A* **364** 244
[28] Zhang C and Yu S 2010 *Phys. Lett. A* **374** 3029
[29] Ai X X, Sun K H and He S B 2014 *Acta Phys. Sin.* **63** 040503 (in Chinese)
[30] Lü J, Zhou T, Chen G and Yang X 2002 *Chaos* **12** 344
[31] Zheng Z, Lü J, Chen G, Zhou T and Zhang S 2004 *Chaos Soliton. Fract.* **20** 277
[32] Yu S, Lü J and Chen G 2007 *IEEE Trans. Circ. Sys. I* **54** 2087
[33] Guo J, Xie G and Wang L 2009 *Chaos Soliton. Fract.* **40** 2160
[34] Chen Q, Hong Y, Chen G and Zhong Q 2006 *Phys. Lett. A* **348** 195
[35] Zhou T and Chen G 2004 *Int. J. Bifurc. Chaos* **14** 1795
[36] Sprott J C 1994 *Phys. Rev. E* **50** R647