Landau damping and frequency-shift of a quadrupole mode in a disc-shaped rubidium Bose–Einstein condensate*

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The damping and frequency-shift in Landau mechanism of a quadrupole mode in a disc-shaped rubidium Bose–Einstein condensate are investigated by using the Hartree–Fock–Bogoliubov approximation. The practical relaxations of the elementary excitations and the orthometric relation among them are taken into account to obtain advisable calculation formula for damping as well as frequency-shift. The first approximation of Gaussian distribution function is employed for the ground-state wavefunction to suitably eliminate the divergence of the analytic three-mode coupling matrix elements. According to these methods, both Landau damping rate and frequency-shift of the quadrupole mode are analytically calculated. In addition, all the theoretical results agree with the experimental ones.

Keywords: Bose–Einstein condensate, Landau damping and frequency-shift, Hartree–Fock–Bogoliubov approximation, Thomas–Fermi approximation

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1. Introduction

Elementary excitation in Bose–Einstein condensate (BEC) is a good subject for studying the physics of a many-body system.[1–18] The damping and frequency-shift, which are the main characters of the elementary excitation, are the basic phenomena of BEC and widely observed in the experiments.[19–26] Although there are a lot of theoretic works on the Landau damping of the collective excitation in BEC,[27–46] the problem is not yet studied and understood adequately. Although the damping is accompanied by the frequency-shift, only in Ref. [44] is the frequency-shift calculated in the study of Landau damping.

The dynamic of a collective mode is described respectively by the theory of two fluid hydrodynamics at high temperature and high density[27–32] and the mean-field theory at low temperature and low density.[33–46] In the framework of the mean-field theory, the Landau damping of a collective mode in BEC trapped in inhomogeneously harmonical potential is studied by Refs. [40]– [44]. In Refs. [40]– [43], a Lorentz distribution function is introduced in the perturbation formula of frequency correction to obtain the calculation formula of the damping rate. However, there are some defects in the method and the method is not fit for the cases of quasi-one-dimensional (Q1D) and quasi-two-dimensional (Q2D) condensate.[45,46] In Refs. [45] and [46], the practical relaxations of elementary excitations are taken into account in the frequency correction formula and the damping rate is calculated in an interactive way. Based on Refs. [45] and [46], the orthometric relation among the relaxations is also taken into account in Ref. [44]. The problem will be carefully discussed in Section 2.3.

Because of the complicatedly inhomogeneous character of the condensates trapped in harmonical potential, the work on Landau damping of Refs. [27]– [40] are all based on numerical simulation. The analytical work can only be carried out by using the Thomas–Fermi approximation (TFA). However, the three-mode coupling matrix element, which is important for calculating damping rate and evaluated by using the solutions of Gross–Pitaevskii (GP) equation for ground state and Bogoliubov–de-Gennes (BdG) equations for elementary excitation, is divergent in TFA.[11] References [41]– [43] put forward a method beyond TFA (bTFA) to eliminate the divergence by introducing a parameter q determined by calculus of variations and realize the analytic calculation of damping. In addition, the method is finally improved in Ref. [44] by fixing the parameter $q = 1$. The problem will be carefully discussed in Section 2.4.

The Landau damping and frequency-shift of a monopole mode in a cigar-shaped rubidium condensate are investigated in Ref. [44] and the theoretical results agree with the experimental ones of Ref. [22]. With the same method, the Landau damping and frequency-shift of a quadrupole mode in a disc-shaped rubidium condensate are investigated in this paper. The theoretical results will be compared with the experimental ones, to explain the famous experiment of Ref. [20] and further examine the theory of Ref. [44].

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2. The Hartree–Fock–Bogoliubov mean field theory for Landau damping and frequency-shift

We adopt the Hartree–Fock–Bogoliubov (HFB) mean field approximation[33] and the analytical eigenfunction set of the BdG equations.[14] We take a brief review in Sections 2.1 and 2.2.

We use the method for eliminating the divergence of the three-mode coupling matrix elements and the calculation formula for damping rate and frequency-shift in Ref. [44]. We undertake careful discussion in Sections 2.3 and 2.4.

2.1. HFB approximation and eigenfunction set of BdG equations

The grand-canonical Hamiltonian of a dilute Bose-condensed atomic gas trapped in an axial external potential $V_{\rm ext}(\boldsymbol{r}) = m\omega_{\rm ho}^2 r^2/2$ with $r^2 = s^2 + \lambda^2 z^2$ and $s^2 = x^2 + y^2$ ($\omega_{\rm ho}$ is the harmonic frequency, λ is the anisotropic parameter, and m is the atomic mass) is

$$\begin{aligned} K &= H - \mu N \\ &= \int {\rm d}\boldsymbol{r}\,\psi^\dagger(\boldsymbol{r},t) H_0 \psi(\boldsymbol{r},t) \\ &\quad + \frac{g}{2}\int {\rm d}\boldsymbol{r}\,\psi^\dagger(\boldsymbol{r},t)\psi^\dagger(\boldsymbol{r},t)\psi(\boldsymbol{r},t)\psi(\boldsymbol{r},t), \end{aligned} \tag{1}$$

where $H_0 = -\hbar^2\nabla^2/(2m) + V_{\rm ext}(\boldsymbol{r}) - \mu$ with μ being the chemical potential, $g = 4\pi\hbar^2 a_{\rm sc}/m$ with $a_{\rm sc}$ being the s-wave scattering length. The equation of motion for Bose field operator $\psi(\boldsymbol{r},t)$ is

$$i\hbar\frac{\partial\psi(\boldsymbol{r},t)}{\partial t} = H_0\psi(\boldsymbol{r},t) + g\psi^\dagger(\boldsymbol{r},t)\psi(\boldsymbol{r},t)\psi(\boldsymbol{r},t). \tag{2}$$

The field operator ψ is decomposed into a condensate part $\phi = \langle\psi\rangle$ and a noncondensed part $\widetilde{\psi}$, $\psi(\boldsymbol{r},t) = \phi(\boldsymbol{r},t) + \widetilde{\psi}(\boldsymbol{r},t)$ and $\langle\widetilde{\psi}\rangle = 0$. The condensate part is also decomposed into a stationary component $\phi_0 = \langle\psi\rangle_0$ and an excited component $\delta\phi$, $\phi(\boldsymbol{r},t) = \phi_0(\boldsymbol{r}) + \delta\phi(\boldsymbol{r},t)$. Where the symbol $\langle\cdots\rangle(\langle\cdots\rangle_0)$ represents nonequilibrium (equilibrium) average. We take the HFB approximation: (i) $\langle\widetilde{\psi}^\dagger\widetilde{\psi}\rangle = \widetilde{n}$ and $\langle\widetilde{\psi}\widetilde{\psi}\rangle = \widetilde{m}$, where $\widetilde{m}(\boldsymbol{r},t)$ and $\widetilde{n}(\boldsymbol{r},t)$ denote respectively the normal and the anomalous thermal particle densities; (ii) $\langle\widetilde{\psi}^\dagger\widetilde{\psi}^\dagger\widetilde{\psi}\widetilde{\psi}\rangle = \langle\widetilde{\psi}^\dagger\widetilde{\psi}^\dagger\widetilde{\psi}\rangle = 0$; (iii) $\langle\widetilde{\psi}^\dagger\widetilde{\psi}^\dagger\widetilde{\psi}\widetilde{\psi}\rangle = 4\widetilde{n}\widetilde{\psi}^\dagger\widetilde{\psi} + \widetilde{m}\widetilde{\psi}^\dagger\widetilde{\psi}^\dagger + \widetilde{m}^*\widetilde{\psi}\widetilde{\psi}$; (iv) $\widetilde{n}(\boldsymbol{r},t) = \widetilde{n}^0(\boldsymbol{r}) + \delta\widetilde{n}(\boldsymbol{r},t)$ and $\widetilde{m}(\boldsymbol{r},t) = \widetilde{m}^0(\boldsymbol{r}) + \delta\widetilde{m}(\boldsymbol{r},t)$, where $\widetilde{n}^0 = \langle\widetilde{\psi}^\dagger\widetilde{\psi}\rangle_0$ and $\widetilde{m}^0 = \langle\widetilde{\psi}\widetilde{\psi}\rangle_0$; (v) $\widetilde{n}^0 = 0$ and $\widetilde{m}^0 = 0$.

Using the above approximation, we get the equation for the condensate wavefunction

$$\begin{aligned} i\hbar\frac{\partial\phi(\boldsymbol{r},t)}{\partial t} &= H_0\phi(\boldsymbol{r},t) + g|\phi(\boldsymbol{r},t)|^2\phi(\boldsymbol{r},t) \\ &\quad + 2g\phi(\boldsymbol{r},t)\delta\widetilde{n}(\boldsymbol{r},t) + g\phi(\boldsymbol{r},t)^*\delta\widetilde{m}(\boldsymbol{r},t). \end{aligned} \tag{3}$$

Disregarding $\delta\widetilde{n}$ and $\delta\widetilde{m}$ in Eq. (3), we otain the GP equation for the stationary distribution of the condensate

$$[H_0 + gn_0(\boldsymbol{r})]\,\phi_0(\boldsymbol{r}) = 0, \tag{4}$$

where $n_0 = \phi_0^2$ is the density of the condensate.

Taking the Bogoliubov transformation $\widetilde{\psi}(\boldsymbol{r},t) = \sum_k[u_k(\boldsymbol{r})\alpha_k^\dagger(t) + v_k^*(\boldsymbol{r})\alpha_k(t)]$ with normalization $\int {\rm d}\boldsymbol{r}[u_k^*(\boldsymbol{r})u_{k'}(\boldsymbol{r}) - v_k(\boldsymbol{r})v_{k'}^*(\boldsymbol{r})] = \delta_{kk'}$ (α_k is the Bose operator satisfying $[\alpha_k^\dagger, \alpha_{k'}] = \delta_{kk'}$) and diagonalizing the Hamiltonian (1), we get the BdG equations for the wavefunction set of the quasi-particles

$$\begin{aligned} \mathscr{L}u_j(\boldsymbol{r}) + gn_0 v_j(\boldsymbol{r}) &= \varepsilon_j u_j(\boldsymbol{r}), \\ \mathscr{L}v_j(\boldsymbol{r}) + gn_0 u_j(\boldsymbol{r}) &= -\varepsilon_j v_j(\boldsymbol{r}), \end{aligned} \tag{5}$$

where $\mathscr{L} = H_0 + 2gn_0$.

From Eq. (3) we can also obtain the equation of motion

$$\begin{aligned} i\hbar\frac{\partial\delta\phi(\boldsymbol{r},t)}{\partial t} &= [H_0 + 2gn_0]\delta\phi(\boldsymbol{r},t) + gn_0\delta\phi^*(\boldsymbol{r},t) \\ &\quad + 2g\phi_0\delta\widetilde{n}(\boldsymbol{r},t) + g\phi_0^*\delta\widetilde{m}(\boldsymbol{r},t) \end{aligned} \tag{6}$$

for the excitation of the condensate.

Ignoring $\delta\widetilde{n}$ and $\delta\widetilde{m}$ in Eq. (6), it is easy to see that the collective excitation in the condensate $\delta\phi(\boldsymbol{r},t) = u_0(\boldsymbol{r})\exp(i\varepsilon_0 t/\hbar)$ and $\delta\phi^*(\boldsymbol{r},t) = v_0(\boldsymbol{r})\exp(i\varepsilon_0 t/\hbar)$ are also determined by the BdG equations, namely $u_0(\boldsymbol{r})$ and $v_0(\boldsymbol{r})$ with ε_0 obey Eq. (5).

Rescaling the variables $\bar{r} = r/R_0$ and $\overline{\nabla} = R_0\nabla$ and introducing $\zeta = \hbar\omega_{\rm ho}/2\mu$, where $R_0 = \sqrt{2\mu/m\omega_{\rm ho}^2}$ is the character radius of the condensate, the equation (4) is transformed into the dimensionless form

$$\zeta^2\sigma(\bar{r}) + \bar{r}^2 - 1 + |\phi_0(\bar{r})/\phi_0(0)|^2 = 0, \tag{7}$$

where $\sigma(\bar{r}) = -[\overline{\nabla}^2\phi_0(\bar{r})]/\phi_0(0)$ is a quantity proportional to the zero-point pressure.

Rescaling the frequency of the elementary excitation $\overline{\omega} = \omega/\omega_{\rm ho}$ with $\omega = \varepsilon/\hbar$ and defining $\phi_j^\pm = u_j \pm v_j$, equation (5) becomes the dimensionless form

$$\begin{aligned} &-\bar{\nabla}^2(1-\bar{r}^2)\phi_j^+ - (1-\bar{r}^2)\sigma\phi_j^+ \\ &+ \frac{\zeta^2}{2}\left[\bar{\nabla}^4 + 3\bar{\nabla}^2\sigma + \sigma\bar{\nabla}^2 + 3\sigma^2\right]\phi_j^+ = 2\bar{\omega}_j^2\phi_j^+, \\ &-(1-\bar{r}^2)\bar{\nabla}^2\phi_j^- - (1-\bar{r}^2)\sigma\phi_j^- \\ &+ \frac{\zeta^2}{2}\left[\bar{\nabla}^4 + \bar{\nabla}^2\sigma + 3\sigma\bar{\nabla}^2 + 3\sigma^2\right]\phi_j^- = 2\bar{\omega}_j^2\phi_j^-. \end{aligned} \tag{8}$$

The bTFA solution of Eq. (7) for the groundstate wavefunction is taken as

$$\phi_0(\bar{r}) = \phi_0(0)(1 - \bar{s}^2 - \lambda^2\bar{z}^2)^{(q+1)/2}, \tag{9}$$

where $\bar{s} = s/R_0$, $\bar{z} = z/R_0$, and $\phi_0(0) = \lambda[N_0/(2\pi R_0^3 B(3/2, 2+q))]^{1/2}$ is a normalized constant with $B(3/2, 2+q)$ being Beta function and $N_0 = R_0^3\int {\rm d}\bar{\boldsymbol{r}}|\Phi_0(\bar{\boldsymbol{r}})|^2$ being the particle number in the condensate. The chemical potential is $\mu = \hbar\omega_{\rm ho}[4\lambda P/B(3/2, 2+q)]^{2/5}/2$, where the dimensionless

interatomic interaction strength $P = N_0 a_{\rm sc}/a_{\rm ho}$ with $a_{\rm ho} = \sqrt{\hbar/m\omega_{\rm no}}$ being the character radius of harmonic oscillator. Here the parameter q will be discussed in Section 2.4.

The bTFA solution of Eq. (8) for the elementary excitations wavefunction

$$\phi^{\pm}_{n_z n_s m}(\bar{\boldsymbol{r}}) = \frac{(\zeta\bar{\omega}_{n_z n_s m})^{\pm 1/2}}{\sqrt{2\pi R_0^3 I_{n_z n_s m}}}(1-\bar{s}^2-\lambda^2\bar{z}^2)^{(q\mp 1)/2} \times \bar{s}^m P^{(2n_s)}_{n_{\rm p}}(\bar{s},\bar{z})\,{\rm e}^{{\rm i}m\varphi}, \tag{10}$$

with $\bar{\omega}_{n_z n_s m}$ being eigenvalue and $I_{n_z n_s m}$ being normalized coefficient. Here $n_{\rm p}$ (= 0, 1, 2, ...) is principal quantum number, n_s (= 0, 1, 2, ..., int[$n_{\rm p}$/2]) is the radial quantum number, $n_z = n_{\rm p} - 2n_s$ is axial quantum number, and m (= 0, ±1, ±2, ...) is azimuthal quantum number. The coupled axial and radial function $P^{(2n_s)}_{n_{\rm p}}$ satisfies a two-dimensional differential equation, which has the solution with the form $P^{(2n_s)}_{n_{\rm p}}(\bar{z},\bar{s}) = \sum_{k=0}^{n_{\rm p}}\sum_{n_s=0}^{{\rm int}[k/2]} b_{k,n_s}\bar{z}^{k-2n_s}\bar{s}^{2n_s}$. The eigenvalues $\bar{\omega}_{n_z n_s m}$ are determined by a close equation

$$-1 = \cfrac{g_0}{1+\cfrac{g_1}{1+\cfrac{g_2}{1+\cdots}}} \qquad \ddots \qquad \cfrac{}{1+\cfrac{g_{Z-2}}{1+0}}, \tag{11}$$

where

$$g_{n_s} = \frac{-4\lambda^2(n_s+1)(n_s+|m|+1)(n_{\rm p}-2n_s-1)(n_{\rm p}-2n_s)}{2\gamma-2|m|(1+q)-4n_s(n_s+|m|+1+q)-\lambda^2(n_{\rm p}-2n_s)(n_{\rm p}-2n_s+1+2q)} \times [2\gamma-2|m|(1+q)-4(n_s+1)(n_s+|m|+2+q) - \lambda^2(n_{\rm p}-2n_s-2)(n_{\rm p}-2n_s-1+2q)]^{-1}, \tag{12}$$

with $Z = 1+{\rm int}[n_{\rm p}/2]$ and $\gamma = \bar{\omega}^2_{n_z n_s m}$. The coefficients b_{k,n_s} are determined a recurrence relation

$$\begin{aligned} &4(n_s+1)(n_s+|m|+1)b_{k+2,n_s+1} \\ &+(k-2n+2)(k-2n_s+1)b_{k+2,n_s} \\ =\;& 4\lambda^2(n_s+1)(n_s+|m|+1)b_{k,n_s+1} \\ &-[2\gamma-2|m|(1+q)-4n_s(n_s+|m|+1+q) \\ &-\lambda^2(k-2n_s)(k-2n_s+1+2q)]b_{k,n_s} \\ &+(k-2n_s+2)(k-2n_s+1)b_{k,n_s-1}. \end{aligned} \tag{13}$$

2.2. Frequency correction of collective mode with damping strength and coupling matrix elements

With the Bogoliubov transformation, we can obtain equations of motion

$$\begin{aligned} {\rm i}\hbar\frac{\partial f_{ij}(t)}{\partial t} &= \langle[\alpha_i^\dagger\alpha_j, K]\rangle, \\ {\rm i}\hbar\frac{\partial g_{ij}(t)}{\partial t} &= \langle[\alpha_i\alpha_j, K]\rangle \end{aligned} \tag{14}$$

reflecting respectively the time evolutions of $\delta\hat{n}$ and $\delta\hat{m}$, where $f_{ij} = \langle\alpha_i^\dagger\alpha_j\rangle - f_i^0$ and $g_{ij} = \langle\alpha_i\alpha_j\rangle$ are respectively the normal and anomalous quasi-particle distribution functions with $f_j^0 = \langle\alpha_j^\dagger\alpha_j\rangle_0 = 1/\left[\exp\left(2\zeta\overline{\omega}_j/\overline{T}\right)-1\right]$ being the equilibrium density of quasi-particle, $\overline{T} = k_{\rm B}T/\mu$ is the dimensionless temperature with $k_{\rm B}$ being the Boltzmann constant.

We are interested in the collective mode with frequency $\overline{\omega}_0$ excited in the condensate. We can get the frequency correction to the collective mode in the Landau mechanism by using the Fourier transformation of Eqs. (6) and (14)

$$\overline{\omega} = \overline{\omega}_0 + \sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{1}{\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j}, \tag{15}$$

where

$$\bar{\gamma}_{ij} = 4\pi(4\pi a_{\rm sc}/a_{\rm ho})^2|\bar{A}_{ij}|^2(f_i^0-f_j^0) \tag{16}$$

is the damping strength rescaling by $\omega^2_{\rm ho}$ with

$$\begin{aligned} \bar{A}_{ij} &= a^3_{\rm ho}\int {\rm d}\boldsymbol{r}\phi_0[u_0(u_iu_j^*+v_iv_j^*+v_iu_j^*) \\ &\quad +v_0(u_iu_j^*+v_iv_j^*+u_iv_j^*)] \\ &= \frac{[4P/B(3/2,2+q)]^{1/10}}{8\pi\lambda^{9/10}[I_0I_iI_j\bar{\omega}_0\bar{\omega}_i\bar{\omega}_j]^{1/2}}\left(\frac{N_0}{P}\right)^{1/2} \\ &\quad \times\int_0^1{\rm d}x\int_0^1{\rm d}yW_0W_iW_j^*F(x,y) \end{aligned} \tag{17}$$

being the three-mode coupling matrix element rescaled by $1/a^3_{\rm ho}$. Here we have defined $x = \bar{s}^2$, $y = \lambda\bar{z}/\sqrt{1-x}$, $W_j = \bar{s}^m P^{(2n_s)}_{n_{\rm p}}(\bar{s},\bar{z})$, and $F(x,y) = 3\xi^2\bar{\omega}_0\bar{\omega}_i\bar{\omega}_j(1-x)^{2q-1/2}(1-y^2)^{2q-1}+(\bar{\omega}_0+\bar{\omega}_i-\bar{\omega}_j)(1-x)^{2q+3/2}(1-y^2)^{2q+1}$.

In the right-hand side of Eq. (15), the first term is the un-corrected frequency of the collective mode, the second term is the frequency correction to the collective mode. The quasi-particle transition satisfying the phase-match condition $\omega_0+\omega_i-\omega_j = 0$ has remarkable contribution to the second term. The quasi-particle transition, in which a quasi-particle with frequency ω_i is turned into another one with frequency ω_j by absorbing a collective mode with frequency ω_0, contributes to the energy correction of the collective mode through the Landau mechanism.

2.3. Calculation formula of Landau damping rate and frequency-shift

In the right-hand side of Eq. (15), which is deduced from the perturbation theory, the first term is the zero-order fre-

quency of the collective mode and the second term is the perturbation correction, which diverges under the condition $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j=0$.

In Refs. [40]–[43], the relaxation of elementary excitation is phenomenologically taken into account, namely equation (15) is rewritten as

$$\begin{aligned}\overline{\omega} &= \overline{\omega}_0+\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+(\overline{\Delta}/2)^2}\\ &\quad+\mathrm{i}\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\Delta}/2}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+(\overline{\Delta}/2)^2}\end{aligned}\tag{18}$$

by replacing $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j$ with $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j-\mathrm{i}\overline{\Delta}/2$ and the imaginary part in Eq. (18) is taken as the Landau damping rate of the collective mode

$$\overline{\gamma}_0=\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\Delta}/2}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+(\overline{\Delta}/2)^2},\tag{19}$$

and the calculation value of the damping rate is found in the region where $\overline{\gamma}_0$ varies slowly with $\overline{\Delta}$. In addition, the transition is divided into the resonance transition and background transition with big and small damping strength respectively and only a small part of background transition satisfying 0.82 $\overline{\omega}_0<\overline{\omega}_{ij}<$1.18 $\overline{\omega}_0$ with $\overline{\omega}_{ij}=\overline{\omega}_j-\overline{\omega}_i$ is taken into account contributing to the damping.

There are some defects in the above calculation method: (i) the significance of Lorentz width $\overline{\Delta}$ is not yet elaborated; (ii) the calculated value of Landau damping rate should be extraordinarily big, if all of the transitions are taken into account. Thus a large approximation is adopted; (iii) it is not fit for Q1D and Q2D condensate, because there is no region that $\overline{\gamma}_0$ varies slowly with $\overline{\Delta}$ in those cases; (iv) the real part in Eq. (18) cannot be taken as frequency-shift, because there is no region that the real part varies slowly with $\overline{\Delta}$ in any cases. In addition, the value of frequency-shift is also extraordinarily big if we calculate it by using Eq. (15), which is just the formula of frequency-shift without considering the relaxation of elementary excitation.

The above method is improved in Ref. [44] as follows.

Considering the practical relaxation of the collective mode $\bar{\omega}_0$, we replace $\overline{\omega}_0$ with $\overline{\omega}_0+\mathrm{i}\overline{\gamma}_0$, in the right-hand side of Eq. (15), where $\overline{\gamma}_0$ is the Landau damping rate of the collective mode rescaled by ω_{ho}, and the equation (15) becomes

$$\begin{aligned}\overline{\omega} &= \overline{\omega}_0+\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_0^2}\\ &\quad+\mathrm{i}\left[\overline{\gamma}_0-\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\gamma}_0}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_0^2}\right].\end{aligned}\tag{20}$$

Zeroing the imaginary part in Eq. (20), we obtain

$$\overline{\gamma}_0=\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\gamma}_0}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_0^2},\tag{21}$$

$$\overline{\omega}-\overline{\omega}_0=\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j}{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_0^2}.\tag{22}$$

Considering the relaxation of the quasi-particle $\bar{\omega}_i\to\bar{\omega}_i+\mathrm{i}\bar{\gamma}_i$, where $\bar{\gamma}_i$ is the Landau damping rate of the quasi-particle $\bar{\omega}_i$, we replace $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j$ with $\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_i^2\right]/\overline{\gamma}_i$ in Eq. (21) and obtain

$$\overline{\gamma}_0=\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\gamma}_0}{\left[\dfrac{(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_i^2}{\overline{\gamma}_i}\right]^2+\overline{\gamma}_0^2}.\tag{23}$$

Considering the relaxation of the quasi-particle $\bar{\omega}_j\to\bar{\omega}_j-\mathrm{i}\bar{\gamma}_j$, where $\bar{\gamma}_j$ is the Landau damping rate of the quasi-particle $\bar{\omega}_j$, we replace $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j$ with $[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_j^2]/\overline{\gamma}_j$ in Eq. (23) and finally provide the formula for calculating the Landau damping rate of the collective mode

$$\overline{\gamma}_0=\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\overline{\gamma}_0}{\dfrac{\left\{\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_j^2\right]^2+\overline{\gamma}_j^2\overline{\gamma}_i^2\right\}^2}{\overline{\gamma}_j^4\overline{\gamma}_i^2}+\overline{\gamma}_0^2}.\tag{24}$$

The relaxations of the elementary excitations are introduced in Eq. (24) and the relation among the relaxations is orthometric, because the eigenfunctions of the elementary excitations are orthometric.

Replacing $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j$ with $\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}^2\right]/\overline{\gamma}$ is equivalent to modifying the mismatch. The width of energy level is induced by relaxation and thus the mismatch is modified. If $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j\ll\overline{\gamma}$, $\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}^2\right]/\overline{\gamma}\to\overline{\gamma}$, the modified mismatch equals exactly to the relaxation although the mismatch of transition is very small; If $\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j=\overline{\gamma}$, $\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}^2\right]/\overline{\gamma}=2\overline{\gamma}$, the modified mismatch equals exactly to twice the mismatch; If $\overline{\gamma}\ll\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j$, $\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}^2\right]/\overline{\gamma}\to\infty$, the transition with very big modified mismatch has very small contribution to the damping because the relaxation of the elementary excitation is very small.

By modifying the mismatch for the same physical consideration, we also provide the formula for calculating the frequency-shift of the collective mode

$$\begin{aligned}&\overline{\omega}-\overline{\omega}_0\\ &=\sum_{ij}\overline{\gamma}_{ij}\frac{1}{\pi}\frac{\dfrac{\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_j^2\right]^2+\overline{\gamma}_j^2\overline{\gamma}_i^2}{\overline{\gamma}_j^2\overline{\gamma}_i}\dfrac{\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j}{|\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j|}}{\dfrac{\left\{\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\overline{\gamma}_j^2\right]^2+\overline{\gamma}_j^2\overline{\gamma}_i^2\right\}^2}{\overline{\gamma}_j^4\overline{\gamma}_i^2}+\overline{\gamma}_0^2}.\end{aligned}\tag{25}$$

2.4. Determination of q

We use elementary excitation eigenvalues of TFA, because the eigenvalues of TFA are found to be close to the experimental ones. We use the elementary excitation eigenfunctions of bTFA, because the divergence of the three-mode coupling matrix element arises from the elementary excitation eigenfunctions of TFA.

The parameter $q > 0$ in bTFA is introduced to eliminate the divergence of the three-mode coupling matrix element. If $q = 0$, the bTFA[14] degenerates into the TFA.[13]

In the TFA, the groundstate wavefunction is solved as $\phi_0(\bar{r}) = \phi_0(0)(1-\bar{r}^2)^{1/2}$ with $\phi_0(0) = \lambda[N_0/(2\pi R_0^3 B(3/2,2)]^{1/2}$ by ignoring the $\sigma(\bar{r})$ in GP Eq. (7), which is a small approximation for large particle number condensate, and therefore the analytic solution of BdG Eq. (8) is obtained. However, the evaluated three-mode coupling matrix element $\bar{A}_{ij}$ is divergent. In the bTFA, the groundstate wavefunction is taken as Eq. (9) with a parameter q, which does not satisfy the Eq. (7) and therefore there is an average error without considering the neglect of $\sigma(\bar{r})$

$$\overline{\delta} = \frac{\int_0^1[(1-\bar{r}^2)^{1+q} - (1-\bar{r}^2)](1-r^2)^{1+q}\bar{r}^2\,\mathrm{d}\bar{r}}{\int_0^1(1-\bar{r}^2)^{1+q}\bar{r}^2\,\mathrm{d}\bar{r}}.$$

The analytic solution of Eq. (8) can also be obtained and the $\bar{A}_{ij}$ is no longer divergent.

Because the average error $\overline{\delta}$ decreases with the decrease of q but the $\bar{A}_{ij}$ (it is divergent at $q = 0$) increases with the decrease of q, a suitable value of q should be fixed theoretically. However, in Refs. [41]–[43], the q is determined by minimizing groundstate energy and therefore completely dependent on the experimental parameters.

In Ref. [44], the method eliminating the divergence is improved as follows.

The parameter is taken as $q = 1$, which is the case closest to the TFA ($\overline{\delta} = 0$) in the cases satisfying the boundary continuity condition of groundstate wavefunction, because the kinetic energy density of condensate $\sim -\phi_0\overline{\nabla}^2\phi_0 \sim -(1+q)(1-\bar{s}^2-\lambda^2\bar{z}^2)^{-1+q}[-2-\lambda^2+\bar{z}^2(2\lambda^2+q\lambda^4)+\bar{s}^2(1+q+\lambda^2)]$ is divergent if $0 < q < 1$, equals to an arbitrary value if $q = 1$, and equals to 0 if $q > 1$, at the boundary of the condensate $1-\bar{s}^2-\lambda^2\bar{z}^2 = 0$. The change of $\bar{A}_{ij}$ with q is continuous although the variation of kinetic energy density with q is mutational at $q = 1$. The $\overline{\delta} \approx 18\%$ for $q = 1$.

Moreover, for $q = 1$ the groundstate wavefunction $\phi_0 \sim 1-\bar{r}^2$ is just the first approximation of Gaussian distribution function $\sim \mathrm{e}^{-\bar{r}^2}$, which is the groundstate wavefunction of the harmonic oscillator. Therefore, in the method of Ref. [44], the harmonic oscillator character of the particle in the magnetic trap is still preserved.

3. Calculations and results

We consider a ^{87}Rb atomic gas ($a_{\mathrm{sc}} = 5.82\times10^{-9}$ m) condensing in a disc-shaped harmonic trap with trapping frequency $\omega_{\mathrm{ho}} = 810$ Hz and anisotropic parameter $\lambda = 2.83$. The atom number in the condensate $N_0 = N\left[1-\left(T/T_{\mathrm{c}}\right)^3\right]$ with the total atom number $N = 6000$ and the BEC transition temperature $T_{\mathrm{c}} = 4.5(\omega_{\mathrm{ho}}/200\pi)(\lambda N)^{1/3}$ nK $= 149.2$ nK.[1] The $q = 1$. The collective mode under study is the quadrupole mode $(n_z, n_s, m) = (0,1,0)$. The frequency of the collective mode is $\omega_0 = 1.797$ (in unit of ω_{ho}). The Bogoliubov amplitudes of the collective mode $u_0 = (\phi^+_{010} + \phi^-_{010})/2$ and $v_0 = (\phi^+_{010} - \phi^-_{010})/2$.

3.1. Landau damping strength

We calculate the damping strength γ_{ij} by using Eq. (16) with Eq. (17).

In Fig. 1 we show the calculation value γ_{ij} (in unit of ω^2_{ho}) of the allowed transitions ω_{ij} ($\omega_{ij} = \omega_j - \omega_i$, in unit of ω_{ho}) at $T = 100$ nK. The transition ω_{ij} is allowed by the selection rules $m_i = m_j$ with m_i and m_j being respectively the azimuthal quantum numbers of the quasi-particle ω_i and ω_j. The arrow in the figure points to the frequency of the collective mode. The position of bars corresponds to allowed transition frequency ω_{ij}, whereas their heights define the calculation value of γ_{ij}.

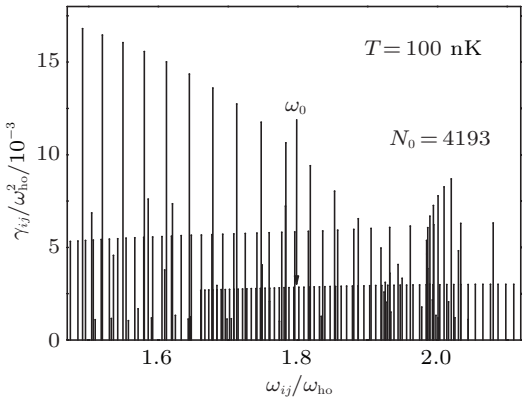


Fig. 1. Histogram of damping strength γ_{ij} as a function of the transition ω_{ij} for the quadrupole mode in the condensate.

We notice that the eigenfunctions of quasi-particles for the levels of large quantum number have very fast oscillations and their maxima are far away from the center of the condensate. Thus the coupling matrix elements for the transition between levels with large quantum numbers are small. In addition, the levels with large quantum number have high energy and hence the Bose occupation factor f_i^0 is small. By a suitable estimation, the levels (0, 0, m), (1, 0, m), (0, 1, m), (2, 0, m), (1, 1, m), (3, 0, m), (0, 2, m), (2, 1, m), (4, 0, m), (1, 2, m), (3, 1, m), (5, 0, m), (0, 3, m), (2, 2, m), (4, 1, m), and (6, 0, m) with m satisfying $\omega_{n_z n_s m} < 1.4\ \mu/\hbar$ are taken into account in our calculation.

3.2. Damping rate and frequency-shift with their temperature dependence

By replacing $\overline{\gamma}_0$ with $\overline{\gamma}$ in the right-hand side of Eq. (24) and taking the approximation that the damping rate is proportional to the frequency, we rewrite the Eq. (24) as

$$\overline{\gamma}_0 = \sum_{ij} \overline{\gamma}_{ij} \frac{1}{\pi} \frac{\overline{\gamma}}{\dfrac{\left\{\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}\right)^2\right]^2+\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}\right)^2\left(\dfrac{\overline{\omega}_i}{\overline{\omega}_0}\overline{\gamma}\right)^2\right\}^2}{\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}\right)^4\left(\dfrac{\overline{\omega}_i}{\overline{\omega}_0}\overline{\gamma}\right)^2}+\overline{\gamma}^2} \tag{26}$$

and calculate the damping rate of the collective mode in an iterative way.

Figure 2 shows the values of γ_0 (in unit of $\omega_{\rm ho}$) versus the values of γ (in unit of $\omega_{\rm ho}$) at $T = 80$, 100, 120 nK, respectively. We can find in Fig. 2 that γ_0 changes with γ and there are three points represented by stars where γ_0 equals to γ. The damping rates of the collective mode at $T = 80$, 100, 120 nK are read to be 0.033, 0.051, 0.078, corresponding to 27, 41, 63 $\rm s^{-1}$, respectively.

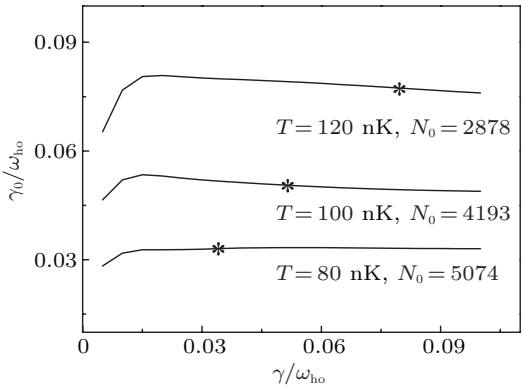


Fig. 2. The γ_0 as a function of γ for the Landau damping.

Equation (26) can be rewritten as

$$\overline{\gamma}_0 = \sum_{ij} \overline{\gamma}_{ij} \frac{1}{\pi} \frac{1}{\dfrac{\overline{\Delta_s''^2}+\overline{\gamma}_0^2}{\overline{\gamma}_0}}, \tag{27}$$

where $\overline{\Delta_s''} = \left(\overline{\Delta_s'^2}+\overline{\gamma}_i^2\right)/\overline{\gamma}_i$ with $\overline{\Delta_s'} = \left(\overline{\Delta_s}^2+\overline{\gamma}_j^2\right)/\overline{\gamma}_j$ and $\overline{\Delta_s} = \overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j$. In Eq. (27), $\overline{\Delta_s'}$ increases rapidly with the increase of $\overline{\Delta_s}$ when $\overline{\Delta_s} > \overline{\gamma}_j$, $\overline{\Delta_s''}$ increases rapidly with the increase of $\overline{\Delta_s'}$ when $\overline{\Delta_s'} > \overline{\gamma}_i$, $\left(\overline{\Delta_s''^2}+\overline{\gamma}_0^2\right)/\overline{\gamma}_0$ increases rapidly with the increase of $\overline{\Delta_s''}$ when $\overline{\Delta_s''} > \overline{\gamma}_0$. Because of the above three reasons, only the transitions with very small mismatch are responsible for the main contribution to the damping rate and the contribution of most other transitions with big mismatch are insignificant.

With the calculation results of the damping rate γ_0, we rewrite Eq. (25) as

$$\overline{\omega}-\overline{\omega}_0 = \sum_{ij} \overline{\gamma}_{ij} \frac{1}{\pi} \frac{\dfrac{\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}_0\right)^2\right]^2+\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}_0\right)^2\left(\dfrac{\overline{\omega}_i}{\overline{\omega}_0}\overline{\gamma}_0\right)^2}{\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}_0\right)^2\left(\dfrac{\overline{\omega}_i}{\overline{\omega}_0}\overline{\gamma}_0\right)} \dfrac{\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j}{|\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j|}}{\dfrac{\left\{\left[(\overline{\omega}_0+\overline{\omega}_i-\overline{\omega}_j)^2+\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}_0\right)^2\right]^2+\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}_0\right)^2\left(\dfrac{\overline{\omega}_i}{\overline{\omega}_0}\overline{\gamma}_0\right)^2\right\}^2}{\left(\dfrac{\overline{\omega}_j}{\overline{\omega}_0}\overline{\gamma}_0\right)^4\left(\dfrac{\overline{\omega}_i}{\overline{\omega}_0}\overline{\gamma}_0\right)^2}+\overline{\gamma}_0{}^2} \tag{28}$$

to calculate the frequency-shift.

In Fig. 3 we show the damping rate γ_0 (in unit of $\rm s^{-1}$) and the frequency-shift (in the form of ω, in unit of $\omega_{\rm ho}$) versus the temperature T (in unit of $T_{\rm c}$). The smooth curves are our theoretical results. The experimental results of Ref. [20] are denoted by the solid triangles and error bars. The magnitude of damping rate and the magnitude and sign of frequency-shift and their temperature dependence of our calculation results are in agreement with the experimental ones.

One can find in Fig. 3 that the theoretic value of the TFA frequency of the quadrupole mode is close to the experimental one, which is the value at $T \to 0$.

The present method of Ref. [44] is more reasonable than the previous one of Refs. [40]– [43], because all the transitions with desirable values of damping strength can be taken into account in the present calculation whereas only a small

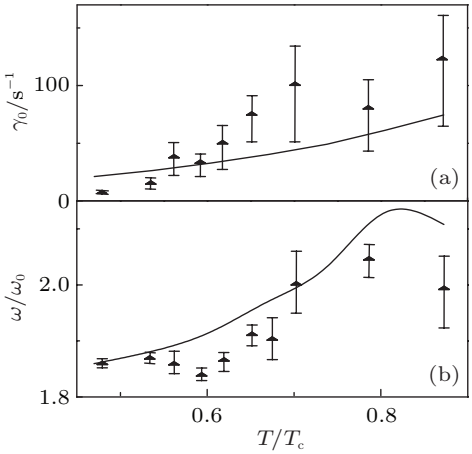


Fig. 3. The Landau damping rate γ_0 (a) and frequency-shift (b) of the collective mode in the condensate as a function of the temperature T.

part of background transitions are taken into account in the previous one. If all of the transitions are taken into account, the previous calculation result for damping should be about 25 times of the present one. If all of the transitions are taken into account, the value of the frequency-shift calculated by using the formula of frequency-shift without considering the relaxation of elementary excitation equation (15) should be about 30 times of the present one.

It is interesting that the γ_0 changes slowly with γ in Fig. 2, which is similar to the Lorentz distribution function changing slowly with the Lorentz width in Refs. [40]–[43], although the present calculation method is different from the previous one. Thus the introducing of the Lorentz distribution function in the previous method is equivalent to taking into account only one relaxation of elementary excitation.

Only one relaxation of elementary excitation is taken into account in the previous method whereas three relaxations of the elementary excitations and the orthometric relation among them are taken into account in the present method. It is simply the reason that the results of the previous method are much larger than the ones of the present method considering all of the transitions.

4. Conclusion

In this paper, a quadrupole mode in a disk-shaped rubidium condensate have been investigated by using the Hartree–Fock–Bogoliubov approximation. The practical relaxations of elementary excitation and the orthometric relation among them are taken into account in the perturbation formula of mean field theory. The divergence of the three-mode coupling matrix element in Thomas–Fermi approximation is eliminated by taking first Gauss distribution function approximation for ground-state wavefunction of condensate. With those reasonable methods, both the damping rate and frequency-shift of the quadrupole mode have been analytically calculated. In addition, our results agreed with the experimental ones. Thus our theoretic method should be helpful to understand the damping and frequency-shift mechanism of elementary excitation in Bose–Einstein condensate.

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