Joint remote preparation of an arbitrary five-qubit Brown state via non-maximally entangled channels*

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(Received 21 November 2013; revised manuscript received 10 March 2014; published online 23 July 2014)

We firstly present a novel scheme for deterministic joint remote state preparation of an arbitrary five-qubit Brown state using four Greenberg–Horme–Zeilinger (GHZ) entangled states as the quantum channel. The success probability of this scheme is up to 1, which is superior to the existing ones. Moreover, the scheme is extended to the generalized case where three-qubit and four-qubit non-maximally entangled states are taken as the quantum channel. We simultaneously employ two common methods to reconstruct the desired state. By comparing these two methods, we draw a conclusion that the first is superior to the second-optimal positive operator-valued measure only taking into account the number of auxiliary particles and the success probability.

Keywords: joint remote state preparation, Brown state, optimal positive operator-valued measure

PACS: 03.67.–a, 03.65.Ud, 03.67.Mn **DOI:** 10.1088/1674-1056/23/9/090307

1. Introduction

Quantum communication has been regarded as one of the core fields of information theory. Quantum teleportation (QT) and remote state preparation (RSP) play a central role in quantum communication. QT allows for the transmission of an unknown quantum state to a distant location with the help of local operations and classical communication. So far, researchers have made dramatic progress.[1–6] Comparing with QT, the RSP presented by Lo,[7] Pati,[8] and Bennett *et al.*,[9] is dedicated to accomplishing the information-transmitted task that one sender transports a known quantum state to one receiver utilizing a shared quantum channel in advance as well as classical communication. RSP not only transmits a known state but also requires less classical communication cost than QT in some special cases.[8] Hence, RSP has gained its popularity with a large number of RSP protocols having been proposed over the past decade.[10–20]

To avoid leaking the full information of desired states, joint remote state preparation (JRSP) protocols were presented. In the JRSP, there are two or more senders located at distant places separately, the information of quantum states to be delivered is secretly split into several pieces in such a way that neither an individual sender nor a subgroup can deduce the desired state. Up to now, the JRSP protocols with respect to one-particle,[21,22] two-particle,[23,24] three-particle,[24,25] and four-particle[26,27] have been extensively studied. Nevertheless, the JRSP protocol of a five-particle Brown state has been rarely investigated. Although Luo *et al.* have proposed a JRSP protocol for an arbitrary five-qubit Brown state[28,29] by using quantum permutation via GHZ states as the quantum channel,[30] the success probability of it is lower. The motivation of this work is to increase the success probability and to further enrich JRSP for five-qubit Brown states.

This paper is organized as follows. In Section 2, we present a novel scheme for JRSP of an arbitrary five-qubit Brown state utilizing four GHZ states as the quantum channel. It is worth noting that the success probability of this scheme is as high as 100%. In Section 3, we conduct further research on the case that non-maximally entangled states are looked upon as the quantum channel. Moreover, we employ two existing approaches to construct the desired state and compare them. In Section 4, we shortly introduce the experimental feasibility and the security of our schemes. Section 5 is devoted to the conclusion.

2. JRSP of an arbitrary five-qubit Brown state with GHZ states

In this section, the JRSP protocol of an arbitrary five-qubit Brown state utilizing GHZ states as the quantum channel is put forward. This protocol is a better job, whose success probability is much higher than that of the protocol in Ref. [30], up to 100%. This protocol is described as follows.

Two senders Alice and Bob, located at two spatially separated places respectively, want to help the remote receiver

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61370194 and 61202082), the Fundamental Research Funds for the Central Universities of China (Grant Nos. BUPT2012RC0219), and the Foundation of Science and Technology of Huawei of China.

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Charlie prepare an arbitrary five-qubit Brown state

$$\begin{aligned}|\Phi\rangle \;=\; & \lambda_0|00101\rangle+\lambda_1 e^{i\theta_1}|10001\rangle \\ & +\lambda_2 e^{i\theta_2}|00110\rangle+\lambda_3 e^{i\theta_3}|10010\rangle \\ & +\lambda_4 e^{i\theta_4}|01000\rangle+\lambda_5 e^{i\theta_5}|11100\rangle \\ & +\lambda_6 e^{i\theta_6}|01011\rangle+\lambda_7 e^{i\theta_7}|11111\rangle, \end{aligned} \tag{1}$$

where λ_j and $\theta_k \in [0,2\pi]$ $(j,k=0,1,\ldots,7)$ are real, $\theta_0=0$, and $\sum_{j=0}^{7}\lambda_j^2=1$. All the information contained in state $|\Phi\rangle$ is $S=\{\lambda_j,\theta_k\}$, which can be divided into $S_1=\{\lambda_j\}$ and $S_2=\{\theta_k\}$ such that S cannot be inferred from either S_1 or S_2, but can from both. Here, we assume that S_1 is known only to Alice, S_2 only to Bob, but none to Charlie. Hence, neither Alice nor Bob alone can help Charlie to reconstruct the desired state $|\Phi\rangle$ only if they agree to collaborate together.

The quantum channel connecting Alice, Bob, and Charlie is composed of one four-particle GHZ state and three three-particle GHZ states, which can be depicted as

$$\begin{aligned}|q\rangle \;=\; & \frac{1}{4}(|000\rangle+|111\rangle)_{123}\otimes(|000\rangle+|111\rangle)_{456} \\ & \otimes(|000\rangle+|111\rangle)_{789}\otimes(|0000\rangle+|1111\rangle)_{10111213}, \end{aligned} \tag{2}$$

where the qubits (1,4,7,10) belong to Alice, (2,5,8,11) to Bob, and (3,6,9,12,13) to Charlie, respectively.

As shown in Fig. 1, the new protocol can be completed by three steps.

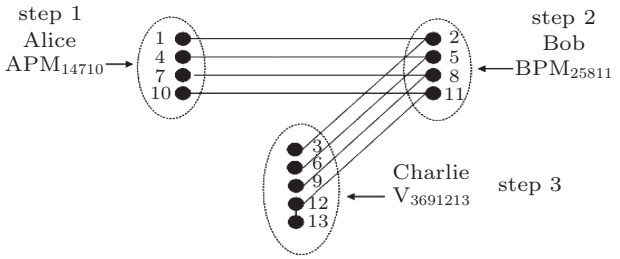


Fig. 1. Each dot denotes a qubit, entangled qubits are connected by solid lines, APM_{14710} as well as BPM_{25811} represent two projective measurements and $V_{3691213}$ stands for appropriate unitary operations.

Step 1 To help Charlie to prepare the desired state $|\Phi\rangle$ remotely, what Alice needs to do is to perform a four-qubit projective measurement on her own particles (1,4,7,10). After Alice's measurement, she proclaims her measurement result via classical channels. By the way, suppose Alice, Bob, and Charlie agree in advance that "0000" corresponds to $|\eta_0\rangle$, "0001" to $|\eta_1\rangle$, …, "1111" to $|\eta_{15}\rangle$, respectively. The measuring basis chosen by Alice are a set of mutually orthogonal basis $\{|\eta_0\rangle,|\eta_1\rangle,\ldots,|\eta_{15}\rangle\}$, which are described as follows:

$$\begin{pmatrix}|\eta_0\rangle\\|\eta_1\rangle\\|\eta_2\rangle\\|\eta_3\rangle\\|\eta_4\rangle\\|\eta_5\rangle\\|\eta_6\rangle\\|\eta_7\rangle\end{pmatrix}=\boldsymbol{H}\begin{pmatrix}|0010\rangle\\|1000\rangle\\|0011\rangle\\|1001\rangle\\|0100\rangle\\|1110\rangle\\|0101\rangle\\|1111\rangle\end{pmatrix},$$

$$\begin{pmatrix}|\eta_8\rangle\\|\eta_9\rangle\\|\eta_{10}\rangle\\|\eta_{11}\rangle\\|\eta_{12}\rangle\\|\eta_{13}\rangle\\|\eta_{14}\rangle\\|\eta_{15}\rangle\end{pmatrix}=\boldsymbol{H}\begin{pmatrix}|0000\rangle\\|1010\rangle\\|0001\rangle\\|1011\rangle\\|0110\rangle\\|1100\rangle\\|0111\rangle\\|1101\rangle\end{pmatrix}, \tag{3}$$

where

$$\boldsymbol{H}=\begin{pmatrix}\lambda_0&\lambda_1&\lambda_2&\lambda_3&\lambda_4&\lambda_5&\lambda_6&\lambda_7\\ \lambda_1&-\lambda_0&\lambda_3&-\lambda_2&\lambda_5&-\lambda_4&\lambda_7&-\lambda_6\\ \lambda_2&-\lambda_3&-\lambda_0&\lambda_1&-\lambda_6&\lambda_7&\lambda_4&-\lambda_5\\ \lambda_3&\lambda_2&-\lambda_1&-\lambda_0&\lambda_7&\lambda_6&-\lambda_5&-\lambda_4\\ \lambda_4&-\lambda_5&\lambda_6&-\lambda_7&-\lambda_0&\lambda_1&-\lambda_2&\lambda_3\\ \lambda_5&\lambda_4&-\lambda_7&-\lambda_6&-\lambda_1&-\lambda_0&\lambda_3&\lambda_2\\ \lambda_6&-\lambda_7&-\lambda_4&\lambda_5&\lambda_2&-\lambda_3&-\lambda_0&\lambda_1\\ \lambda_7&\lambda_6&\lambda_5&\lambda_4&-\lambda_3&-\lambda_2&-\lambda_1&-\lambda_0\end{pmatrix}. \tag{4}$$

Therefore, the quantum channel is rewritten as

$$\begin{aligned}|q\rangle \;=\; & \frac{1}{4}[(|000\rangle+|111\rangle)_{123}\otimes(|000\rangle+|111\rangle)_{456} \\ & \otimes(|000\rangle+|111\rangle)_{789}\otimes(|0000\rangle+|1111\rangle)]_{10111213} \\ =\; & \frac{1}{4}[|\eta_0\rangle_{14710}(\lambda_0|000011000\rangle+\lambda_1|110000000\rangle \\ & +\lambda_2|000011111\rangle+\lambda_3|110000111\rangle \\ & +\lambda_4|001100000\rangle+\lambda_5|111111000\rangle \\ & +\lambda_6|001100111\rangle+\lambda_7|111111111\rangle) \\ & +|\eta_8\rangle_{14710}(\lambda_0|000000000\rangle+\lambda_1|110011000\rangle \\ & +\lambda_2|000000111\rangle+\lambda_3|110011111\rangle \\ & +\lambda_4|001111000\rangle+\lambda_5|111100000\rangle \\ & +\lambda_6|001111111\rangle+\lambda_7|111100111\rangle) \\ & +|\eta_1\rangle_{14710}(\lambda_1|000011000\rangle-\lambda_0|110000000\rangle \\ & +\lambda_3|000011111\rangle-\lambda_2|110000111\rangle \\ & +\lambda_5|001100000\rangle-\lambda_4|111111000\rangle \\ & +\lambda_7|001100111\rangle-\lambda_6|111111111\rangle) \\ & +|\eta_9\rangle_{14710}(\lambda_1|000000000\rangle-\lambda_0|110011000\rangle \\ & +\lambda_3|000000111\rangle-\lambda_2|110011111\rangle \\ & +\lambda_5|001111000\rangle-\lambda_4|111100000\rangle \\ & +\lambda_7|001111111\rangle-\lambda_6|111100111\rangle) \\ & +|\eta_2\rangle_{14710}(\lambda_2|000011000\rangle-\lambda_3|110000000\rangle \\ & -\lambda_0|000011111\rangle+\lambda_1|110000111\rangle \\ & -\lambda_6|001100000\rangle+\lambda_7|111111000\rangle \\ & +\lambda_4|001100111\rangle-\lambda_5|111111111\rangle) \\ & +|\eta_{10}\rangle_{14710}(\lambda_2|000000000\rangle-\lambda_3|110011000\rangle \\ & -\lambda_0|000000111\rangle+\lambda_1|110011111\rangle \\ & -\lambda_6|001111000\rangle+\lambda_7|111100000\rangle \\ & +\lambda_4|001111111\rangle-\lambda_5|111100111\rangle) \end{aligned}$$

$$
\begin{aligned}
&+|\eta_3\rangle_{14710}(\lambda_3|000011000\rangle+\lambda_2|110000000\rangle\\
&-\lambda_1|000011111\rangle-\lambda_0|110000111\rangle\\
&+\lambda_7|001100000\rangle+\lambda_6|111111000\rangle\\
&-\lambda_5|001100111\rangle-\lambda_4|111111111\rangle)\\
&+|\eta_{11}\rangle_{14710}(\lambda_3|000000000\rangle+\lambda_2|110011000\rangle\\
&-\lambda_1|000000111\rangle-\lambda_0|110011111\rangle\\
&+\lambda_7|001111000\rangle+\lambda_6|111100000\rangle\\
&-\lambda_5|001111111\rangle-\lambda_4|111100111\rangle)\\
&+|\eta_4\rangle_{14710}(\lambda_4|000011000\rangle-\lambda_5|110000000\rangle\\
&+\lambda_6|000011111\rangle-\lambda_7|110000111\rangle\\
&-\lambda_0|001100000\rangle+\lambda_1|111111000\rangle\\
&-\lambda_2|001100111\rangle+\lambda_3|111111111\rangle)\\
&+|\eta_{12}\rangle_{14710}(\lambda_4|000000000\rangle-\lambda_5|110011000\rangle\\
&+\lambda_6|000000111\rangle-\lambda_7|110011111\rangle\\
&-\lambda_0|001111000\rangle+\lambda_1|111100000\rangle\\
&-\lambda_2|001111111\rangle+\lambda_3|111100111\rangle)\\
&+|\eta_5\rangle_{14710}(\lambda_5|000011000\rangle+\lambda_4|110000000\rangle\\
&-\lambda_7|000011111\rangle-\lambda_6|110000111\rangle\\
&-\lambda_1|001100000\rangle-\lambda_0|111111000\rangle\\
&+\lambda_3|001100111\rangle+\lambda_2|111111111\rangle)\\
&+|\eta_{13}\rangle_{14710}(\lambda_5|000000000\rangle+\lambda_4|110011000\rangle\\
&-\lambda_7|000000111\rangle-\lambda_6|110011111\rangle\\
&-\lambda_1|001111000\rangle-\lambda_0|111100000\rangle\\
&+\lambda_3|001111111\rangle+\lambda_2|111100111\rangle)\\
&+|\eta_6\rangle_{14710}(\lambda_6|000011000\rangle-\lambda_7|110000000\rangle\\
&-\lambda_4|000011111\rangle+\lambda_5|110000111\rangle\\
&+\lambda_2|001100000\rangle-\lambda_3|111111000\rangle\\
&-\lambda_0|001100111\rangle+\lambda_1|111111111\rangle)\\
&+|\eta_{14}\rangle_{14710}(\lambda_6|000000000\rangle-\lambda_7|110011000\rangle\\
&-\lambda_4|000000111\rangle+\lambda_5|110011111\rangle\\
&+\lambda_2|001111000\rangle-\lambda_3|111100000\rangle\\
&-\lambda_0|001111111\rangle+\lambda_1|111100111\rangle)\\
&+|\eta_7\rangle_{14710}(\lambda_7|000011000\rangle+\lambda_6|110000000\rangle\\
&+\lambda_5|000011111\rangle+\lambda_4|110000111\rangle\\
&-\lambda_3|001100000\rangle-\lambda_2|111111000\rangle\\
&-\lambda_1|001100111\rangle-\lambda_0|111111111\rangle)\\
&+|\eta_{15}\rangle_{14710}(\lambda_7|000000000\rangle+\lambda_6|110011000\rangle\\
&+\lambda_5|000000111\rangle+\lambda_4|110011111\rangle\\
&-\lambda_3|001111000\rangle-\lambda_2|111100000\rangle\\
&-\lambda_1|001111111\rangle-\lambda_0|111100111\rangle)]_{235689111213}. \qquad (5)
\end{aligned}
$$

Step 2 According to Alice's measurement outcome, Bob should choose a corresponding set of mutually orthogonal basis from $\{|\mu_0^{(k)}\rangle,|\mu_1^{(k)}\rangle,\ldots,|\mu_{15}^{(k)}\rangle\}$ to carry out a four-qubit projective measurement on his own particles (2,5,8,11). The corresponding relation of Alice's measurement and the measuring basis selected by Bob can be described as

$$
\begin{aligned}
&|\eta_i\rangle_{14710}\rightarrow\{|\mu_j^{(k)}\rangle_{25811}(k=i,\ i<8,\ j=0,1,\ldots,7)\}\\
&|\eta_i\rangle_{14710}\rightarrow\{|\mu_{j+8}^{(k)}\rangle_{25811}(k=i\mod 8,\ i\geq 8, j=0,1,\ldots,7)\}.
\end{aligned} \qquad (6)
$$

The measurement basis he chose is sixteen sets of mutually orthogonal basis, which are expressed as

$$
\begin{aligned}
\begin{pmatrix}|\mu_0^{(k)}\rangle\\|\mu_1^{(k)}\rangle\\|\mu_2^{(k)}\rangle\\|\mu_3^{(k)}\rangle\\|\mu_4^{(k)}\rangle\\|\mu_5^{(k)}\rangle\\|\mu_6^{(k)}\rangle\\|\mu_7^{(k)}\rangle\end{pmatrix}&=\frac{1}{2\sqrt{2}}\boldsymbol{G}^{(k)}\begin{pmatrix}|0010\rangle\\|1000\rangle\\|0011\rangle\\|1001\rangle\\|0100\rangle\\|1110\rangle\\|0101\rangle\\|1111\rangle\end{pmatrix},\\
\begin{pmatrix}|\mu_8^{(k)}\rangle\\|\mu_9^{(k)}\rangle\\|\mu_{10}^{(k)}\rangle\\|\mu_{11}^{(k)}\rangle\\|\mu_{12}^{(k)}\rangle\\|\mu_{13}^{(k)}\rangle\\|\mu_{14}^{(k)}\rangle\\|\mu_{15}^{(k)}\rangle\end{pmatrix}&=\frac{1}{2\sqrt{2}}\boldsymbol{G}^{(k)}\begin{pmatrix}|0000\rangle\\|1010\rangle\\|0001\rangle\\|1011\rangle\\|0110\rangle\\|1100\rangle\\|0111\rangle\\|1101\rangle\end{pmatrix},
\end{aligned} \qquad (7)
$$

where $k=0,1,\ldots,7$, and $G^{(k)}$ is an 8×8 matrix,

$$
\begin{aligned}
\boldsymbol{G}^{(0)}&=\boldsymbol{G}\left(1,\ r_1,\ r_2,\ r_3,\ r_4,\ r_5,\ r_6,\ r_7\right),\\
\boldsymbol{G}^{(1)}&=\boldsymbol{G}\left(r_1,\ 1,\ r_3,\ r_2,\ r_5,\ r_4,\ r_7,\ r_6\right),\\
\boldsymbol{G}^{(2)}&=\boldsymbol{G}\left(r_2,\ r_3,\ 1,\ r_1,\ r_6,\ r_7,\ r_4,\ r_5\right),\\
\boldsymbol{G}^{(3)}&=\boldsymbol{G}\left(r_3,\ r_2,\ r_1,\ 1,\ r_7,\ r_6,\ r_5,\ r_4\right),\\
\boldsymbol{G}^{(4)}&=\boldsymbol{G}\left(r_4,\ r_5,\ r_6,\ r_7,\ 1,\ r_1,\ r_2,\ r_3\right),\\
\boldsymbol{G}^{(5)}&=\boldsymbol{G}\left(r_5,\ r_4,\ r_7,\ r_6,\ r_1,\ 1,\ r_3,\ r_2\right),\\
\boldsymbol{G}^{(6)}&=\boldsymbol{G}\left(r_6,\ r_7,\ r_4,\ r_5,\ r_2,\ r_3,\ 1,\ r_1\right),\\
\boldsymbol{G}^{(7)}&=\boldsymbol{G}\left(r_7,\ r_6,\ r_5,\ r_4,\ r_3,\ r_2,\ r_1,\ 1\right),
\end{aligned} \qquad (8)
$$

where $r_j=\mathrm{e}^{-\mathrm{i}\theta_j}$ $(j=0,1,\ldots,7)$, $\theta_0=0$, and $\boldsymbol{G}_{(c_1,\,c_2,\,c_3,\,c_4,\,c_5,\,c_6,\,c_7,\,c_8)}$ are specified by

$$
\begin{aligned}
&\boldsymbol{G}_{(c_1,\,c_2,\,c_3,\,c_4,\,c_5,\,c_6,\,c_7,\,c_8)}\\
&=\begin{pmatrix}
c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8\\
c_1 & -c_2 & c_3 & -c_4 & c_5 & -c_6 & c_7 & -c_8\\
c_1 & -c_2 & -c_3 & c_4 & -c_5 & c_6 & c_7 & -c_8\\
c_1 & c_2 & -c_3 & -c_4 & c_5 & c_6 & -c_7 & -c_8\\
c_1 & -c_2 & c_3 & -c_4 & -c_5 & c_6 & -c_7 & c_8\\
c_1 & c_2 & -c_3 & -c_4 & -c_5 & -c_6 & c_7 & c_8\\
c_1 & -c_2 & -c_3 & c_4 & c_5 & -c_6 & -c_7 & c_8\\
c_1 & c_2 & c_3 & c_4 & -c_5 & -c_6 & -c_7 & -c_8
\end{pmatrix}.
\end{aligned} \qquad (9)
$$

After the measurement, Bob informs Charlie of his measurement outcome through a classical channel. Suppose that Alice, Bob, and Charlie agree in advance that cbit sequence 000, 001, 010, 011, 100, 101, 110, and 111 denote the measurement results $|\mu_0^{(k)}\rangle(|\mu_8^{(k)}\rangle)$, $|\mu_1^{(k)}\rangle(|\mu_9^{(k)}\rangle)$, $|\mu_2^{(k)}\rangle(|\mu_{10}^{(k)}\rangle)$, $|\mu_3^{(k)}\rangle(|\mu_{11}^{(k)}\rangle)$, $|\mu_4^{(k)}\rangle(|\mu_{12}^{(k)}\rangle)$, $|\mu_5^{(k)}\rangle(|\mu_{13}^{(k)}\rangle)$, $|\mu_6^{(k)}\rangle(|\mu_{14}^{(k)}\rangle)$, and $|\mu_7^{(k)}\rangle(|\mu_{15}^{(k)}\rangle)$, respectively.

Without loss of generality, suppose that Alice's measurement result is $|\eta_1\rangle_{14710}$, Bob should select the corresponding measuring basis $\{|\mu_0^{(1)}\rangle, |\mu_1^{(1)}\rangle, \ldots, |\mu_7^{(1)}\rangle\}$ to measure his own particles (2,5,8,11). Then, remaining system is described as

$$
\begin{aligned}
&\frac{1}{4}(\lambda_1|000011000\rangle-\lambda_0|110000000\rangle\\
&+\lambda_3|000011111\rangle-\lambda_2|110000111\rangle\\
&+\lambda_5|001100000\rangle-\lambda_4|111111000\rangle\\
&+\lambda_7|001100111\rangle-\lambda_6|111111111\rangle)\\
=&\frac{1}{8\sqrt{2}}[|\mu_0^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle-\lambda_0|10000\rangle\\
&+\lambda_3 e^{i\theta_3}|00111\rangle-\lambda_2 e^{i\theta_2}|10011\rangle\\
&+\lambda_5 e^{i\theta_5}|01000\rangle-\lambda_4 e^{i\theta_4}|11100\rangle\\
&+\lambda_7 e^{i\theta_7}|01011\rangle-\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_1^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle+\lambda_0|10000\rangle\\
&+\lambda_3 e^{i\theta_3}|00111\rangle+\lambda_2 e^{i\theta_2}|10011\rangle\\
&+\lambda_5 e^{i\theta_5}|01000\rangle+\lambda_4 e^{i\theta_4}|11100\rangle\\
&+\lambda_7 e^{i\theta_7}|01011\rangle+\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_2^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle+\lambda_0|10000\rangle\\
&-\lambda_3 e^{i\theta_3}|00111\rangle-\lambda_2 e^{i\theta_2}|10011\rangle\\
&-\lambda_5 e^{i\theta_5}|01000\rangle-\lambda_4 e^{i\theta_4}|11100\rangle\\
&+\lambda_7 e^{i\theta_7}|01011\rangle+\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_3^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle-\lambda_0|10000\rangle\\
&-\lambda_3 e^{i\theta_3}|00111\rangle+\lambda_2 e^{i\theta_2}|10011\rangle\\
&+\lambda_5 e^{i\theta_5}|01000\rangle-\lambda_4 e^{i\theta_4}|11100\rangle\\
&-\lambda_7 e^{i\theta_7}|01011\rangle+\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_4^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle+\lambda_0|10000\rangle\\
&+\lambda_3 e^{i\theta_3}|00111\rangle+\lambda_2 e^{i\theta_2}|10011\rangle\\
&-\lambda_5 e^{i\theta_5}|01000\rangle-\lambda_4 e^{i\theta_4}|11100\rangle\\
&-\lambda_7 e^{i\theta_7}|01011\rangle-\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_5^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle-\lambda_0|10000\rangle\\
&-\lambda_3 e^{i\theta_3}|00111\rangle+\lambda_2 e^{i\theta_2}|10011\rangle\\
&-\lambda_5 e^{i\theta_5}|01000\rangle+\lambda_4 e^{i\theta_4}|11100\rangle\\
&+\lambda_7 e^{i\theta_7}|01011\rangle-\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_6^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle+\lambda_0|10000\rangle\\
&-\lambda_3 e^{i\theta_3}|00111\rangle-\lambda_2 e^{i\theta_2}|10011\rangle\\
&+\lambda_5 e^{i\theta_5}|01000\rangle+\lambda_4 e^{i\theta_4}|11100\rangle\\
&-\lambda_7 e^{i\theta_7}|01011\rangle-\lambda_6 e^{i\theta_6}|11111\rangle)\\
&+|\mu_7^{(1)}\rangle(\lambda_1 e^{i\theta_1}|00100\rangle-\lambda_0|10000\rangle\\
&+\lambda_3 e^{i\theta_3}|00111\rangle-\lambda_2 e^{i\theta_2}|10011\rangle\\
&-\lambda_5 e^{i\theta_5}|01000\rangle+\lambda_4 e^{i\theta_4}|11100\rangle\\
&-\lambda_7 e^{i\theta_7}|01011\rangle+\lambda_6 e^{i\theta_6}|11111\rangle).
\end{aligned}
\tag{10}
$$

Step 3 On the basis of Alice's and Bob's measurement outcomes, Charlie can convert her particles into the desired state $|\Phi\rangle$ by means of suitable unitary operations.

Without generality, suppose that Bob's measurement result is $|\mu_3^{(1)}\rangle_{25811}$, the state of particles (3,6,9,12,13) held by Charlie are transformed into

$$
\begin{aligned}
|\varphi\rangle_{3,6,9,12,13} =& \frac{1}{8\sqrt{2}}(\lambda_1 e^{i\theta_1}|00100\rangle-\lambda_0|10000\rangle\\
&-\lambda_3 e^{i\theta_3}|00111\rangle+\lambda_2 e^{i\theta_2}|10011\rangle\\
&+\lambda_5 e^{i\theta_5}|01000\rangle-\lambda_4 e^{i\theta_4}|11100\rangle\\
&-\lambda_7 e^{i\theta_7}|01011\rangle+\lambda_6 e^{i\theta_6}|11111\rangle).
\end{aligned}
\tag{11}
$$

As displayed in Table 1, Charlie carries out an unitary operation $V_{3691213}^{13}=\sigma_3^x\sigma_3^z\otimes I_6\otimes\sigma_9^x\otimes\sigma_{12}^z\otimes\sigma_{13}^x$ on the particles 3, 6, 9, 12, and 13 as well as a controlled-not (CNOT) operation on particle 6 and particle 13, respectively. Then the receiver Charlie can obtain the desired state

$$
\begin{aligned}
|\Phi\rangle_{3,6,9,12,13} =& \lambda_0|00101\rangle+\lambda_1 e^{i\theta_1}|10001\rangle\\
&+\lambda_2 e^{i\theta_2}|00110\rangle+\lambda_3 e^{i\theta_3}|10010\rangle\\
&+\lambda_4 e^{i\theta_4}|01000\rangle+\lambda_5 e^{i\theta_5}|11100\rangle\\
&+\lambda_6 e^{i\theta_6}|01011\rangle+\lambda_7 e^{i\theta_7}|11111\rangle.
\end{aligned}
\tag{12}
$$

To sum up, for all the 16×8 measurement results of Alice and Bob, the particles possessed by Charlie will always collapse into the state

$$
\begin{aligned}
&\frac{1}{8\sqrt{2}}(\lambda_{i_0} e^{i\theta_{i_0}}|00100\rangle\circ\lambda_{i_1} e^{i\theta_{i_1}}|10000\rangle\\
&\circ\lambda_{i_2} e^{i\theta_{i_2}}|00111\rangle\circ\lambda_{i_3} e^{i\theta_{i_3}}|10011\rangle\\
&\circ\lambda_{i_4} e^{i\theta_{i_4}}|01000\rangle\circ\lambda_{i_5} e^{i\theta_{i_5}}|11100\rangle\\
&\circ\lambda_{i_6} e^{i\theta_{i_6}}|01011\rangle\circ\lambda_{i_7} e^{i\theta_{i_7}}|11111\rangle),
\end{aligned}
\tag{13}
$$

or

$$
\begin{aligned}
&\frac{1}{8\sqrt{2}}(\lambda_{i_0} e^{i\theta_{i_0}}|00000\rangle\circ\lambda_{i_1} e^{i\theta_{i_1}}|10100\rangle\\
&\circ\lambda_{i_2} e^{i\theta_{i_2}}|00011\rangle\circ\lambda_{i_3} e^{i\theta_{i_3}}|10111\rangle\\
&\circ\lambda_{i_4} e^{i\theta_{i_4}}|01100\rangle\circ\lambda_{i_5} e^{i\theta_{i_5}}|11000\rangle\\
&\circ\lambda_{i_6} e^{i\theta_{i_6}}|01111\rangle\circ\lambda_{i_7} e^{i\theta_{i_7}}|11011\rangle),
\end{aligned}
\tag{14}
$$

where $(\lambda_{i_0},\lambda_{i_1},\lambda_{i_2},\lambda_{i_3},\lambda_{i_4},\lambda_{i_5},\lambda_{i_6},\lambda_{i_7})$ is a row vector of H in Eq. (4), $(e^{i\theta_{i_0}}, e^{i\theta_{i_1}}, e^{i\theta_{i_2}}, e^{i\theta_{i_3}}, e^{i\theta_{i_4}}, e^{i\theta_{i_5}}, e^{i\theta_{i_6}}, e^{i\theta_{i_7}})$ is a row vector of $\boldsymbol{G}^{(k)}$ corresponding to the measurement basis of Bob in Eq. (8). Each symbol $\circ$ is "$+$" or "$-$", which is determined by the above two vectors. Note that there are always four "$+$"s and four "$-$"s in Eqs. (13) and (14). We

can recover the desired state $|\Phi\rangle$ by performing some suitable unitary operations such as σ_z, σ_x, and CNOT. That is to say, the total success probability of this protocol is

$$16\times 8\times\left(\frac{1}{8\sqrt{2}}\right)^2=1. \tag{15}$$

Obviously, our protocol is superior to the protocol in Ref. [30], in which only a little part can be restored.

In this scheme, it can be calculated that the success probability and the classical communication cost are 1 and seven bits, respectively.

Table 1. The measurement results (MR) of Alice and Bob, Charlie's appropriate unitary operations ($V^{ij}_{3691213}$) on her particles 3, 6, 9, 12, and 13, and collective unitary operation (U_{613}) with particle 6 as the controlled qubit while particle 13 as the target one.

MR	$V^{ij}_{3691213}$	U_{613}
$\|\eta_1\rangle_{14710}\|\mu_0^{(1)}\rangle_{25811}$	$\sigma_3^x\sigma_3^z\otimes I_6\otimes\sigma_9^x\otimes I_{12}\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_1^{(1)}\rangle_{25811}$	$\sigma_3^x\otimes I_6\otimes\sigma_9^x\otimes I_{12}\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_2^{(1)}\rangle_{25811}$	$\sigma_3^x\otimes\sigma_6^z\otimes\sigma_9^x\otimes\sigma_{12}^z\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_3^{(1)}\rangle_{25811}$	$\sigma_3^x\sigma_3^z\otimes I_6\otimes\sigma_9^x\otimes\sigma_{12}^z\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_4^{(1)}\rangle_{25811}$	$\sigma_3^x\otimes\sigma_6^z\otimes\sigma_9^x\otimes\sigma_{12}^z\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_5^{(1)}\rangle_{25811}$	$\sigma_3^x\sigma_3^z\otimes\sigma_6^z\otimes\sigma_9^x\otimes\sigma_{12}^z\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_6^{(1)}\rangle_{25811}$	$\sigma_3^x\otimes I_6\otimes\sigma_9^x\otimes\sigma_{12}^z\otimes\sigma_{13}^x$	CNOT_{613}
$\|\eta_1\rangle_{14710}\|\mu_7^{(1)}\rangle_{25811}$	$\sigma_3^x\sigma_3^z\otimes\sigma_6^z\otimes\sigma_9^x\otimes I_{12}\otimes\sigma_{13}^x$	CNOT_{613}

3. JRSP of an arbitrary five-qubit Brown state with non-maximally entangled states

In this section, since the maximally entangled state is unstable, we will extend the scheme presented in Section 2 to a more general case where one four-qubit non-maximally entangled state and three three-qubit non-maximally entangled states are taken as the quantum channel. Furthermore, we employ two common methods to recover the desired state $|\Phi\rangle$ and compare these two methods in the number of auxiliary particles and the success probability of the protocol.

Without generality, we suppose the quantum channel shared by Alice, Bob, and Charlie is given by

$$\begin{aligned}|q\rangle \;=\; & (a_1|000\rangle+b_1|111\rangle)_{123}\otimes(a_2|000\rangle\\ & +b_2|111\rangle)_{456}\otimes(a_3|000\rangle+b_3|111\rangle)_{789}\\ & \otimes(a_4|0000\rangle+b_4|1111\rangle)_{10111213},\end{aligned} \tag{16}$$

where a_j and b_j ($j=1,2,3,4$) are real, and the coefficients satisfy $a_j^2+b_j^2=1$. Particles (1,4,7,10) are held by Alice, (2,5,8,11) by Bob, and (3,6,9,12,13) by Charlie, respectively.

Assume the state that Alice and Bob want to help Charlie to remotely prepare is still a five-qubit Brown state $|\Phi\rangle$. As described in the previous scheme, S_1 is known only to Alice, S_2 only to Bob, but none to Charlie.

As shown in Fig. 2, first, Alice carries out a four-qubit projective measurement on her own particles (1,4,7,10) and announces her measurement result via classical channel. Second, on the receipt of the measurement outcome of Alice, Bob selects the corresponding measuring basis to measure the particles (2,5,8,11) in his hand and informs Charlie of his measurement result through a classical channel. Finally, based on the measurement outcomes of Alice and Bob, Charlie can make some appropriate unitary transformations on her own particles (3,6,9,12,13) to reconstruct the desired state $|\Phi\rangle$. Notice that, for each of the 16×8 measurement results of Alice and Bob, Charlie can recover the desired state $|\Phi\rangle$ by performing appropriate unitary operations with certain probability.

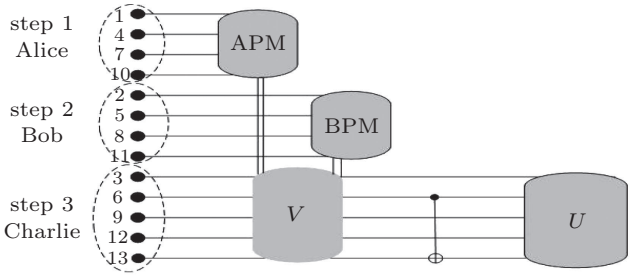


Fig. 2. Quantum circuit of the JRSP protocol of an arbitrary five-bit Brown state. APM and BPM represent two projective measurements, V denotes appropriate unitary operations, $\oplus$ stands for a controlled-not operation, and U is a collective unitary operation U' or optimal positive operator-valued measurement.

To clarify this scheme in detail, without loss of generality, we assume the measurement result of Alice is still $|\eta_1\rangle_{14710}$, Bob selects the corresponding measuring basis $\{|\mu_0^{(1)}\rangle,\ |\mu_1^{(1)}\rangle,\ldots,|\mu_7^{(1)}\rangle\}$ to measure his own particles (2,5,8,11) and obtains the measurement outcome $|\mu_3^{(1)}\rangle_{25811}$. Subsequently, Bob gives notice to Charlie of his measurement result. Corresponding to Alice's and Bob's measurement results, the particles possessed by Charlie have collapsed into the state, as listed in Table 2.

$$\begin{aligned}|\Phi'\rangle \;=\; & \frac{1}{2\sqrt{2}}(a_1a_2b_3a_4\lambda_1\,\mathrm{e}^{\mathrm{i}\theta_1}|00100\rangle-b_1a_2a_3a_4\lambda_0|10000\rangle\\ & -a_1a_2b_3b_4\lambda_3\,\mathrm{e}^{\mathrm{i}\theta_3}|00111\rangle+b_1a_2a_3b_4\lambda_2\,\mathrm{e}^{\mathrm{i}\theta_2}|10011\rangle\\ & +a_1b_2a_3a_4\lambda_5\,\mathrm{e}^{\mathrm{i}\theta_5}|01000\rangle-b_1b_2b_3a_4\lambda_4\,\mathrm{e}^{\mathrm{i}\theta_4}|11100\rangle\\ & -a_1b_2a_3b_4\lambda_7\,\mathrm{e}^{\mathrm{i}\theta_7}|01011\rangle+b_1b_2b_3b_4\lambda_6\,\mathrm{e}^{\mathrm{i}\theta_6}|11111\rangle).\end{aligned} \tag{17}$$

Then Charlie makes some corresponding unitary transformations in Table 1 on her own particles. As a consequence, Charlie readily obtains that the state $|\Phi'\rangle$ has become

$$\begin{aligned}|\Phi''\rangle \;=\; & \frac{1}{2\sqrt{2}}(b_1a_2a_3a_4\lambda_0|00101\rangle+a_1a_2b_3a_4\lambda_1\,\mathrm{e}^{\mathrm{i}\theta_1}|10001\rangle\\ & +b_1a_2a_3b_4\lambda_2\,\mathrm{e}^{\mathrm{i}\theta_2}|00110\rangle+a_1a_2b_3b_4\lambda_3\,\mathrm{e}^{\mathrm{i}\theta_3}|10010\rangle\\ & +b_1b_2b_3a_4\lambda_4\,\mathrm{e}^{\mathrm{i}\theta_4}|01000\rangle+a_1b_2a_3a_4\lambda_5\,\mathrm{e}^{\mathrm{i}\theta_5}|11100\rangle\\ & +b_1b_2b_3b_4\lambda_6\,\mathrm{e}^{\mathrm{i}\theta_6}|01011\rangle+a_1b_2a_3b_4\lambda_7\,\mathrm{e}^{\mathrm{i}\theta_7}|11111\rangle).\end{aligned} \tag{18}$$

Finally, Charlie will take advantage of two common ways to convert the state $|\Phi''\rangle$ into the desired state $|\Phi\rangle$. We go into the specific detail as follows together.

3.1. Method one

In order to construct the desired state $|\Phi\rangle$, Charlie introduces an auxiliary qubit A with the initial state $|0\rangle_A$ and makes a collective unitary transformation U' on her particles (3,6,9,12,13,A) under a set of ordering basis $\{|001010\rangle, |100010\rangle, |001100\rangle, |100100\rangle, |010000\rangle, |111000\rangle, |010110\rangle, |111110\rangle, |001011\rangle, |100011\rangle, |001101\rangle, |100101\rangle, |010001\rangle, |111001\rangle, |010111\rangle$, and $|111111\rangle\}$. Without loss of generality, suppose $|b_1b_2b_3b_4|$ is the largest one in the set $\{|b_1a_2a_3a_4|, |a_1a_2b_3a_4|, |a_1a_2b_3b_4|, |a_1b_2a_3a_4|, |b_1b_2b_3a_4|, |b_1a_2a_3b_4|, |a_1b_2a_3b_4|$, and $|b_1b_2b_3b_4|\}$, then U' is given by

$$\boldsymbol{U}' = \begin{pmatrix} \boldsymbol{D}_1 & \boldsymbol{D}_2 \\ \boldsymbol{D}_2 & -\boldsymbol{D}_1 \end{pmatrix}_{16\times 16}, \tag{19}$$

where $\boldsymbol{D}_i$ $(i = 1, 2)$ is an 8×8 matrix and can be described as

$$\begin{aligned} \boldsymbol{D}_1 &= \mathrm{diag}(d_0, d_1, d_2, d_3, d_4, d_5, d_6, d_7), \\ \boldsymbol{D}_2 &= \mathrm{diag}(\sqrt{1-d_0^2}, \sqrt{1-d_1^2}, \sqrt{1-d_2^2}, \sqrt{1-d_3^2}, \\ &\quad \sqrt{1-d_4^2}, \sqrt{1-d_5^2}, \sqrt{1-d_6^2}, \sqrt{1-d_7^2}), \end{aligned} \tag{20}$$

satisfying

$$\begin{aligned} &d_0 = \frac{b_2b_3b_4}{a_2a_3a_4}, \quad d_1 = \frac{b_1b_2b_4}{a_1a_2a_4}, \quad d_2 = \frac{b_2b_3}{a_2a_3}, \quad d_3 = \frac{b_1b_2}{a_1a_2}, \\ &d_4 = \frac{b_4}{a_4}, \quad d_5 = \frac{b_1b_3b_4}{a_1a_3a_4}, \quad d_6 = 1, \quad d_7 = \frac{b_1b_3}{a_1a_3}. \end{aligned}$$

Therefore, the state of Charlie's particles will evolve as

$$\begin{aligned} &U'(|\Phi''\rangle_{3691213} \otimes |0\rangle_A) \\ &= \frac{1}{2\sqrt{2}}\Big[b_1b_2b_3b_4(\lambda_0|00101\rangle + \lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}|10001\rangle \\ &\quad + \lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}|00110\rangle + \lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}|10010\rangle + \lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}|01000\rangle \\ &\quad + \lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}|11100\rangle + \lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}|01011\rangle \\ &\quad + \lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}|11111\rangle)_{3691213}) \otimes |0\rangle_A \\ &\quad + \left(\sqrt{1-\left(\frac{b_2b_3b_4}{a_2a_3a_4}\right)^2}\lambda_0|00101\rangle \right. \\ &\quad + \sqrt{1-\left(\frac{b_1b_2b_4}{a_1a_2a_4}\right)^2}\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}|10001\rangle \\ &\quad + \sqrt{1-\left(\frac{b_2b_3}{a_2a_3}\right)^2}\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}|00110\rangle \\ &\quad + \sqrt{1-\left(\frac{b_1b_2}{a_1a_2}\right)^2}\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}|10010\rangle \\ &\quad + \sqrt{1-\left(\frac{b_4}{a_4}\right)^2}\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}|01000\rangle \\ &\quad + \sqrt{1-\left(\frac{b_1b_3b_4}{a_1a_3a_4}\right)^2}\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}|11100\rangle \\ &\quad \left. + \sqrt{1-\left(\frac{b_1b_3}{a_1a_3}\right)^2}\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}|11111\rangle)_{3691213} \otimes |1\rangle_A\right]. \end{aligned} \tag{21}$$

In the end, Charlie measures the auxiliary particle A under the basis $\{|0\rangle, |1\rangle\}$. If $|1\rangle$ is probed, her particles will collapse into the trivial state. That is to say, the RSP fails; otherwise, Charlie readily realizes that her particles have accessed to the desired state $|\Phi\rangle$. Therefore, Charlie will achieve success with the probability

$$p = \frac{1}{(2\sqrt{2})^2} \times |b_1b_2b_3b_4|^2 = \frac{1}{8}|b_1b_2b_3b_4|^2. \tag{22}$$

3.2. Method two

For recovering the desired state $|\Phi\rangle$, Charlie introduces three auxiliary particles a, b, and c, which are in the initial state $|000\rangle_{abc}$ and performs three CNOT operations with particles 6, 9, and 12 as the controlled qubits while the auxiliary particles a, b and c as the target ones, respectively. After operating three CNOT operations, the state $|\Phi''\rangle_{3691213}$ will turn into

$$\begin{aligned} |\Phi'''\rangle_{3691213abc} &= b_1a_2a_3a_4\lambda_0|00101010\rangle \\ &\quad + a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}|10001000\rangle \\ &\quad + b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}|00110011\rangle \\ &\quad + a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}|10010001\rangle \\ &\quad + b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}|01000100\rangle \\ &\quad + a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}|11100110\rangle \\ &\quad + b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}|01011101\rangle \\ &\quad + a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}|11111111\rangle. \end{aligned} \tag{23}$$

For the sake of convenience, let $\alpha = b_1a_2a_3a_4$, $\beta = a_1a_2b_3a_4$, $\gamma = b_1a_2a_3b_4$, $\delta = a_1a_2b_3b_4$, $\varepsilon = b_1b_2b_3a_4$, $\tau = a_1b_2a_3a_4$, $\zeta = b_1b_2b_3b_4$, and $\eta = a_1b_2a_3b_4$. This state can be further rewritten as

$$|\Phi'''\rangle_{3691213abc} = \frac{1}{8}\sum_{j=1}^{8}|\Phi_j\rangle_{3691213}|H_j\rangle_{abc}, \tag{24}$$

where

$$\begin{aligned} |\Phi_1\rangle_{3691213} &= (\lambda_0|00101\rangle + \lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}|10001\rangle + \lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}|00110\rangle \\ &\quad + \lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}|10010\rangle + \lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}|01000\rangle \\ &\quad + \lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}|11100\rangle + \lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}|01011\rangle \\ &\quad + \lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}|11111\rangle)_{3691213}, \\ |H_1\rangle_{abc} &= (\alpha|010\rangle + \beta|000\rangle + \gamma|011\rangle + \delta|001\rangle \\ &\quad + \varepsilon|100\rangle + \tau|110\rangle + \zeta|101\rangle + \eta|111\rangle)_{abc}, \\ |\Phi_2\rangle_{3691213} &= (\lambda_0|00101\rangle - \lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}|10001\rangle + \lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}|00110\rangle \end{aligned}$$

$$
\begin{aligned}
&-\lambda_3 e^{i\theta_3}|10010\rangle+\lambda_4 e^{i\theta_4}|01000\rangle\\
&-\lambda_5 e^{i\theta_5}|11100\rangle+\lambda_6 e^{i\theta_6}|01011\rangle\\
&-\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_2\rangle_{abc} &= (\alpha|010\rangle-\beta|000\rangle+\gamma|011\rangle-\delta|001\rangle\\
&+\varepsilon|100\rangle-\tau|110\rangle+\zeta|101\rangle-\eta|111\rangle)_{abc},\\
|\Phi_3\rangle_{3691213} &= (\lambda_0|00101\rangle-\lambda_1 e^{i\theta_1}|10001\rangle-\lambda_2 e^{i\theta_2}|00110\rangle\\
&+\lambda_3 e^{i\theta_3}|10010\rangle-\lambda_4 e^{i\theta_4}|01000\rangle\\
&+\lambda_5 e^{i\theta_5}|11100\rangle+\lambda_6 e^{i\theta_6}|01011\rangle\\
&-\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_3\rangle_{abc} &= (\alpha|010\rangle-\beta|000\rangle-\gamma|011\rangle+\delta|001\rangle\\
&-\varepsilon|100\rangle+\tau|110\rangle+\zeta|101\rangle-\eta|111\rangle)_{abc},\\
|\Phi_4\rangle_{3691213} &= (\lambda_0|00101\rangle+\lambda_1 e^{i\theta_1}|10001\rangle-\lambda_2 e^{i\theta_2}|00110\rangle\\
&-\lambda_3 e^{i\theta_3}|10010\rangle+\lambda_4 e^{i\theta_4}|01000\rangle\\
&+\lambda_5 e^{i\theta_5}|11100\rangle-\lambda_6 e^{i\theta_6}|01011\rangle\\
&-\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_4\rangle_{abc} &= (\alpha|010\rangle+\beta|000\rangle-\gamma|011\rangle-\delta|001\rangle\\
&+\varepsilon|100\rangle+\tau|110\rangle-\zeta|101\rangle-\eta|111\rangle)_{abc},\\
|\Phi_5\rangle_{3691213} &= (\lambda_0|00101\rangle-\lambda_1 e^{i\theta_1}|10001\rangle+\lambda_2 e^{i\theta_2}|00110\rangle\\
&-\lambda_3 e^{i\theta_3}|10010\rangle-\lambda_4 e^{i\theta_4}|01000\rangle\\
&+\lambda_5 e^{i\theta_5}|11100\rangle-\lambda_6 e^{i\theta_6}|01011\rangle\\
&+\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_5\rangle_{abc} &= (\alpha|010\rangle-\beta|000\rangle+\gamma|011\rangle-\delta|001\rangle\\
&-\varepsilon|100\rangle+\tau|110\rangle-\zeta|101\rangle+\eta|111\rangle)_{abc},\\
|\Phi_6\rangle_{3691213} &= (\lambda_0|00101\rangle+\lambda_1 e^{i\theta_1}|10001\rangle-\lambda_2 e^{i\theta_2}|00110\rangle\\
&-\lambda_3 e^{i\theta_3}|10010\rangle-\lambda_4 e^{i\theta_4}|01000\rangle\\
&-\lambda_5 e^{i\theta_5}|11100\rangle+\lambda_6 e^{i\theta_6}|01011\rangle\\
&+\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_6\rangle_{abc} &= (\alpha|010\rangle+\beta|000\rangle-\gamma|011\rangle-\delta|001\rangle\\
&-\varepsilon|100\rangle-\tau|110\rangle+\zeta|101\rangle+\eta|111\rangle)_{abc},\\
|\Phi_7\rangle_{3691213} &= (\lambda_0|00101\rangle-\lambda_1 e^{i\theta_1}|10001\rangle-\lambda_2 e^{i\theta_2}|00110\rangle\\
&+\lambda_3 e^{i\theta_3}|10010\rangle+\lambda_4 e^{i\theta_4}|01000\rangle\\
&-\lambda_5 e^{i\theta_5}|11100\rangle-\lambda_6 e^{i\theta_6}|01011\rangle\\
&+\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_7\rangle_{abc} &= (\alpha|010\rangle-\beta|000\rangle-\gamma|011\rangle+\delta|001\rangle\\
&+\varepsilon|100\rangle-\tau|110\rangle-\zeta|101\rangle+\eta|111\rangle)_{abc},\\
|\Phi_8\rangle_{3691213} &= (\lambda_0|00101\rangle+\lambda_1 e^{i\theta_1}|10001\rangle+\lambda_2 e^{i\theta_2}|00110\rangle\\
&+\lambda_3 e^{i\theta_3}|10010\rangle-\lambda_4 e^{i\theta_4}|01000\rangle\\
&-\lambda_5 e^{i\theta_5}|11100\rangle-\lambda_6 e^{i\theta_6}|01011\rangle\\
&-\lambda_7 e^{i\theta_7}|11111\rangle)_{3691213},\\
|H_8\rangle_{abc} &= (\alpha|010\rangle+\beta|000\rangle+\gamma|011\rangle+\delta|001\rangle\\
&-\varepsilon|100\rangle-\tau|110\rangle-\zeta|101\rangle-\eta|111\rangle)_{abc}. \qquad (25)
\end{aligned}
$$

It is obvious that if the non-orthonormal state $|H_j\rangle_{abc}$ ($j=1,2,\ldots,8$) can be distinguished, Charlie will know exactly the states $|\Phi_j\rangle$ ($j=1,2,\ldots,8$). To discriminate the state $|H_j\rangle_{abc}$ ($j=1,2,\ldots,8$), Charlie can perform an optimal POVM[31,32] on the ancillary particles a, b, and c. The POVM takes the following form:

$$
\boldsymbol{P}_i=\frac{1}{x}|\boldsymbol{M}_i\rangle\langle\boldsymbol{M}_i|,\quad \boldsymbol{P}_9=\boldsymbol{I}-\frac{1}{x}\sum_{i=1}^{8}|\boldsymbol{M}_i\rangle\langle\boldsymbol{M}_i|, \qquad (26)
$$

where

$$
\begin{aligned}
|\boldsymbol{M}_1\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle+\frac{1}{\beta}|000\rangle+\frac{1}{\gamma}|011\rangle+\frac{1}{\delta}|001\rangle\\
&+\frac{1}{\varepsilon}|100\rangle+\frac{1}{\tau}|110\rangle+\frac{1}{\zeta}|101\rangle+\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_2\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle-\frac{1}{\beta}|000\rangle+\frac{1}{\gamma}|011\rangle-\frac{1}{\delta}|001\rangle\\
&+\frac{1}{\varepsilon}|100\rangle-\frac{1}{\tau}|110\rangle+\frac{1}{\zeta}|101\rangle-\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_3\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle-\frac{1}{\beta}|000\rangle-\frac{1}{\gamma}|011\rangle+\frac{1}{\delta}|001\rangle\\
&-\frac{1}{\varepsilon}|100\rangle+\frac{1}{\tau}|110\rangle+\frac{1}{\zeta}|101\rangle-\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_4\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle+\frac{1}{\beta}|000\rangle-\frac{1}{\gamma}|011\rangle-\frac{1}{\delta}|001\rangle\\
&+\frac{1}{\varepsilon}|100\rangle+\frac{1}{\tau}|110\rangle-\frac{1}{\zeta}|101\rangle-\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_5\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle-\frac{1}{\beta}|000\rangle+\frac{1}{\gamma}|011\rangle-\frac{1}{\delta}|001\rangle\\
&-\frac{1}{\varepsilon}|100\rangle+\frac{1}{\tau}|110\rangle-\frac{1}{\zeta}|101\rangle+\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_6\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle+\frac{1}{\beta}|000\rangle-\frac{1}{\gamma}|011\rangle-\frac{1}{\delta}|001\rangle\\
&-\frac{1}{\varepsilon}|100\rangle-\frac{1}{\tau}|110\rangle+\frac{1}{\zeta}|101\rangle+\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_7\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle-\frac{1}{\beta}|000\rangle-\frac{1}{\gamma}|011\rangle+\frac{1}{\delta}|001\rangle\\
&+\frac{1}{\varepsilon}|100\rangle-\frac{1}{\tau}|110\rangle-\frac{1}{\zeta}|101\rangle+\frac{1}{\eta}|111\rangle\Big),\\
|\boldsymbol{M}_8\rangle &= \frac{1}{\sqrt{\xi}}\Big(\frac{1}{\alpha}|010\rangle+\frac{1}{\beta}|000\rangle+\frac{1}{\gamma}|011\rangle+\frac{1}{\delta}|001\rangle\\
&-\frac{1}{\varepsilon}|100\rangle-\frac{1}{\tau}|110\rangle-\frac{1}{\zeta}|101\rangle-\frac{1}{\eta}|111\rangle\Big),
\end{aligned}
$$

$$
\xi=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}+\frac{1}{\delta^2}+\frac{1}{\varepsilon^2}+\frac{1}{\tau^2}+\frac{1}{\zeta^2}+\frac{1}{\eta^2}, \qquad (27)
$$

where $\boldsymbol{I}$ is an identity operator, and the coefficient x, relating to α, β, γ, δ, ε, τ, ζ, and η, can be able to assure $\boldsymbol{P}_9$ to be a positive operator. For exactly determining x, we write $\boldsymbol{P}_1$–$\boldsymbol{P}_8$ in matrix form according to Eqs. (26) and (27), and find out $\boldsymbol{P}_9$ is a diagonal matrix described as

$$\boldsymbol{P}_9 = \begin{pmatrix} 1-\frac{8}{x\xi\alpha^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\frac{8}{x\xi\beta^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\frac{8}{x\xi\gamma^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{8}{x\xi\delta^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-\frac{8}{x\xi\varepsilon^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\frac{8}{x\xi\tau^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-\frac{8}{x\xi\zeta^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\frac{8}{x\xi\eta^2} \end{pmatrix}. \tag{28}$$

Obviously, x should satisfy the inequality $x \geq 8/\xi\mu^2$ because $\boldsymbol{P}_9$ is a positive operator, where μ^2 is the smallest one in the set $\{\alpha^2, \beta^2, \gamma^2, \delta^2, \varepsilon^2, \tau^2, \zeta^2, \eta^2\}$. After performing the previous POVM on the auxiliary particles a, b, and c, Charlie can obtain Φ_i $(i = 1, 2, \ldots, 8)$ with the probability

$$\begin{aligned} \boldsymbol{p}_i &= {}_{3691213abc}\langle \Phi''' | P_i | \Phi''' \rangle_{3691213abc} \\ &= \frac{1}{x\xi}, \quad i = 1, 2, \ldots, 8. \end{aligned} \tag{29}$$

Nevertheless, Charlie may obtain $\boldsymbol{P}_9$'s value with a probability of $1 - 8/x\xi$, under such circumstance Charlie cannot determine which state the particles a, b, and c are in. As proposed before, only if Charlie can accurately determine $|H_j\rangle$ $(j = 1, 2, \ldots, 8)$, can Charlie construct the desired state $|\Phi\rangle$ by preforming some appropriate unitary operations. Hence, when $x = 8/\xi\mu^2$, Charlie will achieve success with the maximum probability $p_{\max}$,

$$p_{\max} = \frac{1}{(2\sqrt{2})^2} \times 8 \times \frac{\mu^2}{8} = \frac{\mu^2}{8}. \tag{30}$$

In conclusion, Charlie employs method one, and she needs to introduce one auxiliary particle and achieves success with the probability $\frac{1}{8}|b_1b_2b_3b_4|^2$. While method two, she needs to introduce three auxiliary particles and achieves success with the probability $\mu^2/8$. Compared with the above two methods, it is evident that Charlie needs to introduce fewer auxiliary particles and achieves higher success probability by means of method one. For each of the 16×8 measurement results of Alice and Bob, Charlie can recover the desired state $|\Phi\rangle$ by means of method one with the probability of $\frac{1}{8}|b_1b_2b_3b_4|^2$. Therefore, the maximal success probability of this scheme is $16|b_1b_2b_3b_4|^2$ and the classical communication cost is seven cbits.

Table 2. Corresponding relationship between the measurement results (MR) of Alice and Bob and the collapsed state $|\Phi'\rangle_{3691213}$ of Charlie.

MR	$\lvert\Phi'\rangle_{3691213}$
$\lvert\eta_1\rangle_{14710}\lvert\mu_0^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle - b_1a_2a_3a_4\lambda_0\lvert 10000\rangle + a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $-b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle + a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle - b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $+a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle - b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle)$
$\lvert\eta_1\rangle_{14710}\lvert\mu_1^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle + b_1a_2a_3a_4\lambda_0\lvert 10000\rangle + a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $+b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle + a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle + b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $+a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle + b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$
$\lvert\eta_1\rangle_{14710}\lvert\mu_2^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle + b_1a_2a_3a_4\lambda_0\lvert 10000\rangle - a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $-b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle - a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle - b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $+a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle + b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$
$\lvert\eta_1\rangle_{14710}\lvert\mu_3^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle - b_1a_2a_3a_4\lambda_0\lvert 10000\rangle - a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $+b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle + a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle - b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $-a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle + b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$
$\lvert\eta_1\rangle_{14710}\lvert\mu_4^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle + b_1a_2a_3a_4\lambda_0\lvert 10000\rangle + a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $+b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle - a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle - b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $-a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle - b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$
$\lvert\eta_1\rangle_{14710}\lvert\mu_5^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle - b_1a_2a_3a_4\lambda_0\lvert 10000\rangle - a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $+b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle - a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle + b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $+a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle - b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$
$\lvert\eta_1\rangle_{14710}\lvert\mu_6^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle + b_1a_2a_3a_4\lambda_0\lvert 10000\rangle - a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $-b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle + a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle + b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $-a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle - b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$
$\lvert\eta_1\rangle_{14710}\lvert\mu_7^{(1)}\rangle_{25811}$	$\frac{1}{2\sqrt{2}}$ $(a_1a_2b_3a_4\lambda_1 \mathrm{e}^{\mathrm{i}\theta_1}\lvert 00100\rangle - b_1a_2a_3a_4\lambda_0\lvert 10000\rangle + a_1a_2b_3b_4\lambda_3 \mathrm{e}^{\mathrm{i}\theta_3}\lvert 00111\rangle$ $-b_1a_2a_3b_4\lambda_2 \mathrm{e}^{\mathrm{i}\theta_2}\lvert 10011\rangle - a_1b_2a_3a_4\lambda_5 \mathrm{e}^{\mathrm{i}\theta_5}\lvert 01000\rangle + b_1b_2b_3a_4\lambda_4 \mathrm{e}^{\mathrm{i}\theta_4}\lvert 11100\rangle$ $-a_1b_2a_3b_4\lambda_7 \mathrm{e}^{\mathrm{i}\theta_7}\lvert 01011\rangle + b_1b_2b_3b_4\lambda_6 \mathrm{e}^{\mathrm{i}\theta_6}\lvert 11111\rangle$

4. Experimental feasibility and security

Now, let us turn to analyze the experimental feasibility of our schemes. Four-particle projective measurement, local six-particle collective unitary transform, and three-particle POVM play a vital role in our two schemes. First, so far projective measurement has added its appeal for scholars. It has turned out to be any projective measurement can be decomposed into a sequence of weak measurement,[33,34] which causes only small changes to original state. Second, it has been verified that any many-particle collective unitary transformation can be decomposed into some single-particle rotation operations as well as a CNOT gate transformation. In fact, single-particle rotating operations together with CONT gate have been investigated by various protocols.[35–38] Finally, POVM has been demonstrated in the frame of all-optics by some scholars and experts.[39,40] Hence, we believe that our schemes may be realized in the realm of current experimental technology.

As for the security of our schemes, we will consider two possible cases: internal and external attacks. We give a brief explanation about why these two protocols are secure against them. With respect to internal attack, if a sender does not cooperate with the other, he cannot obtain any information of the desired state, because the specific information of the desired state is known partially to every sender. With regard to external attack, the adversaries have no knowledge about the desired state, because only classical information corresponding to measurement results of all the senders is transmitted in the channels throughout the entire process.

5. Conclusion

In this paper, we put forward two novel protocols for joint remote preparation of an arbitrary five-qubit Brown state to enrich the research of Brown state. In the first scheme, we employ some basic quantum operations to improve its success probability to 1, which is far more than the existing ones. In the second scheme, two existing methods are used to recover the desired state. Comparing these two methods, we come to a conclusion that the first is superior to the second-optimal positive operator-valued measure in the number of auxiliary particles and the success probability, which will contribute to the further development of quantum information science.

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