Comments on "Distillability sudden death in qutrit–qutrit systems under thermal reservoirs"

Mazhar Ali†

Department of Electrical Engineering, COMSATS Institute of Information Technology, 22060 Abbottabad, Pakistan

(Received 29 December 2013; revised manuscript received 17 February 2014; published online 23 July 2014)

In a recent paper (2012 *Chin. Phys. B* **21** 084205), the authors studied the problem of distillability sudden death. We find that their equation of motion is incorrect and consequently the rest of the paper is wrong. Even apart from starting with a wrong equation of motion, their description of the phenomenon of distillability sudden death is totally misleading and needs to be rectified. To this aim, we show that certain initially prepared free-entangled states become bound-entangled in a finite time under thermal reservoirs. Moreover, in contrast with zero-temperature reservoirs, simple local unitary transformations cannot completely avoid distillability sudden death.

Keywords: distillability, thermal reservoirs, bound entanglement

PACS: 03.65.Yz, 03.67.Mn, 03.65.Ud **DOI:** 10.1088/1674-1056/23/9/090306

1. Introduction

It was shown that certain free-entangled states of qutrit–qutrit systems become non-distillable in a finite time under the influence of either multi-local decoherence[1] and/or under the combination of collective and local dephasing processes.[2] Such behavior has been named distillability sudden death (DSD).[1] This phenomenon was later also studied for qutrit–qutrit systems interacting with zero-temperature reservoirs[3] and under depolarizing noise.[4] Recently, this problem was attempted for qutrit–qutrit systems interacting with thermal reservoirs,[5] however, we find that the authors studied this problem in an unfortunate way. Not only is their basic equation of motion wrong but they also employ the concept in a totally misleading way. The purpose of this paper is to rectify this misconception, not only for the authors but also for other readers. We find that the distillability sudden death may occur in this type of open system as well and simple local unitary transformations cannot avoid this phenomenon. Even a small amount of temperature is sufficient to bring NPT-entangled states into PPT-states whose entanglement may not always be detectable.

2. Model and results

We take a three-level atom in the V configuration with $|e\rangle$, and $|u\rangle$ as the two excited states and a ground state $|g\rangle$. Let γ_e and γ_u be the spontaneous decay rates. There exists quantum interference defined as

$$\beta = \frac{\gamma_e - \gamma_u}{\gamma_e + \gamma_u}. \tag{1}$$

This expression shows that maximum interference is expected when the level $|u\rangle$ is metastable. Details can be found in Ref. [3] and the references therein. For a system of two three-level atoms A and B such that they are separated by a distance which is much larger than the wavelength of radiation. The master equation describing the dissipative part of dynamics is given as[6]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \Lambda\rho, \tag{2}$$

with

$$\begin{aligned}\Lambda\rho \;=\; & \frac{\gamma_e^A}{2}(m_A+1)\left[2\sigma_{ge}^A\rho\sigma_{eg}^A - \sigma_{ee}^A\rho - \rho\sigma_{ee}^A\right] \\ & + \gamma_e^A m_A\left[2\sigma_{eg}^A\rho\sigma_{ge}^A - \sigma_{gg}^A\rho - \rho\sigma_{gg}^A\right] \\ & + \frac{\gamma_u^A}{2}(m_A+1)\left[2\sigma_{gu}^A\rho\sigma_{ug}^A - \sigma_{uu}^A\rho - \rho\sigma_{uu}^A\right] \\ & + \gamma_u^A m_A\left[2\sigma_{ug}^A\rho\sigma_{gu}^A - \sigma_{gg}^A\rho - \rho\sigma_{gg}^A\right] \\ & + \frac{\gamma_e^B}{2}(n_B+1)\left[2\sigma_{ge}^B\rho\sigma_{eg}^B - \sigma_{ee}^B\rho - \rho\sigma_{ee}^B\right] \\ & + \gamma_e^B n_B\left[2\sigma_{eg}^B\rho\sigma_{ge}^B - \sigma_{gg}^B\rho - \rho\sigma_{gg}^B\right] \\ & + \frac{\gamma_u^B}{2}(n_B+1)\left[2\sigma_{gu}^B\rho\sigma_{ug}^B - \sigma_{uu}^B\rho - \rho\sigma_{uu}^B\right] \\ & + \gamma_u^B n_B\left[2\sigma_{ug}^B\rho\sigma_{gu}^B - \sigma_{gg}^B\rho - \rho\sigma_{gg}^B\right],\end{aligned} \tag{3}$$

where m_A and n_B are the mean photon numbers for qutrits A and B, respectively. At this stage, please compare Eq. (3) with the incorrect Eq. (2) of Ref. [5]. For concreteness, we have provided the detailed derivation of this equation in the appendix. For $m_A = n_B = 0$, we have vaccum reservoirs, which have already been studied.[3] The atomic operators are defined

†Corresponding author. E-mail: mazharaliawan@yahoo.com

© 2014 Chinese Physical Society and IOP Publishing Ltd

as

$$\sigma_{ij}^{A} = \sigma_{ij} \otimes \mathbb{I}_3, \quad \sigma_{ij}^{B} = \mathbb{I}_3 \otimes \sigma_{ij}, \quad i,j = e,u,g. \tag{4}$$

For the sake of greater simplicity, we take $\gamma_e^A = \gamma_e^B = \gamma_e$, $\gamma_u^A = \gamma_u^B = \gamma_e$, and $m_A = n_B = m$ in the rest of this paper. The atomic transition operator $\sigma_{ij} = |i\rangle\langle j|$ takes an atom from its state $|j\rangle$ to state $|i\rangle$. We prefer to work with basis $\{|e,e\rangle, |e,u\rangle, |e,g\rangle, |u,e\rangle, |u,u\rangle, |u,g\rangle, |g,e\rangle, |g,u\rangle, |g,g\rangle\}$. The general solution of the master equation (2) is quite lengthy and is not worth providing here.

Bound-entangled states have positive partial transpose (PPT) and therefore are non-distillable. In general, there is not a unique criterion to detect all bound-entangled states, However, one can use the realignment criterion[7,8] to detect certain bound-entangled states. The realignment of a given density matrix is obtained as $(\rho^R)_{ij,kl} = \rho_{ik,jl}$. For a separable state ρ, realignment criterion implies that $\|\rho^R\| \leq 1$. For a PPT-state, the positive value of the quantity $\|\rho^R\| - 1$ can prove the bound-entangled state. Distillability sudden death corresponds to a situation where free-entangled states become non-distillable (PPT-entangled) in a finite time under the influence of decoherence.[1–3] For an example, consider quantum states given as

$$\sigma_\alpha = \frac{2}{7}|\Psi_+\rangle\langle\Psi_+| + \frac{\alpha}{7}\sigma_+ + \frac{5-\alpha}{7}\sigma_-, \tag{5}$$

where $2 \leq \alpha \leq 5$, $|\Psi_+\rangle = (|0,1\rangle + |1,0\rangle + |2,2\rangle)/\sqrt{3}$, $\sigma_+ = (|0,0\rangle\langle 0,0| + |1,2\rangle\langle 1,2| + |2,1\rangle\langle 2,1|)/3$, and $\sigma_- = (|1,1\rangle\langle 1,1| + |2,0\rangle\langle 2,0| + |0,2\rangle\langle 0,2|)/3$. It is known[9] that σ_α is separable for $2 \leq \alpha \leq 3$, bound-entangled for $3 < \alpha \leq 4$, and free-entangled for $4 < \alpha \leq 5$. We take $4 < \alpha \leq 5$. We need to determine the time evolution of an appropriate measure of entanglement for qutrit–qutrit systems. One appropriate measure for free-entangled states (NPT-states) is negativity,[10] which is a measure of a state having negative partial transpose. Negativity $N(\rho)$ is equal to the absolute value of the sum of negative eigenvalues of partial transpose of a state ρ. Once negativity becomes zero, we can easily study time evolution of realignment criterion to determine the existence of bound-entangled states.

The partial transpose of time evolved density matrix $\sigma_\alpha(t)$ can have three possible negative eigenvalues. However, expressions for possible negative eigenvalues are quite involved, therefore, we do not write them. Negativity for these states can be defined as

$$N(\sigma_\alpha(t)) = \max[0,-\eta_1] + \max[0,-\eta_2] + \max[0,-\eta_3], \tag{6}$$

where η_i are the possible negative eigenvalues. Let us now demonstrate that the time evolved density matrix $\sigma_\alpha(t)$ does undergo DSD. This could be most easily demonstrated by checking the entanglement of $\sigma_\alpha(t)$ after it becomes PPT. Let us fix the interference parameter β such that $\gamma_u = 0.5\gamma_e$ throughout this paper. We set the parameter $m = 0.01$. The relation between negativity and the realignment criterion is plotted in Fig. 1 against decay parameter $t\gamma_e$ and for a specific choice of the parameter $\alpha = 4.2$. Figure 1 shows that an initial free-entangled state becomes bound-entangled at a time $t\gamma_e \approx 0.175$. We call this point Γt_N where negativity becomes zero. Entanglement of the PPT-state is verified by the positive value of $\|\sigma^R(t)\| - 1$ in the range of $0.175 \lesssim t\gamma_e \lesssim 0.292$. However, realignment criterion fails to detect possible entanglement after time $t\gamma_e \approx 0.292$. We call this point Γt_R, shown in Fig. 1. Hence, we have demonstrated that free-entangled states exhibit DSD under thermal reservoirs. In comparison, the authors of Ref. [5] have only plotted $\|\sigma^R(t)\| - 1$ (although an incorrect value due to the incorrect equation of motion) without even worrying about the NPT-property of time evolved states, for example, Figs. 1–3 of Ref. [5]. They assume that as they have taken an initial NPT state, so vanishing of $\|\sigma^R(t)\| - 1$ means DSD, which is obviously misleading and meaningless.

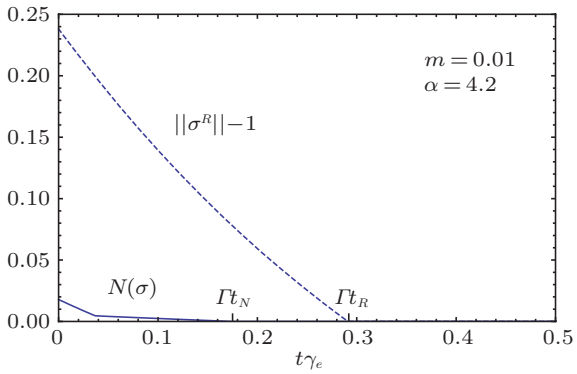


Fig. 1. (color online) Negativity (solid line) and realignment criterion (dashed line) are plotted against decay parameter $t\gamma_e$.

Figure 2 depicts the time evolution of negativity and realignment criterion for an initial quantum state σ_α (Eq. (5)) with $\alpha = 4.5$. For this particular case, negativity becomes zero at $t\gamma_e = \Gamma t_N \approx 0.614$, however, realignment criterion fails to detect entangled states after time $t\gamma_e = \Gamma t_R \approx 0.292$. As there exists no criterion to detect the entanglement of quantum states

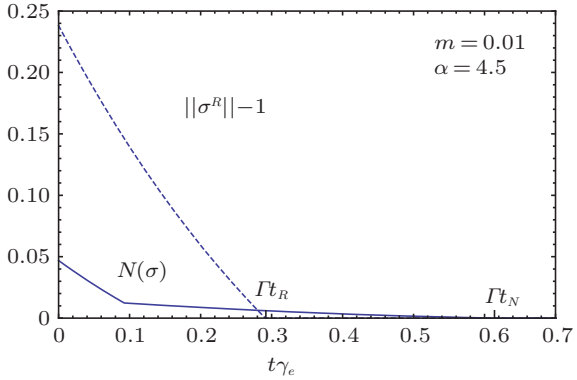


Fig. 2. (color online) Negativity (solid line) and realignment criterion (dashed line) are plotted against decay parameter $t\gamma_e$.

in dimension ≥ 6, therefore, we cannot comment on the entanglement of state $\sigma_\alpha(t)$ after $t\gamma_e \approx 0.614$. Therefore, we are not sure whether DSD has occurred or not. This example also shows the insensitivity of realignment criterion on parameter α. We have seen that although the initial NPT-states (free-entangled) become PPT after a finite time, however we cannot conclude their separability/entanglement immediately.

Consider the ρ_α obtained from Eq. (5) by applying unitary operation $\mathbb{J} = \mathbb{I}_3 \otimes \kappa$, with $\kappa = |0\rangle\langle 1| + |1\rangle\langle 0| + |2\rangle\langle 2|$ as

$$\rho_\alpha = \mathbb{J}\sigma_\alpha\mathbb{J}^\dagger = \frac{2}{7}|\tilde{\Psi}_+\rangle\langle\tilde{\Psi}_+| + \frac{\alpha}{7}\tilde{\sigma}_+ + \frac{5-\alpha}{7}\tilde{\sigma}_-, \tag{7}$$

where $\tilde{\sigma}_+ = (|0,1\rangle\langle 0,1| + |1,2\rangle\langle 1,2| + |2,0\rangle\langle 2,0|)/3$, $\tilde{\sigma}_- = (|1,0\rangle\langle 1,0| + |2,1\rangle\langle 2,1| + |0,2\rangle\langle 0,2|)/3$, and $|\tilde{\Psi}_+\rangle = (|0,0\rangle + |1,1\rangle + |2,2\rangle)/\sqrt{3}$. The possible negative eigenvalues of the partial transpose of time evolved density matrix $\rho_\alpha(t)$ are again quite involved, therefore we do not write them here. In Fig. 3, we have plotted negativity and $\|\rho^R(t)\| - 1$ against parameter $t\gamma_e$ for specific choices of $\alpha = 4.5$ and $m = 0.01$. We observe that realignment criterion fails to detect entanglement after $t\gamma_e \approx 0.281$ whereas negativity becomes zero at $t\gamma_e \approx 1.72$, which is sufficiently large as compared with Fig. 1. This implies that the state $\rho_\alpha(t)$ becomes PPT at finite time. This observation is in contrast with zero-temperature reservoirs where for the same state $\rho_\alpha(t)$, negativity is always non-zero in the range $4 < \alpha \leq 5$.[3] Hence, even a small amount of the mean number of photons is sufficient to bring NPT states to the PPT region. Hence we have seen that simple local unitary transformations cannot completely avoid DSD under thermal reservoirs.

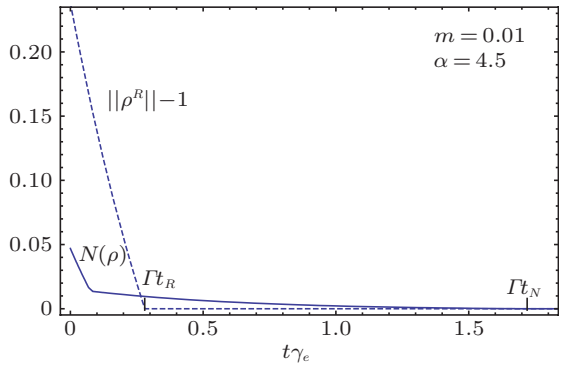


Fig. 3. (color online) N negativity (solid line) and realignment criterion (dashed line) are plotted against decay parameter $t\gamma_e$. The state $\rho_\alpha(t)$ becomes PPT at a finite time of $t\gamma_e \approx 1.72$.

3. Summary

In summary, we have pointed out errors in a published paper. We have also added some new results. We have found that certain free-entangled qutrit–qutrit states become bound-entangled under thermal reservoirs. We have observed that local unitary transformations cannot avoid this phenomenon under thermal reservoirs.

Appendix A: Derivation of Eq. (3)

We consider a system "S" interacting with a reservoir "R". The combined density operator is given as ρ_{SR}. The reduced density matrix ρ_S for a system alone can be obtained by taking a trace over the reservoir degrees of freedom, that is, $\rho_S = \mathrm{Tr}_R(\rho_{SR})$. In the interaction picture, the equation of motion is given as

$$\mathrm{i}\hbar\dot{\rho}_{SR} = [\mathscr{V}, \rho_{SR}(t)]. \tag{A1}$$

After integrating this equation, we get

$$\rho_{SR}(t) = \rho_{SR}(t_\mathrm{i}) - \frac{\mathrm{i}}{\hbar}\int_{t_\mathrm{i}}^{t}[\mathscr{V}(t'), \rho_{SR}(t')]\,\mathrm{d}t'. \tag{A2}$$

Substituting Eq. (A2) back into Eq. (A1), we obtain

$$\begin{aligned}\dot{\rho}_{SR}(t) \;=\; & -\frac{\mathrm{i}}{\hbar}[\mathscr{V}(t), \rho_{SR}(t_\mathrm{i})] \\ & - \frac{1}{\hbar^2}\int_{t_\mathrm{i}}^{t}[\mathscr{V}(t), [\mathscr{V}(t'), \rho_{SR}(t')]]\,\mathrm{d}t'.\end{aligned} \tag{A3}$$

As the interaction between system and the reservoir is weak, therefore we look at the solution of the form $\rho_{SR}(t) = \rho_S(t)\otimes\rho_R(t_\mathrm{i}) + \rho_c(t)$, where for consistency we demand that $\mathrm{Tr}_R(\rho_c(t)) = 0$. Substituting this form into Eq. (A3) and retaining terms upto order $\mathscr{V}^2$, we get

$$\begin{aligned}&\dot{\rho}_S(t) \\ &= -\frac{\mathrm{i}}{\hbar}\mathrm{Tr}_R[\mathscr{V}(t), \rho_S(t_\mathrm{i})\otimes\rho_R(t_\mathrm{i})] \\ &\quad - \frac{1}{\hbar^2}\mathrm{Tr}_R\int_{t_\mathrm{i}}^{t}[\mathscr{V}(t), [\mathscr{V}(t'), \rho_S(t')\otimes\rho_R(t_\mathrm{i})]]\,\mathrm{d}t'.\end{aligned} \tag{A4}$$

Under Markovian assumption, the system density operator $\rho_S(t')$ can be replaced by $\rho_S(t)$. Hence, equation (A4) becomes

$$\begin{aligned}&\dot{\rho}_S(t) \\ &= -\frac{\mathrm{i}}{\hbar}\mathrm{Tr}_R[\mathscr{V}(t), \rho_S(t_\mathrm{i})\otimes\rho_R(t_\mathrm{i})] \\ &\quad - \frac{1}{\hbar^2}\mathrm{Tr}_R\int_{t_\mathrm{i}}^{t}[\mathscr{V}(t), [\mathscr{V}(t'), \rho_S(t)\otimes\rho_R(t_\mathrm{i})]]\,\mathrm{d}t'.\end{aligned} \tag{A5}$$

We consider the radiative decay of three level atom with $|e\rangle$ and $|u\rangle$ as non-degenerate excited states and $|g\rangle$ as the ground state, which is damped by a reservoir with annihilation and creation operators a_k and $a_k^\dagger$ with density frequencies $\nu_k = ck$. The Hamiltonian of the system in the interaction picture and rotating wave approximation is given as

$$\begin{aligned}\mathscr{V}(t) \;=\; & \hbar\sum_k [g_k^{(eg)} a_k^\dagger \sigma_{ge}\,\mathrm{e}^{-\mathrm{i}(\omega_{eg}-\nu_k)t} \\ & + g_k^{(ug)} a_k^\dagger \sigma_{gu}\,\mathrm{e}^{-\mathrm{i}(\omega_{ug}-\nu_k)t}] + \mathrm{H.c},\end{aligned} \tag{A6}$$

where H.c represents hermitian conjugate. This Hamiltonian can be written as

$$\mathscr{V}(t) = \mathscr{V}^{(eg)}(t) + \mathscr{V}^{(ug)}(t), \tag{A7}$$

where

$$\mathscr{V}^{(eg)}(t) = \hbar\sum_k g_k^{(eg)}[a_k^\dagger\sigma_{ge}\,\mathrm{e}^{-\mathrm{i}(\omega_{eg}-\nu_k)t} + \sigma_{eg}a_k\,\mathrm{e}^{\mathrm{i}(\omega_{eg}-\nu_k)t}], \quad (A8)$$

$$\mathscr{V}^{(ug)}(t) = \hbar\sum_k g_k^{(ug)}[a_k^\dagger\sigma_{gu}\,\mathrm{e}^{-\mathrm{i}(\omega_{ug}-\nu_k)t} + \sigma_{ug}a_k\,\mathrm{e}^{\mathrm{i}(\omega_{ug}-\nu_k)t}]. \quad (A9)$$

Substituting this Hamiltonian in Eq. (A4), we obtain

$$\begin{aligned}\dot\rho_S(t) =& -\mathrm{i}\sum_k g_k^{(eg)}\langle a_k^\dagger\rangle[\sigma_{ge},\rho_S(t_\mathrm{i})]\,\mathrm{e}^{-\mathrm{i}(\omega_{eg}-\nu_k)t}\\
&-\int_{t_\mathrm{i}}^t \mathrm{d}t'\sum_{k,k'} g_k^{(eg)}g_{k'}^{(eg)}\\
&\times\{[\sigma_{ge}\sigma_{ge}\rho_S(t')-2\sigma_{ge}\rho_S(t')\sigma_{ge}+\rho_S(t')\sigma_{ge}\sigma_{ge}]\\
&\times\mathrm{e}^{-\mathrm{i}(\omega_{eg}-\nu_k)t-\mathrm{i}(\omega_{eg}-\nu_{k'})t'}\langle a_k^\dagger a_{k'}^\dagger\rangle\\
&+[\sigma_{gg}\rho_S(t')-\sigma_{eg}\rho_S(t')\sigma_{ge}]\\
&\times\mathrm{e}^{-\mathrm{i}(\omega_{eg}-\nu_k)t+\mathrm{i}(\omega_{eg}-\nu_{k'})t'}\langle a_k^\dagger a_{k'}\rangle\\
&+[\sigma_{ee}\rho_S(t')-\sigma_{ge}\rho_S(t')\sigma_{eg}]\\
&\times\mathrm{e}^{\mathrm{i}(\omega_{eg}-\nu_k)t-\mathrm{i}(\omega_{eg}-\nu_{k'})t'}\langle a_k a_{k'}^\dagger\rangle\}+\mathrm{H.c.}\\
&-\mathrm{i}\sum_k g_k^{(ug)}\langle a_k^\dagger\rangle[\sigma_{gu},\rho_S(t_\mathrm{i})]\,\mathrm{e}^{-\mathrm{i}(\omega_{ug}-\nu_k)t}\\
&-\int_{t_\mathrm{i}}^t \mathrm{d}t'\sum_{k,k'} g_k^{(ug)}g_{k'}^{(ug)}\\
&\times\{[\sigma_{gu}\sigma_{gu}\rho_S(t')-2\sigma_{gu}\rho_S(t')\sigma_{gu}+\rho_S(t')\sigma_{gu}\sigma_{gu}]\\
&\times\mathrm{e}^{-\mathrm{i}(\omega_{ug}-\nu_k)t-\mathrm{i}(\omega_{ug}-\nu_{k'})t'}\langle a_k^\dagger a_{k'}^\dagger\rangle\\
&+[\sigma_{gg}\rho_S(t')-\sigma_{ug}\rho_S(t')\sigma_{gu}]\\
&\times\mathrm{e}^{-\mathrm{i}(\omega_{ug}-\nu_k)t+\mathrm{i}(\omega_{ug}-\nu_{k'})t'}\langle a_k^\dagger a_{k'}\rangle\\
&+[\sigma_{uu}\rho_S(t')-\sigma_{gu}\rho_S(t')\sigma_{ug}]\\
&\times\mathrm{e}^{\mathrm{i}(\omega_{ug}-\nu_k)t-\mathrm{i}(\omega_{ug}-\nu_{k'})t'}\langle a_k a_{k'}^\dagger\rangle\}+\mathrm{H.c.} \quad (A10)\end{aligned}$$

For the thermal reservoir, the density operator is the multi-mode extension of the thermal operator, that is

$$\rho_R=\prod_k\left[1-\exp\left(-\frac{\hbar\nu_k}{k_\mathrm{B}T}\right)\right]\exp\left(-\frac{\hbar\nu_k a_k^\dagger a_k}{k_\mathrm{B}T}\right), \quad (A11)$$

where k_B is the Boltzmann constant and T is the temperature. It can be easily shown that

$$\begin{aligned}&\langle a_k\rangle=\langle a_k^\dagger\rangle=0,\\
&\langle a_k^\dagger a_{k'}\rangle=\bar n_k\delta_{kk'},\\
&\langle a_k a_{k'}^\dagger\rangle=(\bar n_k+1)\delta_{kk'},\\
&\langle a_k a_{k'}\rangle=\langle a_k^\dagger a_{k'}^\dagger\rangle=0, \quad (A12)\end{aligned}$$

where the mean thermal boson number is

$$\bar n_k=\frac{1}{\exp\left(\dfrac{\hbar\nu_k}{k_\mathrm{B}T}\right)-1}. \quad (A13)$$

Substituting various expectation values from Eq. (A12) into Eq. (A10), we obtain

$$\begin{aligned}\dot\rho(t) =& -\int_{t_\mathrm{i}}^t \mathrm{d}t'\sum_k (g_k^{(eg)})^2\Big\{[\sigma_{gg}\rho(t')-\sigma_{eg}\rho(t')\sigma_{ge}]\\
&\times\mathrm{e}^{-\mathrm{i}(\omega_{eg}-\nu_k)(t-t')}\bar n_k+[\sigma_{ee}\rho(t')-\sigma_{ge}\rho(t')\sigma_{eg}]\\
&\times\mathrm{e}^{\mathrm{i}(\omega_{eg}-\nu_k)(t-t')}(\bar n_k+1)\Big\}+\mathrm{H.c.}\\
&-\int_{t_\mathrm{i}}^t \mathrm{d}t'\sum_k (g_k^{(ug)})^2\Big\{[\sigma_{gg}\rho(t')-\sigma_{ug}\rho(t')\sigma_{gu}]\\
&\times\mathrm{e}^{-\mathrm{i}(\omega_{ug}-\nu_k)(t-t')}\bar n_k+[\sigma_{uu}\rho(t')-\sigma_{gu}\rho(t')\sigma_{ug}]\\
&\times\mathrm{e}^{\mathrm{i}(\omega_{ug}-\nu_k)(t-t')}(\bar n_k+1)\Big\}+\mathrm{H.c.} \quad (A14)\end{aligned}$$

We have dropped the subscript "S" with no confusion to describe the density operator of our three level atom. Finally, one can carry out the standard procedure of the Weisskopf–Wigner theory of spontaneous emission to solve the above equation and obtain

$$\begin{aligned}&\dot\rho(t)\\
=&-\bar n\frac{\gamma_e}{2}[\sigma_{gg}\rho(t)-\sigma_{eg}\rho(t)\sigma_{ge}]\\
&-(\bar n+1)\frac{\gamma_e}{2}[\sigma_{ee}\rho(t)-\sigma_{ge}\rho(t)\sigma_{eg}]+\mathrm{H.c.}\\
&-\bar n\frac{\gamma_u}{2}[\sigma_{gg}\rho(t)-\sigma_{ug}\rho(t)\sigma_{gu}]\\
&-(\bar n+1)\frac{\gamma_u}{2}[\sigma_{uu}\rho(t)-\sigma_{gu}\rho(t)\sigma_{ug}]+\mathrm{H.c.} \quad (A15)\end{aligned}$$

Hence we have completed our derivation with an obvious extension for two atoms, as described in Eq. (3).

References

[1] Song W, Chen L and Zhu S L 2009 *Phys. Rev. A* **80** 012331
[2] Ali M 2010 *Phys. Rev. A* **81** 042303
[3] Ali M 2010 *J. Phys. B: At. Mol. Phys.* **43** 045504
[4] Khan S and Khan M K 2011 *J. Mod. Opt.* **58** 918
[5] Huang J, Fang M F, Yang B Y and Liu X 2012 *Chin. Phys. B* **21** 084205
[6] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)
[7] Chen K and Wu L A 2003 *Quantum. Inf. Comput.* **3** 193
[8] Rudolph O 2005 *Quantum. Inf. Process.* **4** 219
[9] Horodecki P, Horodecki M and Horodecki R 1999 *Phys. Rev. Lett.* **82** 1056
[10] Vidal G and Werner R F 2002 *Phys. Rev. A* **65** 032314