Unsuitable use of spin and pseudospin symmetries with a pseudoscalar Cornell potential

L. B. Castro[a)b)†] and A. S. de Castro[c)]

[a)] *Departamento de Física, CFM Universidade Federal de Santa Catarina*, 88040-900, *CP.* 476, *Florianópolis - SC, Brazil*
[b)] *Departamento de Física, Universidade Federal do Maranhão Campus Universtitário do Bacanga*, 6580-805, *São Luís, MA, Brazil*
[c)] *Departamento de Física e Química, Campus de Guaratinguetá Universidade Estadual Paulista*, 12516-410, *Guaratinguetá - SP, Brazil*

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The concepts of spin and pseudospin symmetries has been used as mere rhetorics to decorate the pseudoscalar potential [*Chin. Phys. B* **22** 090301 (2013)]. It is also pointed out that a more complete analysis of the bound states of fermions in a pseudoscalar Cornell potential has already been published elsewhere.

Keywords: Dirac equation, pseudoscalar Cornell potential, spin symmetry, pseudospin symmetry

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In 2004, de Castro[1] found the analytic solutions of the Dirac equation for a pseudoscalar linear plus Coulomb-like potential

$$mc\omega x+\frac{\hbar cg}{x}, \tag{1}$$

where ω and g are real parameters. The author of Ref. [1] solved the problem under a Sturm–Liouville perspective for a large range of parameters but the possibility of isolated solutions from the Sturm–Liouville perspective was neglected.[2] In a recent article, Hamzavi and Rajabi[3] have found the exact solutions of the Dirac equation with a pseudoscalar Cornell potential

$$-\frac{a}{x}+bx, \tag{2}$$

under the spin and pseudospin symmetry limits. The purpose of this work is to clarify that Hamzavi and Rajabi failed to perceive the possibility of isolated solutions, as did by de Castro in Ref. [1]. Moreover, Hamzavi and Rajabi incurred in a clear lack of understanding of spin and pseudospin symmetries and truly presented the solutions for a pure pseudoscalar coupling for a more restrict range of parameters than in Ref. [1].

Spin and pseudospin symmetries are SU(2) symmetries of a Dirac equation with vector and scalar potentials realized when the difference between the potentials or their sum is a constant.[4] The upper (lower) component of the Dirac spinor satisfies a Schrödinger-like equation in the case of spin (pseudospin) symmetry and the resulting spectrum is independent of the orientation of the spin (see, e.g., Ref. [5]). Furthermore, the spin–orbit and the Darwin terms disappear from the second-order equations for the upper and lower components of the Dirac spinor under the spin and pseudospin conditions in such a way that spin-1/2 and spin-0 particles have equivalent spectra.[6] If the difference between the vector and scalar potentials or their sum is not a constant, there is no spin or pseudospin symmetry. Spin and pseudospin symmetries are also broken if the potential is contaminated by other sorts of couplings (e.g., pseudoscalar or tensor couplings).[7] In fact, there has been a continuous interest for solving the Dirac equations in the four-dimensional space-time as well as in lower dimensions by assuming that the vector potential has the same magnitude as the scalar potential.[1–54] In 1+1 dimensions, it is usual to say that the system has spin or pseudospin symmetries if the potentials are limited to vector and scalar Lorentz structures with the same magnitude. Despite the absence of spin effects, many attributes of the real spin or pseudospin symmetries in 3+1 dimensions are preserved. In the presence of time-independent interactions, the most general (1+1)-dimensional time-independent Dirac equation for a fermion of rest mass m and momentum p reads

$$\left(\sigma_1\hat{p}+\sigma_3 m+\frac{\boldsymbol{I}+\sigma_3}{2}V_\Sigma+\frac{\boldsymbol{I}-\sigma_3}{2}V_\Delta+\sigma_2 V_{\rm p}\right)\psi=E\psi, \tag{3}$$

where E is the energy of the fermion, $V_\Sigma=V_{\rm v}+V_{\rm s}$, and $V_\Delta=V_{\rm v}-V_{\rm s}$. The subscripts for the terms of potential denote their properties under a Lorentz transformation: v for the time component of the two-vector potential, s for the scalar potential, and p for the pseudoscalar potential. The σ_i are the Pauli matrices, and $\boldsymbol{I}$ is the 2×2 unit matrix. Hamzavi and Rajabi[3] considered an equation for the upper component of the Dirac spinor with $\Sigma=0$, $\Delta=C_{\rm s}=0$, and $E\neq-m$ in Subsection 3.1, and considered an equation for the lower component of the Dirac spinor with $\Delta=0$, $\Sigma=C_{\rm ps}=0$, and $E\neq+m$ in Subsection 3.2. Therefore, in both circumstances, they took into account $V_{\rm v}=V_{\rm s}=0$. Indeed, in one word, they in fact

†Corresponding author. E-mail: luis.castro@pgfsc.ufsc.br

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considered a pure pseudoscalar potential in such a way that for $E \neq \mp m$

$$\psi_{\mp} = -\frac{\mathrm{i}}{E \pm m}\left(\frac{\mathrm{d}\psi_{\pm}}{\mathrm{d}x} \mp V_{\mathrm{p}}\psi_{\pm}\right), \tag{4}$$

$$-\frac{\mathrm{d}^2\psi_{\pm}}{\mathrm{d}x^2} + \left(V_{\mathrm{p}}^2 \pm \frac{\mathrm{d}V_{\mathrm{p}}}{\mathrm{d}x}\right)\psi_{\pm} = \left(E^2 - m^2\right)\psi_{\pm}. \tag{5}$$

On the other hand, for $E = \mp m$ one has

$$\psi_{\pm} = N_{\pm}\,\mathrm{e}^{\pm v(x)}, \tag{6}$$

$$\psi_{\mp} = \left[N_{\mp} - \mathrm{i}N_{\pm}I_{\pm}(x)\right]\mathrm{e}^{\mp v(x)}, \tag{7}$$

where N_+ and N_- are normalization constants, and

$$v(x) = \int^{x} \mathrm{d}y V_p(y),$$

$$I_{\pm}(x) = \pm 2m\int^{x} \mathrm{d}y\,\mathrm{e}^{\pm 2v(y)}. \tag{8}$$

In summary, the solutions of the Sturm–Liouville problem with a pseudoscalar linear plus Coulomb-like potential (or Cornell potential if one likes) has already been discussed in Ref. [1], and the isolated solutions in Ref. [2]. Furthermore, the rhetoric is great but there is no spin or pseudospin symmetries in the system approached in Ref. [3] because of the complete absence of vector and scalar couplings.

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