Homotopic mapping solitary traveling wave solutions for the disturbed BKK mechanism physical model*

Zhou Xian-Chun(周先春)[a)b)c)†], Shi Lan-Fang(石兰芳)[d)], Han Xiang-Lin(韩祥临)[e)], and Mo Jia-Qi(莫嘉琪)[f)‡]

[a)] College of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing 210044, China
[b)] Jiangsu Technology and Engineering Center for Meteorological Sensor Network, Nanjing University of Information Science and Technology, Nanjing 210044, China
[c)] Jiangsu Key Laboratory of Meteorological Observation and Information Processing, Nanjing University of Information Science and Technology, Nanjing 210044, China
[d)] College of Mathematics and Statistics, Nanjing University of information Science and Technology, Nanjing 210044, China
[e)] Department of Mathematics, Huzhou Teachers College, Huzhou 313000, China
[f)] Department of Mathematics, Anhui Normal University, Wuhu 241003, China

(Received 14 January 2014; revised manuscript received 16 April 2014; published online 24 July 2014)

Using the trial equation method, a Broer–Kau–Kupershmidt (BKK) mechanism physical model is obtained, and the exact and approximate solitary traveling wave solutions are found. As an example, it is pointed out that the solitary traveling wave approximate solutions have better accurate degree by using the homotopic mapping theory.

Keywords: nonlinear, solitary, traveling wave

PACS: 02.30.Mv **DOI:** 10.1088/1674-1056/23/9/090204

1. Introduction

Nonlinear evolution equation has very important applications in physics. How to solve the nonlinear evolution equation is an important field, where Broer-Kau-Kupershmidt (BKK) mechanism system is a physical model. More new methods have been established, for example, Wang and Zhang presented a further improved F-expansion method (improved Jacobi elliptic function method),[1] Fan and Zhang gave a homogeneous balance method,[2] and similarity transformation method (see Refs. [3]–[5]), and more scholars have studied the method of hyperbolic tangent function, the homogeneous equilibrium method, the method of Jacobi elliptic function, the trial equation method, the (G'/G) developed method, and the method of auxiliary equation, etc. They have done a great deal of work on the theory of nonlinear evolution equations. The asymptotic method for nonlinear theory of the BKK mechanism system is a new method.[6,7] Many approximate methods have been developed and refined, including the method of averaging, the boundary layer method, the methods of matched asymptotic expansion, and the multiple scales.[8,14] Using the differential inequalities and other methods, the authors also considered a class of nonlinear problems using the approximate analytic methods.[20–30] In this paper, using a simple approximate method, we consider a class of nonlinear disturbed BKK mechanism physical models and obtain exact and approximate solutions.

2. BKK mechanism disturbed physical model

Consider the following nonlinear evolution equations for the BKK mechanism disturbed physical model:

$$u_{ty}+u_{xxy}+2(uu_x)_y+2v_{xx}=f(u,v), \tag{1}$$

$$v_t+v_{xx}+2(uv)_x=g(u,v), \tag{2}$$

where f and g are the disturbed terms. The above BKK mechanism disturbed model describes a nonlinear dispersion gravitational wave model in the horizontal direction of shallow sea.

We first consider the typical BKK mechanism non-disturbed physical model

$$u_{ty}+u_{xxy}+2(uu_x)_y+2v_{xx}=0, \tag{3}$$

$$v_t+v_{xx}+2(uv)_x=0. \tag{4}$$

In Refs. [6] and [15], using the trial equation method, the non-disturbed BKK mechanism model (Eqs. (3) and (4)) were studied. Introducing transformations

$$\xi=x-\frac{(a_1t+a_2)}{b_1}\ln(b_1y+b_2),\quad \tau=t,\quad v=\omega(\xi,\tau),$$

$$u=\frac{\theta(\xi,\tau)}{x}+\frac{1}{2}\ln(b_1y+b_2)\,, \tag{5}$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 141275062 and 1202106), the Specialized Research Fund for the Doctoral Program of Higher Education, China (Grant No. 20123228120005), the Jiangsu Sensor Network and Modern Meteorological Equipment Preponderant Discipline Platform, China, the Natural Science Foundation from the Universities of Jiangsu Province, China (Grant No. 13KJB170016), the Advance Research Foundation in Nanjing University of Information Science and Technology of China (Grant No. 20110385), and the Natural Science Foundation of Zhejiang Province, China (Grant No. LY13A010005).

†Corresponding author. E-mail: zhouxc2008@163.com

‡Corresponding author. E-mail: mojiaqi@mail.ahnu.edu.cn

© 2014 Chinese Physical Society and IOP Publishing Ltd

where $(a_1t+a_2)(b_1y+b_2)\neq 0$, and a_i and b_i $(i=1,2)$ are the arbitrary constants. Then from Eq. (5), we can obtain the reduced equations of the typical BKK mechanism system (3) and (4) as follows:[17]

$$\theta_{\tau\xi}+2\theta_{\xi\xi}-\theta_{\tau\tau\tau}+2\theta_{\xi}^2+2\theta\theta_{\xi\xi}-2\omega_{\xi\xi}=0, \quad (6)$$

$$\omega_{\tau}+\omega_{\xi\xi}+2\omega\theta_{\xi}+2\theta\omega_{\xi}=0. \quad (7)$$

From Eqs. (6) and (7), using the G/G' developed method and the auxiliary function Riccati developed method, we can obtain the following solitary solution $\bar{U}$, $\bar{V}$ of the typical BKK mechanism model (3) and (4) (as $\lambda^2-4\mu>0$):[17]

$$\begin{aligned}&\tilde{U}(t,x,y)\\ &=-\lambda+\frac{l}{2}+\frac{2-\lambda l}{\mu}\\ &\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right),\end{aligned} \quad (8)$$

$$\begin{aligned}&\tilde{V}(t,x,y)\\ &=D+\frac{D+l}{\mu}\\ &\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right)^2.\end{aligned} \quad (9)$$

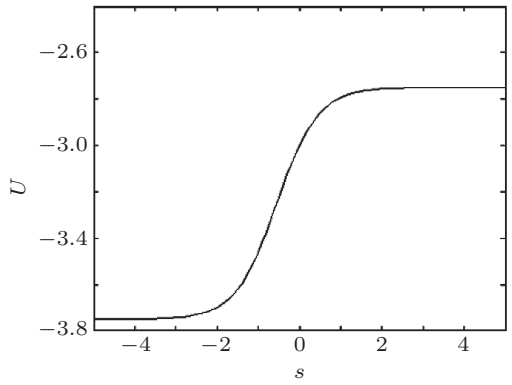


Fig. 1. Solitary traveling wave curves of U. $\lambda=3$, $\mu=2$, $l=1C_1=2$, $C_2=1$, $D=2$.

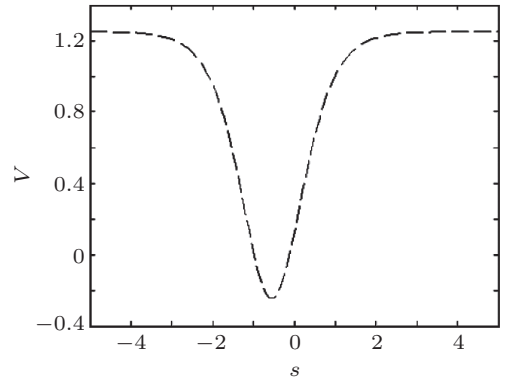


Fig. 2. Solitary traveling wave curves of V. $\lambda=3$, $\mu=2$, $l=1C_1=2$, $C_2=1$, $D=2$.

In Eqs. (8) and (9),

$$s=x-lt-\frac{(a_1t+a_2)}{b_1}\ln(b_1y+b_2),$$

and D, C_1, C_2, $C_1C_2\neq 0$ are the arbitrary constants. The solitary traveling wave curves of the solutions for the BKK mechanism model (3) and (4) are shown in Figs. 1 and 2.

3. Homotopic mapping solutions of disturbed model

From the BKK mechanism of disturbed physical model, we cannot find exact solitary solutions using the finite terms for the elementary functions. Therefore, we use the homotopic mapping method, and obtain approximate analytic solutions for the BKK mechanism disturbed physical model (1) and (2).

Firstly, introducing a set of the following homotopic mappings on $R^2\times[0,1]\to R$:

$$\begin{aligned}H_1[u,v,p]&=L_1[u,v]-L_1[\bar{u},\bar{v}]+p[L_1[\bar{u},\bar{v}]\\&\quad+2(uv_x)_y+2v_{xx}-f(u,v)],\end{aligned} \quad (10)$$

$$\begin{aligned}H_2[u,v,p]&=L_2[v]-L_2[\bar{v}]+p[L_2[\bar{v}]\\&\quad+2(uv)_x-g(u,v)],\end{aligned} \quad (11)$$

where p is an artificial parameter,[31,32] $\bar{u}$ and $\bar{v}$ are the initial approximate solution of the original model (1) and (2), and linear operators $L_1[u,v]$ and $L_2[v]$ are

$$L_1[u,v]=u_{ty}+u_{xxy}+2v_{xx},\quad L_2[v]=v_t+v_{xx}.$$

Let

$$U=\sum_{i=0}^{\infty}u_ip^i,\quad V=\sum_{i=0}^{\infty}v_ip^i. \quad (12)$$

Obviously, from the homotopic mappings (10) and (11), $H_i(u,v,1)=0$ $(i=1,2)$ and the BKK mechanism disturbed physical model (1) and (2) are the same. Thus the solutions $U(t,x,y,1)$, $V(t,x,y,1)$ of $H_i(u,v,s)=0$ $(i=1,2)$ as $s\to 1$ are the exact solution of the BKK mechanism disturbed physical system (1) and (2). $U_{m\text{app}}=\sum_{i=0}^{m}u_i$, $V_{m\text{app}}=\sum_{i=0}^{m}v_i$ are the mth-oder approximate analytic solutions of the BKK mechanism disturbed physical model (1) and (2).

From the homotopic mappings (10) and (11), substituting Eq. (12) into Eqs. (10) and (11), developing nonlinear terms in p and equating coefficients of the same power of p for $H_i(u,v,p)=0$ $(i=1,2)$, from the coefficients in p^0 for $H_i(u,v,p)=0$ $(i=1,2)$, we have

$$L_1[u_0,v_0]=L_1[\bar{u},\bar{v}], \quad (13)$$

$$L_2[v_0]=L_2[\bar{v}]. \quad (14)$$

For $\lambda^2-4\mu>0$, from Eqs. (8), (9), (13), and (14), we now select $\bar{u}$ and $\bar{v}$ as the solutions of the typical BKK mechanism non-disturbed physical model (3) and (4). Thus from system (13) and (14), we have

$$u_0=U_{0\text{app}}=\tilde{U},\quad v_0=V_{0\text{app}}=\tilde{V}. \tag{15}$$

Substituting Eq. (12) into Eqs. (10) and (11), from the coefficients in p^1 for $H_i(u,v,p)=0$ $(i=1,2)$, we have

$$L_1[u_1,v_1]=f(u_0,v_0), \tag{16}$$
$$L_2[v_1]=g(u_0,v_0). \tag{17}$$

Assuming traveling wave transformation as $z=t+cx+dy$, we can obtain a set of traveling wave solutions of the linear system (16) and (17) with the zero initial condition selected as

$$\begin{aligned}u_1 &= \frac{1}{c^2d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1} f(u_0(\xi),v_0(\xi))\\ &\quad\times\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2,\end{aligned} \tag{18}$$

$$\begin{aligned}v_1 &= \frac{1}{c^2}\int_0^z\int_0^{\xi_1} g(u_0(\xi),v_0(\xi))\\ &\quad\times\exp\left(-\frac{(\xi-z)}{c^2}\right)\mathrm{d}\xi\,\mathrm{d}\xi_1,\end{aligned} \tag{19}$$

where u_0 and v_0 represent Eqs. (15) and (19) respectively.

Thus, from Eqs. (12), (15), (18), and (19), and let $p=1$, we obtain a set of the first-order approximate solitary traveling wave solutions of the BKK mechanism disturbed physical model (1) and (2) (i.e., $U_{1\text{app}}$ and $V_{1\text{app}}$) as

$$\begin{aligned}&U_{1\text{app}}(z)-\lambda+\frac{l}{2}+\frac{2-\lambda l}{\mu}\\ &\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}\,s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right)\\ &+\frac{1}{c^2d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1} f(u_0(\xi),v_0(\xi))\\ &\times\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2,\end{aligned} \tag{20}$$

$$\begin{aligned}&V_{1\text{app}}(z)\\ &=D+\frac{D+l}{\mu}\\ &\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right)^2\\ &+\frac{1}{c^2}\int_0^z\int_0^{\xi_1} g(u_0(\xi),v_0(\xi))\\ &\times\exp\left(-\frac{(\xi-z)}{c^2}\right)\mathrm{d}\xi\,\mathrm{d}\xi_1,\end{aligned} \tag{21}$$

where $z=t+cx+dy$ and

$$s=x-lt-\frac{(a_1t+a_2)}{b_1}\ln(b_1y+b_2).$$

Substituting Eq. (12) into Eqs. (10) and (11), from the coefficients in p^2 for $H_i(u,v,p)=0$ $(i=1,2)$, we have

$$\begin{aligned}L_1[u_2,v_2] &= f_u(u_0,v_0)u_1+f_v(u_0,v_0)\,v_1\\ &\quad+2(u_0v_{1x}+u_1v_{0x})_y+2v_{1xx},\end{aligned} \tag{22}$$

$$\begin{aligned}L_2[v_2] &= g_u(u_0,v_0)u_1+g_v(u_0,v_0)v_1\\ &\quad+2(u_0v_1+u_1v_0)_x.\end{aligned} \tag{23}$$

We can obtain a set of traveling wave solutions of the linear system (22) and (23) as

$$\begin{aligned}u_2 &= \frac{1}{c^2d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1}[f_u(u_0,v_0)u_1+f_v(u_0,v_0)\,v_1\\ &\quad+2c(u_0v_{1\xi}+u_1v_{0\xi})+2c^2v_{1\xi\xi}]\\ &\quad\times\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2,\end{aligned} \tag{24}$$

$$\begin{aligned}v_2 &= \frac{1}{c^2}\int_0^z\int_0^{\xi_1}[g_u(u_0,v_0)u_1+g_v(u_0,v_0)v_1\\ &\quad+2c(u_1v_0+u_0v_1)_\xi]\exp\left(-\frac{\xi-z}{c^2}\right)\mathrm{d}\xi\,\mathrm{d}\xi_1,\end{aligned} \tag{25}$$

where u_0, v_0, u_1, and v_1 represent Eqs. (15), (18), and (19) respectively.

Thus, from Eqs. (12), (15), (18), (19), (24), and (25), and let $p=1$, we obtain a set of second-order approximate solitary traveling wave solutions of the BKK mechanism disturbed physical model (1) and (2) (i.e., $U_{2\text{app}}$ and $V_{2\text{app}}$) as

$$\begin{aligned}&U_{2\text{app}}(z)-\lambda+\frac{l}{2}+\frac{2-\lambda l}{\mu}\\ &\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right)\\ &+\frac{1}{c^2d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1} f(u_0(\xi),v_0(\xi))\\ &\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\\ &+\frac{1}{c^2d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1}[f_u(u_0,v_0)u_1+f_v(u_0,v_0)\,v_1\\ &+2c(u_0v_{1\xi}+u_1v_{0\xi})-2c^2v_{1\xi\xi}]\\ &\times\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2,\end{aligned} \tag{26}$$

$$\begin{aligned}&V_{2\text{app}}(z)\\ &=D+\frac{D+l}{\mu}\\ &\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right)^2\\ &+\frac{1}{c^2}\int_0^z\int_0^{\xi_1} g(u_0(\xi),v_0(\xi))\exp\left(-\frac{(\xi-z)}{c^2}\right)\mathrm{d}\xi\,\mathrm{d}\xi_1\\ &+\frac{1}{c^2}\int_0^z\int_0^{\xi_1}[g_u(u_0,v_0)u_1+g_v(u_0,v_0)v_1\\ &+2c(u_1v_0+u_0v_1)_\xi]\exp(-\frac{\xi-z}{c^2})\mathrm{d}\xi\,\mathrm{d}\xi_1,\end{aligned} \tag{27}$$

where $z = t + cx + dy$ and

$$\tau = t, s = x - lt - \frac{(a_1 t + a_2)}{b_1} \ln(b_1 y + b_2).$$

Using the homotopic mapping method, substituting Eq. (12) into Eqs. (10) and (11), from the coefficients $H_i(u, v, p) = 0$ $(i = 1, 2)$, we can obtain u_i and v_i $(i = 1, 2, \ldots)$, respectively. From obtained u_i and v_i $(i = 1, 2, \ldots)$ and the transformations $z = t + cx + dy$,

$$s = x - lt - \frac{(a_1 t + a_2)}{b_1} \ln(b_1 y + b_2),$$

we can obtain $U(t, x, y, 1)$, $V(t, x, y, 1)$.

4. Example

Now we consider a simple BKK mechanism disturbed physical model: its disturbed terms are $f = \varepsilon \exp u$, $g = \varepsilon \exp(-v)$ $(0 < \varepsilon \ll 1)$. Thus, we have

$$u_{ty} + u_{xxy} + 2(uu_x)_y + 2v_{xx} = \varepsilon \exp u, \tag{28}$$

$$v_t + v_{xx} + 2(uv)_x = \varepsilon \exp(-v). \tag{29}$$

Using the homotopic mappings (10) and (11), we select a set of solitary hyperbolic function solutions (U, V) for $\lambda^2 - 4\mu > 0$ of the BKK mechanism disturbed physical model (28) and (29).

From Eqs. (8), (9), and (15), we can obtain the zeroth-order approximate solution, i.e., u_0 and v_0:

$$u_0 = U_{0\text{app}} = -\lambda + \frac{l}{2} + \frac{2 - \lambda l}{\mu} \times \left(-\frac{\lambda}{2} + \frac{C_1 \sinh\sqrt{\lambda^2 - 4\mu}s + C_2 \cosh\sqrt{\lambda^2 - 4\mu}s}{C_1 \cosh\sqrt{\lambda^2 - 4\mu}\, s + C_2 \sinh\sqrt{\lambda^2 - 4\mu}s} \right), \tag{30}$$

$$v_0 = V_{0\text{app}} = D + \frac{D + l}{\mu} \times \left(-\frac{\lambda}{2} + \frac{C_1 \sinh\sqrt{\lambda^2 - 4\mu}s + C_2 \cosh\sqrt{\lambda^2 - 4\mu}s}{C_1 \cosh\sqrt{\lambda^2 - 4\mu}s + C_2 \sinh\sqrt{\lambda^2 - 4\mu}s} \right)^2. \tag{31}$$

From Eqs. (18), (19), (30), and (30), we can obtain

$$u_1 = \frac{\varepsilon}{c^2 d} \int_0^z \int_0^{\xi_2} \int_0^{\xi_1} \times \exp\left[-\left(\frac{1}{c^2}(\xi - \xi_2) - u_0(\xi) \right) \right] \mathrm{d}\xi\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2, \tag{32}$$

$$v_1 = \frac{\varepsilon}{c^2} \int_0^z \int_0^{\xi_1} \exp\left(-\left(\frac{(\xi - z)}{c^2} + v_0(\xi) \right) \right) \mathrm{d}\xi\, \mathrm{d}\xi_1, \tag{33}$$

where u_0 and v_0 are Eqs. (30) and (32), respectively.

Thus, from Eqs. (12), (26), (27), (32), and (33), and let $p = 1$, we obtain a set of the first-order approximate solitary traveling wave solutions of the BKK mechanism disturbed physical model (28) and (29) (i.e., $U_{1\text{app}}$ and $V_{1\text{app}}$) as

$$U_{1\text{app}} = -\lambda + \frac{l}{2} + \frac{2 - \lambda l}{\mu} \times \left(-\frac{\lambda}{2} + \frac{C_1 \sinh\sqrt{\lambda^2 - 4\mu}s + C_2 \cosh\sqrt{\lambda^2 - 4\mu}s}{C_1 \cosh\sqrt{\lambda^2 - 4\mu}\, s + C_2 \sinh\sqrt{\lambda^2 - 4\mu}s} \right) + \frac{\varepsilon}{c^2 d} \int_0^z \int_0^{\xi_2} \int_0^{\xi_1} \times \exp\left[-\left(\frac{1}{c^2}(\xi - \xi_2) - u_0(\xi) \right) \right] \mathrm{d}\xi\, \mathrm{d}\xi_1\, \mathrm{d}\xi_2, \tag{34}$$

$$V_{1\text{app}} = D + \frac{D + l}{\mu} \times \left(-\frac{\lambda}{2} + \frac{C_1 \sinh\sqrt{\lambda^2 - 4\mu}s + C_2 \cosh\sqrt{\lambda^2 - 4\mu}s}{C_1 \cosh\sqrt{\lambda^2 - 4\mu}s + C_2 \sinh\sqrt{\lambda^2 - 4\mu}s} \right)^2 + \frac{\varepsilon}{c^2} \int_0^z \int_0^{\xi_1} \exp\left(-\left(\frac{(\xi - z)}{c^2} + v_0(\xi) \right) \right) \mathrm{d}\xi\, \mathrm{d}\xi_1, \tag{35}$$

where

$$s = x - lt - \frac{(a_1 t + a_2)}{b_1} \ln(b_1 y + b_2).$$

As $\lambda = 3$, $\mu = 2$, $C_1 = 2$, $C_2 = 1$, $D = 2$, and $\varepsilon = 0.1$, the approximate curves of Eqs. (34) and (35) of solitary approximate solutions for the BKK mechanism model (28) and (29) are plotted in Figs. 3 and 4 respectively.

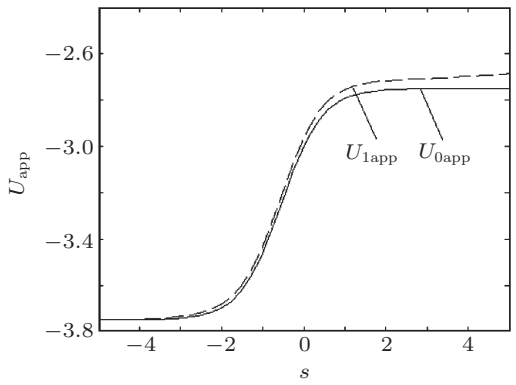


Fig. 3. Approximate curves of solitary U. $\lambda = 3$, $\mu = 2$, $C_1 = 2$, $C_2 = 1$, $D = 2$, $\varepsilon = 0.1$.

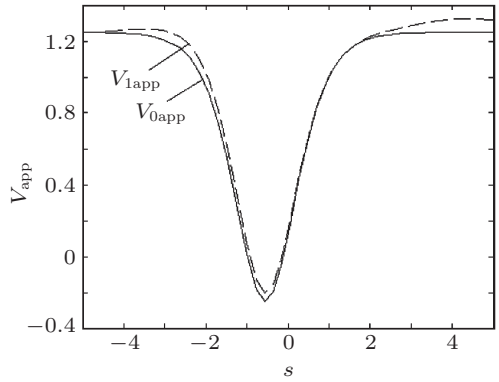


Fig. 4. Approximate curves of solitary V. $\lambda = 3$, $\mu = 2$, $C_1 = 2$, $C_2 = 1$, $D = 2$, $\varepsilon = 0.1$.

Applying the transformation $z = t + cx + dy$ to Eqs. (38) and (39), we obtain a set of the first-order approximate hyperbolic function solitary traveling wave analytic solutions.

From Eqs. (20), (21), and (30)–(33), we can obtain u_2 and v_2 as

$$\begin{aligned} u_2 &= \frac{1}{c^2 d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1}[\varepsilon\exp(-u_0)u_1 \\ &\quad +2c(u_0 v_{1\xi}+u_1 v_{0\xi})-2dv_{1\xi\xi}] \\ &\quad \times\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2, \qquad (36) \\ v_2 &= \frac{1}{c^2}\int_0^z\int_0^{\xi_1}[-\varepsilon\exp(-v_0)v_1 \\ &\quad +2c(u_1 v_0+u_0 v_1)_\xi]\exp\left(-\frac{(\xi-z)}{c^2}\right)\mathrm{d}\xi\,\mathrm{d}\xi_1, \qquad (37) \end{aligned}$$

where u_0, v_0, u_1, and v_1 are Eqs. (30)–(33), respectively.

Thus, from Eqs. (12), (15), (18), (19), (24), (25), (30)–(33), (36), and (37), selecting $p = 1$, we have

$$\begin{aligned} &U_{2\mathrm{app}} \\ &= -\lambda+\frac{l}{2}+\frac{2-\lambda l}{\mu} \\ &\quad\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}\,s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right) \\ &\quad+\frac{\varepsilon}{c^2 d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1} \\ &\quad\times\exp[-(\frac{1}{c^2}(\xi-\xi_2)-u_0(\xi))]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2 \\ &\quad+\frac{1}{c^2 d}\int_0^z\int_0^{\xi_2}\int_0^{\xi_1}[-\varepsilon\exp(-u_0)u_1 \\ &\quad+2c(u_0 v_{1\xi}+u_1 v_{0\xi})-2dv_{1\xi\xi}] \\ &\quad\times\exp\left[-\frac{1}{c^2}(\xi-\xi_2)\right]\mathrm{d}\xi\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2, \qquad (38) \\ &V_{2\mathrm{app}} \\ &= D+\frac{D+l}{\mu} \\ &\quad\times\left(-\frac{\lambda}{2}+\frac{C_1\sinh\sqrt{\lambda^2-4\mu}s+C_2\cosh\sqrt{\lambda^2-4\mu}s}{C_1\cosh\sqrt{\lambda^2-4\mu}s+C_2\sinh\sqrt{\lambda^2-4\mu}s}\right)^2 \\ &\quad+\frac{\varepsilon}{c^2}\int_0^z\int_0^{\xi_1}\exp\left(-\left(\frac{(\xi-z)}{c^2}+v_0(\xi)\right)\right)\mathrm{d}\xi\,\mathrm{d}\xi_1 \\ &\quad+\frac{1}{c^2}\int_0^z\int_0^{\xi_1}[\varepsilon\exp(-v_0)v_1+2c(u_1 v_0+u_0 v_1)_\xi] \\ &\quad\times\exp\left(-\frac{(\xi-z)}{c^2}\right)\mathrm{d}\xi\,\mathrm{d}\xi_1, \qquad (39) \end{aligned}$$

where

$$\eta = x - lt - \frac{(a_1 t+a_2)}{b_1}\ln(b_1 y+b_2).$$

Applying transformation $z = t + cx + dy$ to Eqs. (38) and (39), we obtain a set of the second-order approximate hyperbolic function solitary traveling wave analytic solution.

Sequentially, using the same method, we can obtain higher-order approximate traveling wave analytic solution. We can also prove that the exact solutions U_{exa} and V_{exa} of the BKK mechanism small disturbed physical model (28) and (29) and mth-order approximate analytic solitary traveling wave solution $U_{m\mathrm{app}}$ and $V_{m\mathrm{app}}$ hold the following precision using the fixed point theorem:[18,19]

$$U_{exa} = U_{m\mathrm{app}} + O(\varepsilon^m), \quad V_{exa} = V_{m\mathrm{app}} + O(\varepsilon^m), \quad 0 < \varepsilon << 1.$$

From the above analysis, it is easy to know that the solitary traveling wave approximate solution of the BKK mechanism disturbed physical model by using the homotopic mapping method has a good precision.

5. Summary

The nonlinear evolution equations of BKK mechanism disturbed physical model can describe many complicated natural phenomena. Hence, for many physical phenomena, we need to reduce the basic BKK mechanism disturbed physical model. We solve them using the approximate method. In this paper, the homotopic mapping method is a simple and valid method. First, we construct a set of homotopic mappings, obtain an iteration relation, and select initial approximation. Then we can have the solution of the nonlinear BKK mechanism disturbed physical model using the approximation theory.

The obtained approximate solution of the BKK mechanism nonlinear disturbed physical model by using the homotopic mapping method, is dependent on the initial approximation. In this paper, we select the exact solution of the typical BKK mechanism physical model. It is very natural. In this way, we can faster obtain higher-order approximate solution.

The homotopic mapping method is an approximate analytic method, which differs from general numerical method. From the approximate analytic expansions of solution for the BKK mechanism disturbed physical model, using the homotopic mapping method we can also execute the continuous analytic operations, such as differential operation, integral operation, etc. Thus, we can further study the qualitative and quantitative behaviors of the BKK mechanism disturbed physical model.

References

[1] Wang D S and Zhang H Q 2005 *Chaos Soliton. Fract.* **25** 601
[2] Fan E G and Zhang H Q 1998 *Phys. Lett. A* **246** 403
[3] Wang D S, Hu S H, Hu J P and Liu W M 2010 *Phys. Rev. A* **81** 025604
[4] Yan D, Chang J J, Hamner C, Hoefer M, Kevrekidis P G, Engels P, Achilleos V, Frantzeskakis D J and Cuevas J 2012 *J. Phys. B* **45** 115301
[5] Wang D S, Zeng X and Ma Y Q 2012 *Phys. Lett. A* **376** 3067
[6] Lü D Z 2005 *Acta Phys. Sin.* **54** 4501 (in Chinese)
[7] Wu J P 2011 *Chin. Phys. Lett.* **28** 060207
[8] Liang L W, Li X D and Li Y X 2009 *Acta Phys. Sin.* **58** 2159 (in Chinese)
[9] Chen H T and Zhang H Q 2004 *Chaos Soliton. Fract.* **20** 765
[10] Lü D Z, Cui Y Y, Liu C H and Zhang Y 2010 *Acta Phys. Sin.* **59** 6793 (in Chinese)

[11] Zuo J M and Zhang Y M 2011 *Chin. Phys. B* **20** 010205
[12] Pang J, Jin L H and Zhao Q 2012 *Acta Phys. Sin.* **61** 140201 (in Chinese)
[13] Clarkson P A and Kruskal M D 1989 *Math. Phys.* **30** 2201
[14] Xin X P, Liu X Q and Zhang L L 2011 *Chin. Phys. Lett.* **28** 1
[15] Trikia H and Wazwazb A 2009 *Appl. Math. Comput.* **214** 370
[16] Wang T T and Yu J Q 2011 *Jounal of Liaocheng University* **24** 1 (in Chinese)
[17] Li N and Liu X Q 2013 *Acta Phys. Sin.* **62** 160203 (in Chinese)
[18] de Jager E M and Jiang F R 1996 *The Theory of Singular Perturbation* (Amsterdam: North-Holland Publishing Co.)
[19] Barbu L and Morosanu G 2007 *Singularly Perturbed Boundary-Value Problem* (Basel: Birkhauserm Verlag AG)
[20] Zhou X C, Lin W T, Lin Y H and Mo J Q 2012 *Acta Phys. Sin.* **61** 240202 (in Chinese)
[21] Zhou X C, Lin W T, Lin Y H, Yao J S and Mo J Q 2012 *Acta Phys. Sin.* **61** 110207 (in Chinese)
[22] Shi L F, Zhou X C and Mo J Q 2011 *Acta Phys. Sin.* **60** 110205 (in Chinese)
[23] Shi L F and Mo J Q 2013 *Acta Phys. Sin.* **62** 040203 (in Chinese)
[24] Shi L F, Lin W T, Lin Y H and Mo J Q 2013 *Acta Phys. Sin.* **62** 010201 (in Chinese)
[25] Han X L, Zhao Z J, Cheng R J and Mo J Q 2013 *Acta Phys. Sin.* **62** 040203 (in Chinese)
[26] Mo J Q, Lin W T and Lin Y H 2011 *Chin. Phys. B* **20** 070205
[27] Mo J Q 2011 *Acta Phys. Sin.* **60** 090203 (in Chinese)
[28] Mo J Q, Cheng R J and Ge H X 2011 *Acta Phys. Sin.* **60** 050204 (in Chinese)
[29] Mo J Q 2011 *Acta Phys. Sin.* **60** 030203 (in Chinese)
[30] Mo J Q 2011 *Commun. Theor. Phys.* **55** 387
[31] Liao S J 2004 *Beyond Perturbation: Introduction to the Homotopy Analysis Method* (New York: CRC Press Co.)
[32] He J H 2002 *Approximate Nonlinear Analytical Methods in Engineering and Sciences* (Zhengzhou: Henan Science and Technology Press) (in Chinese)