Magnetic interaction in the metamaterial/magnet system

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The stability of the magnetic dipole and the metamaterial with negative permeability are investigated. Analytical expressions of the interaction force and stiffness of the magnetic line and metamaterial with negative permeability are derived. The repulsive force between the magnetic line and the metamaterial exceeds the value of the maximum force in the magnet-superconductor system.

Keywords: metamaterial, interaction energy, negative permeability

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1. Introduction

Recently, metamaterials have become the target of several theoretical and experimental studies due to their unusual physical properties such as the negative permittivities, the negative permeabilities, and the negative refractive indexes.

The potential applications of the metamaterials cover wide range of fields such as perfect lens,[1] high resolution imaging, invisible cloak,[2] magnetic and quantum levitation (repulsive Casimir force).[3] For instance, in magnetic levitation applications, Rangelov calculated the interaction force between a magnetic dipole and a semi-infinite space of a meta material. He found that the maximum levitation force attained in the case of materials with negative permeability is higher than that in the case of superconducting levitation.[4] Moreover, Coffey studied the interaction force between a point magnet and a metamaterial of finite thickness and pointed out that the force is stronger than that in the case of the well-known magnetic levitation.[5] The above two studies indicate the promising potential of using the metamaterials for constructing frictionless levitation devices of high efficiency.

Urzhumov *et al.* calculated the magnetic force between a metamaterial disk and a circular coil. They found that the force is enhanced and its enhancement is related to the magnetic surface resonance.[6]

The previous studies were on the assumption that the magnetic field frequency is low, and hence metamaterial was considered as an object of homogeneous permeability.

The dependence of relative magnetic permeability on frequency for a certain material is given by[7]

$$\frac{\mu}{\mu_o} = 1 - \frac{F\omega^2}{\omega^2 + \mathrm{i}\gamma\omega - \omega_{\mathrm{res}}^2}, \tag{1}$$

where F is the magnetic oscillator strength or filling factor, ω_{res} is the magnetic plasma frequency, and γ is the dissipative coefficient. As shown in Eq. (1), the value of permeability is taken to be negative sign when the frequency of the magnetic field is different from zero and within a specific band of frequencies.

In this study, we investigate the stability of a magnetic point dipole placed above a semi-infinite space of a metamaterial with negative permeability. The interaction force acting on the magnetic line located above a metamaterial as well as the stability of the proposed system are analyzed.

2. Calculations

In our calculations, the assumption of low frequency magnetic field is considered and hence the magnetostatic limit is applied. The metamaterial is assumed to be a homogeneous medium where the wavelength corresponding to ω is larger than the dimensions of the building unit cell of the metamaterial.

2.1. Magnetic dipole and metamaterial

Consider a magnetic dipole moment embedded into a medium with permeability μ_1 placed at a distance z above another medium with permeability μ_2 filling the half space $z < 0$. The dipole moment makes an angle θ with respect to the surface of the metamaterial.

Using the magnetostatic image method, the magnitude of the moment of the magnetic dipole image m' is given by[8,9]

$$m' = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} m. \tag{2}$$

The dipole–dipole interaction method is used to simulate the interaction between the magnetic dipole and the metamaterial. Therefore, the interaction energy between the magnetic dipole m and its induced magnetic dipole image m' in the medium μ_2 is given by

$$U = \frac{\mu_1}{4\pi} m_1^2 \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \frac{(1 + \sin^2\theta)}{z^3}, \tag{3}$$

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where z is the distance between the magnetic dipole and its image.

The value of vertical stiffness k of the system is calculated by taking the second derivative of the potential energy with respect to the height coordinate z and shown below

$$k = \frac{3\mu_1}{\pi} m_1^2 \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \frac{(1+\sin^2\theta)}{z^5}. \tag{4}$$

2.2. Magnetic line and metamaterial

Consider a magnetic line of length h placed along the z axis parallel to a medium of the permeability μ_2 filling the half space $z < 0$.

The magnetic line is assumed to have a uniform magnetic moment per unit length and its magnetization direction makes an angle θ with respect to the z axis. The magnetic line is modeled as a continuous distribution of magnetic dipole moments. Each magnetic dipole element $\mathrm{d}m_1 = (m_1/h)\mathrm{d}z$ induces a magnetic dipole image of magnetic moment

$$\mathrm{d}m_2 = \frac{\mu_2 - \mu_2}{\mu_2 + \mu_1} \mathrm{d}m_1.$$

The interaction between the magnetic line and the metamaterial is simulated as the interaction between the magnetic line and its image. The interaction energy is given by

$$U = -\int \mathrm{d}m_2 \cdot \boldsymbol{H}, \tag{5}$$

where H is the magnetic field produced by the magnetic line, which can be calculated by modeling the magnetic line as continuous distribution of magnetic dipole moments with $\mathrm{d}m_1 = (m_1/h)\mathrm{d}z$ and integrating $\mathrm{d}H$ over the length (h) of the magnet.

The interaction force is calculated by taking the minus derivative of the potential energy with respect to the height r:

$$F = \frac{M_{\mathrm{PM}}^2 \mu_1}{2\pi} \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \frac{(r((h^2+r^2)^{3/2} - r^3)\cos^2\theta + h^2(2h^2+3r^2)\sin^2\theta)}{r^3(h^2+r^2)^{3/2}} \tag{6}$$

and the vertical stiffness is given by

$$K = \frac{M_{\mathrm{PM}}^2 \mu_1}{4\pi} \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \frac{(4r(h^2+r^2)^{5/2}\cos^2\theta + 6h^2 + 15h^2r^4 - 2r^6 - (h^2+r^2)(6h^4+9h^2r^2+2r^4)\cos 2\theta)}{r^4(h^2+r^2)^{5/2}}, \tag{7}$$

where $M_{\mathrm{PM}} = m_1/h$.

It is worth mentioning that the sign of the value of the stiffness K indicates the stability of the levitated system. The positive (negative) value of K means the levitated system is stable (unstable).

3. Results and discussion

Figure 1 shows the interaction energy between the magnetic dipole and the metamaterial system as a function of the orientation of the magnetic dipole moment θ for three different values of relative permeability ($\mu = -2$, -0.5, and 0). It is clear that the interaction energy is minimum when the magnetic moment is oriented parallel to the surface of the metamaterial ($\theta = 0^\circ$ and 180°), indicating that a configuration of stable equilibrium of the system is obtained. The sign of the interaction energy depends on the permeability of the metamaterial. For $\mu = 0$ and -0.5, the interaction forces are repulsive and the values of the vertical stiffness, Eq. (4), are always positive which means that the system is stable and the most stable orientation occurs at $\theta = 0^\circ$.

While for $\mu = -2$ the interaction force is attractive and therefore no stable levitation is expected. Figure 2 shows the interaction force between the magnetic line and the metamaterial as a function of relative permeability μ_{r} at $\theta = 90^\circ$ (the vertical configuration). The figure shows that the interaction force can be repulsive (with positive value) or attractive (with negative sign), depending on the value of the relative permeability μ_{r} . For the positive values of relative permeability μ_{r} $(0 < \mu_{\mathrm{r}} < 1)$ the force is repulsive and its maximum value occurs at $\mu_{\mathrm{r}} = 0$ which represents the diamagnetic material case; for the negative values of the relative permeability in the range $-1 < \mu_{\mathrm{r}} < 0$, the repulsive force exceeds the value of the maximum force for the case of $\mu_{\mathrm{r}} = 0$ which is an interesting feature, which can be concluded from the figure. This feature may be considered as a promising result of potential application in magnetic levitation phenomena at room temperature without the need of superconducting material.[10]

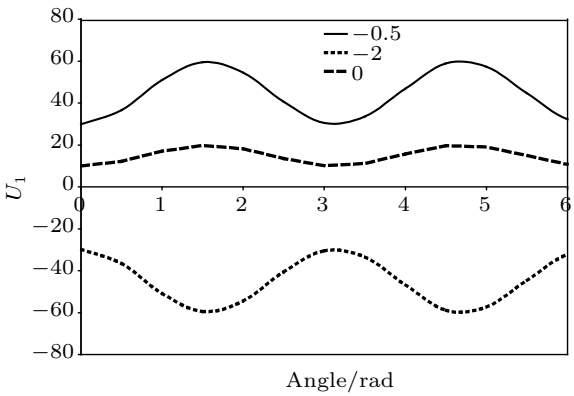


Fig. 1. Interaction potential (in $\mu_1 m_1^2$) as a function of the orientation angle at different values of relative permeability μ_{r} (-0.5, 0, and -2.0). The height of the dipole $z = 0.2$ cm.

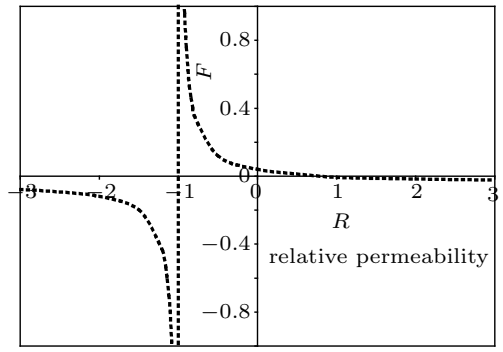


Fig. 2. Interaction force between the magnetic line and the meta material (in $\mu_1 m_1^2$) as a function of relative permeability μ_r. The height of the magnetic line $r = 0.2$ cm with $h = 10$ cm and $\theta = 90°$.

Finally, for negative relative permeability with $\mu_r < -2$ the interaction force becomes attractive force. It is worth mentioning that the existence of the infinite force (the singularity at $\mu_r = -1$) is due to the neglect of the imaginary part of the permeability which has a considerable importance when the values of the real parts are close to each other.

Figure 3 shows the force acting on the magnetic line of length h (10 cm) as a function height r. The orientation of magnetic dipole is taken to be parallel to the metamaterial surface which has a relative permeability of -0.5.

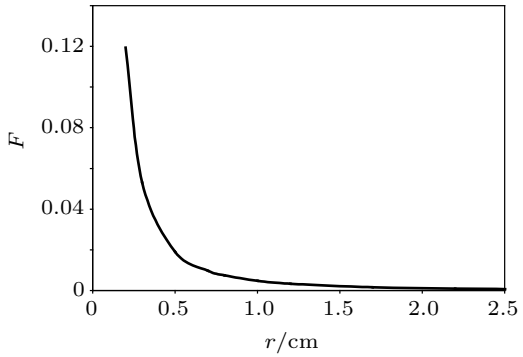


Fig. 3. Interaction force (in $\mu_1 m_1^2$) between the magnetic line and the metamaterial (in $\mu_1 m_1^2$) as a function of height levitation r with $h = 10$ cm and $\theta = 0°$.

To investigate the stability of the magnetic line above the metamaterial with permeability of -0.5, we plot the vertical stiffness as a function of magnetic moment orientation θ. As can be seen in Fig. 4, the values of the stiffness are positive for all angles, indicating stable states of the system. In conclusion, the most stable configuration occurs when the direction of magnetization of the magnetic line is parallel to the material surface.

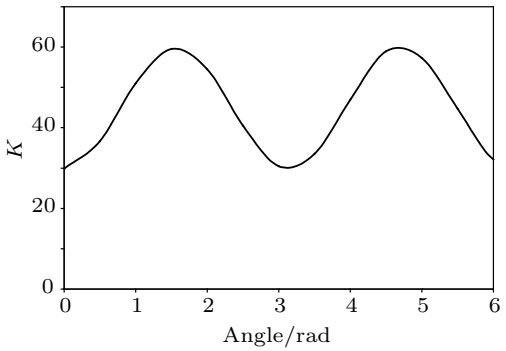


Fig. 4. Vertical stiffness (in $\mu_1 m_1^2$) as a function of orientation angle at relative permeability $\mu_r = -0.5$ with $r = 0.2$ cm and $h = 10$ cm.

4. Conclusion

We investigated the stability of the magnetic dipole placed above a metamaterial with negative permeability by studying the vertical stiffness of the system. We found that the parallel orientation of the magnetic moment presents a most stable state.

The magnetic interaction between a magnetic line of length h and the metamaterial with negative permeability is analyzed using the image method, and analytical expressions of the interaction force and vertical stiffness are derived. Depending on the value of the permeability, the interaction force can be repulsive or attractive. In the case $|\mu| < 1$ the force is repulsive and its value exceeds the value of the levitation force in magnet-superconductor levitation system.

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