# The spin dynamics of the random transverse Ising chain with a double-Gaussian disorder*

Liu Zhong-Qiang(刘中强)†, Jiang Su-Rong(姜素蓉), and Kong Xiang-Mu(孔祥木)

*College of Physics and Engineering, Qufu Normal University, Qufu* 273165, *China*

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The dynamical properties of one-dimensional random transverse Ising model (RTIM) with a double-Gaussian disorder is investigated by the recursion method. Based on the first twelve recurrences derived analytically, the spin autocorrelation function (SAF) and associated spectral density at high temperature were obtained numerically. Our results indicate that when the standard deviation $\sigma_J$ (or $\sigma_B$) of the exchange couplings $J_i$ (or the random transverse fields $B_i$) is small, no long-time tail appears in the SAF. The spin system undergoes a crossover from a central-peak behavior to a collective-mode behavior, which is the dynamical characteristics of RTIM with the bimodal disorder. However, when $\sigma_J$ (or $\sigma_B$) is large enough, the system exhibits similar dynamics behaviors to those of the RTIM with the Gaussian disorder, i.e., the system exhibits an enhanced central-peak behavior for large $\sigma_J$ or a disordered behavior for large $\sigma_B$. In this instance, SAFs exhibit a similar long-time tail, i.e., $C(t) \sim t^{-2}$ for large $t$. Similar properties are obtained when $J_i$ (or $B_i$) satisfy the double-exponential distribution or the double-uniform distribution. Besides, when both the standard deviations and the mean values of the exchange couplings are small, the effects of the Gaussian random bonds may drive the system undergo two crossovers from a triplet state to a doublet state, and then to a collective-mode state.

**Keywords:** random transverse Ising model, spin autocorrelation function, spectral density, long-time tail

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## 1. Introduction

The time-dependent behavior of quantum spin systems has been the subject of theoretical studies for quite a long time. However, most literature deals with the dynamic correlation functions of the pure quantum spin systems. It is found that the spin autocorrelation functions (SAFs) of these pure systems are associated with the temperature. The transverse correlation functions show a power-law behavior at $T = 0$,[1–3] an exponential behavior at $0 < T < \infty$,[4,5] and a Gaussian behavior at $T = \infty$,[6–9] respectively. As to the longitudinal correlation functions, Niemeijer[10] found that SAF of $XY$ spin chain is a damped oscillation function which may be expressed by the square of the Bessel function at $T = \infty$, but it has a more complex expression for other temperature.[11] For the spin-1/2 van der Waals model, Dekeyser and Lee[12] pointed out that the power-law long-time tails are found in the transverse component only at high temperature.

Only recently, since the disorder effects besides the effects of the temperature has been shown to affect the dynamics behavior of spin systems in a drastic way providing a very rich area of investigation, more and more attention has been paid to the random quantum spin systems, especially on the low-dimensional systems, such as the random transverse Ising model (RTIM),[13–20] $XY$ chain,[21–24] $XX$ and $XXZ$ chain,[25] one-dimensional Blume–Capel model,[26] etc. The known results indicate that the disorder effects may drastically slow down the behavior of the disordered averaged spin–spin correlation function,[25] including the SAFs. For example, as the couplings vary, the Gaussian behavior of $xx$-relaxation of the single impurity in $XY$ chain slows down to the stretched exponential-like relaxation at $T = \infty$.[27] However, as far as we know, although the short-time behaviors of the SAFs have been reported,[15–19,21–23] the long-time behaviors of them are still not understood. An important issue in this work is to explore whether it is possible to realize a situation in which one may obtain the long-time behavior of SAFs of the random transverse Ising chain, where the exchange couplings and the transverse fields are independent random variable satisfying a double-Gaussian distribution. The answer is positive. Moreover, the spectral density of SAF, reflecting the dynamical properties of spin systems, was also calculated in this work. Calculational results exhibit more details of the dynamics of RTIM.

The remainder of this paper is organized as follows. The RTIM with the double-Gaussian disorder and the recursion method will be introduced in Section 2. Section 3 presents the calculational results of the SAFs and the corresponding spectral densities for two types models: a random-bonds model and a random-fields model. The related discussion is also given in this section. Finally, the summary is presented in Section 4.

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†Corresponding author. E-mail: phyzhqliu@163.com

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## 2. Model and the recursion method

### 2.1. RTIM with the double-Gaussian disorder

One-dimensional (1D) RTIM is one of simplest but nontrivial example of the random quantum spin chains. It is defined by the following Hamiltonian:

$$H=-\frac{1}{2}\sum_i J_i\sigma_i^x\sigma_{i+1}^x-\frac{1}{2}\sum_i B_i\sigma_i^z, \tag{1}$$

where $1/2$ is the constant number introduced for the convenience of the derivation. $\sigma_i^x$, $\sigma_i^z$ denote Pauli matrices at site $i$. The exchange couplings $J_i$ (or the transverse fields $B_i$) are independent random variables obeying the distribution $\rho(J_i)$ [or $\rho(B_i)$]. This model has a well-known physical interpretation in connection not only with some quasi-one-dimensional hydrogen-bonded ferroelectric crystals like $Cs(H_{1-x}D_x)_2PO_4$, $PbH_{1-x}D_xPO_4$, etc.,[28–31] but also with Ising spin glasses $LiHo_{0.167}Y_{0.833}F_4$.[32] Besides, this model may capture vital features of the neutron scattering experiment in $LiHoF_4$.[20,33]

In this work, we mainly investigate the dynamics of RTIM with the double-Gaussian disorder, i.e., $J_i$ or $B_i$ are independent variables, which satisfy respectively a double-Gaussian distribution

$$\rho(\beta_i)=p\rho_1(\beta_i)+(1-p)\rho_2(\beta_i), \tag{2}$$

where

$$\rho_1(\beta_i)=\frac{1}{\sqrt{2\pi}\sigma_\beta}\exp\left[-(\beta_i-\beta_{1m})^2/2\sigma_\beta^2\right] \tag{3}$$

and

$$\rho_2(\beta_i)=\frac{1}{\sqrt{2\pi}\sigma_\beta}\exp\left[-(\beta_i-\beta_{2m})^2/2\sigma_\beta^2\right] \tag{4}$$

are the standard Gaussian distributions, wherein $\beta_{1m}$ ($J_{1m}$ or $B_{1m}$) and $\beta_{2m}$ ($J_{2m}$ or $B_{2m}$) denote the corresponding mean values of the random variables $\beta_i$ ($J_i$ or $B_i$), $\sigma_\beta$ ($\sigma_J$ or $\sigma_B$) is the standard deviation of them. The physical sense of Eq. (2) is that the random variable $\beta_i$ satisfies $\rho_1(\beta_i)$ with probability $p$ and $\rho_2(\beta_i)$ with probability $(1-p)$ respectively. The mean value and the standard deviation of the random variables $\beta_i$ obeying distribution $\rho(\beta_i)$ are respectively $p\beta_{1m}+(1-p)\beta_{2m}$ and $\sigma_{\beta S}=\sqrt{p(1-p)(\beta_{1m}-\beta_{2m})^2+\sigma_\beta^2}$. Since $\sigma_{\beta S}$ is an increasing function of $\sigma_\beta$, $\sigma_\beta$ is used to denote the standard deviation of the random variable unless specified otherwise in the following contents.

Obviously, the cases discussed in Refs. [15] and [16] are two special cases of the RTIM with the double-Gaussian disorder. When $\sigma_\beta\to 0$, equation (2) becomes a bimodal distribution

$$\rho(\beta_i)=p\delta(\beta_i-\beta_{1m})+(1-p)\delta(\beta_i-\beta_{2m}). \tag{5}$$

When $p=1$ (or $p=0$), equation (2) becomes a standard Gaussian distribution. Of course, $\rho_1(\beta_i)$ and $\rho_2(\beta_i)$ may be other distributions, such as an exponential distribution or a uniform distribution. In this way, $\rho(\beta_i)$ becomes a double-exponential distribution or a double-uniform distribution.

### 2.2. Recursion method

The basic tool employed in this work is the recursion method,[34] which is very powerful in the study on many-body dynamics.[9,15,16,21–23,35–46] It may help us obtain the SAF and associated spectral density. We will review this method briefly as follows.

For RTIM, the time evolution of $\sigma_j^x(t)$ is governed by the Liouville equation

$$\mathrm{d}\sigma_j^x(t)/\mathrm{d}t=\mathrm{i}L\sigma_j^x(t), \tag{6}$$

where $L$ is the Liouville operator defined by $LA=[H,A]\equiv HA-AH$. $H$ denote the Hamiltonian of the spin system. Then the solution of Eq. (6) can be formally given as

$$\sigma_j^x(t)=\sum_{\nu=0}^{\infty}a_\nu(t)f_\nu, \tag{7}$$

where $\{f_\nu\}$ is a complete orthogonal sequence in the Hilbert space $S$, $\{a_\nu(t)\}$ is a set of time-dependent real functions. They respectively satisfy two recurrence relations,[34] i.e.,

$$f_{\nu+1}=\mathrm{i}Lf_\nu+\Delta_\nu f_{\nu-1},\quad \nu\geq 0, \tag{8}$$

and

$$\Delta_{\nu+1}a_{\nu+1}(t)=-\mathrm{d}a_\nu(t)/\mathrm{d}t+a_{\nu-1}(t),\quad \nu\geq 0, \tag{9}$$

where $f_{-1}\equiv 0$, $a_{-1}(t)\equiv 0$, the coefficients (recurrences) are defined as

$$\Delta_\nu=(f_\nu,f_\nu)/(f_{\nu-1},f_{\nu-1}) \tag{10}$$

with $\Delta_0\equiv 1$. $(f_\nu,f_\nu)$ is the inner product of the basis vectors in the Hilbert space $S$, which include both the statistical and random averages.[9] The recurrences $\Delta_\nu$ play an important role in the construction of the spin autocorrelation function and the corresponding spectral function. Subsequently, we will give their construction process respectively.

For RTIM, $f_0=\sigma_j^x$, the average SAF is defined by

$$C(t)=\overline{\left\langle\sigma_j^x(t)\,\sigma_j^x\right\rangle}. \tag{11}$$

It can be written as the form of moment expansion

$$C(t)=\sum_{k=0}^{\infty}\mu_{2k}t^{2k} \tag{12}$$

with

$$\mu_{2k}=\frac{(-1)^k}{(2k)!}\overline{\mathrm{Tr}\sigma_j^x\left[H,\left[H,\ldots,\left[H,\sigma_j^x\right]\ldots\right]\right]}, \tag{13}$$

where $\mu_{2k}$ is the $2k$-th moment of the spin autocorrelation function. The relations among the recurrences and moments are determined by[34]

$$\mu_{2k}^{(\nu-1)} = \Delta_{\nu-1}\mu_{2k}^{(\nu)} + \frac{\Delta_{\nu-1}}{\Delta_{\nu-2}}\mu_{2k-2}^{(\nu-2)}, \tag{14}$$

where $\mu_{2k}^{(-1)} \equiv 0$, $\mu_{2k}^{(0)} \equiv \mu_{2k}$, $\Delta_\nu \equiv \mu_{2\nu}^{(\nu)}$, $(k = 1,2,...,K,\ \nu = k,k-1,...,1)$. Supposing that the first $n$ recurrences have been obtained, the first $2n$ moments may be deduced by Eq. (14). And then the autocorrelation functions $C(t)$ can be determined by the Padé approximant using these known moments.

Applying the Laplace transformation $a_\nu(z) = \int_0^{+\infty} \mathrm{e}^{-zt} a_\nu(t)\,\mathrm{d}t$ ($z = \varepsilon + \mathrm{i}\omega$ is a complex number, $\varepsilon > 0$) to the recurrence relation (9), we have

$$\begin{aligned} &\Delta_{\nu+1}a_{\nu+1}(z) - \delta_{\nu,0} = -z a_\nu(z) + a_{\nu-1}(z), \\ &\nu = 0,1,2,\ldots. \end{aligned} \tag{15}$$

From Eq. (15), $a_0(z)$ is an infinite continued fraction as described below

$$a_0(z) = 1/\left(z + \Delta_1/\left\{z + \Delta_2/\left[z + \Delta_3/(z+\cdots)\right]\right\}\right). \tag{16}$$

The spectral density $\Phi(\omega)$ that may be measured directly in experiments is defined as the Fourier transformation of SAF, i.e.,

$$\Phi(\omega) = \int_0^{+\infty} C(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t. \tag{17}$$

In fact, $\Phi(\omega)$ is determined directly by Eq. (16)

$$\Phi(\omega) = \lim_{\varepsilon\to 0} \mathrm{Re}a_0(z)\,|_{z=\varepsilon+\mathrm{i}\omega}\,. \tag{18}$$

Although only a finite number of recurrences can be deduced analytically, they play vital roles in the whole study process. On the one hand, since the first $2k$ moments may be expressed by the first $k$ recurrences,[34] SAF can be determined by constructing the Padé approximant using these known moments. On the other hand, one can determine the spectral density of the SAF by employing the known recurrences and so-called Gaussian terminator.[15,34,44,47] Due to the first 12 recurrences have been deduced analytically, we extract more relevant information from them in this paper than we have done in our previous work[16] in which only the first 9 recurrences have been obtained analytically.

## 3. Results and discussion

As mentioned above, we have deduced the first twelve basic vectors (See the complementary material) and their associated recurrences. To ensure the accuracy of our results, we compare our results with the first nine recurrences given in Ref. [15]. We find our results are completely identical with aforementioned ones for the same parameters.

In this section, to investigate the effects of the double-Gaussian disorder on the dynamics of RTIM, we will mainly focus our attention on both the spin autocorrelation function $C(t)$ and the associated spectral density $\Phi(\omega)$. Besides, we will restudy in detail the effects of the Gaussian disorder on the dynamics of RTIM. To our surprise, several new phenomena occur as shown in the following.

### 3.1. The double-Gaussian disorder on the dynamics of RTIM

To study the effects of double-Gaussian disorder on the dynamics of RTIM, in this subsection two typical random spin models are studied: the double-Gaussian random-bonds model and the double-Gaussian random-fields model, respectively.

#### 3.1.1. The double-Gaussian random-bonds model

In this model, the exchange couplings $J_i$ are independent and satisfy respectively the double-Gaussian distributions defined by Eq. (2), meanwhile the transverse fields $B_i$ remain unaltered. To compare our results with the previous results presented in Ref. [15], we set the same parameters as those in it: $J_{1m} = 1.0$, $J_{2m} = 0.4$, and $B_i = B = 1$ which fixes the energy scale, and $\sigma_J$ varies from 0.01 to 2. $C(t)$ and $\Phi(\omega)$ are plotted in Figs. 1 and 2, respectively. Each inset in Fig. 2 gives the recurrences $\Delta_\nu$ $(\nu = 1,\ldots,12)$ for the same parameters in each related figure, and the lines in it are just a guide to the eye.

As $\sigma_J \to 0$, the double-Gaussian distribution turns into the bimodal distribution. Taking $\sigma_J = 0.01$ for example, as shown in Figs. 1(a) and 2(a), the dynamics of the system undergoes a crossover from a central-peak behavior to a collective-mode behavior as $p$ decreases from 1 to 0. These are highly consistent with those given in Ref. [15]. Although we cannot obtain $C(t)$ for large time, comparison shows that figure 1(a) presents a better improvement in SAFs for short time than those given by Florencio, *et al.*[15] Comparing all the figures in Figs. 1 and 2, one finds that as $\sigma_J$ increases from 0.01 to 2, the damped oscillations of the SAFs diminish in magnitude, meanwhile the recurrences for the same parameters tend to fall into a straight line. In this process, the spectral weights at $\omega = 0$ are enhanced, and the peaks' locations of the spectral lines move towards $\omega = 0$ gradually (e.g., curves with $p = 0$). The solid curves $p = 1$ in Fig. 2 illustrate how an almost pure Gaussian profile of $\Phi(\omega)$ evolves into a curve with some additional structure which consists of a central peak with increasing height and decreasing width. The above analysis indicates that the collective-mode behavior becomes weaker and weaker, while the dynamics becomes increasingly dominated by a central-peak behavior as $\sigma_J$ increases. When $\sigma_J$ are large enough [see Figs. 1(d) and 2(d)], the crossover vanishes, and the system exhibits an enhanced central-peak behavior. These results are in agreement with those in our previous work.[16]

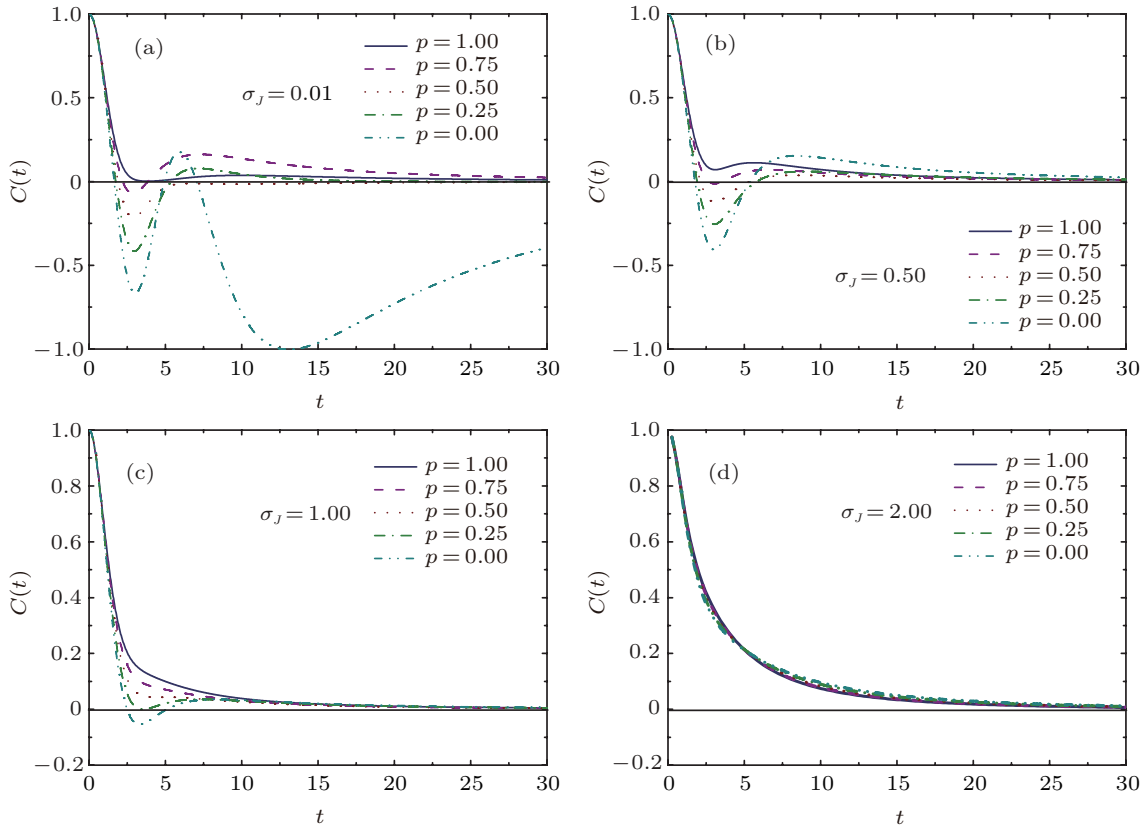



**Fig. 1.** (color online) Spin autocorrelation functions $C(t)$ of RTIM with the double-Gaussian random bonds for different $\sigma_J$: (a) $\sigma_J = 0.01$; (b) $\sigma_J = 0.50$; (c) $\sigma_J = 1.00$; (d) $\sigma_J = 2.00$.

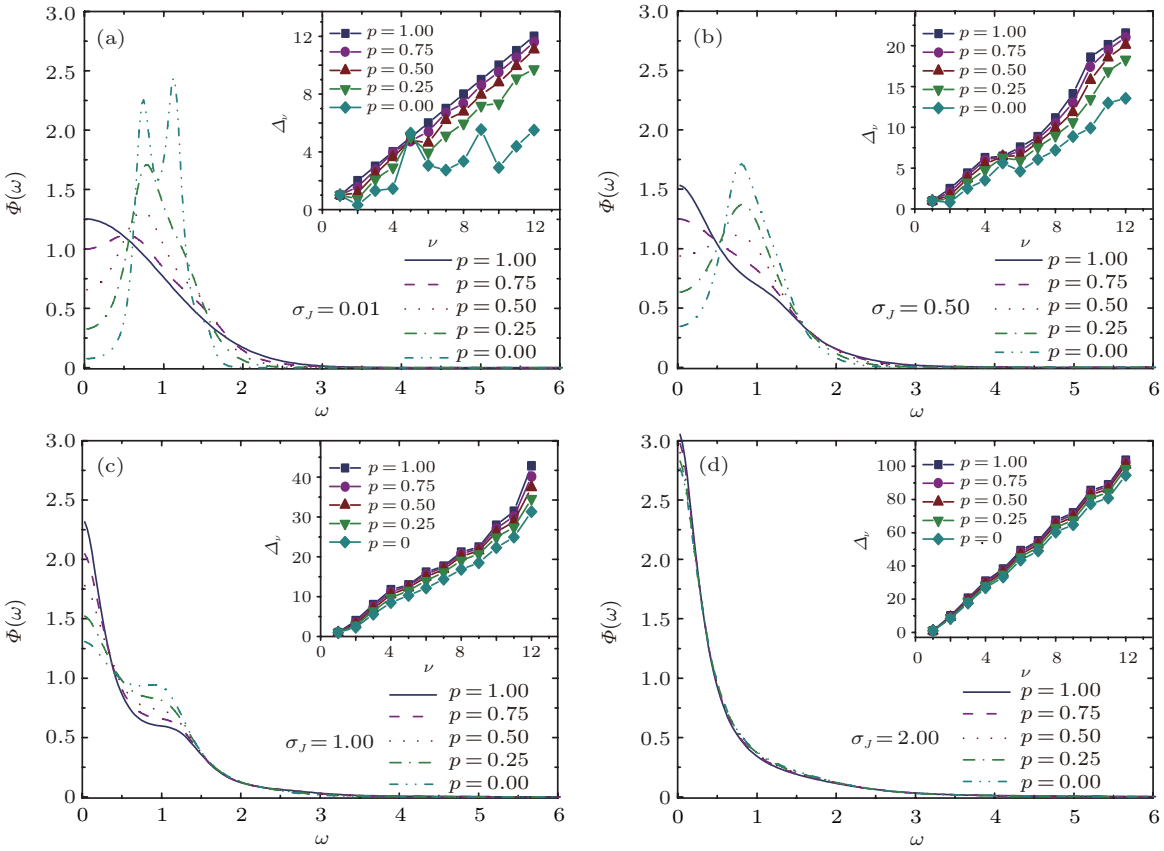



**Fig. 2.** (color online) The associated spectral densities for the same parameters as in Fig. 1. The insets show the recurrences $\Delta_\nu$ ($\nu = 1, \cdots, 12$). The lines in the insets are just a guide to the eye. For small $\sigma_J$ (e.g., $\sigma_J \leq 0.01$), (a) recovers $\Phi(\omega)$ of RTIM with the bimodal distribution. As $\sigma_J$ increases, the crossover from a central-peak behavior to a collective-mode behavior vanishes gradually. The system exhibits an enhanced central-peak behavior only when $\sigma_J$ is large enough.

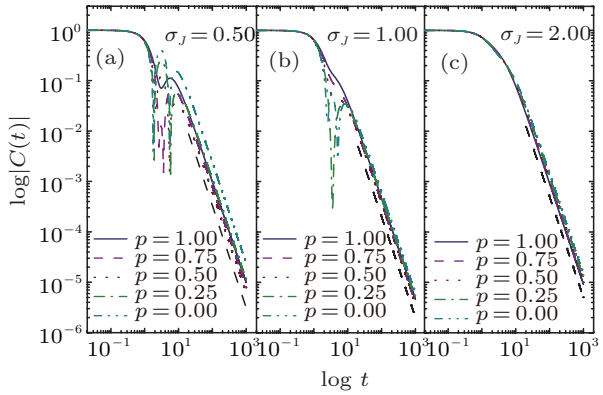



**Fig. 3.** (color online) The log–log plot of $|C(t)|$ versus $t$ for different values of $p$ and $\sigma_J$. Black dashed line in each figure represents the power-law decay $t^{-2}$.

The infrared (i.e. low-frequency) singularity in the spectral density signals the SAF should manifest itself a characteristic quantum-diffusion long-time tail.[48] We find, as expected, long-time tails exist in the SAFs when $\sigma_J$ are large enough. Figure 3 gives the log–log plot of the absolute values of $C(t)$ versus time. The solid lines $p = 1$ in Figs. 1 and 3 show how a Gaussian behavior of $C(t)$, due to the disorder effects, slows down to a power-law relaxation. Calculation results indicate that $C(t) \sim t^{-2}$ for $t > 20$. This result is identical with the pure $XY$ interaction case ($R = 0$) of the spin-1/2 van der Waals model.[12] In fact, the $XY$ model and the transverse Ising model are dynamically equivalent to the spin van der Waals model.[9] These systems are indicated by the same geometry of their respective dynamics Hilbert spaces. Finally, we should like to emphasize that our success in obtaining the long-time tails of the SAFs is owing to the fact that the true long-time behavior is only nebulously encoded in the first few continued-fraction coefficients $\Delta_\nu$.[48]

### 3.1.2. The double-Gaussian random-fields model

In the random-fields model, the random transverse fields $B_i$ are independent and satisfy the double-Gaussian distributions defined by Eq. (2), meanwhile the exchange couplings are uniform (i.e., $J_i = J = 1$). For convenience to compare the results in this paper with the previous results,[15] we set $B_{1m} = 0.6$, $B_{2m} = 1.4$, and $\sigma_B$ varies from 0.01 to 2. The SAFs and the corresponding spectral densities are shown in Figs. 4 and 5 respectively. The insets within respectively give the double-logarithmic plots of the absolute values of $C(t)$ versus time $t$ and the associated recurrences $\Delta_\nu$ ($\nu = 1, \ldots, 12$) for the same parameters in each figure.

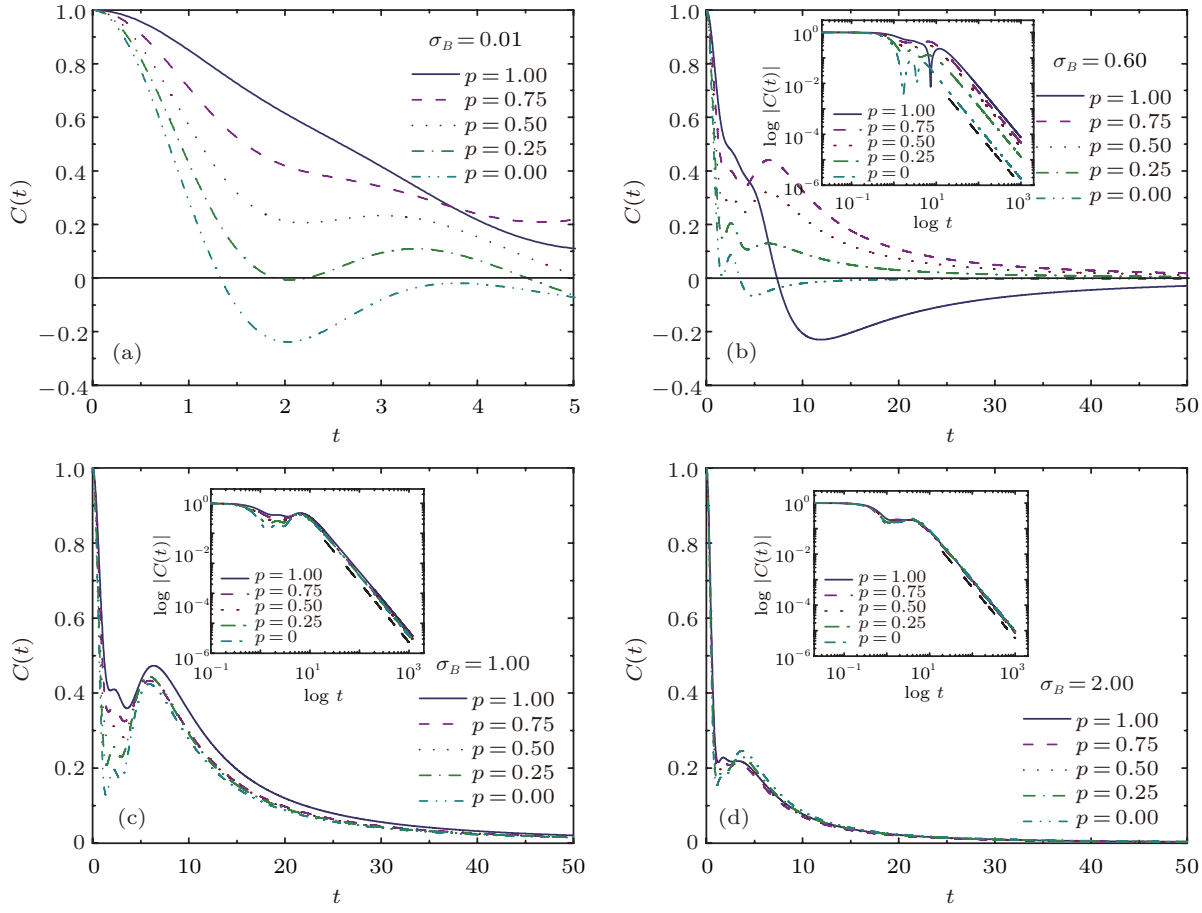



**Fig. 4.** (color online) SAFs of RTIM with the double-Gaussian random fields. When $\sigma_B$ is small ($\sigma_B = 0.01$), (a) recovers the results presented in Ref. [15]. The inset presents log–log plot of $|C(t)|$ versus $t$. The black dashed lines in the insets represent the power laws $t^{-2}$.

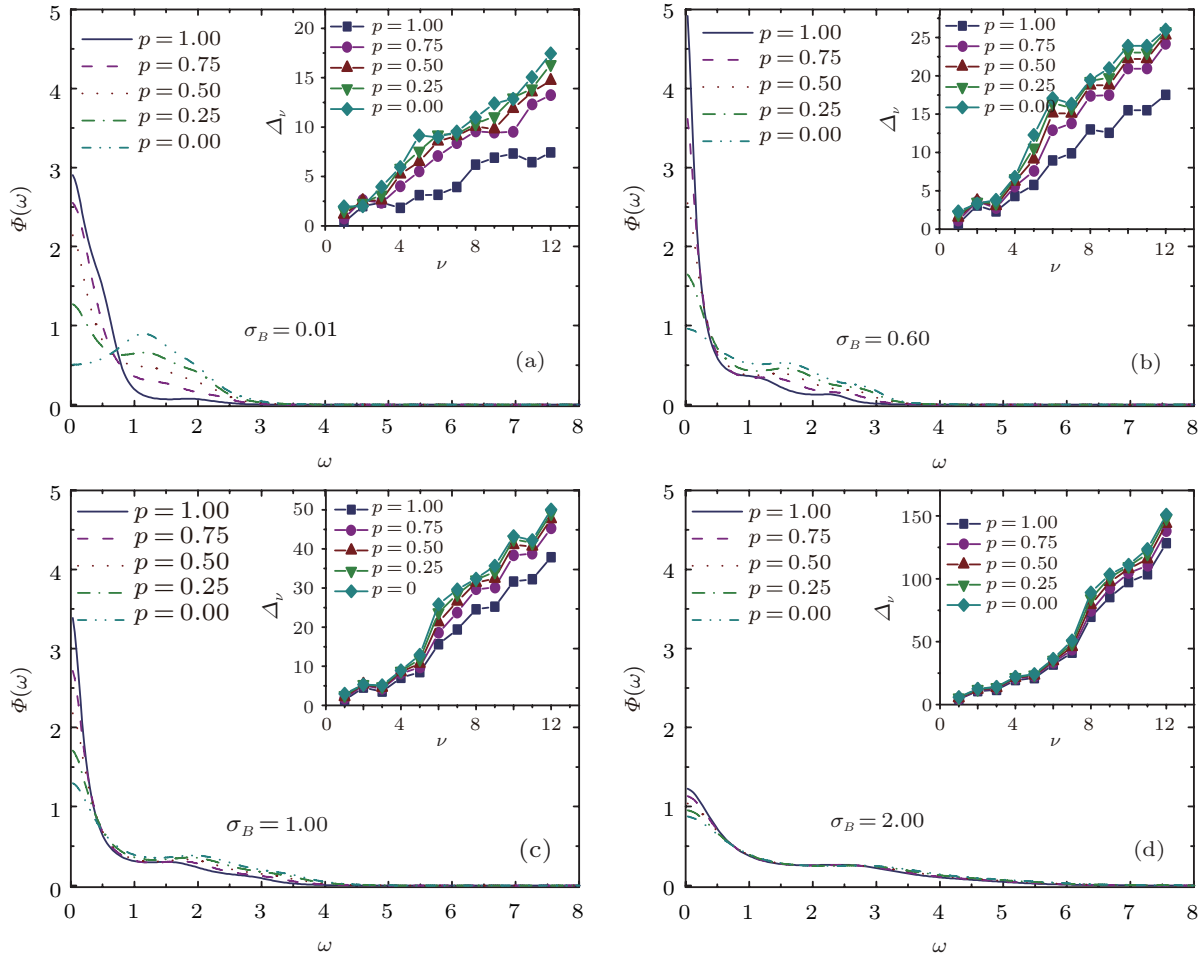



**Fig. 5.** (color online) The corresponding spectral densities for the same parameters as in Fig. 4. For small $\sigma_B$ ($\sigma_B = 0.01$), the system moves from a collective mode dynamics to a central peak type of dynamics as $p$ varies from 0 to 1. However, the system is in a disordered state when $\sigma_B$ is large enough ($\sigma_B = 2.00$).

For small $\sigma_B$ (e.g., $\sigma_B \leq 0.10$), the system undergoes a crossover from a central-peak behavior to a collective-mode behavior as $p$ decreases from 1 to 0, as shown in Figs. 4(a) and 5(a). This result is consistent with that obtained by Florencio *et al.*[15] However, as $\sigma_B$ increases, this crossover disappears, see Figs. 5(b), 5(c), and 5(d). Meanwhile, the SAFs exhibit an interesting phenomenon. From Fig. 4(b) one finds that the decays of long-time tails in curves $p = 0$ and $p = 1$ are proportional to $-t^{-2}$, and those in other lines are proportional to $t^{-2}$. When $\sigma_B$ is large enough (e.g., $\sigma_B = 1.00$), SAFs fall into three successive phases. In the initial phase ($0 < t < 1$), the decay of $C(t)$ is Gaussian regime. In the third phase ($t > 20$), the long-time tails of SAFs obey the power-law decay $t^{-2}$. There is a cross-over time between the Gaussian regime and the power-law regime. In this cross-over, the oscillation of $C(t)$ implies that the $x$ component of the spin may be coherently coupled to some mode with small fluctuation. Thus, slow decay may emerge. This is consistent well with the analysis given in Ref. [12]. The strong peaks at $\omega = 0$ also signal the existence of the long-time tails in the SAFs. The cross-over time becomes shorter as $\sigma_B$ is increased further. While more and more high-frequency modes are involved in the corresponding spectral densities. The system is in a disordered state when $\sigma_B$ is large enough.

### 3.2. The Gaussian disorder on the dynamics of RTIM

In this subsection, the effects of the Gaussian disorder on the dynamics of RTIM is restudied. In our previous work[16] since only up to the first 18 moments used to obtain the spin autocorrelation function were deduced, we merely uncover its dynamics properties partly. In this paper, extracting more information from the first 24 moments, we found some new dynamical behaviors of this system.

#### 3.2.1. The Gaussian random-bonds model

The exchange couplings $J_i$ are independent and satisfy the Gaussian distribution defined by Eq. (2) with $p = 1$ and $\beta_1 = J$, while the transverse fields $B_i$ remain unaltered ($B_i = B = 1$). Obviously, $J$ and $\sigma_J$ respectively denote the mean value and the standard deviation of $J_i$. To illustrate SAF and its corresponding spectral density clearly, we set that: $J$ varies from 0.0 to 4.0, and $\sigma_J$ from 0.1 to 2.0. Figure 6 gives the SAF for some cases with different $\sigma_J$. Compared with Fig. 1 in Ref. [16], figure 6 gives an apparent improvement in the SAF: it presents more credible results of the SAF for a long period of time. Figures 6(a) and 7(a) show that for small $\sigma_J$, the system with

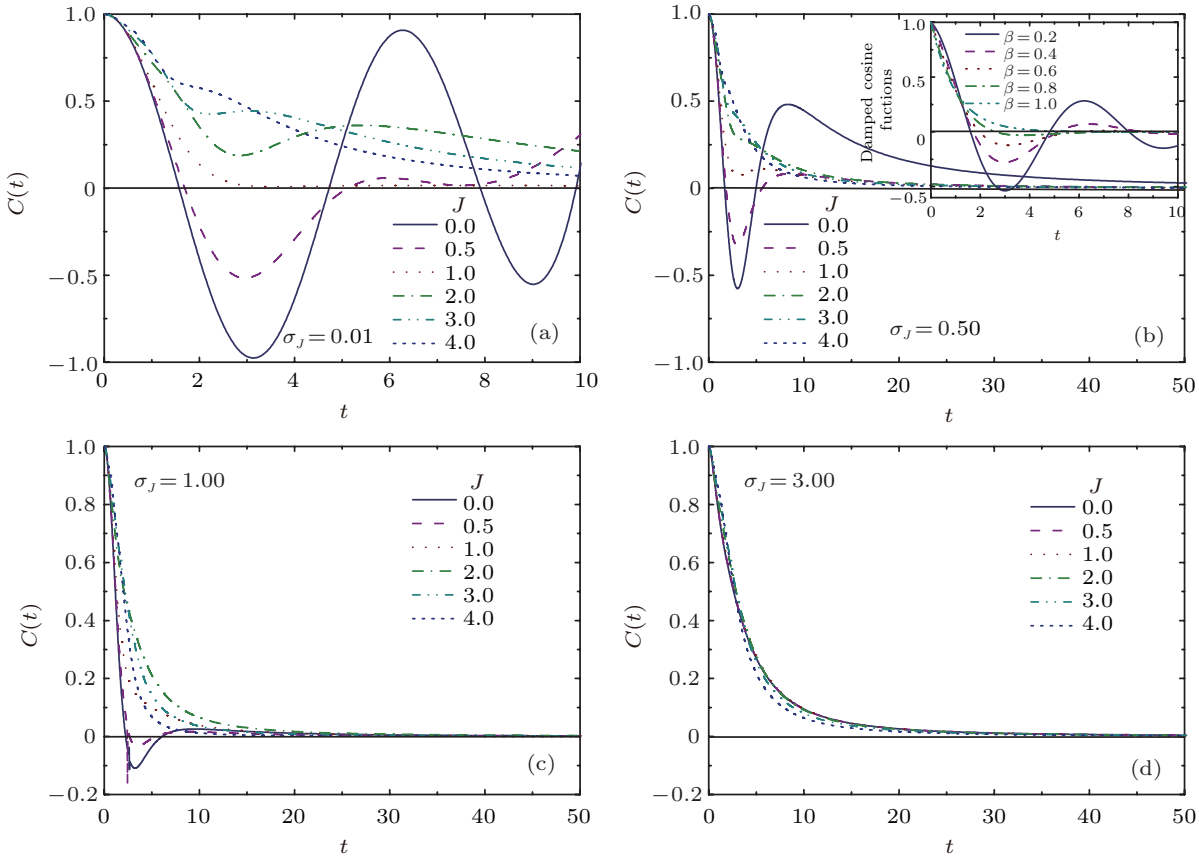



**Fig. 6.** (color online) SAFs of RTIM with the Gaussian random bonds for different $\sigma_B$: (a) $\sigma_B = 0.10$; (b) $\sigma_B = 0.50$; (c) $\sigma_B = 1.00$; (d) $\sigma_B = 3.00$.

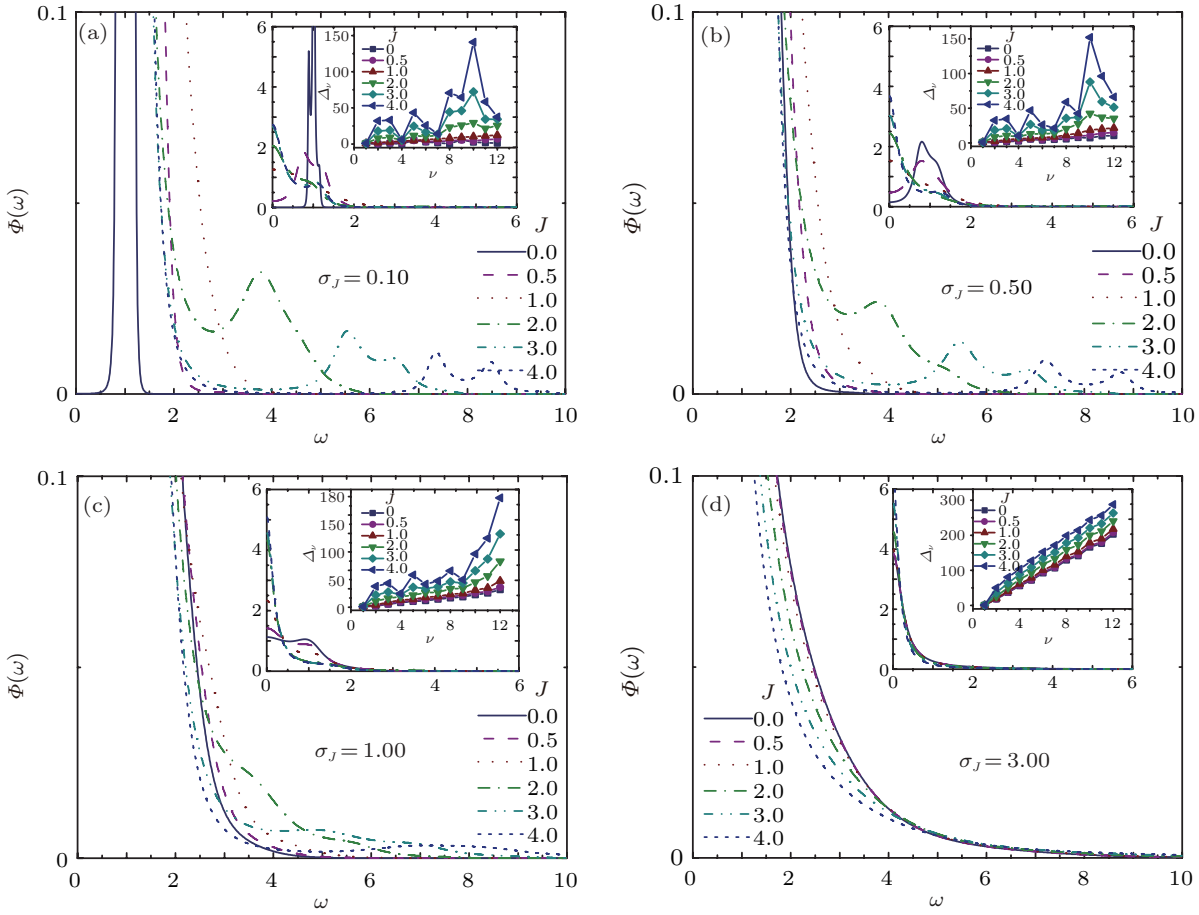



**Fig. 7.** (color online) The high-frequency modes (local excitations) of $\omega = 2J$ in RTIM with the Gaussian random bonds. The insets show the low energy part and the corresponding recurrences.

increasing $J$ undergoes two crossovers: from the precession of independent free spins in an external transverse magnetic field ($J = 0.0$) to the collective-mode behavior ($J = 0.5$), and then to a central-peak one ($J = 4.0$). It is noteworthy that although the curve with $J = 0.0$ in Fig. 6(a) is similar to a damped cosine curve, the curves with different $J$ is different distinctly from the damped cosine curves $D(t) = \exp(-\beta t)\cos(\sqrt{1-\beta^2}t)$ with different $\beta$. Figure 6(a) and its inset illustrate this viewpoint clearly. Calculations indicate that the SAF of system can also show a same power-law long-time tail just as the case for the corresponding double-Gaussian disorder. Figure 7 plots close-ups of the spectral densities for the same parameters in Fig. 6. The insets in Fig. 7 exhibit the panorama of $\Phi(\omega)$ and $\Delta_\nu$. We find that for small $\sigma_J$, the corresponding spectral densities have peaks around $\omega = 2J$, as shown in Figs. 7(a) and 7(b). Moreover, the peaks split into two peaks when $J$ is large enough. As $\sigma_J$ increases, the amplitudes of the peaks decrease and the gaps between two peaks vanish, see Fig. 7(c). Finally, these peaks disappear, see Fig. 7(d).

To further explore the Gaussian random-bonds disorder on the dynamics of RTIM, we investigate in detail the cases in which both $J$ and $\sigma_J$ are small. The mean value varies from 0.0 to 0.6, and the standard deviation from 0.01 to 0.30. Our calculations show that the spectral densities of the SAFs exhibit several unexpected phenomena, as shown in Fig. 8. From Fig. 8(a), we find the system stays in a triplet state when $J = 0.0$. In this instance, the almost free spin exhibits a Larmor precession in an external transverse magnetic field. As $J$ increases, the spectral density splits into three peaks. The gaps among them become larger and larger as $J$ increases further. However, when $J$ is large enough (e.g. $J = 0.4$), the peak located on high frequency vanishes. Then the height of two peaks decrease and their widths broaden further as $J$ increases further. Now the system shows a continuous collective-mode spectral line. Since the strong exchange couplings bind all spins as a whole, they exhibit a collective motion in an external magnetic field. A similar phenomenon may be observed when $\sigma_J$ increases while $J$ remains unaltered (such as $J = 0.1$). When $\sigma_J$ is large (e.g., $\sigma_J = 0.2, 0.3$), the system undergoes a crossover from doublet-like state to collective-mode behavior as $J$ increases from 0.0 to 0.6.

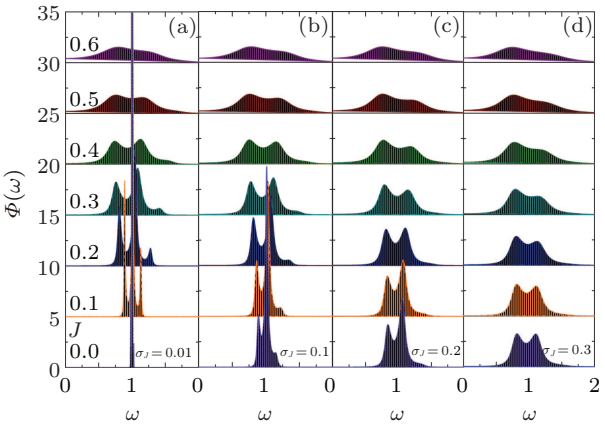
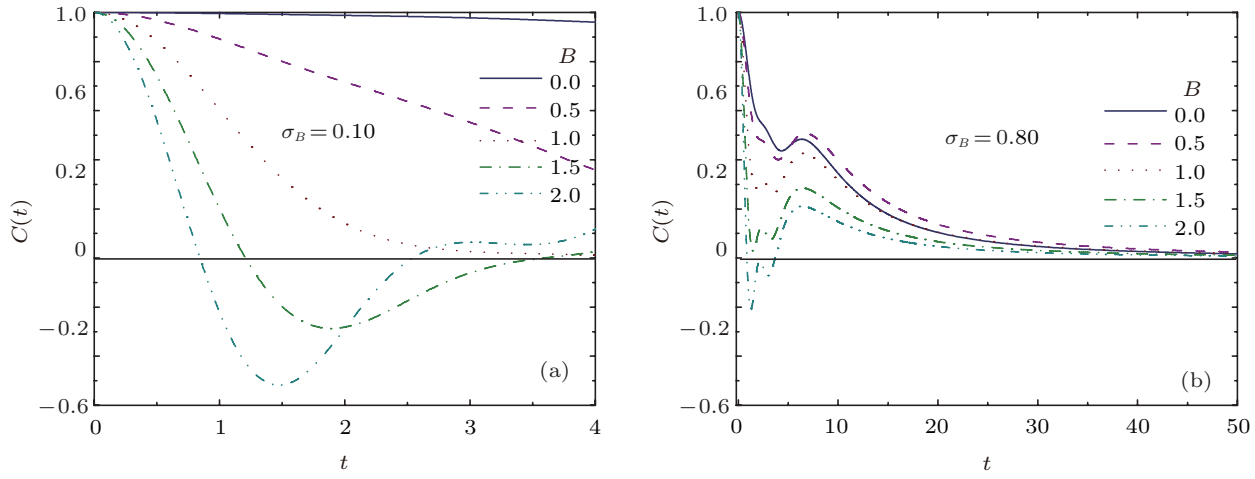



**Fig. 8.** (color online) The spectral densities of the autocorrelation functions for small $J$ and $\sigma_J$: (a) $\sigma_J = 0.01$; (b) $\sigma_J = 0.10$; (c) $\sigma_J = 0.20$; (d) $\sigma_J = 0.30$.

### 3.2.2. The Gaussian random-fields model

In this case, the random transverse fields $B_i$ are independent and satisfy the Gaussian distributions defined by Eq. (2) with $p = 1$ and $\beta_1 = B$, while the exchange couplings are uniform ($J_i = J = 1$). The calculational results of the $C(t)$ and $\Phi(\omega)$ are shown in Figs. 9 and 10, respectively. Comparing Fig. 9 in this paper with Fig. 3 in Ref. [16], one may find that the improvement of $C(t)$ is not remarkable when $\sigma_B$ is small (e.g., $\sigma_B = 0.10$). However, when $\sigma_B$ is large enough (e.g., $\sigma_B \geq 0.80$), the long-time tails of the SAFs appear. Our calculation proves that the system exhibits the same long-time behavior as the random-fields case with the double-Gaussian distribution, i.e., $C(t) \sim t^{-2}$ for $t > 20$. Figure 10 illustrates that the system exhibits a similar dynamics behavior to that shown in Fig. 5.



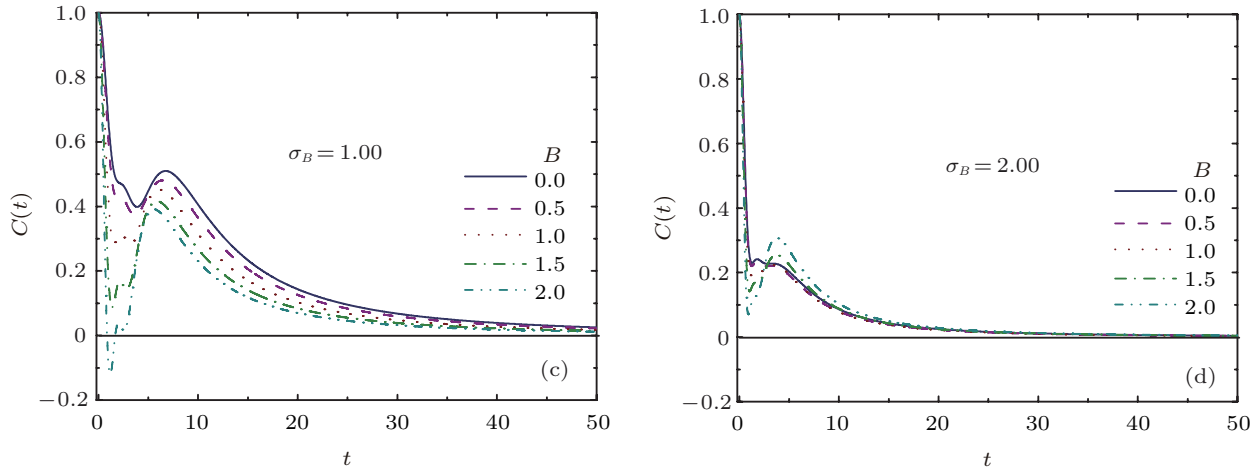



**Fig. 9.** (color online) $C(t)$ of RTIM with the Gaussian random fields for different $\sigma_B$: (a) $\sigma_B = 0.10$; (b) $\sigma_B = 0.80$; (c) $\sigma_B = 1.00$; (d) $\sigma_B = 2.00$.

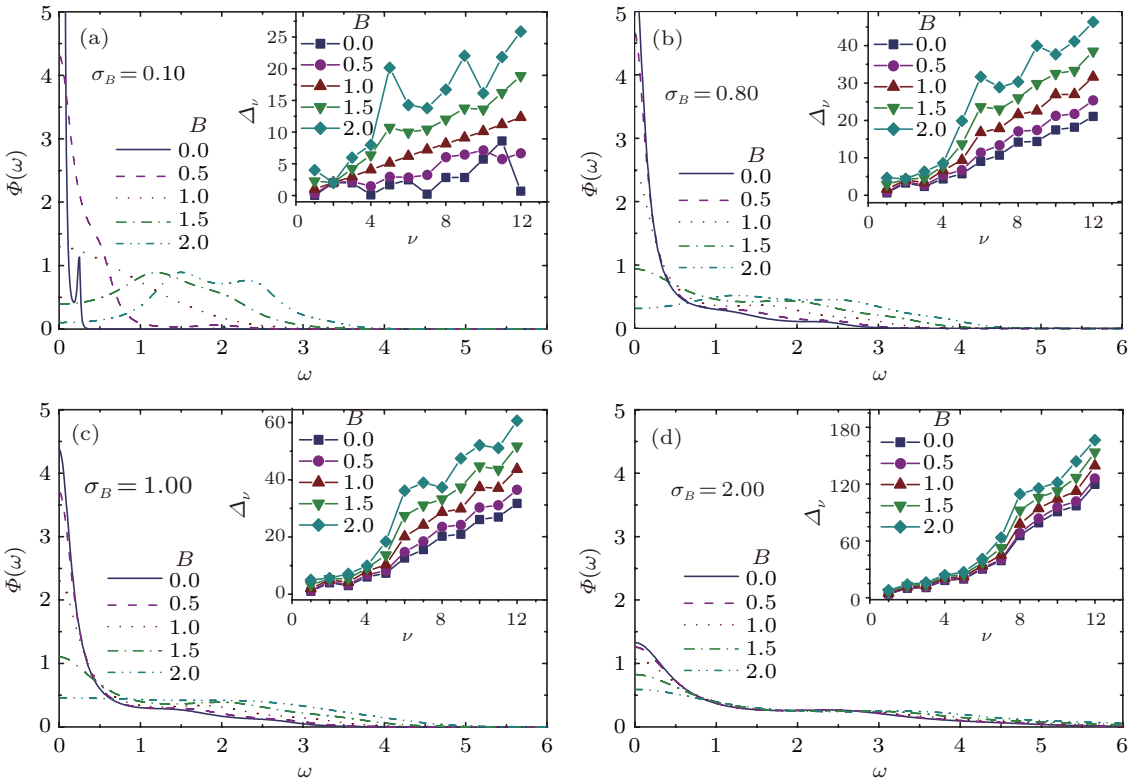



**Fig. 10.** (color online) The corresponding spectral densities for the same parameters as in Fig. 9.

## 4. Summary

In this paper, we have studied the dynamics of 1D RTIM with the double-Gaussian disorder, a generalized random quantum spin model, at the high-temperature by the recursion method. This generalized model may recover the dynamics of the RTIM both with the bimodal disorder and that with the Gaussian disorder.

The calculational results indicate that the dynamics of the RTIM with the double-Gaussian disorder is affected by the competition between the spin interactions and the transverse fields, and the competitive result depends on many factors, such as deviations $\sigma_J$ (or $\sigma_B$) and probability $p$. Generally speaking, when $\sigma_J$ (or $\sigma_B$) is small, the dynamics of the system undergoes a crossover from a central-peak behavior to a collective-mode one as $p$ (or the mean value of $J_i$ or $B_i$) varies. No long-time tail in SAF is obtained. As $\sigma_J$ (or $\sigma_B$) increases, the crossover vanishes gradually, and the power-law decay appears in the SAFs when the standard deviation is large enough. For the case of large $\sigma_J$ the system exhibits a central-peak behavior, but it shows a disordered behavior for the case of large $\sigma_B$. One may wonder if these spin dynamics features are universal or not. Therefore, we have studied the dynamics of the RTIM with other types of disorder, in which the variables independently satisfy the double-exponential distribution, and the double-uniform distribution. Calculational results indicate

that these systems show a similar dynamical behavior to those of the RTIM with the double-Gaussian disorder.

The restudy of the RTIM with Gaussian disorder indicates that besides the similar dynamical properties to those of RTIM with a double-Gaussian disorder summarized in the above paragraph, we find that the system exhibits some new characteristics. There are local excitations around $\omega = 2J$ besides central peak mode. Therefore, it might follow that each spin in this system fluctuates not freely at all, but with very short-ranged and very weak correlation still existing at $T = \infty$. In addition, under appropriate conditions the effects of the Gaussian disorder may drive the system to undergo the crossovers from a triplet state to the doublet-like state, and then to collective-mode state.

As to the long-time behavior of SAF, one may ask the following question: since it appears when $\sigma_J$ or $\sigma_B$ is large enough, is it independent of the special distributions which $J_i$ or $B_i$ obey? To answer this question, besides the aforementioned distributions, we also calculated two cases, in which $J_i$ or $B_i$ satisfy a uniform distribution, an exponential distribution. Calculational results show that independent of the type of disorder, as long as $\sigma_J$ (or $\sigma_B$) is large enough, the SAFs in 1D RTIM decay as $t^{-2}$ for large time due to quantum spin diffusion.

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