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Exchange interaction between vortex and antivortex*

Liu Yan(柳 艳)†, Li Hua-Nan(李化南), Hu Yong(胡 勇), and Du An(杜 安)

College of Sciences, Northeastern University, Shenyang 110819, *China*

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The dynamic properties of interacting vortex–antivortex pairs in thin film are studied by analytical calculations. Analytical expressions for the magnetization vector distribution of vortex–antivortex pairs and the trivortex states are given. The magnetic states of the vortices are treated as having rigid structures, i.e., the vortex maintains its spin distribution when moving. The trajectories of the vortex cores are calculated by the Thiele's equation. It is found that the vortex–antivortex pair rotates around each other when they have opposite polarities, however, vortex and antivortex cores move along straight lines when they have the same polarity. The frequency of the rotation decreases with increasing the distance between the two cores of vortex–antivortex pair, and it has a lower value when a third vortex is introduced.

Keywords: vortex–antivortex, exchange interaction, Thiele's equation

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1. Introduction

The competition between the exchange energy and magnetostatic energy in a nanodot favors an in-plane curling magnetization and forces magnetization to form an out-of-plane core in the center, called vortex or antivortex, which is an important phenomenon in many fields of condensed matter physics.[1–5] The vortex offers various opportunities for potential applications such as magnetic random access memory, high-density patterned recording medias, and ultrasmall magnetic field sensors. The vortex state can be described in terms of three quantities: (i) winding number (q) which determines a vortex ($q = 1$) or an antivortex ($q = -1$), (ii) core polarization (p) that determines the out-of-plane component of the magnetization to point up ($p = 1$) or down ($p = -1$), (iii) chirality (C) which determines the in-plane curling direction to be counterclockwise ($C = 1$) or clockwise ($C = -1$). The vortex structure can appear as a steady-state in circular disks or other patterns,[6–9] and the antivortex is observed typically on crosstie walls or asteroid-shaped nanodots.[10–13]

The interaction between vortices is a crucial issue that has been studied in the past few years.[14–16] Analytical study of the interaction between two vortices with opposite polarities indicates that the pair moves roughly as two interacting electrons, and the two vortices actually rotate around each other. The dynamic motion of vortices is attributed to the interaction energy between them, but the interaction energy includes Skyrme-like energy, which does not correspond to any reasonable realistic interaction.[17] As for the interaction of vortex–antivortex pair, Komineas pointed out that it behaves as a rotating vortex dipole because of its nonzero angular momentum based on the topology.[18] Moreover, the interaction between vortices has been proposed as a two-dimensional (2D) Coulomb force.[19] However, according to the dynamics theory the motion of the vortex should be excited by an external force which is induced by the magnetic energy of the system.[20] Which specific energy contributes to the dynamical motion of the interacting vortex–antivortex pair is still unclear. In this paper, we develop an analytical model to describe how exchange interaction between vortex and antivortex influences the dynamic properties of the vortex–antivortex pair. Indeed, Kuepper *et al.* have proposed that the interaction between vortex and antivortex results from the indirect coupling between vortex cores mediated by the exchange interaction with the adjacent domain walls when they investigated the dynamics and their mutual interaction of vortex and antivortex in crosstie walls.[21] Besides the crosstie walls, the vortex–antivortex pair can also be observed in the nanodots. The polarization of vortex core can be switched by applying an external field or current via nucleation, displacement, and annihilation of vortex–antivortex pair, wherein the interaction between vortex and antivortex plays an important role.[22–24] Therefore, we use the exchange energy to explain the dynamic properties of the vortex–antivortex pair and the trivortex state appearing in the reversal process of the vortex core.

2. Model

First, we consider the magnetization distribution of vortices in a thin film with a thickness of the order of exchange length $L_{\rm ex} = \sqrt{2A/M_{\rm s}^2}$, where A is the exchange constant, and $M_{\rm s}$ is the saturation magnetization. The nanodot is simplified

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†Corresponding author. E-mail: liuyanphys@mail.neu.edu.cn

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into a 2D model, due to the magnetization $\boldsymbol{m}(\boldsymbol{r})$ with unit amplitude only as a function of position in the disk plane (x–y plane). It is convenient to use a complex function $\omega(\zeta,\bar{\zeta})$ to define the magnetization component at point (x,y) as[6,17]

$$m_x = \frac{\omega+\bar{\omega}}{1+\omega\bar{\omega}},\quad m_y = \frac{1}{\mathrm{i}}\frac{\omega-\bar{\omega}}{1+\omega\bar{\omega}},\quad m_z = \frac{1-\omega\bar{\omega}}{1+\omega\bar{\omega}}, \tag{1}$$

where m_x, m_y, and m_z are the three components of the magnetization, $\bar{\omega}$ is the conjugate of ω, and $\zeta = X - X_0$, $\bar{\zeta}$ is the conjugate of ζ, in which $\boldsymbol{X} = x + \mathrm{i}y$ and $\boldsymbol{X}_0 = x_0 + \mathrm{i}y_0$, with x_0 and y_0 denoting the coordinates of the vortex core or the antivortex core. It has been mentioned in Ref. [6] that for a single vortex, the complex function ω has the form

$$\omega = \begin{cases} f(\zeta), & |f(\zeta)| \le 1, \\ f(\zeta)/|f(\zeta)|, & |f(\zeta)| > 1, \end{cases} \tag{2}$$

where $f(\zeta)$ is an appropriate analytical function.[6] Here, we assume that for a vortex configuration, we have $f_{V^\uparrow}(\zeta) = \mathrm{i}C\zeta/a$ and $\bar{f}_{V^\downarrow}(\zeta) = a/\mathrm{i}C\zeta$, and for an antivortex configuration, we have $\bar{f}_{\bar{V}^\uparrow}(\zeta) = f_{V^\uparrow}(\zeta)$ and $f_{\bar{V}^\downarrow}(\zeta) = \bar{f}_{V^\downarrow}(\zeta)$, with a being the radius of the vortex core. The subscripts $V^\uparrow$ and $V^\downarrow$ denote a vortex with polarization $p = 1$ and a vortex with polarization $p = -1$ respectively. The bar over the V denotes an antivortex. The complex functions for a $V^\uparrow - \bar{V}^\downarrow$ pair and a $V^\downarrow - \bar{V}^\downarrow$ pair are written as $\omega = -\mathrm{i}\omega_{V^\uparrow}\omega_{\bar{V}^\downarrow}$ and $\omega = \mathrm{i}\omega_{V^\downarrow}\omega_{\bar{V}^\downarrow}$ respectively. For the case of a $V^\uparrow - \bar{V}^\downarrow - V^\downarrow$ trivortex state, which usually appears when a vortex reverses its polarization or crosstie walls, $\omega = \omega_{V^\uparrow}\omega_{\bar{V}^\downarrow}\omega_{V^\downarrow}$.

The total magnetic energy of the system consists of exchange, magnetostatic, and anisotropy energies. The interaction between the vortex and antivortex becomes obvious only when their distance is small, so the magnetostatic energy is much smaller than the exchange energy. On the other hand, we do not restrict the vortex–antivortex pair in a confined dot, so there is no magnetostatic energy originating from the side charge. In addition, we assume that the system is of a soft magnetism, the anisotropy can be neglected. Thus, only the exchange energy of the interacting $V^\uparrow - \bar{V}^\downarrow$ pair is taken into account. The exchange energy is given by

$$W_{\mathrm{ex}} = \iint w_{\mathrm{ex}}\,\mathrm{d}x\mathrm{d}y, \tag{3}$$

where w_{ex} is the energy density, and

$$w_{\mathrm{ex}} = \sum_{i=x,y,z}(\nabla m_i)^2. \tag{4}$$

Here, the energy is normalized by AL, where L is the thickness of the nanodot.

Under the force of the effective interaction energy the vortices will be excited, the dynamics of such rigid vortices can be well described by using Thiele approach. Therefore, Thiele's equation is used to calculate the trajectories of the vortex cores. The Thiele's equation is:[20]

$$-\boldsymbol{G}\times\dot{\boldsymbol{X}}_i - D\dot{\boldsymbol{X}}_i + \frac{\partial W(\boldsymbol{X}_i,\boldsymbol{X}_j,\boldsymbol{X}_k)}{\partial \boldsymbol{X}_i} = 0, \tag{5}$$

where $\boldsymbol{X}_i = (x_i, y_i)$ is the coordinate of vortex core, the i, j, k denote $V^\uparrow$, $V^\downarrow$, and $\bar{V}^\downarrow$ respectively. The first term on the left-hand side in Eq. (5) is the gyroscopic force, where $\boldsymbol{G} = -G\hat{\boldsymbol{z}}$ with $G = 2\pi qpLM_{\mathrm{s}}/\gamma$. The force acts on a moving vortex in the same way as a magnetic field influencing a charged particle by the Lorentz force. The second term is the drag force with damping constant $D = -\alpha\pi M_{\mathrm{s}}L(2+\ln(R/a))/\gamma$, where γ is the gyromagnetic ratio, α is the Gilbert damping constant, and a is the core radius.[21] The third term of Eq. (5) is the effective force, where $W(\boldsymbol{X}_i,\boldsymbol{X}_j,\boldsymbol{X}_k) = W_{\mathrm{eff}}(\boldsymbol{X}_i,\boldsymbol{X}_j,\boldsymbol{X}_k)$. For a $V^\uparrow - \bar{V}^\downarrow$ pair with a distance of d, the effective interaction energy between the vortex and antivortex core is defined as $W_{\mathrm{eff}} = W_{\mathrm{ex}}(d) - W_{\mathrm{ex}}(2a)$, where $W(d)$ and $W(2a)$ are the vortex exchange energies between vortex and antivortex at a distance d and $2a$ respectively.

In the following calculations, we choose and study the Permalloy film and the parameters of saturation magnetization $M_{\mathrm{s}} = 8.6\times10^5$ A/m and exchange constant $A = 13\times10^{-12}$ J/m. The core radius is assumed to be $a = 1.2L_{\mathrm{ex}}$. The calculations are performed by numerically solving Eq. (5) through using the fourth order Runge–Kutta method.

3. Results and discussion

Figure 1(a) shows the effective interaction energy of the $V^\uparrow - \bar{V}^\downarrow$ pair as a function of $d/2a$. If the two cores are not overlapped, i.e., $d > 2a$, W_{eff} initially increases logarithmically, and then almost reaches a stable value when $d/2a \ge 4$, while W_{eff} decreases with $d/2a$ for the case of $d \le 2a$. Note that W_{eff} is almost stable when the distance between the two cores reaches about $9.6L_{\mathrm{ex}}$. It means that the interaction between the two cores can almost be neglected when the distance is larger than the critical value. The motion trajectories of the vortex and antivortex cores are obtained by solving Eq. (5). Figure 1(b) shows the trajectories of $V^\uparrow$ and $\bar{V}^\downarrow$, respectively, with the initial core coordinates $\boldsymbol{X}_{V^\uparrow} = -12.5$ nm and $\boldsymbol{X}_{\bar{V}^\downarrow} = 12.5$ nm for a weak damping constant $\alpha = 0.05$. Both $V^\uparrow$ and $\bar{V}^\downarrow$ move from their initial core positions counterclockwise with orbits like helix under the influence of the exchange part of the force. Meanwhile, the distance between them decreases because of the dissipative field. The dissipation damps the possible circular motion of the vortex. Figure 2 shows the variations of distance between $V^\uparrow$ and $\bar{V}^\downarrow$ with t for different damping constants. It is found that the distance d decreases with time, and it decreases much faster with increasing the damping constant. Here, we use Δt to denote the duration from the initial moment to the moment when the

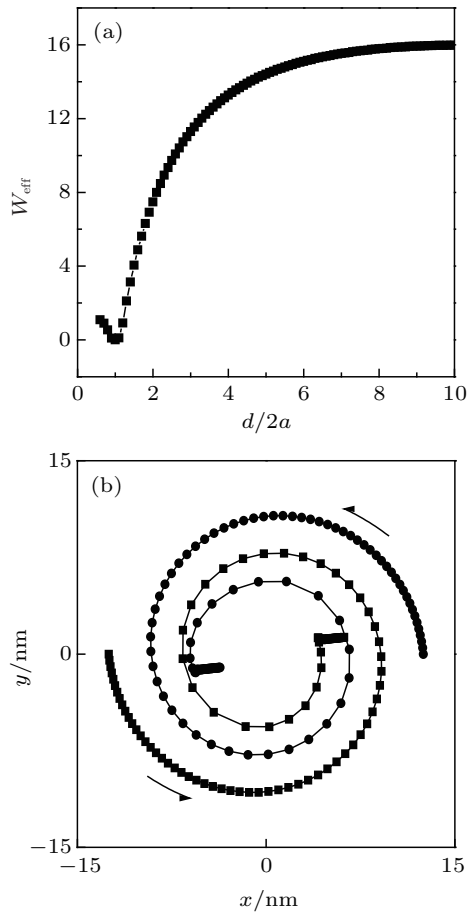


Fig. 1. (a) Energy of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair as a function of the $d/2a$, (b) the trajectories of the $V^{\uparrow}$ and $\bar{V}^{\downarrow}$ of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair with $d=25$ nm, where the square dots and the circle dots denote the trajectories of the $V^{\uparrow}$ and $\bar{V}^{\downarrow}$, respectively, and the arrows show the rotation direction.

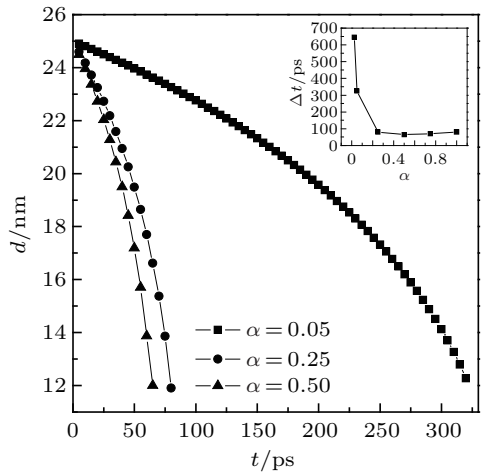


Fig. 2. Time-dependent distances between the $V^{\uparrow}$ and $\bar{V}^{\downarrow}$ during the rotation with different damping constants, where the inset shows the duration Δt as a function of α.

distance between the two cores decreases to $2a$. It is found that Δt initially decreases fast with increasing α and reaches its minimum at $\alpha=0.5$ before increasing slowly as shown in the inset in Fig. 2. The sense of the rotation of the vortex–antivortex pair with opposite polarities is determined by the topological charge G. The rotation sense is counterclockwise for $V^{\uparrow}-\bar{V}^{\downarrow}$ pair, however, clockwise for $V^{\downarrow}-\bar{V}^{\uparrow}$ pair. When the two cores are overlapped, simulation results show that vortex and antivortex with opposite polarities will annihilate accompanied by a violent burst of spin waves when they meet each other.[11,22] However, the distance between the two cores is almost stable according to our calculations. Therefore, the behavior of the vortex–antivortex pair with overlapped cores cannot be fully described by our model, wherein only exchange energy is taken into account. In addition, G of the overlapped cores is complex, and it cannot be described by $G=2\pi qpLM_{\rm s}/\gamma$.

Figure 3(a) shows the effective interaction energy of the $V^{\downarrow}-\bar{V}^{\downarrow}$ pair as a function of $d/2a$. $W_{\rm eff}$ of the pair is a continuous function, and it increases with increasing $d/2a$. Therefore, the dynamic motion of the $V^{\downarrow}-\bar{V}^{\downarrow}$ pair in the case of overlapped cores is consistent with that in the case of their separation from each other. Different from the rotating motion of the vortex–antivortex pair with the opposite polarities, the vortex and antivortex with the same polarity are no longer act as a rotating pair. Figure 3(b) shows the trajectories of the $V^{\downarrow}$ and $\bar{V}^{\downarrow}$ of the $V^{\downarrow}-\bar{V}^{\downarrow}$ pair with the initial cores coordinates $\boldsymbol{X}_{V^{\downarrow}}=-12.5$ nm and $\boldsymbol{X}_{\bar{V}^{\downarrow}}=12.5$ nm. The initial condition of this pair is quite different from the condition of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair, the trajectories of them are lines, and the distance between them decreases until they dissipate quietly.

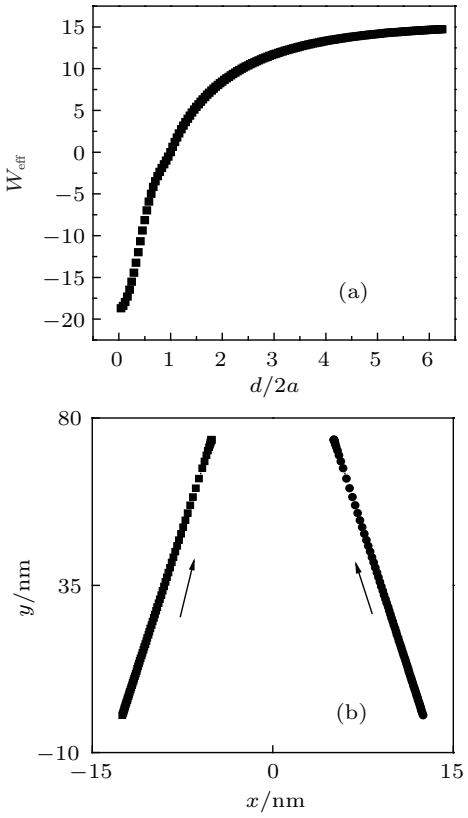


Fig. 3. Energy of the $V^{\downarrow}-\bar{V}^{\downarrow}$ pair as a function of the $d/2a$, (b) the trajectories of the $V^{\downarrow}$ and $\bar{V}^{\downarrow}$ of the $V^{\downarrow}-\bar{V}^{\downarrow}$ pair with $d=25$ nm, where the square dots and the circle dots denote the trajectories of the $V^{\downarrow}$ and $\bar{V}^{\downarrow}$, respectively, and the arrows show the moving direction.

The presence of a third vortex influences the dynamic behavior of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair. Figure 4 shows the motion trajectories of the $V^{\uparrow}$, $\bar{V}^{\downarrow}$, and $V^{\downarrow}$ of $V^{\uparrow}-\bar{V}^{\downarrow}-V^{\downarrow}$ state with the initial coordinate $\boldsymbol{X}_{V^{\downarrow}}=-25$ nm, $\boldsymbol{X}_{\bar{V}^{\downarrow}}=0$ nm, and $\boldsymbol{X}_{V^{\uparrow}}=-25$ nm. By the interaction between the three vortex, the $V^{\uparrow}$ and $\bar{V}^{\downarrow}$ rotate around and approach to each other gradually, but the $V^{\downarrow}$ moves along a curve and more slowly than $V^{\uparrow}$ and $\bar{V}^{\downarrow}$.

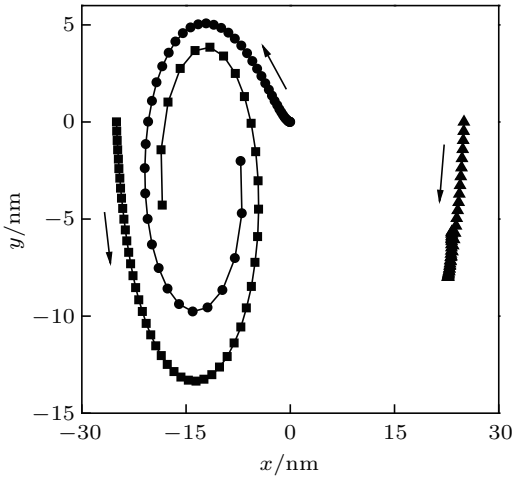


Fig. 4. Trajectories of $V^{\uparrow}$, $\bar{V}^{\downarrow}$, and $V^{\downarrow}$ in the interacting $V^{\uparrow}-\bar{V}^{\downarrow}-V^{\downarrow}$ state, where the square dots, circle dots, and the triangular dots denote the trajectories of the $V^{\uparrow}$, $\bar{V}^{\downarrow}$, and $V^{\downarrow}$ respectively, and the arrows show the moving direction.

The rotation frequency of the vortex–antivortex pair is dependent on the distance between the two cores. The variations of frequency of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair with the distance between them for the cases of Fig. 1(b) and Fig. 4 are shown in Fig. 5. It is found that the frequency decreases with increasing the distance between $V^{\uparrow}$ and $\bar{V}^{\downarrow}$. Compared with the frequency of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair, the frequency of the $V^{\uparrow}-\bar{V}^{\downarrow}$ in $V^{\uparrow}-\bar{V}^{\downarrow}-V^{\downarrow}$ state shows a lower value when the distance between $V^{\uparrow}$ and $\bar{V}^{\downarrow}$ is larger than 19 nm. But the frequencies shows no difference for the two cases as the distance between $V^{\uparrow}$ and $\bar{V}^{\downarrow}$ is much smaller than the distance between of the $\bar{V}^{\downarrow}$ and $V^{\downarrow}$.

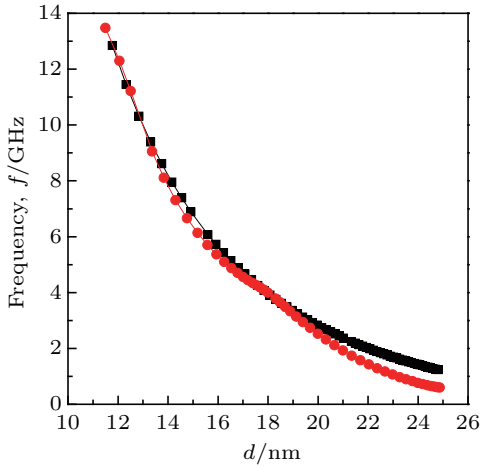


Fig. 5. (color online) Variations of rotation frequency of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair as a function of the distance d, where the square dots and the circle dots indicate the cases of the $V^{\uparrow}-\bar{V}^{\downarrow}$ pair and the $V^{\uparrow}-\bar{V}^{\downarrow}-V^{\downarrow}$ state, respectively.

It means that the presence of the third vortex will slow down the rotating behavior of $V^{\uparrow}-\bar{V}^{\downarrow}$ pair when the distance between them is comparable to that between $\bar{V}^{\downarrow}$ and $V^{\downarrow}$.

4. Conclusions

The exchange energy is used to explain the interaction between vortices. It is found that the exchange energy plays an important role in the dynamic motion of the pair when the vortex core and the antivortex core are not overlapped. The vortex–antivortex pair with opposite polarities rotates around each other. The rotation frequency increases with the decreasing of the distance between them. Under the influence of the third vortex, the frequency of the pair becomes lower. For the vortex and antivortex pair with identical polarity, the two vortex cores will move along straight lines. The vortex–antivortex pair can exist in thin film as a part of the cross-tie wall, and it is also important in the magnetization dynamics in nanodots and domain wall motions in nanostripes. Our results can be used to explain these dynamic behaviors.

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