Heralded linear optical quantum Fredkin gate based on one auxiliary qubit and one single photon detector*

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(Received 20 November 2013; revised manuscript received 15 January 2014; published online 20 June 2014)

Linear optical quantum Fredkin gate can be applied to quantum computing and quantum multi-user communication networks. In the existing linear optical scheme, two single photon detectors (SPDs) are used to herald the success of the quantum Fredkin gate while they have no photon count. But analysis results show that for non-perfect SPD, the lower the detector efficiency, the higher the heralded success rate by this scheme is. We propose an improved linear optical quantum Fredkin gate by designing a new heralding scheme with an auxiliary qubit and only one SPD, in which the higher the detection efficiency of the heralding detector, the higher the success rate of the gate is. The new heralding scheme can also work efficiently under a non-ideal single photon source. Based on this quantum Fredkin gate, large-scale quantum switching networks can be built. As an example, a quantum Beneš network is shown in which only one SPD is used.

Keywords: quantum Fredkin gate, linear optics, quantum switching network

PACS: 42.50.Ex, 03.67.Hk **DOI:** 10.1088/1674-1056/23/8/084207

1. Introduction

Quantum information science aims to use the principle of quantum mechanism and unique quantum mechanical effects to enhance the performance in the transmission, processing, and storage of information. A single photon is one of the carriers of quantum information due to its low decoherence rate and easier manipulation in the single photon level. The single photon has been widely used in quantum communication (e.g. quantum key distribution[1]), quantum computing (e.g. quantum logical gate[2,3]), quantum metrology,[4,5] lithography,[6] etc. In multi-user quantum communication networks and quantum computing, the quantum Fredkin gate plays an important role. By using a quantum Fredkin gate, multiple users can be interconnected to build a large-scale quantum communication network. With the development of integrated optics, large-scale optical quantum circuits or optical quantum computers, in which the quantum Fredkin gate is one of the important gates, can be constructed.[7,8]

The quantum Fredkin gate is also known as a controlled swap gate, that is, if the control bit is in state $|1\rangle$, then the two target qubits swap their states and otherwise, they remain in their initial states if the control bit is in state $|0\rangle$. A quantum Fredkin gate can be built by three CNOT gates,[9] or can be built directly.[10] There are two types of optical methods to implement quantum Fredkin gates. One is a linear optical method by which the Fredkin gate is created by linear optical components; the other is a nonlinear optical method by which the nonlinear effect of optical components is used.

The scheme and experiments with the linear optical method are developed quickly. In Ref. [11], Fiurášek proposed a quantum Fredkin gate by simulating the Kerr medium in Milburn's optical Fredkin gate[12] with a linear interferometer and controlled phase gates. It needs six ancillary photons with the success probability 4.1×10^{-3}. Gong *et al.* constructed a heralded Fredkin gate by using four CNOT gates and linear optical components,[10] and they also simplified the method into a post-selection one, which operates on the coincidence basis and required ancillary photons in a maximally entangled state. In this method, several interferometers needed to be stabilized simultaneously, and the success of the Fredkin gate is conditional on three-fold coincidence detection in the output modes. Fiurášek proposed a linear optical heralded quantum Fredkin gate based on a partial SWAP gate and controlled-z gates,[13] and the suggested setup requires a Mach–Zehnder interferometer and six auxiliary photons forming maximally entangled EPR pairs. Then they simplified the design and gave a scheme operating in the coincidence basis which required one entangled EPR pair and four partially polarizing beam splitters. In the post-selected scheme (operating in coincidence basis),[10,13] the photons at the outputs are consumed and are not available for further processing unless a quantum no-demolition detector is used.

The nonlinear optical quantum Fredkin gate is based on the nonlinear effect. Huang and Kumar proposed an interaction-free quantum optical Fredkin gate using a $\chi^{(2)}$ $LiNbO_3$ microdisk cavity based on the optically-induced quantum-zeno effect.[14] In this scheme, photon loss is minimized due to the elimination of the direct physical coupling between the signal and the pump waves. The quantum noise

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61372076 and 61301171), the 111 Project (Grant No. B08038), and the Fundamental Research Funds for the Central Universities, China (Grant Nos. K5051301059 and K5051201021).

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level is also lowered, owing to the weak background scattering in the $\chi^{(2)}$ crystalline cavity. Recently, Hu *et al.* proposed a method to overcome the quantum fluctuations of pump waves by exploiting stimulated Raman scattering in the implementation of the quantum optical Fredkin gate.[15] In the nonlinear scheme, a pump wave is required. Lin proposed a scheme, with the success rate 1/133, which is based on the cross-Kerr effect between the photons and the ancillary state.[16] Shao *et al.* proposed a scheme that is based on the dispersive atom–cavity interaction. By modulating the cavity frequency and the atomic transition frequency, an effective form of nonlinear interaction between photons in the three-mode cavity can be obtained.[17] In addition, Wang *et al.* also proposed a scheme to implement a fermionic quantum Fredkin gate for spin qubits with the aid of charge detection.[18]

In this paper, we focus on the linear optical quantum Fredkin gate. In the existing heralding scheme,[10,13] several flag detectors are used to indicate the success of the quantum Fredkin gate while they have no photon count. But our analysis results show that for the non-perfect single photon detector (SPD), the lower the detector efficiency, the higher the ratio of successful heralding is, which can be seen in Section 2. Although the result, no matter whether the quantum Fredkin gate is successful, can be determined by coincidence counts in the inputs and outputs, this heralding way may not be a good choice for on-line testing of each Fredkin gate of a large-scale quantum network. In this paper, we modify the heralding way in which the higher the detection efficiency of the heralding detector, the higher the success rate of the quantum Fredkin gate is. Compared with the heralded scheme proposed in Ref. [10], our scheme requires only one detector, of which the success rate is enhanced as the detector efficiency increases. Compared with the heralded scheme in Ref. [13], our scheme does not require any interferometer nor entangled auxiliary photons. We also show that the large quantum switching network can be built with this Fredkin gate by using only one SPD, e.g. the quantum Beneš network.

The rest of the paper is organized as follows. In Section 2, a heralded linear optical quantum Fredkin gate is presented and its success rate under a low-efficient detector is analyzed. The success rate for input pulses with a non-ideal single photon source (SPS) is also analyzed. In Section 3, a quantum Fredkin gate with a new heralding scheme is proposed and its success rate is discussed. Based on the quantum Fredkin gate, we build a quantum Beneš network by using only one SPD. In Section 4, we conclude the paper.

2. Heralded linear optical quantum Fredkin gate

Here a qubit is implemented by the photon with special polarization direction. The state $|0\rangle$ is prepared by the horizontal polarization of a photon, denoted by $|H\rangle$ and the state $|1\rangle$ is prepared by the vertical polarization direction of a photon, denoted by $|V\rangle$. So the operation for an optical quantum bit is converted into the operation of the polarization of the photon. The two input quantum bits of a Fredkin gate are $|\psi_{a_1}\rangle = \beta_{1h}|H\rangle_{a_1} + \beta_{1v}|V\rangle_{a_1}$ and $|\psi_{a_2}\rangle = \beta_{2h}|H\rangle_{a_2} + \beta_{2v}|V\rangle_{a_2}$, where β_{1h}, β_{1v}, β_{2h}, and β_{2v} are arbitrary complexes, and $|\beta_{1h}|^2 + |\beta_{1v}|^2 = 1$, $|\beta_{2h}|^2 + |\beta_{2v}|^2 = 1$, a_1 and a_2 denote different space modes. So the combined state of the two input photons is

$$\begin{aligned}|\psi_{\text{in}}\rangle &= |\psi_{a_1}\rangle|\psi_{a_2}\rangle \\ &= (\beta_{1h}|H\rangle_{a_1} + \beta_{1v}|V\rangle_{a_1})(\beta_{2h}|H\rangle_{a_2} + \beta_{2v}|V\rangle_{a_2}) \\ &= \beta_{1h}\beta_{2h}|H\rangle_{a_1}|H\rangle_{a_2} + \beta_{1h}\beta_{2v}|H\rangle_{a_1}|V\rangle_{a_2} \\ &\quad + \beta_{1v}\beta_{2h}|V\rangle_{a_1}|H\rangle_{a_2} + \beta_{1v}\beta_{2v}|V\rangle_{a_1}|V\rangle_{a_2}. \end{aligned} \quad (1)$$

2.1. Success rate of heralded linear optical quantum Fredkin gate under ideal SPS

A typical quantum Fredkin gate is shown in Fig. 1.[10] It can be in a "cross" state or "through" state controlled by four controlled-Not (CNOT) gates. The "cross" state means that the two input qubits are swapped, the qubit at $b_{1\text{out}}$ is in state $|\psi_{a_2}\rangle$ and the qubit at $b_{2\text{out}}$ is in state $|\psi_{a_1}\rangle$. The "through" state means that after the two input qubits pass through the Fredkin gate, the qubit at $b_{1\text{out}}$ is in state $|\psi_{a_1}\rangle$ and likewise, the qubit at $b_{2\text{out}}$ is in state $|\psi_{a_2}\rangle$.

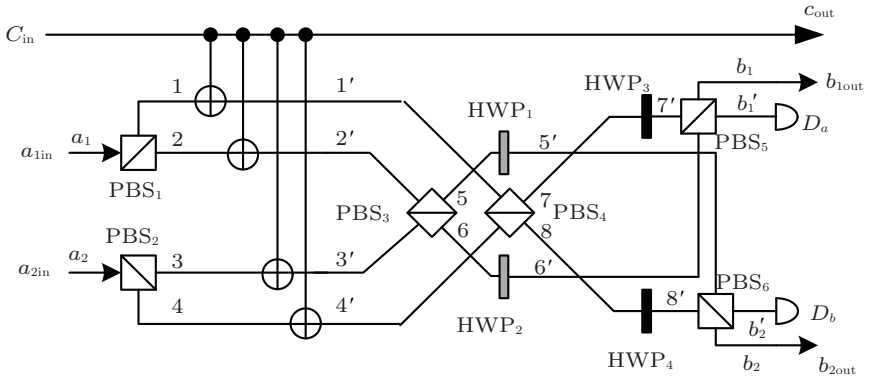


Fig. 1. A linear optical implementation of a quantum Fredkin gate.[10]

In Fig. 1, D_a and D_b are two single photon detectors. The CNOT gate can make the polarization of the input photon rotate 90°, that is, $|H\rangle \to |V\rangle$ and $|V\rangle \to |H\rangle$. HWP_1 and HWP_2 are 22.5°-tilted half-wave plates. HWP_3 and HWP_4 are 67.5°-tilted half-wave plates. The functions of HWP_1 and HWP_2 are shown as follows:

$$\mathrm{HWP}_1,\mathrm{HWP}_2 \to \begin{cases} |H\rangle \to \dfrac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \\ |V\rangle \to \dfrac{1}{\sqrt{2}}(|H\rangle - |V\rangle). \end{cases}$$

The functions of HWP_3 and HWP_4 are shown as follows:

$$\mathrm{HWP}_3,\mathrm{HWP}_4 \to \begin{cases} |H\rangle \to \dfrac{1}{\sqrt{2}}(|V\rangle - |H\rangle), \\ |V\rangle \to \dfrac{1}{\sqrt{2}}(|H\rangle + |V\rangle). \end{cases}$$

When the photons are transmitted through polarization beam splitters PBS_1 and PBS_2, the state is

$$\begin{aligned} |\psi_{1,2,3,4}\rangle \;=\; & \beta_{1h}\beta_{2h}|H\rangle_2|H\rangle_3 + \beta_{1h}\beta_{2v}|H\rangle_2|V\rangle_4 \\ & + \beta_{1v}\beta_{2h}|V\rangle_1|H\rangle_3 + \beta_{1v}\beta_{2v}|V\rangle_1|V\rangle_4. \end{aligned} \quad (2)$$

If $|c_{\mathrm{in}}\rangle = |1\rangle = |V\rangle$, the state of the photons transmitted through CNOT gates, PBS_3 and PBS_4 is

$$\begin{aligned} |\psi_{5,6,7,8}\rangle \;=\; & \beta_{1h}\beta_{2h}|V\rangle_5|V\rangle_6 + \beta_{1h}\beta_{2v}|V\rangle_5|H\rangle_7 \\ & + \beta_{1v}\beta_{2h}|H\rangle_8|V\rangle_6 + \beta_{1v}\beta_{2v}|H\rangle_8|H\rangle_7. \end{aligned} \quad (3)$$

The state of photons transmitted through HWP_1, HWP_2, HWP_3, and HWP_4 is

$$\begin{aligned} |\psi_{5',6',7',8'}\rangle \;=\; & \frac{1}{2}[\beta_{1h}\beta_{2h}(|H\rangle'_5 - |V\rangle'_5)(|H\rangle'_6 - |V\rangle'_6). \\ & + \beta_{1h}\beta_{2v}(|H\rangle'_5 - |V\rangle'_5)(|V\rangle'_7 - |H\rangle'_7) \\ & + \beta_{1v}\beta_{2h}(|V\rangle'_8 - |H\rangle'_8)(|H\rangle'_6 - |V\rangle'_6) \\ & + .\beta_{1v}\beta_{2v}(|V\rangle'_8 - |H\rangle'_8)(|V\rangle'_7 - |H\rangle'_7)]. \end{aligned} \quad (4)$$

The state of photons after PBS_5 and PBS_6 is

$$\begin{aligned} & |\psi_{b_1,b'_1,b_2,b'_2}\rangle \\ = \; & \frac{1}{2}[\beta_{1h}\beta_{2h}(|H\rangle_{b_2} - |V\rangle_{b'_2})(|H\rangle_{b_1} - |V\rangle_{b'_1}) \\ & + \beta_{1h}\beta_{2v}(|H\rangle_{b_2} - |V\rangle_{b'_2})(V\rangle_{b_1} - |H\rangle_{b'_1}) \\ & + \beta_{1v}\beta_{2h}(|V\rangle_{b_2} - |H\rangle_{b'_2})(|H\rangle_{b_1} - |V\rangle_{b'_1}) \\ & + \beta_{1v}\beta_{2v}(|V\rangle_{b_2} - |H\rangle_{b'_2})(|V\rangle_{b_1} - |H\rangle_{b'_1})]. \end{aligned} \quad (5)$$

If no photon arrives at either of the SPDs (D_a and D_b), the output state is

$$\begin{aligned} |\psi_{\mathrm{out}}\rangle \;=\; & \beta_{1h}\beta_{2h}|H\rangle_{b_2}|H\rangle_{b_1} + \beta_{1h}\beta_{2v}|H\rangle_{b_2}|V\rangle_{b_1} \\ & + \beta_{1v}\beta_{2h}|V\rangle_{b_2}|H\rangle_{b_1} + \beta_{1v}\beta_{2v}|V\rangle_{b_2}|V\rangle_{b_1}. \end{aligned} \quad (6)$$

In this case, the quantum Fredkin gate is in the "cross" state. Otherwise, if $|c_{\mathrm{in}}\rangle = |0\rangle = |H\rangle$ and no photon arrives at D_a or at D_b, the output state is

$$\begin{aligned} |\psi_{\mathrm{out}}\rangle \;=\; & \beta_{1h}\beta_{2h}|H\rangle_{b_1}|H\rangle_{b_2} + \beta_{1h}\beta_{2v}|H\rangle_{b_1}|V\rangle_{b_2} \\ & + \beta_{1v}\beta_{2h}|V\rangle_{b_1}|H\rangle_{b_2} + \beta_{1v}\beta_{2v}|V\rangle_{b_1}|V\rangle_{b_2}. \end{aligned} \quad (7)$$

In this case, the quantum Fredkin gate is in the "through" state. Let η and p_d denote the efficiency and the dark count of SPD, respectively. The probability p_{11} with which no photon is counted by D_a or by D_b is

$$\begin{aligned} p_{11} \;&=\; \frac{1}{4}[(1-\eta)^2 + 2(1-\eta)(1-p_d) + (1-p_d)^2] \\ &=\; \frac{1}{4}(2-\eta-p_d)^2. \end{aligned} \quad (8)$$

Since $p_d \ll \eta$, then

$$p_{11} \approx \frac{1}{4}(2-\eta)^2. \quad (9)$$

From Eq. (9) it follows that the probability p_{11} decreases quickly with the detector efficiency η increasing. That is, the lower the detector efficiency, the higher the value of p_{11} is. This is a strange result. Only when the quantum efficiency η of SPD is 100% and the dark count is not taken into account, will the success rate of the quantum Fredkin gate be $p_{11} = 1/4$. Note that the success rate here means the probability with which the Fredkin gate is heralded as performing a successful operation. Actually, the probability includes a false heralding probability because of the low-efficient SPD. So, the fidelity of the gate is smaller than 1.

2.2. Success rate of heralded linear optical quantum Fredkin gate under non-ideal SPS

Nowadays, there is no ideal single photon source that can be applied to practical quantum communication systems. An attenuated laser is often used to serve as a quasi-single photon source in which the photon number of a pulse is Poisson-distributed. By using this kind of source and the scheme shown in Fig. 1, the success rate could further worsen. We analyze this scenario briefly.

Firstly, we assume that there are two photons in each input pulse. The input states at $a_{1\mathrm{in}}$ and $a_{2\mathrm{in}}$ are $|\psi_{a_1}\rangle = (\beta_{1h}|H\rangle_{a_1} + \beta_{1v}|V\rangle_{a_1})(\beta_{1h}|H\rangle_{a_1} + \beta_{1v}|V\rangle_{a_1})$ and $|\psi_{a_2}\rangle = (\beta_{2h}|H\rangle_{a_2} + \beta_{2v}|V\rangle_{a_2})(\beta_{2h}|H\rangle_{a_2} + \beta_{2v}|V\rangle_{a_2})$. So, the joint input state is

$$\begin{aligned} & |\psi_{\mathrm{in}}\rangle \\ = \; & \beta_{1h}^2\beta_{2h}^2|HH\rangle_{a_1}|HH\rangle_{a_2} + \beta_{1h}^2\beta_{2h}\beta_{2v}|HH\rangle_{a_1}|HV\rangle_{a_2} \\ & + \beta_{1h}^2\beta_{2h}\beta_{2v}|HH\rangle_{a_1}|VH\rangle_{a_2} + \beta_{1h}^2\beta_{2v}^2|HH\rangle_{a_1}|VV\rangle_{a_2} \\ & + \beta_{1h}\beta_{1v}\beta_{2h}^2|HV\rangle_{a_1}|HH\rangle_{a_2} + \beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|HV\rangle_{a_1}|HV\rangle_{a_2} \\ & + \beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|HV\rangle_{a_1}|VH\rangle_{a_2} + \beta_{1h}\beta_{1v}\beta_{2v}^2|HV\rangle_{a_1}|VV\rangle_{a_2} \\ & + \beta_{1h}\beta_{1v}\beta_{2h}^2|VH\rangle_{a_1}|HH\rangle_{a_2} + \beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|VH\rangle_{a_1}|HV\rangle_{a_2} \\ & + \beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|VH\rangle_{a_1}|VH\rangle_{a_2} + \beta_{1h}\beta_{1v}\beta_{2v}^2|VH\rangle_{a_1}|VV\rangle_{a_2} \\ & + \beta_{1v}^2\beta_{2h}^2|VV\rangle_{a_1}|HH\rangle_{a_2} + \beta_{1v}^2\beta_{2h}\beta_{2v}|VV\rangle_{a_1}|HV\rangle_{a_2} \\ & + \beta_{1v}^2\beta_{2h}\beta_{2v}|VV\rangle_{a_1}|VH\rangle_{a_2} + \beta_{1v}^2\beta_{2v}^2|VV\rangle_{a_1}|VV\rangle_{a_2}. \end{aligned} \quad (10)$$

If $|c_{\mathrm{in}}\rangle = |1\rangle = |V\rangle$, then the output state is

$$
\begin{aligned}
|\psi_{b_1,b_1',b_2,b_2'}\rangle \;=\; & \frac{1}{4}[\beta_{1h}^2\beta_{2h}^2(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1h}^2\beta_{2h}\beta_{2v}(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1h}^2\beta_{2h}\beta_{2v}(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1h}^2\beta_{2v}^2(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2h}^2(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2v}^2(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2h}^2(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1h}\beta_{1v}\beta_{2v}^2(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_2}-|V\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1v}^2\beta_{2h}^2(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1v}^2\beta_{2h}\beta_{2v}(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|H\rangle_{b_1}-|V\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'}) \\
& +\beta_{1v}^2\beta_{2h}\beta_{2v}(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|H\rangle_{b_1}-|V\rangle_{b_1'}) \\
& +\beta_{1v}^2\beta_{2v}^2(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_2}-|H\rangle_{b_2'})(|V\rangle_{b_1}-|H\rangle_{b_1'})(|V\rangle_{b_1}-|H\rangle_{b_1'})]. \qquad (11)
\end{aligned}
$$

If no photon arrives at D_a or D_b, the output state can be given as

$$
\begin{aligned}
& |\psi_{\rm out}\rangle \\
= \; & \frac{1}{4}[\beta_{1h}^2\beta_{2h}^2|HH\rangle_{b_2}|HH\rangle_{b_1}+\beta_{1h}^2\beta_{2h}\beta_{2v}|HH\rangle_{b_2}|HV\rangle_{b_1} \\
& +\beta_{1h}^2\beta_{2h}\beta_{2v}|HH\rangle_{b_2}|VH\rangle_{b_1}+\beta_{1h}^2\beta_{2v}^2|HH\rangle_{b_2}|VV\rangle_{b_1} \\
& +\beta_{1h}\beta_{1v}\beta_{2h}^2|HV\rangle_{b_2}|HH\rangle_{b_1}+\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|HV\rangle_{b_2}|HV\rangle_{b_1} \\
& +\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|HV\rangle_{b_2}|VH\rangle_{b_1}+\beta_{1h}\beta_{1v}\beta_{2v}^2|HV\rangle_{b_2}|VV\rangle_{b_1} \\
& +\beta_{1h}\beta_{1v}\beta_{2h}^2|VH\rangle_{b_2}|HH\rangle_{b_1}+\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|VH\rangle_{b_2}|HV\rangle_{b_1} \\
& +\beta_{1h}\beta_{1v}\beta_{2h}\beta_{2v}|VH\rangle_{b_2}|VH\rangle_{b_1}+\beta_{1h}\beta_{1v}\beta_{2v}^2|VH\rangle_{b_2}|VV\rangle_{b_1} \\
& +\beta_{1v}^2\beta_{2h}^2|VV\rangle_{b_2}|HH\rangle_{b_1}+\beta_{1v}^2\beta_{2h}\beta_{2v}|VV\rangle_{b_2}|HV\rangle_{b_1} \\
& +\beta_{1v}^2\beta_{2h}\beta_{2v}|VV\rangle_{b_2}|VH\rangle_{b_1}+\beta_{1v}^2\beta_{2v}^2|VV\rangle_{b_2}|VV\rangle_{b_1}]. \qquad (12)
\end{aligned}
$$

In this case, the quantum Fedkin gate is in the "cross" state. If $|c_{\rm in}\rangle=|0\rangle=|H\rangle$ and no photon arrives at D_a or D_b, then the output state is the same as the input state shown in Eq. (10). In this case, the quantum Fredkin gate is in the "through" state.

We also obtain that the probability, p_{22}, with which no photon arrives at D_a or D_b (the quantum Fredkin gate is in the "cross" or "through" state), can be given as

$$
\begin{aligned}
p_{22} \;=\; & \frac{1}{16}[(1-p_d)^2+4(1-p_d)(1-\eta)+2(1-p_d)(1-\eta)^2 \\
& +4(1-\eta)^2+4(1-\eta)^3+(1-\eta)^4]. \qquad (13)
\end{aligned}
$$

Secondly, if the number of photons in a pulse at $a_{1\rm in}$ is m and the number of photons in a pulse at $a_{2\rm in}$ is n, then the probability, p_{mn}, with which no photon arrives at D_a or D_b, is

$$
\begin{aligned}
p_{mn} \;=\; & \frac{1}{2^{m+n}}\Bigg[(1-p_d)^2+(1-p_d)\sum_{m_1=1}^{m}C_m^{m_1}(1-\eta)^{m_1} \\
& +(1-p_d)\sum_{n_1=1}^{n}C_n^{n_1}(1-\eta)^{n_1} \\
& +\sum_{m_1=1}^{m}\sum_{n_1=1}^{n}C_m^{m_1}C_n^{n_1}(1-\eta)^{m_1}(1-\eta)^{n_1}\Bigg]. \qquad (14)
\end{aligned}
$$

We assume that the photon number in the input pulses at $a_{1\rm in}$ is Poisson-distributed with mean μ_1, the photon number in the input pulse at $a_{2\rm in}$ is Poisson-distributed with mean μ_2, and their photon numbers are independently distributed. So under non-ideal SPS and low efficient SPD the probability p, with which no photon arrives at D_a or D_b, is

$$
\begin{aligned}
p \;=\; & \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}p_{mn}\cdot\frac{\mu_1^m}{m!}\cdot\frac{\mu_2^n}{n!}\,{\rm e}^{-(\mu_1+\mu_2)}-(1-p_d)^2\,{\rm e}^{-(\mu_1+\mu_2)} \\
= \; & \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\frac{1}{2^{m+n}}\Bigg[(1-p_d)^2+(1-p_d)\sum_{m_1=1}^{m}C_m^{m_1}(1-\eta)^{m_1} \\
& +(1-p_d)\sum_{n_1=1}^{n}C_n^{n_1}(1-\eta)^{n_1} \\
& +\sum_{m_1=1}^{m}\sum_{n_1=1}^{n}C_m^{m_1}C_m^{m_1}(1-\eta)^{m_1}(1-\eta)^{m_1}\Bigg] \\
& \times\frac{\mu_1^m}{m!}\cdot\frac{\mu_2^n}{n!}\,{\rm e}^{-(\mu_1+\mu_2)}-(1-p_d)^2\,{\rm e}^{-(\mu_1+\mu_2)}. \qquad (15)
\end{aligned}
$$

In Eq. (15), we define $C_0^k=0, k>0$. From Eqs. (13)–(15), we can obtain that the lower the detector efficiency η, the higher the probability p is. This is also a strange result.

3. Modified heralded linear optical quantum Fredkin gate

3.1. Linear optical quantum Fredkin gate with modified heralding circuit

We modify the heralding circuit of the quantum Fredkin gate as shown in Fig. 2. In this heralding scheme, the photons at b_1' are transmitted through two paths: the photons with horizontal polarization pass through PBS_7 and PBS_9; the photons with vertical polarization pass through PBS_7, delay line DL_1 and PBS_9. The photons at b_2' are also transmitted through two paths: the photons with horizontal polarization pass through PBS_8 and PBS_{10}; the photons with vertical polarization pass through PBS_8, delay line DL_2 and PBS_{10}.

In Fig. 2, the time delay of DL_1 is equal to the one of DL_2 and longer than twice the time delay of DL_3. The half wave plates HWP_5, HWP_6, and HWP_7 are designed to rotate the polarization direction by 90° and they can work or not according to the control signal (this can be implemented by a dynamic polarization controller). The applications of time delay lines and HWPs can make the photons detected by only one SPD in a sequence. There are four scenarios when photons pass through PBS_{11}. (i) The horizontally polarized photons from b_1' pass through PBS_{11}, HWP_6 which does not work, PBS_{12} and HWP_7 which works by rotating the horizontal polarization to vertical polarization. (ii) The horizontally polarized photons from b_2' pass through PBS_{11}, HWP_5 which works by rotating the horizontal polarization to the vertical polarization, DL_3, PBS_{12}, and HWP_7 which does not work. (iii) The vertically polarized photons from b_2' pass through PBS_{11}, HWP_6 which works by rotating the vertical polarization to the horizontal polarization, PBS_{12} and HWP_7 which works by rotating the horizontal polarization to the vertical polarization. (iv) The vertically polarized photons from b_1' pass through PBS_{11}, HWP_5 which does not work, DL_3, PBS_{12}, and HWP_7 which does not work. So, all the photons which can arrive at the output port of HWP_7 must be vertically polarized. That is, they are qubits denoted by $|V\rangle$ or $|1\rangle$. Then, these photons are connected to the control port of a CNOT gate with $|1\rangle$ as the target qubit. So, if one or more photons arrive at b_1' or b_2', the detector D_c will be not ticked. Otherwise, if no photon arrives at b_1' or b_2', the detector D_c will be ticked. So the detector D_c can be used to indicate the success of the quantum Fredkin gate. The Fredkin gate succeeds if D_c is ticked. The higher the efficiency of D_c, the higher the efficiency of heralding is.

If the photon number in a pulse at $a_{1\text{in}}$ is m and the photon number in a pulse at $a_{2\text{in}}$ is n, then the probability p_{mn}, with which the detector D_c is ticked, can be given as

$$p_{mn} = \frac{1}{2^{m+n}}\eta + \left(1 - \frac{1}{2^{m+n}}\right) p_d,$$
$$m, n = 0, 1, 2, \ldots, \quad \text{but} \quad m = n \neq 0. \tag{16}$$

So the mean count rate p_{new} can be given as

$$\begin{aligned} p_{\text{new}} &= \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} p_{mn} \cdot \frac{\mu_1^m}{m!} \cdot \frac{\mu_2^n}{n!} \mathrm{e}^{-(\mu_1+\mu_2)} - \eta\, \mathrm{e}^{-(\mu_1+\mu_2)} \\ &= \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} \left[\frac{1}{2^{m+n}}\eta + \left(1 - \frac{1}{2^{m+n}}\right) p_d\right] \\ &\quad \times \frac{\mu_1^m}{m!} \cdot \frac{\mu_2^n}{n!} \mathrm{e}^{-(\mu_1+\mu_2)} - \eta\, \mathrm{e}^{-(\mu_1+\mu_2)} \\ &= \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} \left[(\eta - p_d) \cdot \frac{(\mu_1/2)^m}{m!} \cdot \mathrm{e}^{-\mu_1} \cdot \frac{(\mu_2/2)^n}{n!} \cdot \mathrm{e}^{-\mu_2} \right. \\ &\quad \left. + p_d \cdot \frac{\mu_1^m}{m!} \cdot \frac{\mu_2^n}{n!} \mathrm{e}^{-(\mu_1+\mu_2)}\right] - \eta\, \mathrm{e}^{-(\mu_1+\mu_2)} \\ &= (\eta - p_d)\, \mathrm{e}^{-(\mu_1+\mu_2)/2} + p_d - \eta\, \mathrm{e}^{-(\mu_1+\mu_2)}. \end{aligned} \tag{17}$$

Let $\mu_1 = \mu_2 = \mu$. Since $p_d \ll \eta$, the mean count rate p_{new} can be approximately given as

$$p_{\text{new}} \approx \eta \left(\mathrm{e}^{-\mu} - \mathrm{e}^{-2\mu}\right). \tag{18}$$

In this case, p_{new} is just as the success rate of the quantum Fredkin gate. The probability p_{new} increases linearly with the increase of detector efficiency η, as shown in Fig. 3. The

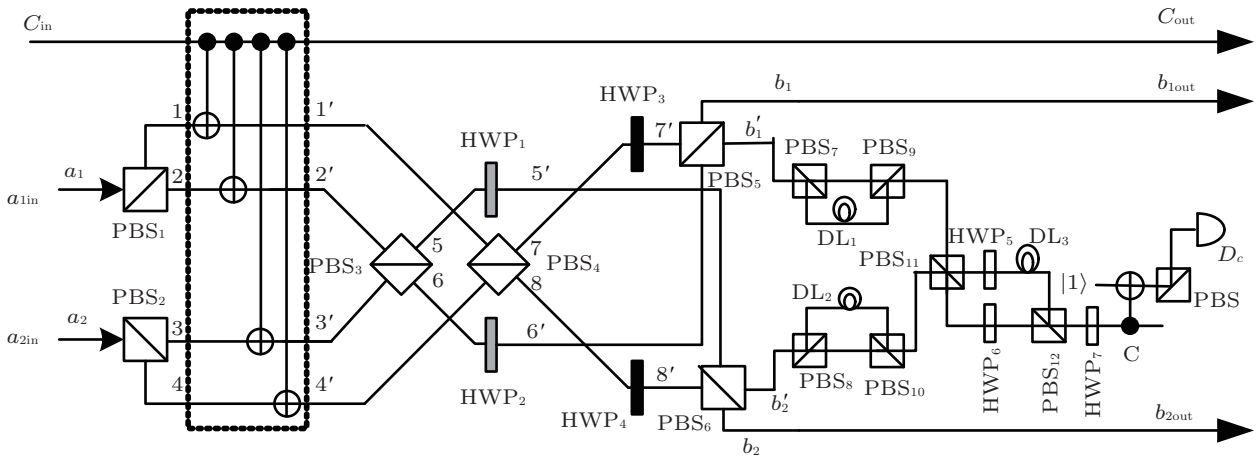


Fig. 2. Modified heralded quantum Fredkin gate.

higher the average photon number, the higher the success rate is. When $\mu = 0.1$ and the detector is ideal (detection efficiency is 100%), the success rate of the Fredkin gate is about 0.085.

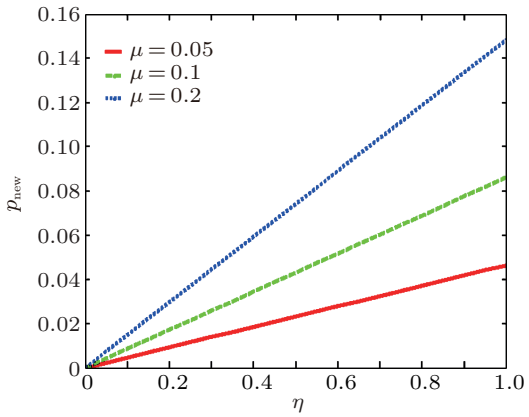


Fig. 3. (color online) Variations of success rate $p_{\rm new}$ with detector efficiency η of modified quantum Fredkin gate at three different values of mean μ.

In Fig. 2, the CNOT gate can be implemented without entangled ancilla,[19] or can be implemented by integrated photonics.[20]

3.2. Quantum Beneš network built with quantum Fredkin gate

Based on the modified heralded quantum Fredkin gate shown in Fig. 2, a quantum switch network can be built. The Fredkin gate is the basic switch unit with three inputs, three outputs, and one flag, as shown in Fig. 4(a). A quantum Beneš network is shown in Fig. 4(b). The symbol "&" denotes a six-qubit quantum "AND" gate, which can be created using conditional NOT with six control qubits a combination of several CNOT gates.

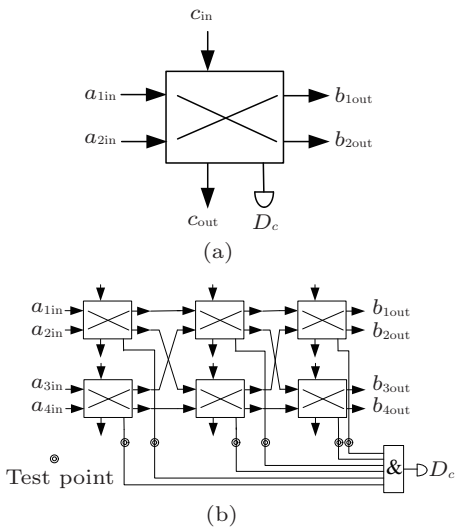


Fig. 4. Quantum Beneš network built with a quantum Fredkin gate, showing (a) quantum Fredkin gate and (b) quantum Beneš network.

In Fig. 4, the quantum Beneš network needs only one SPD. We also present the test points. So whether each Fredkin gate works well can be checked easily.

4. Conclusions

In this paper, we proposed a linear optical quantum Fredkin gate in which a new heralding quantum circuit with one SPD is used and an auxiliary qubit $|1\rangle$ is added. In this scheme, the success rate of the quantum Fredkin gate is proportional to the efficiency of the heralding detector. With this improved heralding scheme, the gate can be easily used to build a large-scale quantum switching network. It does not require an interferometer nor entangled auxiliary photons. It should be mentioned that according to the gate fidelity defined as in Ref. [21], the fidelity of the gate shown in Fig. 1 is $1/(2-\eta)^2$ for $p_d = 0$ and the ideal single photon source, while the fidelity of the gate shown in Fig. 2 is nearly 1. Our further studies are to make this experiment and to investigate how to implement it by integrated optics technology.

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