Large phase shift of spatial soliton in lead glass by cross-phase modulation in pump-signal geometry*

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We investigate the co-propagation of a strong pump beam and a weak signal beam in lead glass, and find that the large phase shift of the strongly nonlocal spatial optical soliton (SNSOS) can be realized via cross-phase modulation. The theoretical study suggests a synchronous propagation of the pump SNSOS and the signal SNSOS under the required initial condition. A π-phase shift of the signal SNSOS is experimentally obtained by changing the power of the pump SNSOS by about 13 mW around the soliton critical power, which agrees qualitatively with our theoretical prediction. The ratio of the phase shift rate of the signal SNSOS to that of the pump SNSOS shows a close match to the reciprocal of the ratio between their wavelengths.

Keywords: strongly nonlocal spatial solitons, large phase shift, lead glass

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1. Introduction

Nonlocality plays an important role in many areas of nonlinear optics.[1–5] With intrinsically infinite nonlocality, lead glass is one of the most promising candidates to serve as the propagation media for the strongly nonlocal spatial optical soliton (SNSOS) predicted by Snyder and Mitchell.[6] High-order solitons,[7] vortex solitons,[4] and surface solitons[8] have been realized or predicted in $(1+2)$-dimensional lead glass since the nonlocal nonlinear response can suppress soliton transverse instabilities; long-range interactions between solitons[9] as well as soliton and boundaries[10–12] have also been carried out in lead glass due to the fact that the interactions are mediated by the light-induced refractive index, which is enlarged by the nonlocal response. Recently, Shou *et al*. investigated the large phase shift of SNSOS in lead glass,[13] which is also one of the most important characteristics resulting from the nonlocality. This is the first theoretical and experimental study on the SNSOS phase difference, which is often regarded as just a parameter[9,14–17] in the study of the soliton interaction. Controlled variation of soliton phases is principally important, because it makes it possible to control the optical signal based on the interference principle, and to encode, transmit, and store information using soliton dynamical lattices.[18]

The theoretical prototype of Shou's work was addressed by Guo *et al*.[19,20] Based on the phenomenological Gaussian response function, Guo predicted a large phase shift rate of SNSOS per unit propagation distance, which is $(w_m/w_0)^2$ times larger than the phase shift of the local counterpart, where w_m and w_0 are the characteristic lengths of the response function and the beam width, respectively. In actual strongly nonlocal media, lead glass, Shou estimated that the result is much smaller, but the phase shift rate of the soliton is still more than one order larger than that in the local media. They conducted the experiment in lead glass to obtain a π-phase shift by changing the soliton power around its critical power.[13]

However, in principle, Shou's "one-soliton scheme" cannot be used in an all-optical control system in which the phase or intensity of the signal beam is adjusted by another control (pump) beam via, for example, cross-phase modulation. In this paper, a π-phase shift of the signal SNSOS is obtained by adjusting the pump SNSOS power with the aid of the cross-phase modulation between the two SNSOSs. Although the power of the signal SNSOS is much smaller than its critical power, trapping in the pump-soliton-induced waveguide, its phase shift can be sensitively and linearly modulated by the power of the pump SNSOS. The ratio of the phase shift rate of the signal SNSOS to that of the pump SNSOS is approximately determined by the ratio between their wavelengths.

2. The solution of the coupled equations

The system we study is described by the nonlocal nonlinear Schrödinger equations for the two slowly varying light field amplitudes A_p and A_s coupled to the steady-state heat transfer equation. In cylindrical coordinates for z-axis symmetry, the three coupled equations can be expressed:[21]

$$2ik_\mathrm{p}\frac{\partial A_\mathrm{p}}{\partial Z}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial A_\mathrm{p}}{\partial R}\right)+2k_\mathrm{p}^2\frac{\Delta N_\mathrm{p}}{n_{0\mathrm{p}}}A_\mathrm{p}=0, \quad (1a)$$

$$2ik_\mathrm{s}\frac{\partial A_\mathrm{s}}{\partial Z}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial A_\mathrm{s}}{\partial R}\right)+2k_\mathrm{s}^2\frac{\Delta N_\mathrm{s}}{n_{0\mathrm{s}}}A_\mathrm{s}=0, \quad (1b)$$

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$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) = -\frac{1}{\kappa}(\alpha_{\rm p}I_{\rm p} + \alpha_{\rm s}I_{\rm s}), \tag{1c}$$

where $I_{\rm p,s} = \mid A_{\rm p,s} \mid^2$ is the light intensity for the pump and signal beams respectively. $\alpha_{\rm p,s}$ and κ are, respectively, the absorption coefficient and the thermal conductivity. $\Delta N_{\rm p,s} = \beta_{\rm p,s}(T - T_0)$, where T_0 is the fixed temperature at the boundary and $\beta_{\rm p,s}$ is the thermal–optical coefficient.

In the case of $I_{\rm s} \ll I_{\rm p}$ we only consider the light energy contributed by the pump beam. Simplifying Eq. (1) in a dimensionless form, we can obtain,

$$\mathrm{i}\frac{\partial a_{\rm p}}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial a_{\rm p}}{\partial r}\right) + 4\Delta n a_{\rm p} = 0, \tag{2a}$$

$$\mathrm{i}\frac{\partial a_{\rm s}}{\partial z} + \frac{1}{\mu r}\frac{\partial}{\partial r}\left(r\frac{\partial a_{\rm s}}{\partial r}\right) + 4\mu\rho\Delta n a_{\rm s} = 0, \tag{2b}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Delta n}{\partial r}\right) = -\mid a_{\rm p} \mid^2, \tag{2c}$$

where $r = R/w_{0\rm p}$, $z = Z/(2k_{\rm p}w_{0\rm p}^2)$, $a_{\rm p,s} = A_{\rm p,s}/A_0$, $A_0^2 = 2n_{0\rm p}\kappa/(\alpha_{\rm p}\beta_{\rm p}k_{\rm p}^2w_{0\rm p}^4)$, $\Delta n = k_{\rm p}^2w_{0\rm p}^2\Delta N_{\rm p}/(2n_{0\rm p})$, $\mu = k_{\rm s}/k_{\rm p} = \lambda_{\rm p}/\lambda_{\rm s}$, and $\rho = n_{0\rm p}\beta_{\rm s}/(n_{0\rm s}\beta_{\rm p})$. It is indicated by Eqs. (2b) and (2c) that the signal beam propagates in a pump-beam-induced refractive index distribution, say a waveguide. Therefore, the propagation behavior of the signal beam is mainly determined by that of the pump beam.

According to Snyder's method,[6] the nonlinear refractive index can be expanded in a Taylor series of which only the first two terms are kept. This indicates a light-induced refractive index acting as a parabolic waveguide in the vicinity of the pump beam center

$$\Delta n = \Delta n^{(0)} - r^2\Delta n^{(2)}. \tag{3}$$

Assuming that the solutions of Eq. (2) are in the forms of Gaussian functions

$$a_{\rm p,s} = \frac{\sqrt{p_{0\rm p,0s}}\exp[\mathrm{i}\,\theta_{\rm p,s}(z)]}{\sqrt{\pi}w_{\rm p,s}(z)}\exp\left[-\frac{r^2}{2w_{\rm p,s}^2(z)} + \mathrm{i}c_{\rm p,s}(z)r^2\right], \tag{4}$$

where $p_{0\rm p,0s} = \int \mid a_{\rm p,s}(x',y') \mid^2 \mathrm{d}x'\mathrm{d}y'$ is the normalized light power. By integrating Eq. (2c) twice, we obtain $\Delta n^{(0)} = p_{0\rm p}/(4\pi)\{\Gamma[0, r_0^2/w_{\rm p}^2(z)] + \ln[r_0^2/w_{\rm p}^2(z)] + \gamma\}$,[13] where Γ is the Gamma function, $\gamma = 0.58$ is Euler's constant, and r_0 is the normalized radius of the cross section of the glass rod.

$\theta_{\rm p,s}(z)$ in Eq. (4) is the phase shift of the pump beam and the signal beam, respectively. We can directly obtain the phase shift associated with $\Delta n^{(0)}$, called the zero-order phase shift to be

$$\theta_{\rm p}^{(0)}(z) = \Delta n^{(0)}z, \quad \theta_{\rm s}^{(0)} = \mu\rho\,\theta_{\rm p}^{(0)}. \tag{5}$$

It is obvious that the zero-order phase shifts of the pump and signal beams are proportional to the pump power and the propagation distance. The total phase shift can be rewritten as, according to the two terms of the nonlinear index designated in Eq. (3),

$$\theta_{\rm p,s}(z) = \theta_{\rm p,s}^{(0)}(z) + \theta_{\rm p,s}^{(2)}(z). \tag{6}$$

Following the method in Ref. [19], by substituting Eq. (3)–(6) into Eq. (2), we can obtain the evolution equations of the beam widths and the second-order phase shifts of the pump and signal beams,

$$\frac{\mathrm{d}^2w_{\rm p}}{\mathrm{d}z^2} = \frac{4}{w_{\rm p}^3} - \frac{4p_{0\rm p}}{\pi w_{\rm p}}, \tag{7a}$$

$$\frac{\mathrm{d}^2w_{\rm s}}{\mathrm{d}z^2} = \frac{4}{\mu^2w_{\rm s}^3} - \frac{4\rho\,p_{0\rm p}w_{\rm s}}{\pi w_{\rm p}^2}, \tag{7b}$$

$$\frac{\mathrm{d}\theta_{\rm p}^{(2)}}{\mathrm{d}z} + \frac{2}{w_{\rm p}^2} = 0, \tag{7c}$$

$$\frac{\mathrm{d}\theta_{\rm s}^{(2)}}{\mathrm{d}z} + \frac{2}{\mu w_{\rm s}^2} = 0. \tag{7d}$$

When there is not much difference between the wavelengths of the two beams, $\rho \approx 1$. Thereby equation (7b) becomes

$$\frac{\mathrm{d}^2w_{\rm s}}{\mathrm{d}z^2} = \frac{4}{\mu^2w_{\rm s}^3} - \frac{4p_{0\rm p}w_{\rm s}}{\pi w_{\rm p}^2}. \tag{8}$$

Applying the replacement of $w_{\rm s}$ with $(1/\sqrt{\mu})w_{\rm p}$, equation (8) degenerates into Eq. (7a) and $\theta_{\rm s}^{(2)}(z)$ has the same evolution equation with $\theta_{\rm p}^{(2)}(z)$ revealed in Eqs. (7c) and (7d). The analytical solutions can be deduced only under the initial condition $w_{0\rm s} = (1/\sqrt{\mu})w_{0\rm p}$, when the pump and signal beams propagate synchronously,[21]

$$w_{\rm s}(z) = \frac{1}{\sqrt{\mu}}w_{\rm p}(z) = \frac{1}{\sqrt{\mu}}[\sigma + (1-\sigma)\cos(bz)], \tag{9}$$

$$\begin{aligned}\theta_{\rm s}^{(2)}(z) &= \theta_{\rm p}^{(2)}(z)\\ &= -\frac{2}{2\sigma-1}\left\{\frac{(1-\sigma)\sin(bz)}{\sigma+(1-\sigma)\cos(bz)}\right.\\ &\quad\left. - \frac{2\sigma}{\sqrt{2\sigma-1}}\left[\arctan\left(\sqrt{(2\sigma-1)}\tan\frac{bz}{2}\right) + \pi\right]\right\},\end{aligned} \tag{10}$$

where $\sigma = \sqrt{p_{\rm c}/p_{0\rm p}}$, $b = 2\sqrt{2}/\sigma^2$. The critical power $p_{\rm c} = \pi$ for the pump beam to propagate in the form of a soliton can be easily obtained based on Eq. (7a) assuming $w_{\rm p}(z) = 1$ and the right-hand side equals zero. The signal beam also forms a soliton in the "soliton-waveguide", although its power is far less than its own critical power.

Figure 1 exhibits the synchronous propagation of the pump and signal beams pumped by different input powers. Under the required initial condition, the pump and signal beams propagate as breathers with the same oscillation period when the pump power is less than the critical power and as solitons when the pump power is equal to the critical power. Nevertheless, we broadly call them SNSOSs when the pump

power is not far from the soliton critical power. Figure 2 shows the comparison between the analytical results and the numerical results of the phase shifts of the pump and signal SNSOSs.

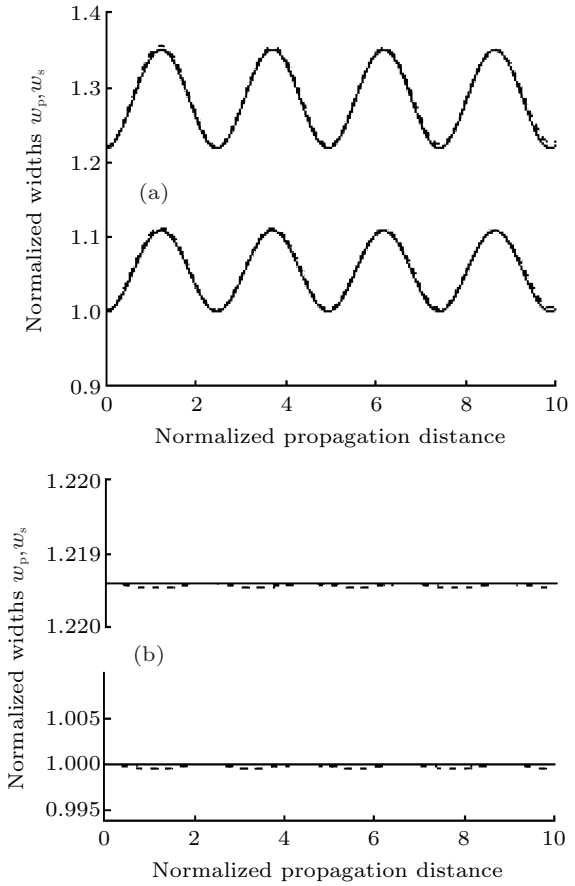


Fig. 1. The beam width changes of the signal (upper) and pump (lower) SNSOSs along with the normalized propagation distance in the case of $w_{0s} = (1/\sqrt{\mu})w_{0p}$ and different pump powers of $0.9p_c$ (left) and p_c (right). Solid and dashed lines are, respectively, the analytical and numerical results.

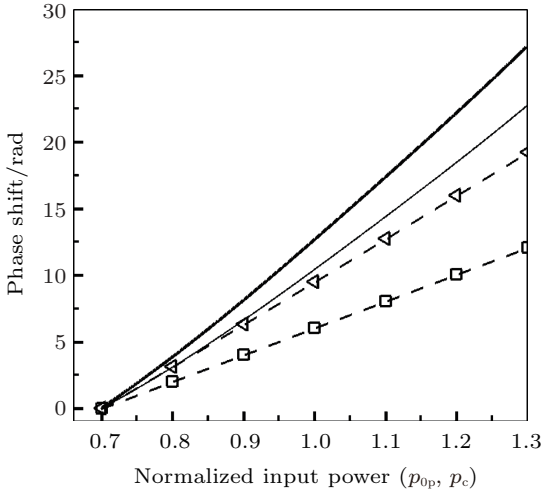


Fig. 2. Phase shifts of pump and signal SNSOSs versus p_{0p}/p_c. Solid thick and thin lines are, respectively, the analytical results for the pump and signal SNSOSs in the case of $r_0 = 200$ and $z = 2$ (also the case in the following experiment). Triangles and squares are, respectively, the numerical results for the pump and signal SNSOSs in the same case.

The disagreements between the analytical results and the numerical results are mainly attributed to the limited expanding of the nonlinear index. Ma *et al.* also numerically obtained a bigger change rate of the propagating constant with power than the analytical result when they investigated the surface soliton in lead glass.[22] The ratio of the phase shift rate (i.e. the slope of the corresponding curve) of the pump SNSOS to that of the signal SNSOS is 1.56 indicated by the numerical results and 1.21 by the analytical results, respectively.

3. Experiment in lead glass

We carry out the experiment in a lead glass rod, relying on the principle of the Mach–Zehnder interferometer to measure the phase shift of the signal SNSOS modulated by the power of the pump SNSOS. The glass rod has a radius of 7.5 mm and a length of 60 mm. Its absorption coefficient changes strongly with the wavelength, which is of the value of 0.07 cm^{-1} at the pump wavelength 532 nm and 0.03 cm^{-1} at the signal wavelength 790 nm. The sketch map of the experimental setup is detailed in Fig. 3. The pump beam of vertical polarization is launched from a coherent Verdi V12 laser and the signal beam of horizontal polarization is produced by a tunable, single-frequency ring Ti:sapphire laser MBR 110. The pump and signal beams are focused onto the center of the head face of the glass rod by lenses L_1 and L_2 with different beam widths of 75 μm and 90 μm, respectively. B_1 is a polarizing cube beamsplitter, which allows the signal beam to transmit and the pump beam to reflect into the Mach–Zehnder interferometer, respectively. The pump beam is absorbed totally by the filter F_1 on one arm after coaxially propagating through the lead glass with the signal beam. On the other arm, the pump beam is absorbed by another filter F_2 just after being reflected by the non-polarizing cube beamsplitter B_2. The beam spot on the end face of the glass rod is imaged by lens L_3 onto the CCD where it interferences with a large-diameter beam spot acting as a phase reference. By tuning the attenuator TA, the interference fringes move along with the change of the pump power. This indicates the modulation of the phase shift of the signal SNSOS by the power of the pump SNSOS. The phase shift of the pump SNSOS modulated by the power of its own can be obtained in the same way but after removing F_1 and F_2 and blocking the signal beam. The rest of the details of the experimental setup can be found in Ref. [13].

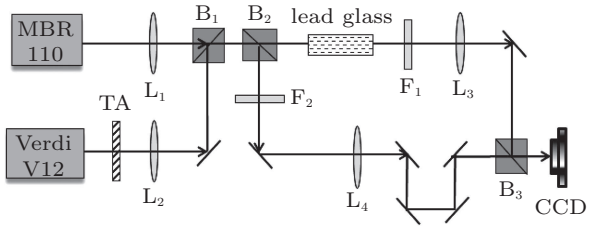


Fig. 3. Experimental setup. TA: tunable attenuator; B_1: polarizing cube beamsplitter; B_2, B_3: non-polarizing cube beamsplitters; F_1, F_2: filters; L_1, L_2, L_3, L_4: lenses.

By tuning the input pump power from 190 mW to 340 mW around its critical power 260 mW,[13] which is determined by measuring the power–beam–width curve,[23] we obtain the phase shifts of the pump and the signal SNSOSs as functions of the input power normalized by the critical power in lead glass. Reference [13] gives a detailed demonstration of how to obtain the phase shift according to the movement of the interference fringes captured by CCD. As shown in Fig. 4, linear fits to the experimental data are good, whose slopes are, respectively, 90.7 rad/p_c and 61.5 rad/p_c. This means the π-phase shift of the signal SNSOS can be realized by changing the pump power about 13 mW around the critical soliton power. The "half-wave" change of the pump power is only 5% of the critical power, therefore the signal beam almost maintains the soliton form during the change. It is reasonable that the phase shift rate of the pump SNSOS in the present "two-soliton scheme" shows a close match to the phase shift rate of the SNSOS in the "one-soliton scheme",[13] since the power of the signal SNSOS does not have an effect on the phase shift of the pump SNSOS. Obviously, the phase shift rate obtained in our experiment are much bigger than the theoretical prediction. This large disagreement between the experiment result and the theoretical prediction, which is based on the thermal nonlinearity mode described by the Poisson equation Eq. (1c), have been observed in almost all the experimental studies on the SNSOS carried out in lead glass. Reference [13] presents a detailed explanation.

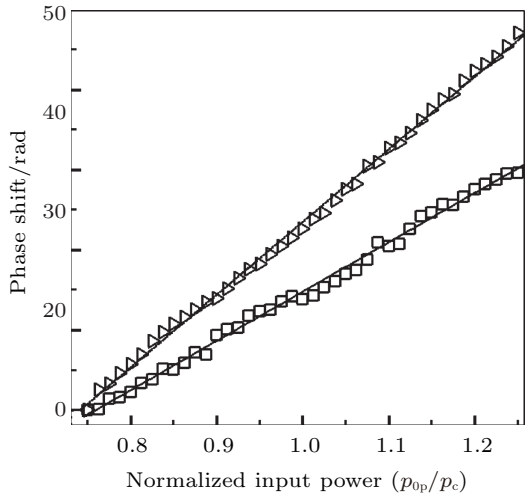


Fig. 4. Experimental phase shifts of pump (triangles) and signal (squares) SNSOSs versus p_{0p}/p_c. The wavelengths of the pump and signal SNSOSs are, respectively, 532 nm and 790 nm. The solid curves are the linear fits.

The ratio between the shift rates of the two SNSOSs with different wavelengths in Fig. 4 is 1.47, which is a close match to the reciprocal of the ratio between the two wavelengths 1.48. Figure 5 shows the phase shifts of the soliton pair under the condition that the beam with a wavelength of 790 nm serves as the pump, while the beam with a wavelength of 532 nm acts as the signal. The absorption coefficient α at 790 nm is so small that the critical soliton power is too high to be reached. Therefore, we cannot conduct the power change around the critical power and the input pump power in Fig. 5 is not normalized. The experimental data in Fig. 5 suggest, nevertheless, the phase modulation sensitivities in the present case since the modulations are linear. The ratio between the two slopes of the fitting lines is 1.45, which is also in close proximity to the reciprocal of the wavelength ratio. These two experimental results on the dependence of the phase shift rate on the wavelength do not quite coincide with the theoretical estimation visualized in Fig. 2. We can, notwithstanding, come to a conclusion based on the experimental result that, for the SNSOS in lead glass, the ratio of the self-phase modulation sensitivity to the cross-phase modulation sensitivity is a fixed value, say the reciprocal of the ratio between the corresponding wavelengths. The dependence of the phase modulation sensitivity on the degree of the nonlocality is discussed in Ref. [13].

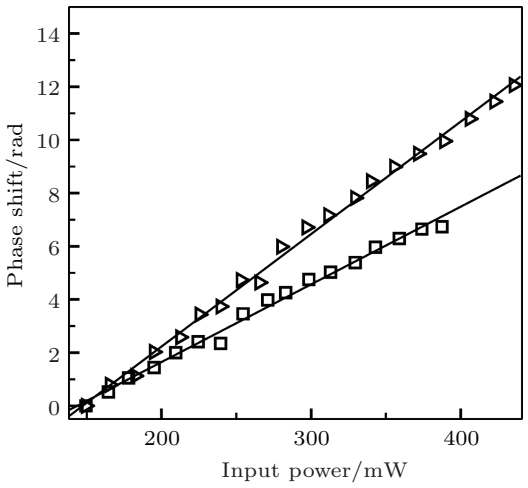


Fig. 5. Experimental phase shifts of pump (triangles) and signal (squares) SNSOSs versus the pump power. The wavelengths of the pump and signal SNSOSs are, respectively, 790 nm and 532 nm. The solid curves are the linear fits.

4. Conclusion

In conclusion, we investigated the large phase shift of the SNSOS in lead glass by cross-phase modulation in pump-signal geometry. A π-phase shift of the signal soliton is obtained by tuning the power of the pump beam around its critical power by about 13 mW. This linear modulation sensitivity is generally high and shows a dependence on the wavelength, i.e., the ratio of the phase shift rate of the signal SNSOS to that of the pump SNSOS is equal to the reciprocal of the ratio between the corresponding wavelengths. This is an indication that higher modulation sensitivity can be realized by means of choosing the higher-wavelength beam as the pump beam. The present experimental scheme provides a possible way of realizing integrated all-optical switching.

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