Relation between Airy beams and triple-cusp beams*

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Airy beams and triple-cusp beams are two kinds of accelerating beams. The propagation characteristics and internal topological structures of accelerating Airy beams are well understood because of the developed mathematical theory about Airy function. However, limited information is available about the optical characteristics of accelerating triple-cusp beams. In this work, the relationship between Airy beams and triple-cusp beams is examined theoretically and experimentally. Results reveal some important optical characteristics of triple-cusp beams based on the optical characteristics of Airy beams. These findings are expected to provide a foundation for future applications of triple-cusp beams.

Keywords: Airy beams, triple-cusp beams, beam characteristics

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1. Introduction

Since Airy beams were first experimentally generated,[1] the so-called accelerating beams have attracted considerable attention. Transversally accelerating Airy beams each exhibit a curved parabolic ballistic trajectory in their points of maximum intensity during the propagation in free space.[2] Hence, accelerating beams can be regarded as a new unique light source of energy and probe that can be widely used in many scientific applications. For example, accelerating Airy beams can be regarded as new sources of energy in light–matter interactions such as vacuum-curved electron acceleration,[3] curved filament,[4,5] curved plasma channel generation,[6] and supercontinuum generation.[7] In addition, accelerating Airy beams can also be used in the field of optical interconnect and optical manipulation. For example, these beams were exploited to trap, sort, mix, measure, manipulate, and transport cells and microparticles precisely by following parabolic trajectories in areas where an ordinary point-blank beam cannot arrive.[2,8]

For generating Airy beams, some methods are given by different research teams, although the nature of generating Airy beams is similar. The common methods include phase modulation,[1,9,10] direct-space phase-only modulation,[11] and nonlinear modulation.[12] The phase modulation method is relatively mature since accelerating Airy beams are first generated using such a method.

In 2010, based on catastrophe theory, another family of finite-energy accelerating beams was theorematically introduced.[13] Such beams possess an odd number of high-intensity accelerating peaks, hence they were named "accelerating regular polygon beams". In 2012, an optical triple-cusp beam, which is one of the accelerating regular polygon beams, was experimentally generated for the first time by our research teams.[14] Although both Airy beams and triple-cusp beams are accelerating beams, they do not belong to the same family. For example, their most optical characteristics, such as beam topology structure and the number of main lobes, are not similar. In the past, researchers have very well understood the propagation characteristics and internal topological structures of Airy beams.[1–12] One of the main reasons behind really understanding Airy beams is the availability of mature mathematical theory regarding Airy function. In fact, in early 1838, Airy function was introduced when George Biddel Airy started calculating light intensity in the neighbourhood of a caustic. Therefore, definitions, properties, and applications of the Airy function have been widely and in depth investigated in some fields of natural science, including classical physics, quantum physics, and optics.[1,15,16] Compared with the knowledge of accelerating Airy beams, we know little about the optical characteristics of accelerating regular polygon beams, since these beams were introduced in 2010. Although the accelerating regular polygon beams are derived from the catastrophe theory,[17] a quantitative demonstration of optical similarity to Airy beams is still difficult because of the mathematical difficulties associated with the catastrophe theory. However, in subsequent research, we find some inherent relationships between the accelerating Airy beams and accelerating triple-cusp beams in terms of the phase distribution and optical characteristics. Based on the relation between Airy beams and triple-cusp beams, we can determine the optical characteristics of triple-cusp beams. The research results have the potential for revealing the nature of accelerating triple-cusp

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beams.

2. Theory

Schematics of the phase modulation method that generates accelerating beams are presented in Fig. 1. Using Fresnel approximation, and assuming that the arbitrary phase mask is placed in the front focal plane of the Fourier transform lens, the field amplitude $g(x_0, y_0)$ in the front plane of the lens is defined as

$$\begin{aligned} & g(x_0,y_0) \\ = & \frac{k\cdot\exp(\mathrm{i}kf)}{2\pi\mathrm{i}f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(\xi,\eta) \\ & \times\exp\left\{\frac{\mathrm{i}k}{2f}[(x_0-\xi)^2+(y_0-\eta)^2]\right\}\mathrm{d}\xi\,\mathrm{d}\eta, \end{aligned} \tag{1}$$

where (ξ, η) and (x_0, y_0) are the generalized transversal coordinates in the front focal plane and front lens plane, respectively. $k = 2\pi/\lambda$ is the wavenumber.

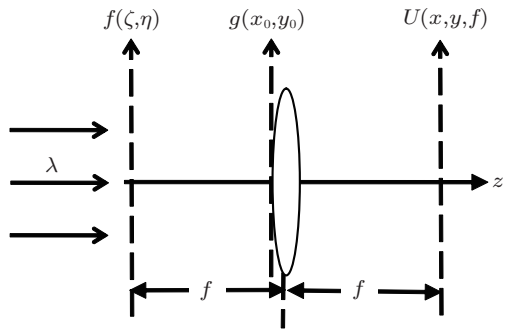


Fig. 1. Schematic illustration of the phase modulation method that generates accelerating beams.

The phase transform expression of an achromatic Fourier transforming lens is expressed as

$$t(x_0,y_0)=\exp\left[-\frac{\mathrm{i}k}{2f}(x_0^2+y_0^2)\right]. \tag{2}$$

Then, the three-dimensional (3D) complex amplitude distribution $U(x,y,z)$ near the back focal plane of the lens is written as

$$\begin{aligned} & U(x,y,z) \\ = & \frac{k\cdot\exp(\mathrm{i}kz)}{2\pi\mathrm{i}z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x_0,y_0)\cdot t(x_0,y_0) \\ & \times\exp\left\{\frac{\mathrm{i}k}{2z}[(x-x_0)^2+(y-y_0)^2]\right\}\mathrm{d}x_0\,\mathrm{d}y_0, \end{aligned} \tag{3}$$

where, (x,y,z) are respectively the generalized transversal and longitudinal coordinates in the out-plane near the back focal plane of the lens.

Substituting Eqs. (1) and (2) into Eq. (3), and performing mathematics calculations, one can obtain

$$\begin{aligned} & U(x,y,z) \\ = & \frac{k\cdot\exp[\mathrm{i}k(z+f)]}{2\pi\mathrm{i}f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(\xi,\eta) \\ & \times\exp\left[-\frac{\mathrm{i}k(z-f)}{2f^2}(\xi^2+\eta^2)-\mathrm{i}\frac{k}{f}(x\xi+y\eta)\right]\mathrm{d}\xi\,\mathrm{d}\eta. \end{aligned} \tag{4}$$

By relating the diffraction integrals to the catastrophe polynomials,[15,17,18] the phase expression that generates triple-cusp beams is given by

$$\phi(\xi,\eta)=\beta[\xi^3/3+\eta^3/3-(\xi^2\eta+\xi\eta^2)], \tag{5}$$

where β is a constant parameter of the phase mask that determines the maximum phase deviation. When input beams are collimated, the optical field distribution of the phase mask surface can be written as

$$f(\xi,\eta)=\exp[\mathrm{i}k\phi(\xi,\eta)]. \tag{6}$$

Substituting Eq. (5) into Eq. (6), which subsequently results in substituting those equations with Eq. (4), the analytical expression of a triple-cusp beam 3D complex amplitude distribution can be obtained. Ignoring the cross term $\xi^2\eta+\xi\eta^2$ of Eq. (5), equation (4) becomes an expression of the 3D amplitude distribution of Airy beams.[15] Based on such an expression, although we cannot clearly obtain the optical characteristics of triple-cusp beams, certain relations and difference in amplitude distribution between Airy beams and triple-cusp beams can be acquired. From the phase distribution of generating Airy beams and triple-cusp beams, the wavefronts of both beams contain the cubic phases. The difference is that the wavefront of generated triple-cusp beams contains cross-term phase besides the cubic phase. The feature may be used to design different phase masks to generate triple-cusp beams and Airy beams. Since triple-cusp beams and Airy beams have certain relationships, we can demonstrate the optical characteristics of a triple-cusp beam if the optical characteristics of Airy beams are given. Thus, we first present the propagation characteristics of Airy beams in the next section.

3. Characteristics of Airy beams

Ignoring the cross term $-(\xi^2\eta+\xi\eta^2)$ in Eq. (5) and if $z=f$, equation (4) is written as,[15,19]

$$\begin{aligned} U(x,y,f) &= \frac{k\cdot\exp(2\mathrm{i}kf)}{2\pi\mathrm{i}f}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[\mathrm{i}k\beta(\xi^3+\eta^3)/3] \\ &\quad\times\exp\left[-\mathrm{i}\frac{k}{f}(x\xi+y\eta)\right]\mathrm{d}\xi\,\mathrm{d}\eta \\ &= \frac{k\cdot\exp(2\mathrm{i}kf)}{2\pi\mathrm{i}f}\int_{-\infty}^{\infty}\exp\left[\mathrm{i}\left(k\beta\xi^3/3-\frac{kx}{f}\xi\right)\right]\mathrm{d}\xi \\ &\quad\times\int_{-\infty}^{\infty}\exp\left[\mathrm{i}\left(k\beta\eta^3/3-\frac{ky}{f}\eta\right)\right]\mathrm{d}\eta \\ &= \frac{2\pi\mathrm{i}k^{1/3}\cdot\exp(2\mathrm{i}kf)}{\beta^{2/3}f}\mathrm{Ai}(-k^{2/3}\beta^{-1/3}x/f) \end{aligned}$$

$$\times \mathrm{Ai}(-k^{2/3}\beta^{-1/3}y/f), \tag{7}$$

where $\mathrm{Ai}(\cdot)$ denotes the Airy function. Equation (7) shows that the amplitude distribution $U(x,y,f)$ in the back focal plane (Fourier transform plane) of the lens is the Airy distribution function when neglecting the cross term in Eq. (5). If $z > f$, let $\Delta z = z - f$ (Δz is the distance from the focal plane to the measured plane along the optic axis). Equation (4) is written as

$$\begin{aligned} & U(x,y,z) \\ = & \frac{k\cdot\exp[\mathrm{i}k(z+f)]}{2\pi i f}\times\int_{-\infty}^{\infty}\mathrm{d}\xi\int_{-\infty}^{\infty}\mathrm{d}\eta \\ & \times\exp\left\{\mathrm{i}\left[k\beta(\xi^3+\eta^3)/3-\frac{k\Delta z}{2f^2}(\xi^2+\eta^2)\right.\right. \\ & \left.\left.-\frac{kx}{f}\xi-\frac{ky}{f}\eta)\right]\right\} \\ = & \frac{2\pi\mathrm{i}k^{1/3}}{\beta^{2/3}f}\exp\left\{\mathrm{i}[k(z+f)-\frac{k^2(x+y)\Delta z}{2\beta f^3}-\frac{k^3\Delta z^3}{6\beta^2 f^6}]\right\} \\ & \times\mathrm{Ai}\left(-\frac{k^{2/3}x}{\beta^{1/3}f}-\frac{k^{2/3}\Delta z^2}{4\beta^{4/3}f^4}\right) \\ & \times\mathrm{Ai}\left(-\frac{k^{2/3}y}{\beta^{1/3}f}-\frac{k^{2/3}\Delta z^2}{4\beta^{4/3}f^4}\right). \end{aligned} \tag{8}$$

In Eqs. (7) and (8), the complex coefficient is not important because it does not influence the intensity distribution according to $I = U^2$. Therefore, we usually neglect the complex coefficient. In Eqs. (7) and (8), let

$$\delta x, \delta y = \beta^{1/3}k^{-2/3}f, \tag{9}$$

where δx is closely related to the main lobes width of Airy beams. According to the definition of an Airy function, we can deduce the diameter expression of the full width at half maximum (FWHM) of the lobes of Airy beams

$$\Delta x, \Delta y = 1.630\cdot\delta x, 1.630\cdot\delta y = 1.630\cdot\beta^{1/3}k^{-2/3}f. \tag{10}$$

Based on the definition of optical intensity, equations (7) and (8) show that the beams are still maintained as an Airy distribution after the propagation of Δz in free space, which demonstrates the non-diffractive propagation characteristics of Airy beams. However, ideal Airy beams cannot be experimentally observed because the square of an Airy function is not integrable ($\int \mathrm{Ai}^2(x)\,\mathrm{d}x \to \infty$), which is just similar to ideal plane wave beams and Bessel beams. Ideal plane wave beams and Bessel beams both contain an infinite energy because the squares of the cosine function and the Bessel function are not integrable either. In fact, experimentally generated Airy beams are doomed to finite-energy Airy beams, i.e., truncated Airy beams. Finite-energy Airy beams are weak diffraction beams,[1,19] hence the experimental FWHM of the main lobes of Airy beams is larger than the theoretical prediction from Eq. (10). However, equation (10) is still important because we can use the equation to determine the parameters that determine the FWHM of the main lobes of Airy beams.

Moreover, equations (7) and (8) also show that the central maxima of Airy beams deflect as Δz^2, which means transversally accelerating propagation. Such results are in agreement with previous research results.[1–12,16,17,20,21] Since the $|\mathrm{Ai}(w)|$ has a peak at $w = -1.018$, we can also deduce the propagation trajectory of the maximum intensity of Airy beams based on Eq. (8). The result is given by

$$x, y = -1.018\frac{\beta^{1/3}f}{k^{2/3}}+\frac{\Delta z^2}{4\beta f^3}, \tag{11}$$

where $-1.018\beta^{1/3}f/k^{2/3}$ is the initial position and $1/4\beta f^3$ is the theoretical acceleration a_A of ideal Airy beams. Equation (11) shows that the intensity maximum at the central lobe propagates along a curved path in free space. Moreover, equation (11) also shows that the intensity maximum of the main lobes of Airy beams in the focal plane is not located on the optical axis, which deviates from $-1.018\beta^{1/3}f/k^{2/3}$. Such a deviation is worthy of attention when we use the Airy beams in light–matter interaction experiments, although the degree of deviation is very small. Substituting Eq. (9) into Eq. (11), a_A can also be written as,[1,20]

$$a_{\mathrm{A}} = x_0^{-3}k^{-2}/4. \tag{12}$$

Considering that the experimental FWHM of the main lobes for Airy beams is larger than the theoretical prediction from Eq. (10), the experimental acceleration of finite energy Airy beams is smaller than the theoretical acceleration of infinite energy Airy beams presented in Eq. (11). Hence, we suggest that equations (10) and (11) give the theoretical minimum size limit of lobes of Airy beams and their corresponding maximum acceleration limit.

Based on Eqs. (10) and (11), the FWHMs of both lobes and the propagating trajectory of the central maxima of Airy beams are determined by k, β, and f. With regard to the monochromatic beam, k is a constant. Therefore, we can control the lobe width and acceleration of Airy beams by changing the phase controlling parameter β and the focal length f of the Fourier transforming lens. The decrease in β and f increases the transverse acceleration and reduces the lobe area of Airy beams. Such results are significant because the acceleration is the most important optical characteristic of accelerating beams. The lobe FWHM is also an important physical parameter of Airy beams because this value determines the peak intensity of Airy beams. Equations (10) and (11) also show that the influence of focal length f is greater than the controlling parameter β for the lobe width and the acceleration of Airy beams. Such theoretical results indicate that we should change the focal length f of the Fourier transform lens of instead of changing the controlling parameter β in the phase mask when we select the acceleration of lobes of Airy beams in scientific research. In addition, it is readily known that a smaller main lobe of Airy beams results in a greater acceleration of Airy

beams. Since the lobe width δx (δy) is directly proportional to $\beta^{1/3} f$ and the acceleration a_A is inversely proportional to βf^3, the lobe of Airy beams decreases by n times and causes the acceleration a_A to increase by n^3 times.

With regard to triple-cusp beams, directly obtaining any clear physical conclusion based on Eq. (4) is difficult. Hence, we should further establish the connection between Airy beams and triple-cusp beams based on experimental results and the numerical simulation presented below.

4. Relationship between Airy beams and triple-cusp beams

By relating diffraction integrals and one catastrophe polynomial to controlling parameter β, we generate accelerating Airy beams and accelerating triple-cusp beams based on the same optical system, as shown in Fig. 2. The corresponding principle schematic plot is shown in Fig. 1. Based on the practical physical parameter, numerically simulated results are first presented in Fig. 3. In this experiment, since the phase modulated method of generating Airy beams is to encode the phase of the desired beams in an input collimating beam (He–Ne laser beam, $\lambda = 632.8$ nm),[1,9,10,22] the main task is to construct the wavefront based on Eq. (5). The gray level image represents the phase mask that generates optical Airy beams and triple-cusp beams, as shown in Fig. 2. Afterward, phase modulator elements (i.e., spatial light modulator) are used to load the corresponding gray level image. Apparently, the phase mask between generating Airy beams and generating triple-cusp beams is different. Based on Eq. (5), we find that the main difference between the two phase masks is that the phase mask of generating triple-cusp beams has a cross term besides the cubic phase. We respectively encode two phase masks onto a spatial light modulation. Then, a well-aligned scientific charge-coupled-device (CCD) camera is used to observe the Airy beams and triple-cusp beams. This CCD camera slides along an optical rail to record the optical intensity photographs in different observation planes. The observation distances are referred to as the distances from the focal plane ($z = 0$ cm) of the lens to the observation plane. The transverse displacement value of the main lobes of the accelerating beams is measured from the optical axis of the lens to the main lobes, which can be used to calculate the acceleration of the accelerating beams. Airy beams and triple-cusp beams are then generated when an optical Fourier transform is performed on the modulated wavefront using a 1.2-m achromatic Fourier transforming lens. Since we generate Airy beams and triple-cusp beams using the same optical system, we consider that the Airy beams and triple-cusp beams have the same finite optical energy. Both are finite energy accelerating beams, hence both beams must be weakly diffracted.

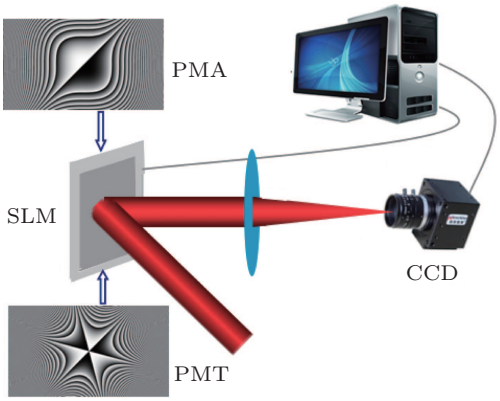


Fig. 2. (color online) Experimental setup for generating optical Airy beams and triple-cusp beams. SLM: spatial light modulation; PMA: phase mask of generating Airy beams; PMT: phase mask of generating triple-cusp beams; CCD: charge-coupled device.

Because of the finite modulating accuracy of the spatial light modulator (Holoeye PLUTO, with an 8.0-μm pitch for each pixel) and the finite measuring accuracy of the scientific CCD camera (Microvision 130FC, with a 5.4-μm pitch for each pixel), the experimental results of Fig. 3 demonstrate some inevitable deviations from the theoretically simulated prediction. Since there are measurement errors, we also study optical characteristics based on the numerically simulated results. In fact, the numerical method is an important tool for studying the optical characteristics of some optical beams.[23,24] Of course, figure 3 shows that the difference between the experimental and theoretical results is very small, hence both the theoretical and experimental results of Airy beams and triple-cusp beams are basically the same.

With regard to each kind of accelerating beam, the main lobe (intensity maximum areas) of each beam is mainly used as a tool in scientific research. The main lobes of accelerating beams are the most important part of the whole beam because the main lobes have high optical intensities compared with their peripheral areas. Hence, in the present paper, we mainly focus on the optical characteristics and propagation characteristics of the main lobes of two accelerating beams. Like the main lobe of Airy beams, each cusp lobe of optical triple-cusp beams can be approximately regarded as curved focal beams with very long focal depth for some applications. However, an optical triple-cusp beam that we produced has three main lobes, whereas one Airy beam has only one single main lobe.

We first observe the shapes of the main lobes of Airy beams and triple-cusp beams based on Fig. 3. The main lobes of Airy beams for the entire experiments are basically circular spots.[1,2,25,26] A triple-cusp beam in the focal plane is one strong deltoid patch surrounded by weakly symmetrical interference fringes, which is the focus. Moving from the focal plane, the central deltoid patch begins to symmetrically split into three distinct cusped lobes, which is named the marginal focus. Apparently, figures 3(g) and 3(h) can be regarded as "indeed" triple-cusp beams at a certain distance away from the focal plane. When $\Delta z = 2$ cm, each main lobe

of triple-cusp beams is approximately a round spot, as shown in Fig. 3(g). During the propagation, we find that the shapes of the main lobes are more and more flat so that they can be approximately regarded as an ellipse spot (shown in Fig. 3(h)). Hence, the main lobes of Airy beams are more structurally stable and have a higher quality than the main lobes of triple-cusp beams. Moreover, figures 3(c)–3(d) and 3(g)–3(h) indicate that triple-cusp beams are also weak diffraction beams because the lobe area of triple-cusp beams slowly increases from 38.23 μm×41.11 μm to 28.42 μm×200.12 μm after a 30-mm propagation. This area approximately increases by 3.62 times. In the same propagating distance, the lobe area of Airy beams only increases from 47.22 μm×47.65 μm to 49.61 μm×50.39 μm, which means that this area only increases by 1.11 times. Apparently, the marginal focus of triple-cusp beams is also comparatively stable.

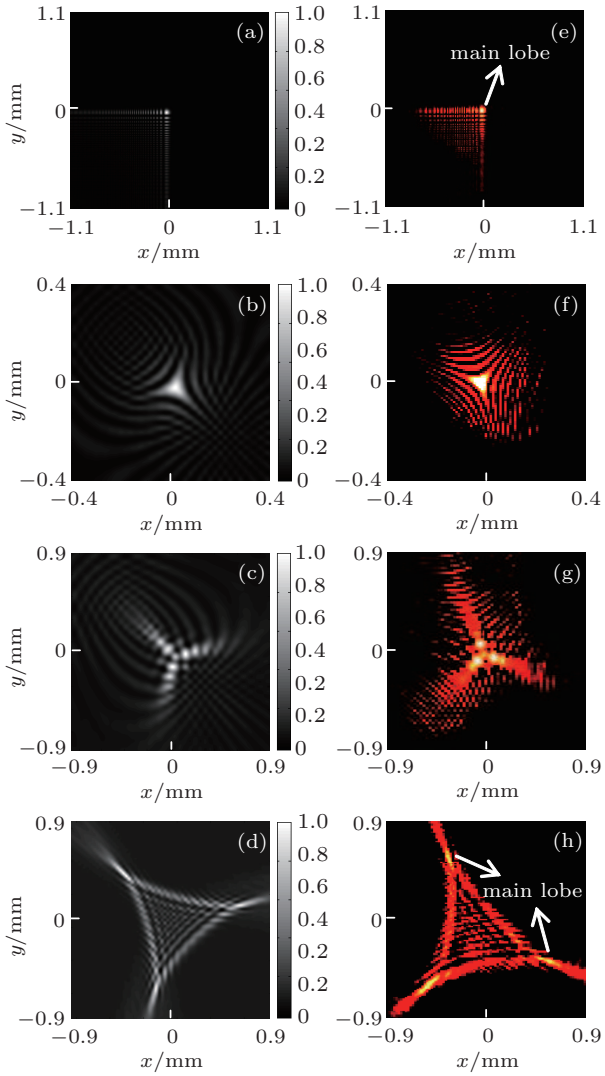


Fig. 3. (color online) Theoretical simulation and experimental intensity distributions ($\beta = 1$, $f = 1.2$ m). Stimulation of Airy beam (a) at $\Delta z = 0$ cm, stimulations of triple-cusp beam at (b) $\Delta z = 0$ cm, (c) $\Delta z = 2$ cm, and (d) $\Delta z = 5$ cm. Corresponding experimental results with the same distances are shown in panel (e) to panel (h).

Subsequently, we discuss the influences of β and f on the "triplet" lobe (i.e., trigon patch in the focal plane of the lens) of triple-cusp beams. As shown in Fig. 4, the distance between two cusps of a trigon patch ($\Delta z = 0$) and the lobe FWHM diameter of triple-cusp beams ($\Delta z > 0$) are directly proportional to $\beta^{1/3}$. The influences on triple-cusp beams from the Fourier transform lens with different values of focal length f are also similar to that of Airy beams, i.e., f is directly proportional to the patch size or lobe size of triple-cusp beams. In conclusion, the FWHM of a trigon patch or lobes of triple-cusp beams is directly proportional to $\beta^{1/3} f$. Further research shows that the distance Δd between any two vertices of a deltoid patch of triple-cusp beams in the focal plane is always twice the diameter of lobes of Airy beams in the focal plane. Hence, based on Eq. (10), this vertex distance can be written as

$$\Delta d = 3.260 \cdot \beta^{1/3} k^{-2/3} f. \tag{13}$$

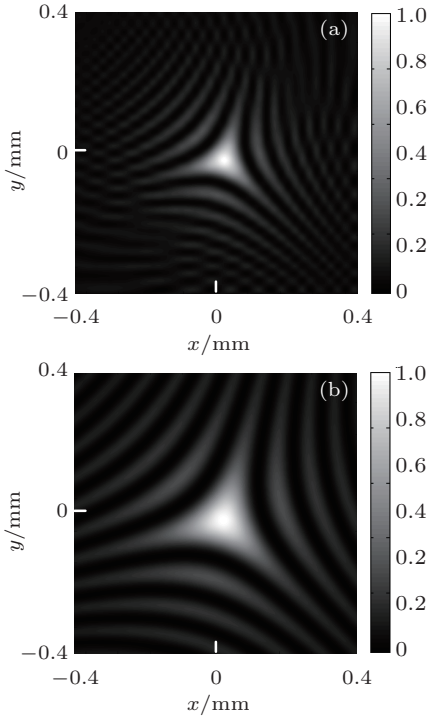


Fig. 4. (color online) Theoretical simulation results of the triple-cusp beams at $\Delta z = 0$ cm, (a) $\beta = 2$, $f = 1.2$ m, and (b) $\beta = 2$, $f = 2$ m.

Since the deltoid patch of triple-cusp beams in the focal plane determines the optical feature and propagating characteristics of triple-cusp beams when $\Delta z > 0$, equation (13) is an important expression to describe the optical characteristics of triple-cusp beams.

Based on Eq. (11), the acceleration trajectory of ideal Airy beams can be calculated. With regard to triple-cusp beams, we cannot analytically calculate the acceleration trajectory of lobes. Hence, we need to use numerical methods. As shown in Fig. 5, we first calculate the deviation value from the optical axis for two accelerating beams in a range from $\Delta z = 3$ cm to 10 cm, where the triple-cusp beam with three distinct cusped lobes is found. Afterward, we measure the deviation value from the optical axis for two accelerating beams in accordance with the propagation distance. Figure 5 shows

that the intensity maximum of the Airy beam theoretically deflects by 1.45 mm and experimentally deflects by 1.34 mm when $\Delta z = 10$ cm. Apparently, the practical acceleration of Airy beams is smaller than the theoretical prediction because of the finite energy. Under the same conditions (especially having the same input energy), each intensity maximum of the optical triple-cusp beams theoretically and experimentally deflects by approximately 2.41 mm and 2.20 mm from the optic axis for the same propagation distance respectively. According to the experimental measurements, we find that although both local intensity maxima propagate throughout the parabolic trajectory, the acceleration of the cusp point of the optical triple-cusp beams exceeds that of the Airy beam's main lobe. Further comparison reveals that the theoretical and experimental lateral shifts of lobes of triple-cusp beams are respectively 1.662 and 1.641 times those of the main lobes of Airy beams in the same physical conditions. Moreover, we find that the acceleration a_{T} of each main lobe of "indeed" triple-cusp beams is always 1.65 times the acceleration a_{A} of main lobes of Airy beams in different propagation distances. This finding is observed no matter how the controlling parameter β and the focal length f of the Fourier transform lens are changed. Hence, based on the acceleration expression of Airy beams, we can present the theoretical acceleration expression of triple-cusp beams as

$$a_{\mathrm{T}} = 1.65 a_A = 0.41 \beta^{-1} f^{-3}. \tag{14}$$

In summary, we find that the optical self-bending triple-cusp beams have a larger lateral acceleration than Airy beams under the same physical parameters. Therefore, the use of optical accelerating triple-cusp beams for particle manipulation and light–matter interactions may be another substantial research field. Moreover, we also find that the influences of β and f on acceleration a_{T} of each main lobe of triple-cusp beams are the same as those of Airy beams, i.e., the acceleration of each lobe of triple-cusp beams is inversely proportional to βf^3. Hence, we can also control the acceleration of triple-cusp beams by changing the phase mask of the controlling parameter β and the focal length f of the Fourier transforming lens. Substituting Eq. (13) into Eq. (14), a_{A} can also be written as

$$a_{\mathrm{T}} = 14.20 k^{-2} \Delta d^{-3}. \tag{15}$$

Equation (15) shows that the vertex distance Δd of a deltoid patch of triple-cusp beams in the focal plane determines the acceleration a_{T} of triple-cusp beams.

Finally, there is a certain relationship between Airy beams and triple-cusp beams. The common cubic phase confirms that both Airy beams and triple-cusp beams are accelerating beams, and the cross-term phase determines the difference between the two kinds of beams.

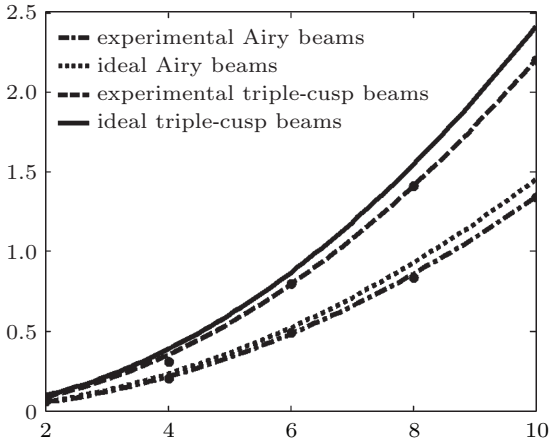


Fig. 5. Accelerating trajectories of one main lobe of Airy beams and triple-cusp beams under the same physical parameters ($\beta = 1$, $f = 1.2$ m). Dash–dot line and dashed line respectively represent the fitted ballistic trajectories of one main lobe of an Airy beam and an optical triple-cusp beam based on experimental results. Black dots represent the experimentally measured values. Corresponding theoretical predicting trajectories are represented by the dotted line and the solid line, respectively.

5. Conclusions

According to theoretical analysis, numerical stimulation, and experimental observation, we present the connection and difference of optical structure and beam propagation characteristics between accelerating Airy beams and accelerating triple-cusp beams. The main difference between two accelerating beams is that triple-cusp beams have more sampling points (lobes) than Airy beams. In addition, the acceleration of the triple-cusp beam lobe is greater than that of Airy beams in the same conditions, although the lobes of Airy beams are more structurally stable and have a higher quality than those of triple-cusp beams. The main conclusion is that the controlling parameter β and focal length f of the Fourier-transforming lens influence the optical topological structure, as well as the propagating characteristics of Airy beams and triple-cusp beams under the same rule. These two kinds of beams have a close relationship in the same conditions. Hence, we can acquire the optical characteristics of triple-cusp beams based on the beam characteristics of Airy beams, as well as the relationship between Airy beams and triple-cusp beams. Our research results provide a foundation for revealing the nature of accelerating triple-cusp beams, and show the potential applications of triple-cusp beams.

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