Evacuation of pedestrians from a hall by game strategy update*

Wang Hao-Nan(王浩楠)[a)c)], Chen Dong(陈 栋)[a)], Pan Wei(盘 薇)[a)],
Xue Yu(薛 郁)[a)†], and He Hong-Di(何红弟)[b)]

[a)] *Institute of Physics Science and Engineering, Guangxi University, Nanning* 530004, *China*
[b)] *Logistics Research Center, Shanghai Maritime University, Shanghai* 200135, *China*
[c)] *Science and Technology Institute of Guangxi Zhuang Autonomous Region, Nanning* 530006, *China*

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In this paper, a cellular automaton model considering game strategy update is proposed to study the pedestrian evacuation in a hall. Pedestrians are classified into two categories, i.e., cooperators and defectors, and they walk to an exit according to their own strategy change. The conflicts that two or three pedestrians try to occupy the same site at the same time are investigated in the Game theory model. Based on it, the relationship between the pedestrian flow rate and the evacuation time as well as the variation of cooperative proportion against evacuation time is investigated from the different initial cooperative proportions under the influence of noise. The critical value of the noise is found when there is a small number of defectors in the initial time. Moreover, the influences of the initial cooperative proportion and strength of noise on evacuation are discussed. The results show that the lower the initial cooperative proportion as well as the bigger the strength of noise, the longer the time it takes for evacuation.

Keywords: pedestrian, evacuation, game, strategy

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1. Introduction

In recent decades, much attention has been paid to the investigation of pedestrian evacuation, not only for its great practical significance, but also for its exhibiting remarkable complicated phenomena, such as self-organization, the "faster-is-slow" effect, "freezing by heating", herd behavior, etc.[1–5] Many scholars in different fields have devoted themselves to the studying the behaviors of pedestrian dynamics. Many typical models, such as social force model, magnetic force model, fluid dynamics model, etc.,[1–15] have been proposed for pedestrian dynamics, especially for the evacuation problem of panicking pedestrians. Among them, the Game theory model is verified as being a sophisticated model, which can discover the pedestrian dynamics from the microscopic view.

Game theory is always a concern to decision makers from heads of state to citizens, especially since mathematician Von Neumann and economist Morgenstern finished the work: "Theory of Games and economic behavior", which is commonly considered as the milestone of the classical game theory. In the 1950s, American mathematician John Nash put forward a vital concept "Nash Equilibrium" with its main idea that individual strategy evolution will tend to a balanced state in the two or more people's game, in which all individuals will adopt the strategy of Nash Equilibrium meanwhile without any individual obtaining a higher payoff via a one-side strategy update. At present, game theory has been introduced into the research of pedestrian traffic and has aroused more researchers' attention.[16]

Helbing and Molnar[4] proposed a social force model to investigate the dynamical features of a panicked escape. They found that the congestion of pedestrians was formed in the congestion arch. Nagatani,[6] Tajima and Natagani[7] studied indoor pedestrian evacuation and discovered that the dynamic phase transitions could lead to congestion by using the lattice gas model. Szabo and Hauert,[17] Szabo *et al.*[18] investigated a prisoner's dilemma game with a slightly different continuous-time evolution, indicating that the transitions from the active state-coexistence of defectors and cooperators to the absorbing ones exhibited universal behavior. Perc[19] introduced the evolutionary game into the BML traffic model to study pedestrian traffic in a public square. In this model, each pedestrian walked forward according to their own payoff obtained by the game between pedestrians, in which the evolutionary prisoner's dilemma and snowdrift game was considered. More recently, Zhang *et. al.*[8] and Xiong *et al.*[9] simulated pedestrian counter flow by adopting an optimal path-choice strategy and explored the mechanism of the lane formation phenomenon. Sun *et al.*[10] discussed the phase transition in random walking with the evolutionary game. Xie *et al.*[15] introduced the boycott strength into the strategy update process and discussed its effect on the evacuation efficiency.

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†Corresponding author. E-mail: yuxuegxu@gxu.edu.cn

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These studies provided a good understanding of the pedestrian evacuation from different views. However, in these models, the effects of the game strategies update on pedestrian evacuation were seldom discussed, though it can reflect the escaping characteristics of a walker in a real panic situation. Hence, in this paper, the pedestrian flow dynamics in a hall is investigated by using the cellular automation model (CA) with a game strategy update.

The rest of this paper is organized as follows. In Section 2, a cellular automaton model with a biased-walker is proposed and described in detail by introducing the defector and cooperator in the evolutionary game. Numerical simulations under different initial cooperative proportions and the corresponding analyses are presented in Section 3. Conclusions are drawn in Section 4.

2. Model

The evacuation of pedestrians is studied on a two-dimensional square lattice $(L \times L)$ in a hall. Each lattice is equivalent to an actual size of 0.4 m $\times$ 0.4 m as shown in Fig. 1. The exit W is located in the center of the upper boundary and the hall is divided into three parts: the left-hand side, the center, and the right-hand side.

At each time step, each pedestrian is allowed to stay at its current position or to move to one of the neighboring lattices, given that the destination lattice is not occupied by another pedestrian. All pedestrians, as biased-random walkers, should update their sites and strategies at the same time and move toward the only exit W without going backward. Hence, for a pedestrian, there are eight choices for choosing their sites. Figure 2(a) illustrates the transition probabilities on the left-hand side of the exit in the hall for a pedestrian located at $(\mathrm{i}x, \mathrm{i}y)$. The black circle indicates a pedestrian while the black cross circle indicates that the site is occupied by another pedestrian.

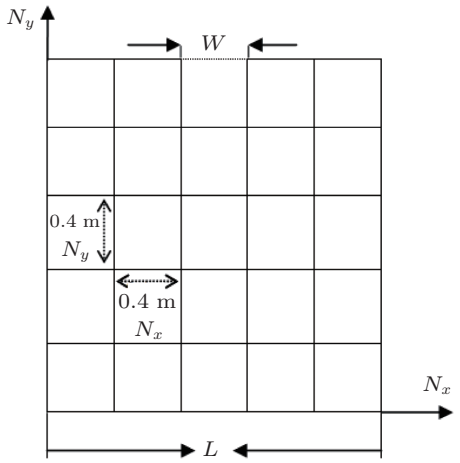


Fig. 1. Sketch of the hall.

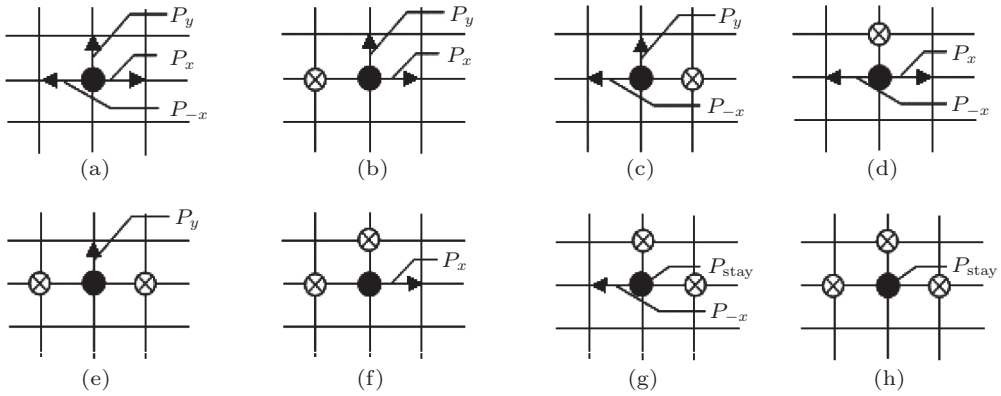


Fig. 2. Illustration of the transition probabilities on the left-hand side of the exit in the hall for a pedestrian located at $(\mathrm{i}x, \mathrm{i}y)$.

As is well known, the exit (W) is located in the center of the hall and all pedestrians take the location of the exit as the final goal. According to the lattice gas model describing the movement of the pedestrian in a hall,[7] the corresponding transition probabilities on the left and the right of the exit are different. When a pedestrian $(\mathrm{i}x, \mathrm{i}y)$ located on the left-hand side of the exit in the hall $(\mathrm{i}x < x_1 \ (x_1 = L/2 - W/2 + 1))$ moves toward his object, the transition probabilities of eight situations in Fig. 2(a) are shown as follows:

1) $P_x = D_x + (1-D)/2$, $P_y = D_y + (1-D)/2$, $P_{-x} = 0$ for configurations (a) and (b);

2) $P_y = 1$, $P_{-x} = 0$, $P_x = 0$ for configurations (c) and (e);

3) $P_x = 1$, $P_{-x} = 0$, $P_y = 0$ for configurations (d) and (f);

4) $P_{-x} = k$, $P_{\text{stay}} = 1 - k$, $P_y = 0$, $P_x = 0$ for configuration (g);

5) $P_{\text{stay}} = 1$, $P_y = 0$, $P_x = 0$, $P_{-x} = 0$ for configuration (h).

Here, P_x is the transition probability to the right while P_y is the transition probability to the top. P_{-x} is the transition probability to the left. As the exit is placed on the right, the transition probabilities to the left for all pedestrians are set to

be zero. D is the drift pointing to the exit,

$$D_x = \frac{D|\mathrm{i}x - x_0|}{|\mathrm{i}x - x_0| + |\mathrm{i}y - y_0|}$$

is the drift in the x component,

$$D_y = \frac{D|\mathrm{i}y - y_0|}{|\mathrm{i}x - x_0| + |\mathrm{i}y - y_0|}$$

is the drift in the y component. k is a parameter of transition probability ($0 \le k \le 1$), which can be regarded as the average of all walkers' memories. (x_0, y_0) is the coordinate of the center of the exit.

If a pedestrian $(\mathrm{i}x, \mathrm{i}y)$ is located on the right-hand side of the exit in the hall $(\mathrm{i}x \ge x_2\ (x_2 = L/2 + W/2 + 1))$, the transition probability to the right is set to be zero.

The corresponding transition probabilities in Fig. 3 are changed as follows:

1) $P_{+x} = 0$, $P_y = D_y + (1-D)/2$,$P_{-x} = D_{-x} + (1-D)/2$ for configurations (a) and (c);

2) $P_y = 1$, $P_x = 0$, $P_{-x} = 0$ for configurations (b) and (e);

3) $P_{-x} = 1$, $P_{+x} = 0$, $P_y = 0$ for configurations (d) and (g);

4) $P_{+x} = k$, $P_{\mathrm{stay}} = 1 - k$, $P_y = 0$, $P_{-x} = 0$ for configuration (f);

5) $P_{\mathrm{stay}} = 1$, $P_y = 0$, $P_x = 0$, $P_{-x} = 0$ for configuration (h),

where

$$D_{-x} = \frac{D|\mathrm{i}x - x_0|}{|\mathrm{i}x - x_0| + |\mathrm{i}y - y_0|}$$

due to the symmetry with respect to the left-hand side of the exit. If a pedestrian $(\mathrm{i}x, \mathrm{i}y)$ is located in the center of the exit in the hall $(\mathrm{i}x \ge x_3\ (x_3 = L/2))$, the transition probabilities are changed as follows:

1) $P_x = 0$, $P_y = D$, $P_{-x} = 0$ for configurations (a), (b), (c) (e);

2) $P_x = 1/2$, $P_y = 0$, $P_{-x} = 1/2$ for configuration (d);

3) $P_x = 1$, $P_y = 0$, $P_{-x} = 0$ for configuration (f);

4) $P_x = 0$, $P_y = 0$, $P_{-x} = 1$ for configuration (g);

5) $P_x = 0$, $P_y = 0$, $P_{-x} = 0$, $P_{\mathrm{stay}} = 1$ for configuration (h).

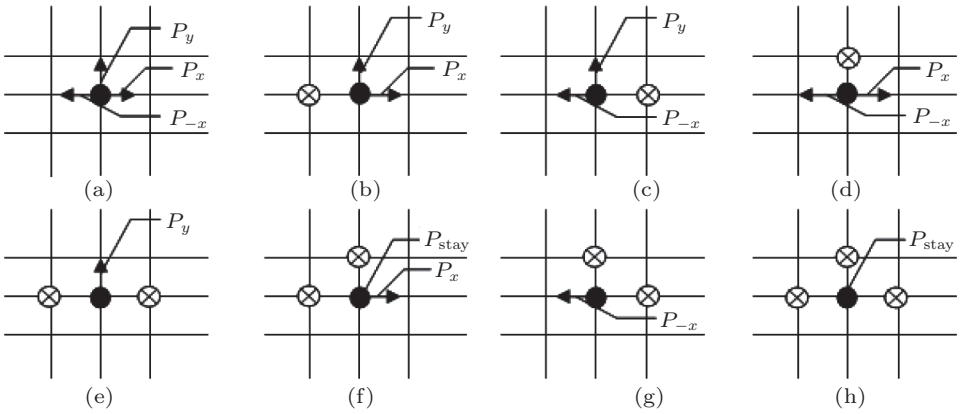


Fig. 3. Illustration of the transition probabilities on the right-hand side of the exit in the hall for a pedestrian located at $(\mathrm{i}x, \mathrm{i}y)$.

Each pedestrian is allowed to walk towards the position of the exit with no back steps and move along the wall. They are deleted on the lattice once they arrive at the exit. In order to study in depth the interaction among pedestrians, the pedestrians are assumed to be classified into two types, i.e., cooperators and defectors. Additionally, the snowdrift game model is introduced to discover the complicated interaction among the conflict individuals. Each walker interacts with all his neighbors and gets the payoff following the payoff matrix in Table 1.

Table 1. Payoff matrix of the game.

Payoff	Cooperator	Defector
Cooperator	0.5, 0.5	0, 1
Defector	1, 0	0, 0

As shown in Fig. 4, supposing that the exit is located on the upper right side of site C. Since both pedestrians A and B want to occupy site C and go forward, a game starts. When both A and B are cooperators, each one has an equal chance of accessing. If A is a cooperator while B is not, then B will occupy site C while A will stay where it is. The worst case is that neither of them is a cooperator and both of them remain at the site at this time step. The conflict happens when both of them try to occupy the same site at the same time.

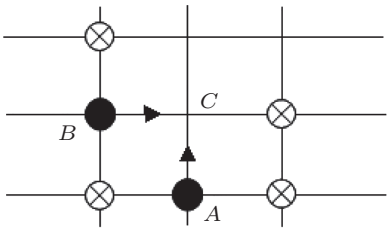


Fig. 4. Game between A and B for site C.

Generally, pedestrians have their own way of thinking and get used to following the ones who gain the most. With this consideration, the game with a strategy update is suggested to be introduced into the evacuation. Hence there is some probability that pedestrians can change their strategies in the evacuation process. The corresponding probability[20] is given by

$$P[c(c)=c]=1, \quad P[c(c)=d]=0, \tag{1}$$

$$P[c(d)=d]=P(c-d)=\frac{1}{1+\mathrm{e}^{(-\beta(U_d-U_c))}},$$

$$P[c(d)=c]=1-P(c-d), \tag{2}$$

$$P[d(d)=c]=P(d-c)=\frac{1}{1+\mathrm{e}^{(-\beta(U_c-U_d))}},$$

$$P[d(d)=d]=1-P(d-c), \tag{3}$$

$$P[d(c)=d]=1, \quad P[d(c)=c]=0, \tag{4}$$

where $U_c = f_c(t) \times pc_{\max}$, $U_d = (1-f_c(t)) \times pd_{\max}$, $f_c(t) = n_c(t)/n_{\mathrm{all}}(t)$, and $0 \le f_c(t) \le 1$. Here $pc_{\max}$ ($pd_{\max}$) means the maximum benefit that a cooperator (defector) can join the game. $n_c(t)$ and $n_{\mathrm{all}}(t)$ are the numbers of cooperators and pedestrians (cooperators and defectors) in the hall at time step (t) respectively. Equation (1) means that the encounter of two cooperators will not affect their original strategies. Equation (2) indicates that when a cooperator meets a defector, he will have a probability of $P(c-d)$ to become a defector in the next run, but he also has a possibility of $(1-P(c-d))$ to stay where he is. Equation (3) shows the probability when two defectors come together in a similar way. Equation (4) reveals such a fact that after playing the game with a cooperator, a defector would never give up its strategy as a betrayer.

In this paper, parameters $pc_{\max}$ and $pd_{\max}$ are set to be 0.5 and 1 respectively. Parameter β indicates the noise of environmental factors in the hall. The larger the value of β, the bigger the chance for cooperators to change into defectors is. Otherwise, cooperators will make judgments rationally, which means that they are less likely to become defectors.

3. Simulations and results

Simulations are carried out on a square lattice for a hall of scale 200×200 and the initial density of pedestrian $\rho = 0.3$. Initially, the pedestrians are randomly distributed in the hall, numbered and selected to be cooperators with the probability of $f_c(0)$. The drift coefficient is set as $D = 0.5$ and the parameter is set to be $k = 0.2$. For each kind of exit size, the simulation is performed with 50 independent runs in different initial configurations. It should be noted that the flow rate is obtained on average every 50 times. The rules of updating are as follows:

(i) Each pedestrian takes an empty site around him as the goal site. If there is no site available, he should remain at the site this time.

(ii) If there is only one candidate that competes for the goal site, this pedestrian win by default.

(iii) When there are more than two candidates competing for the same goal site, the winner of the game obtains it while others hold on. For the case without a winner, all candidates remain at their own site where they are.

(iv) The loser will change its game strategy according to Eqs. (1)–(4).

(v) The one who gets into the range of the exit will be removed from the hall.

(vi) All pedestrians update in parallel.

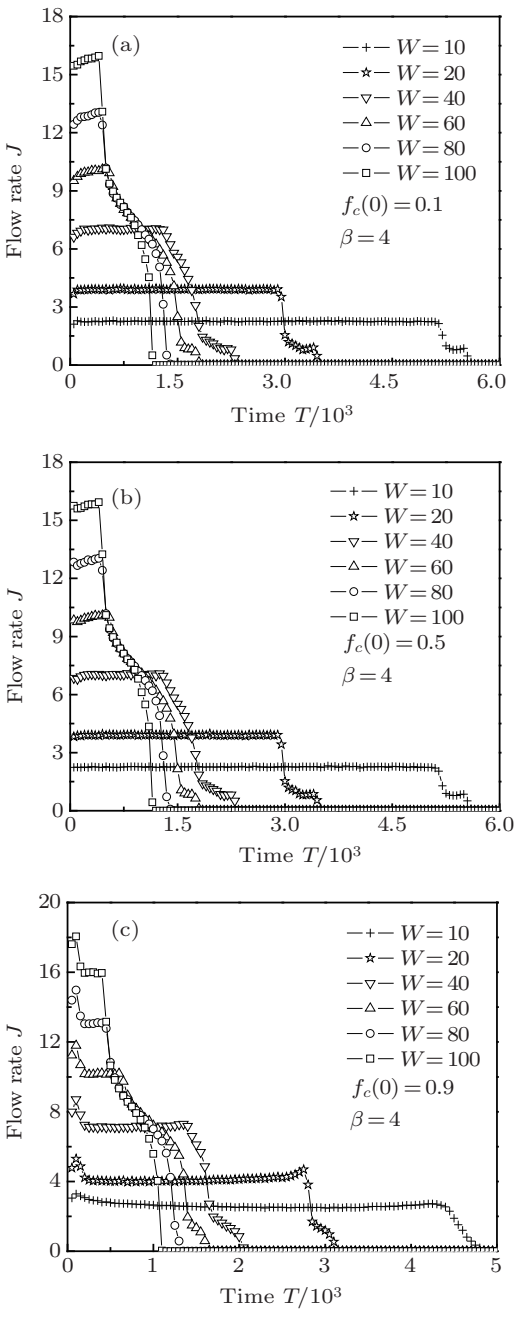


Fig. 5. Pedestrian flow rates against time for $f_c(0) = 0.1$ (a), 0.5 (b), and 0.9 (c) under $\beta = 4$ and different values of W.

With these rules, the simulation is performed. Figure 5 shows the variation of the pedestrian flow rate against the evac-

uation time, while figure 6 presents the corresponding cooperative proportion against evacuation time. It can be found that the pedestrian flow rates in Figs. 5(a) and 5(b) are smaller than those in Fig. 5(c). It can be judged that as the initial cooperative proportions are not high enough, most of the pedestrians would be cooperators rather than defectors. As the initial cooperative proportion increases, the game between cooperators takes effect in pedestrian movement and most of the cooperators would change into defectors after meeting defectors. The cooperative proportion decreases as the defective proportion increases. As time goes on, they reach their saturation states respectively and keep these states for a long period. When the bulk of defectors get out of the hall successively, the defective proportion begins to decrease while the cooperative proportion begins to increase. The intersection appears again at time $T = 1600$ (Fig. 6(c)).

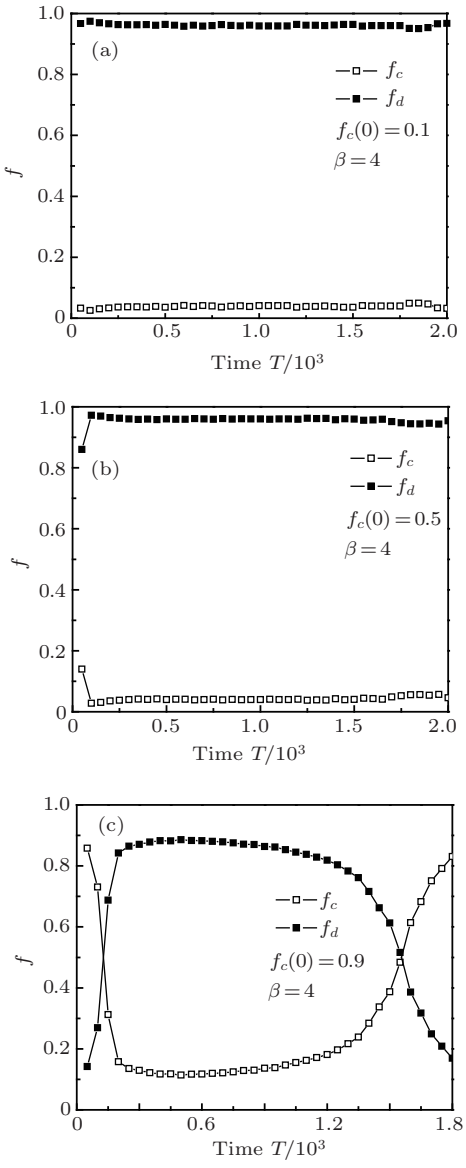


Fig. 6. The cooperative proportions and the defective proportions against time T for $f_c(0) = 0.1$ (a), 0.5 (b), and 0.9 (c) under $\beta = 4$ and $W = 40$.

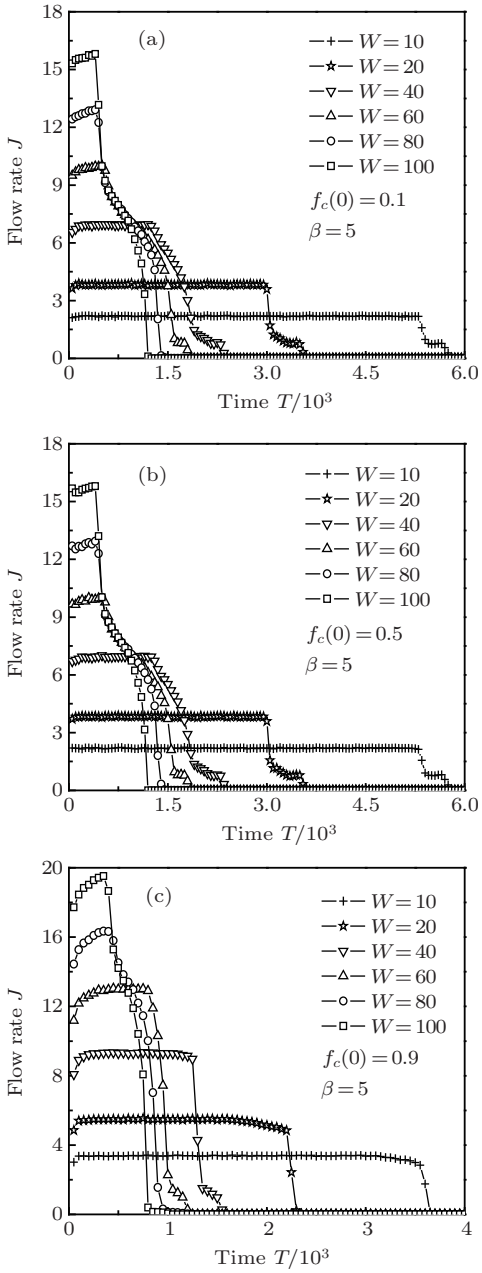


Fig. 7. Flow rates J versus time T for (a) $f_c(0) = 0.1$ (a), 0.5 (b), and 0.9 (c) under $\beta = 5$ and different values of W.

Similarly, the variation of the pedestrian flow rate with the evacuation time for $\beta = 5$ is shown in Fig. 7. Compared with Fig. 5, figure 7 shows some new findings. The evacuation time in Fig. 7(c) is much shorter than that in Fig. 5(c) when the size of the exit is large enough. Additionally, the variation of the corresponding cooperative proportion with time is plotted in Fig. 8. The intersections which appear in Fig. 6(c) disappear in Fig. 8(c), implying that the parameter of β is obviously influenced in the Game. When the cooperative proportion U_c is high enough, the value of

$$P(c-d) = \frac{1}{1 + \mathrm{e}^{(-\beta(U_d - U_c))}}$$

will become small and a cooperator would be less likely to

change his game strategy. Since a game between cooperators could enhance the flow rate, the evacuation time is shortened. Hence, the critical noise value of $\beta = 5$ is obtained when most of the pedestrians are cooperators in the initial time.

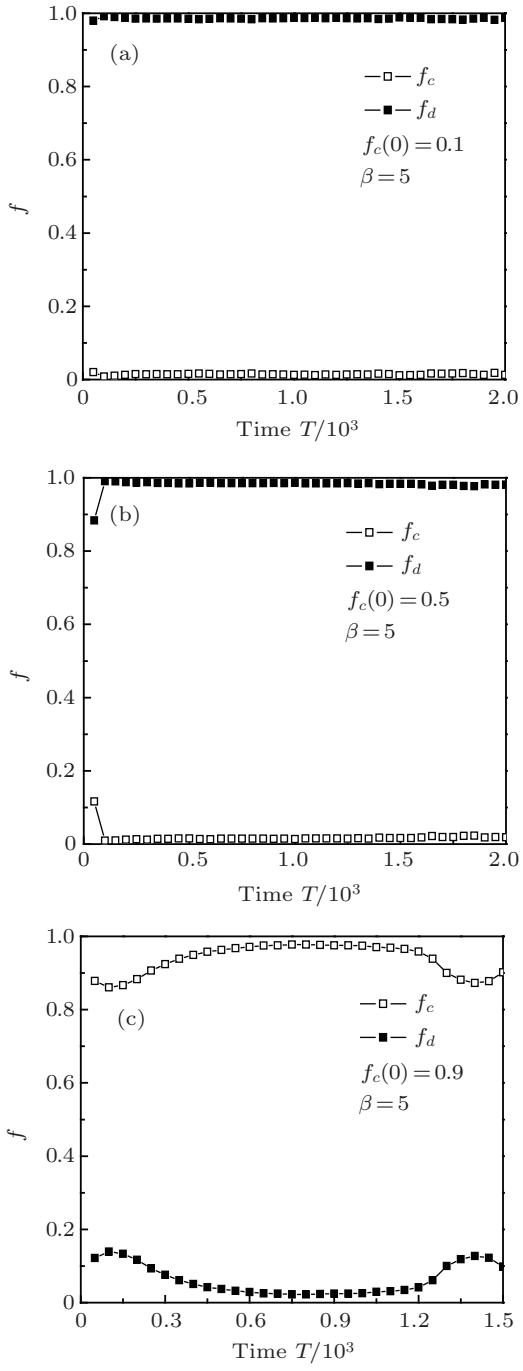


Fig. 8. Cooperator proportions and the defector proportions versus time T for $f_c(0) = 0.1$ (a), 0.5 (b), and 0.9 (c) under $\beta = 5$ and $W = 40$.

In addition, figure 9 shows the plots of pedestrian flow rate against evacuation time. From the figure, it can be clearly found that the larger the value of β, the longer the evacuation time needed will be. In turn, it can be concluded that the game between cooperators could effectively enhance the pedestrian flow rate.

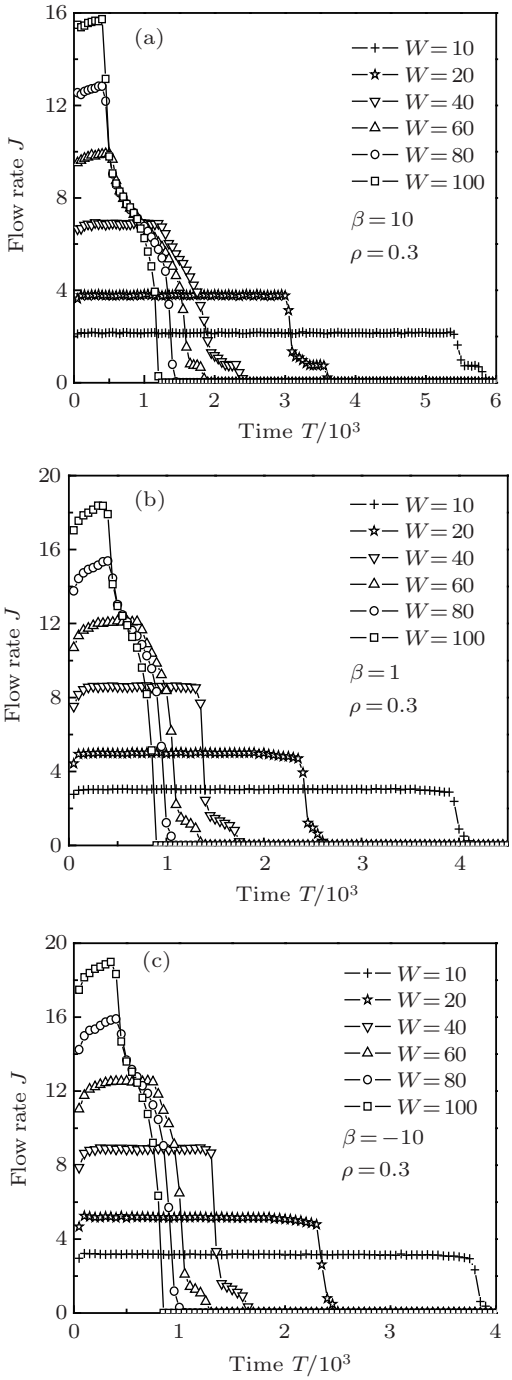


Fig. 9. Pedestrian flow rates against time for $\beta = 10$ (a), 1 (b), and -10 (c) under $f_c(0) = 0.5$ and different values of W.

Figure 10 presents the patterns formed when pedestrians go to the exit for $\rho = 0.3$, $\beta = 1.0$, and $t = 50$, 200, 600, and 1200. At the beginning, the pedestrians move slowly and there appears to be no congestion near the exit (Fig. 10(a)). As the time goes on, two peaks are created by the introduction of parameter k (Fig. 10(b)), indicating that most of the pedestrians do not get stuck in the corner. When the time increases, the peaks become smaller (Fig. 10(c)) and then disappear finally (Fig. 10(d)).

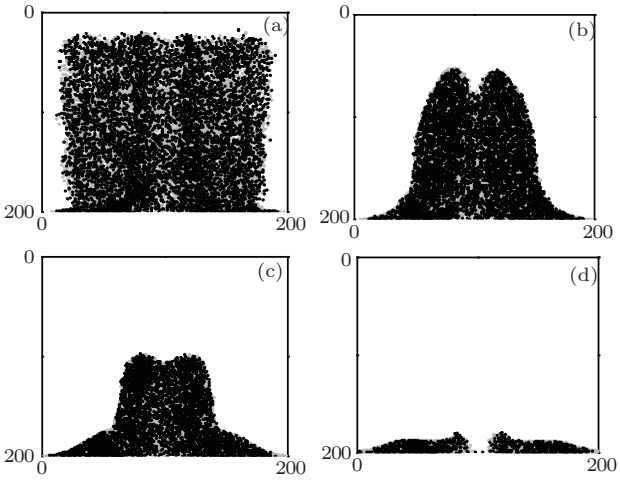


Fig. 10. Patterns formed when pedestrians go to the exit of size $W = 40$ where the initial density of pedestrians is $\rho = 0.3$ at different time steps: (a) $t = 50$, (b) $t = 200$, (c) $t = 600$, and (d) $t = 1200$.

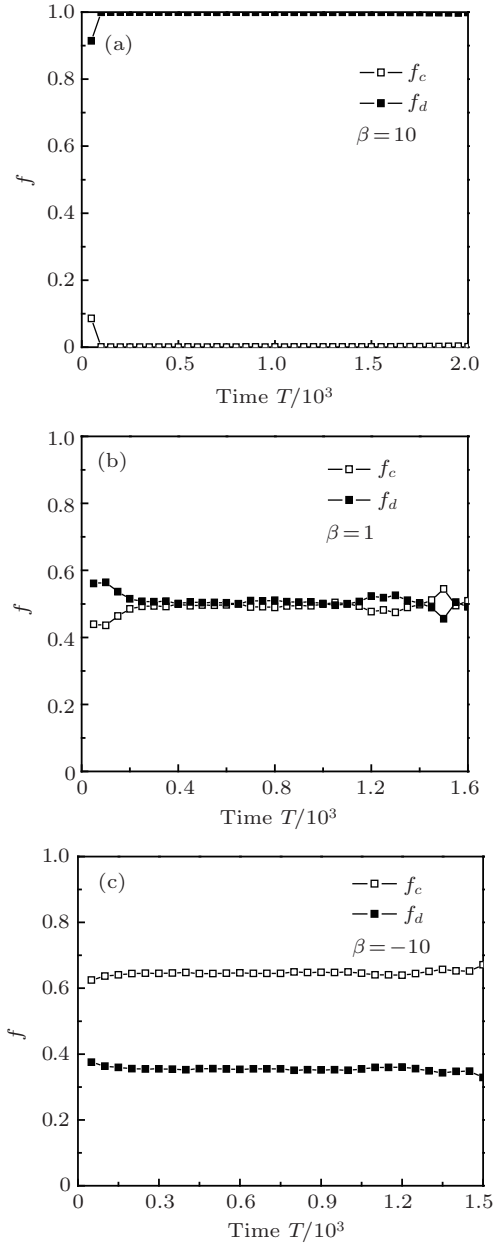


Fig. 11. Cooperator proportions and the defector proportions against time T for $\beta = 10$ (a), 1 (b), and -10 (c) under $W = 40$.

In order to investigate how cooperative proportion changes with time, plots of the cooperator proportion and the defector proportion against time are provided in Fig. 11. All the cooperators quickly change into defectors when the value of β reaches 10, i.e., $\beta = 10.0$. For the case of $\beta = 1.0$, the cooperative proportion remains unchanged. When the noise is as small as $\beta = -10.0$, the cooperative proportion will reach a stable proportion of about 0.65. It can be concluded that the noise could affect the evacuation time through influencing the pedestrian flow rate. The bigger the value of the noise, the shorter the evacuation time is.

Furthermore, the scaling behavior of the saturated pedestrian flow rate against the size of exit, as well as the total evacuation time against the size of exit, is studied. Figure 12 shows the log–log plot of the saturated pedestrian flow rate and evacuation time $T_{\rm all}$ against exit size for $\beta = -10.0$, 1.0, and 10.0.

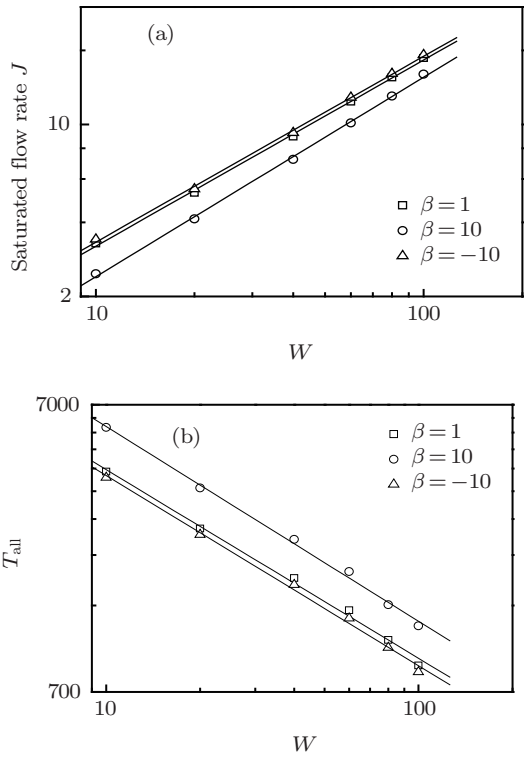


Fig. 12. The log–log plots of saturated flow rate J and evacuation time $T_{\rm all}$ against exit size W for $\beta = -10.0$, 1.0, and 10.0.

From the figures, the saturated flow rate J can be scaled as follows:

$$T_{\rm all} \propto \begin{cases} W^{0.755\pm0.002}, & \beta = -10.0, \\ W^{0.757\pm0.012}, & \beta = 1.0, \\ W^{0.810\pm0.014}, & \beta = 10.0. \end{cases} \quad (5)$$

Compared with the scaling saturated flow rate of a granular flow,[7] the scaling exponents in a pedestrian flow are smaller.

The scaling behavior of the total evacuation time against the size of exit is also obtained and shown in Fig. 12(b). Sim-

ilarly, the scaling relations are as follows:

$$T_{\text{all}} \propto \begin{cases} W^{-0.662\pm0.020}, & \beta = -10.0, \\ W^{-0.657\pm0.022}, & \beta = 1.0, \\ W^{-0.678\pm0.020}, & \beta = 10.0. \end{cases} \tag{6}$$

4. Conclusion

In this paper, the evacuation of pedestrians in a hall is studied based on the CA model by introducing the game theory with an update feature. The payoff matrix of the game model represents the complicated interactions among the conflict pedestrians. It is found that each pedestrian interacts with all of its neighbors, and chooses a neighbor to occupy the site in the game strategy update manner. Besides the size of the door, two other factors, namely the initial cooperative proportion and the noise, are numerically investigated and found to have an obvious influence on the evacuation time. A critical noise value is obtained via simulations for various values of noise. In addition, the size of the exit is found to be highly dependent on the saturated flow rate and the total evacuation time.

Hence, the initial cooperative proportion and the noise play dominant roles in the process of evacuation. The initial small cooperative proportion and the large noise could result in a long time for an evacuation in a hall. Results also indicate that besides building a bigger sized door, establishing a cooperative relationship between pedestrians in order and removing disturbance from the surrounding environment are effective ways to avoid an evacuation crisis.

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