Entanglement concentration for W-type entangled coherent states*

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An entangled coherent state (ECS) is one type of entanglement, which is widely discussed in the application of quantum information processing (QIP). In this paper, we propose an entanglement concentration protocol (ECP) to distill the maximally entangled W-type ECS from the partially entangled W-type ECS. In the ECP, we adopt the balanced beam splitter (BS) to make the parity check measurement. Our ECP is quite different from the conventional ECPs. After performing the ECP, not only can we obtain the maximally entangled ECS with some success probability, but also we can increase the amplitude of the coherent state. Therefore, it is especially useful in long-distance quantum communication, if the photon loss is considered.

Keywords: quantum communication and computation, entanglement concentration, coherent state

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1. Introduction

Quantum entanglement plays a very important role in quantum information processing (QIP). For instance, quantum entanglement can be used in quantum teleportation,[1] quantum key distribution (QKD),[2] quantum secure direct communication,[3–5] and other quantum communication protocols.[6–12] On the other hand, in quantum computation, the entanglement should be created first, such as the linear optics quantum computation,[13] one-way quantum computation,[14] and so on.[15–17]

Generally speaking, there are two types of entanglement. The first one is encoded in discrete variables, such as the polarization of photons, the spin of electrons, and so on. The second one is encoded in continuous variables. An entangled coherent state (ECS) is the entanglement encoded in continuous variables, which are attracting more and more attention.[18,19] The ECSs in free-traveling optical fields have been widely studied and found to be useful in performing tasks such as quantum teleportation,[20–23] quantum computation,[24] entanglement purification,[25,26] quantum error corrections,[27,28] quantum repeaters,[29] and so on.[30–33] On the one hand, many conventional schemes use single-mode coherent state superpositions (CSSs) as qubits and two-mode ECSs as quantum channels.[34] On the other hand, Nguyen recently[30] studied an optimal QIP via W-type ECS and found that there exists a quantum information protocol which can only be done with W-type ECS. The W-type ECSs are the necessary resources in the remote symmetric entanglement, which allow two distant parties to share a symmetric entangled state. It can also be used to test quantum nonlocality without inequality.[35,36] The violation of the Bell inequalities for the W-type ECS is greater than that for any states involving two spin-1/2 particles.[36,37]

In practical applications of ECSs, the maximally ECSs are usually the necessary resources. The maximally entangled W-type ECS has the form of

$$|\Psi\rangle_{abc} = N_0[|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c + |\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c + |-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c], \quad (1)$$

where $N_0 = [3(1+2\mathrm{e}^{-4|\alpha|^2})]^{-1/2}$.

However, when the maximally entangled states prepared for practical applications suffer from the interaction influence from the environment, the quality of the entanglement will inevitably be degraded. It will make the maximally entangled state become a mixed state or a partially entangled state. For the purpose of obtaining the maximally entangled state from a pure partially entangled state, the concept of entanglement concentration has been proposed.[38]

The first entanglement concentration protocol (ECP) with collective measurement was proposed by Bennett *et al.*, which is referred to as the Schimidt projection method.[38] However, this ECP is difficult to operate in experiments. In 1999, Bose *et al.* proposed the ECP based on entanglement swapping.[39] To improve the efficiency of the ECP proposed by Bose *et al.*, Shi *et al.* developed the ECP subsequently.[40] In 2001, Zhao *et al.* and Yamamoto *et al.* proposed the ECPs based on linear optical elements, respectively.[41,42] In recent years, vari-

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ous ECPs have been proposed, such as the ECPs based on the cross-Kerr nonlinearity,[43–56] quantum dot and optical cavity systems,[57,58] atoms,[59] and so on.[60–65]

Though many ECPs were proposed and discussed, unfortunately, no protocol discusses the entanglement concentration for W-type ECSs. Inspired by the previous works of the W-type ECSs and the protocols for entanglement concentration, we propose two ECPs for the W-type ECSs. In the first ECP, we only prepare an ancillary single coherent state, and use the beam splitter (BS) to perform the parity check measurement for the coherent state. In the second ECP, taking a more general situation into consideration, we describe the concentration for the W-type ECS with arbitrary coefficients.

This paper is organized as follows. In Section 2, we will describe the ECP for a special W-type ECS assisted with single coherent states. In Section 3, we will describe the ECP for the arbitrary coefficient W-type ECS. In Section 4, we will present a discussion and summary.

2. Single coherent state assisted entanglement concentration for W-type ECS

As shown in Fig. 1, Alice, Bob, and Charlie share the partially entangled ECS of the form

$$|\psi\rangle_{abc} = N_1[\gamma|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c + \delta(|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c + |-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c)]. \tag{2}$$

Here, we let γ and δ be a real number for simplicity, and the normalized coefficient $N_1 = [\gamma^2 + 2\delta^2 + (4\gamma\delta + 2\delta^2)\,\mathrm{e}^{-4|\alpha|^2}]^{-1/2}$. Alice, Bob, and Charlie own the coherent state denoted as a, b, and c, respectively. In the ECP, Charlie first prepares an ancillary single coherent state in the spatial mode d of the form

$$|\psi\rangle_d = N_2[\gamma|\alpha\rangle_d + \delta|-\alpha\rangle_d], \tag{3}$$

where $N_2 = [\gamma^2 + \delta^2 + 2\gamma\delta\,\mathrm{e}^{-2|\alpha|^2}]^{-1/2}$. Therefore, the whole system $|\psi\rangle_{abc1} \otimes |\psi\rangle_{d_1}$ can be written as

$$\begin{aligned}
&|\Psi\rangle_{abc_1d_1}\\
&= |\psi\rangle_{abc_1} \otimes |\psi\rangle_{d_1}\\
&= N_1N_2[[\gamma|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_{c_1}\\
&\quad + \delta(|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_{c_1} + |-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_{c_1})]\\
&\quad \otimes[\gamma|\alpha\rangle_{d_1} + \delta|-\alpha\rangle_{d_1}]]\\
&= N_1N_2[\gamma\delta(|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}\\
&\quad + |\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_{c_1}|\alpha\rangle_{d_1}\\
&\quad + |-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_{c_1}|\alpha\rangle_{d_1}) + \gamma^2|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_{c_1}|\alpha\rangle_{d_1}\\
&\quad + \delta^2(|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}\\
&\quad + |-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1})].
\end{aligned} \tag{4}$$

Then, they let the photons in spatial modes c_1 and d_1 pass through the 50:50 beam splitter (BS). The BS is used to transform the two different coherent states $|\alpha\rangle$ and $|\beta\rangle$ to the states (the function of BS is denoted as B_{12})

$$B_{12}|\alpha\rangle|\beta\rangle \longrightarrow \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle\left|\frac{\alpha-\beta}{\sqrt{2}}\right\rangle. \tag{5}$$

In the ECP, there are four different possibilities for the photons in spatial modes c_1 and d_1 incident onto the BS. In detail, it can be described as

$$|\alpha\rangle_{c_1}|\alpha\rangle_{d_1} \longrightarrow |\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f_1}, \tag{6}$$
$$|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1} \longrightarrow |0\rangle_{e_1}|\sqrt{2}\alpha\rangle_{f_1}, \tag{7}$$
$$|-\alpha\rangle_{c_1}|\alpha\rangle_{d_1} \longrightarrow |0\rangle_{e_1}|-\sqrt{2}\alpha\rangle_{f_1}, \tag{8}$$
$$|-\alpha\rangle_{c_1}|-\alpha\rangle_{d_1} \longrightarrow |-\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f_1}. \tag{9}$$

After the coherent states passing through the BS1, the state in Eq. (4) evolves to

$$\begin{aligned}
|\Psi\rangle_{abe_1f1} = {} & N_1N_2[\gamma\delta(|\alpha\rangle_a|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f1}\\
& + |\alpha\rangle_a|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f1}\\
& + |-\alpha\rangle_a|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f1})\\
& + \gamma^2|\alpha\rangle_a|\alpha\rangle_b|0\rangle_{e_1}|-\sqrt{2}\alpha\rangle_{f1}\\
& + \delta^2(|\alpha\rangle_a|-\alpha\rangle_b|0\rangle_{e_1}|\sqrt{2}\alpha\rangle_{f1}\\
& + |-\alpha\rangle_a|\alpha\rangle_b|0\rangle_{e_1}|\sqrt{2}\alpha\rangle_{f1})].
\end{aligned} \tag{10}$$

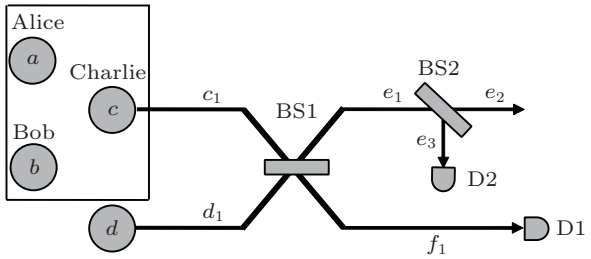


Fig. 1. The schematic diagram showing the principle of the proposed ECP for W-type ECS. Alice, Bob, and Charlie share the ECS, and own the coherent states denoted as a, b, and c, respectively. First, Charlie will prepare an ancillary single coherent state. The BS1 is used to perform a parity check measurement. The second BS (i.e. BS2) is used to obtain the W-type maximally entangled coherent state.

From Eq. (10), one can find that the items $|\alpha\rangle_a|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f_1}$, $|\alpha\rangle_a|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f_1}$, and $|-\alpha\rangle_a|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{e_1}|0\rangle_{f_1}$ lead the spatial mode f_1 to have no photon, while other items lead the spatial mode e_1 to have no photon. Therefore, they can divide the states in Eq. (10) into two parts. By selecting the cases that the spatial mode f_1 has no photon, they can get

$$\begin{aligned}
|\Psi\rangle_{abe_1} = {} & N[|\alpha\rangle_a|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{e_1} + |\alpha\rangle_a|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{e_1}\\
& + |-\alpha\rangle_a|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{e_1}].
\end{aligned} \tag{11}$$

We can find that the state in Eq. (11) has the same form with that in Eq. (1). The difference between the two states is that the amplitude of the coherent state in spatial mode e_1 in Eq. (11) is $\sqrt{2}$ times larger than that in spatial mode c of Eq. (1). For the purpose of obtaining the state in Eq. (1), we

should adopt another beam splitter (BS2). Then when we let the photons in spatial mode e_1 pass through BS2, the state evolves to

$$\begin{aligned}|\Psi\rangle_{abe_2e_3} &= N[|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_{e_2}|-\alpha\rangle_{e_3}\\ &+|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_{e_2}|\alpha\rangle_{e_3}\\ &+|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_{e_2}|\alpha\rangle_{e_3}].\end{aligned} \tag{12}$$

From Eq. (12), in order to obtain the maximally W-type ECS, they should detect the coherent state in e_2 mode and make the $|\pm\alpha\rangle$ undistinguishable. In this way, they should resort to the quantum nondemolition measurement.[66,67] Finally, after performing the measurement, they can get the W-type maximally ECS with the same form of Eq. (1). Actually, in a practical application, it is not necessary to decrease the amplitude of the concentrated coherent state. As pointed out by Ralph *et al.*,[26] the photon loss is also the main obstacle in the quantum communication. Therefore, they should increase the amplitude before application, which provides this protocol with another exciting advantage.

3. Entanglement concentration for arbitrary coefficient W-type ECS

In the above section, we proposed an ECP for the special W-type ECS of the form in Eq. (2). It is straightforward to extend the above protocol to the case of arbitrary partially entangled ECS of the form

$$\begin{aligned}|\phi\rangle_{abc} &= N_4[\gamma|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c+\delta|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c\\ &+\varepsilon|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c].\end{aligned} \tag{13}$$

Here, we let the coefficients γ, δ, and ε be real numbers. N_4 satisfies $N_4=[\gamma^2+\delta^2+\varepsilon^2+2(\gamma\delta+\delta\varepsilon+\varepsilon\gamma)\,\mathrm{e}^{-4|\alpha|^2}]^{-1/2}$. As shown in Fig. 2, Alice and Charlie both prepare the ancillary single coherent states in the spatial modes d and e of the form

$$|\phi\rangle_d = N_5[\varepsilon|\alpha\rangle_d+\delta|-\alpha\rangle_d], \tag{14}$$

$$|\phi\rangle_e = N_6[\gamma|\alpha\rangle_e+\delta|-\alpha\rangle_e], \tag{15}$$

respectively. Here, $N_5=[\varepsilon^2+\delta^2+2\varepsilon\delta\,\mathrm{e}^{-2|\alpha|^2}]^{-1/2}$ and $N_6=[\gamma^2+\delta^2+2\gamma\delta\,\mathrm{e}^{-2|\alpha|^2}]^{-1/2}$. The whole system can be written as

$$\begin{aligned}|\Phi\rangle_{abcde} &= |\phi\rangle_{abc}\otimes|\phi\rangle_d\otimes|\phi\rangle_e\\ &= |\phi\rangle_{abc}\otimes N_5N_6[\gamma\varepsilon|\alpha\rangle_d|\alpha\rangle_e+\delta\varepsilon|\alpha\rangle_d|-\alpha\rangle_e\\ &+\gamma\delta|-\alpha\rangle_d|\alpha\rangle_e+\delta^2|-\alpha\rangle_d|-\alpha\rangle_e]\\ &= N_4N_5N_6[\gamma^2\varepsilon|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c|\alpha\rangle_d|\alpha\rangle_e\\ &+\gamma^2\delta|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c|-\alpha\rangle_d|\alpha\rangle_e\\ &+\delta^3|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c|-\alpha\rangle_d|-\alpha\rangle_e\\ &+\gamma\varepsilon^2|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c|\alpha\rangle_d|\alpha\rangle_e\\ &+\delta\varepsilon^2|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c|\alpha\rangle_d|-\alpha\rangle_e\\ &+\gamma\delta^2(|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c|-\alpha\rangle_d|-\alpha\rangle_e\\ &+|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c|-\alpha\rangle_d|\alpha\rangle_e)\\ &+\delta^2\varepsilon(|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c|\alpha\rangle_d|-\alpha\rangle_e\\ &+|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c|-\alpha\rangle_d|-\alpha\rangle_e)\\ &+\gamma\delta\varepsilon(|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c|\alpha\rangle_d|-\alpha\rangle_e\\ &+|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c|\alpha\rangle_d|\alpha\rangle_e\\ &+|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c|-\alpha\rangle_d|\alpha\rangle_e)].\end{aligned} \tag{16}$$

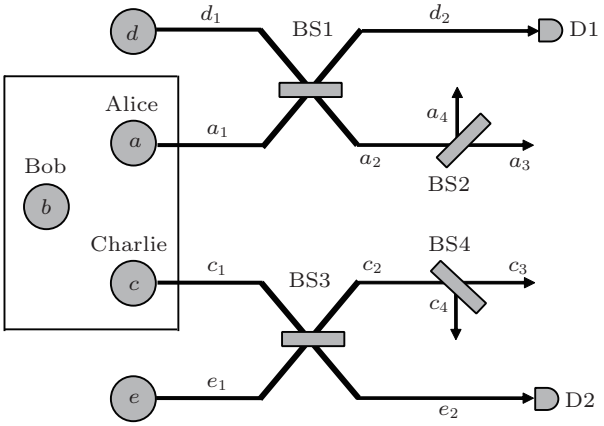


Fig. 2. (color online) The schematic diagram showing the principle of the proposed ECP for arbitrary coefficient W-type entangled coherent state. Alice, Bob, and Charlie share the ECS, and own states a, b, and c, respectively. Alice and Charlie prepare the ancillary single coherent states d and e, respectively. BSs are required for the purpose to obtain the W-type maximally entangled ECS.

Then they let the photons in spatial modes a_1 and d_1 pass through the BS1 and the photons in spatial modes c_1 and e_1 pass through the BS3. They can obtain the state described as

$$\begin{aligned}&|\Phi\rangle_{a_2bc_2d_2e_2}\\ &= N_4N_5N_6[\gamma^2\varepsilon|\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|0\rangle_{d_2}|-\sqrt{2}\alpha\rangle_{e_2}\\ &+\gamma^2\delta|0\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|-\sqrt{2}\alpha\rangle_{e_2}\\ &+\delta^3|0\rangle_{a_2}|-\alpha\rangle_b|0\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}\\ &+\gamma\varepsilon^2|0\rangle_{a_2}|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|-\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{e_2}\\ &+\delta\varepsilon^2|0\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|-\sqrt{2}\alpha\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}\\ &+\gamma\delta^2(|0\rangle_{a_2}|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{e_2}\\ &+|0\rangle_{a_2}|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{e_2})\\ &+\delta^2\varepsilon(|\sqrt{2}\alpha\rangle_{a_2}|-\alpha\rangle_b|0\rangle_{c_2}|0\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}\\ &+|-\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|0\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2})\\ &+\gamma\delta\varepsilon(|\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{c_2}|0\rangle_{d_2}|0\rangle_{e_2}\\ &+|\sqrt{2}\alpha\rangle_{a_2}|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|0\rangle_{d_2}|0\rangle_{e_2}\\ &+|-\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|0\rangle_{d_2}|0\rangle_{e_2})].\end{aligned} \tag{17}$$

The photons in spatial modes d_2 and e_2 can be $|0\rangle$, $|\sqrt{2}\alpha\rangle$ or $|-\sqrt{2}\alpha\rangle$. They select the cases that both the spatial modes d_2 and e_2 have no photon. From Eq. (17), the state can be written as

$$|\Phi\rangle_{a_2bc_2} = N[|\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{c_2}$$

$$+|\sqrt{2}\alpha\rangle_{a_2}|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}$$
$$+|-\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}]. \quad (18)$$

We can find that the state in Eq. (18) has the same form as the state in Eq. (1), except that the amplitude of the photons in spatial modes a_2 and c_2 is different to that in Eq. (1). For the purpose of getting the state in Eq. (1), they let the photons in spatial modes a_2 and c_2 pass through BS2 and BS4, respectively. In this way, the state evolves to

$$\begin{aligned}|\Phi\rangle_{a_3a_4bc_3c_4} &= N[|\alpha\rangle_{a_3}|\alpha\rangle_{a_4}|\alpha\rangle_b|-\alpha\rangle_{c_3}|-\alpha\rangle_{c_4}\\ &+|\alpha\rangle_{a_3}|\alpha\rangle_{a_4}|-\alpha\rangle_b|\alpha\rangle_{c_3}|\alpha\rangle_{c_4}\\ &+|-\alpha\rangle_{a_3}|-\alpha\rangle_{a_4}|\alpha\rangle_b|\alpha\rangle_{c_3}|\alpha\rangle_{c_4}]. \quad (19)\end{aligned}$$

From Eq. (19), they could detect the coherent states in spatial modes a_4 and c_4 to get the maximally W-type ECS in Eq. (1). Here, they should also use the QND measurement to ensure that the states $|\pm\alpha\rangle$ are indistinguishable.

4. Discussion and summary

So far, we have explained our ECPs for the W-type ECS. We first discussed a special W-type ECS of the form $|\psi\rangle_{abc} = N_1[\gamma|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c+\delta(|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c+|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c)]$ in Eq. (2). Then we discussed the ECP for an arbitrary W-type ECS. It is shown that the first ECP can be regarded as a special case of the second one. We can calculate the success probability of both ECPs. In the first ECP, the total success probability is $P_1 = 3|N_1N_2\gamma\delta|^2$. In the second ECP, the success probability is $P_2 = 3|N_4N_5N_5\gamma\delta\varepsilon|^2$. In Figs. 3 and 4, we show the success probabilities with numerical simulation. In Fig. 3, we calculated the success probability altered with $N_1\gamma$. Interestingly, different from the conventional ECPs, the α will affect the success probability. We calculated thesuccess probability with $\alpha = 0.5$, 1, and 2, as shown in Fig. 3. In the second ECP, we calculated the success probability altered with both $N_4\delta$ and $N_4\gamma$.

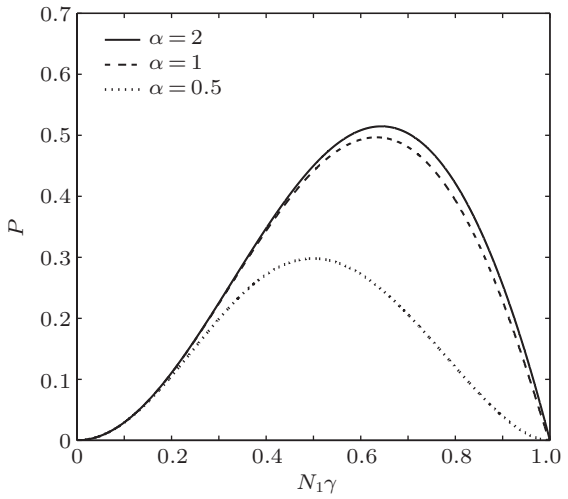


Fig. 3. The schematic diagram showing the success probability of the first ECP. The variable α is the amplitude of the coherent state. Here, we take $\alpha = 0.5, 1, 2$.

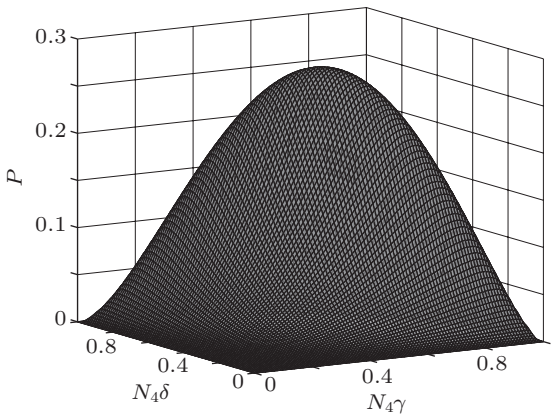


Fig. 4. The schematic diagram showing the success probability of the second ECP. The α is the amplitude of the coherent state. Here, we let the $\alpha = 2$.

From the above discussion, it is obvious to find that the BS plays a significant role in the two ECPs. In fact, from Eqs. (6)–(9), the BS is used to make the parity check measurement for the coherent states. That is to say, after passing through the BS, we can distinguish the even parity states $|\alpha\rangle|\alpha\rangle$ and $|-\alpha\rangle|-\alpha\rangle$ from the odd parity states $|\alpha\rangle|-\alpha\rangle$ and $|-\alpha\rangle|\alpha\rangle$, for the even parity states will lead the spatial mode f_1 to have no photon and the odd parity states will lead spatial mode e_1 to have no photon. This feature of the BS can help to complete the task of entanglement concentration. We should point out that only the state $|0\rangle$ leaves no photon. The coherent state $|\alpha\rangle$ also has a possibility of having no photon. However, the error possibility is very small which can be neglected, even if α is large. For example, the probability $|\langle 0|\alpha\rangle|^2$ is 1.1×10^{-7} for $\alpha = 2$.

In conclusion, we have described two ECPs for W-type ECS. We first discussed the concentration of the special W-type ECS. It is shown that the whole concentration can be completed with the help of one coherent state. We extended this method to deal with the partially entangled ECS with arbitrary coefficients. With the help of two single coherent states, one can obtain the maximally W-type ECS with some probability. We also calculated the total success probability in each protocol. We hope that these ECPs are useful in current QIPs based on the ECSs.

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